Lifetime Labor Supply and Human Capital Investments

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Abstract

We study a model of the choice of lifetime labor supply—or career length, a measure of the quantity of labor—and human capital investments—a measure of the quality of labor. We use the model to study the effects of policies and shocks on both the quantity and quality of labor supply. We find that unanticipated shocks have non-monotonic effects on effective labor supply and that the impact on career length depends on the age of the worker at the time of the shock. We calibrate the model and conduct two types of experiments. First, we find that, for some European countries, a switch to the U.S. style retirement system produces large gains in output per worker in the long run. We then quantify the impact on the U.S. of an increase in the full retirement age by two years (a seven-percent increase in output per worker), an increase in life expectancy by two years (a four-percent drop in output per worker), and the elimination of social security (a 45-percent increase in output per worker). Overall, we find that the lifetime labor supply (quantity of labor) is relatively inelastic with respect to the type of experiments that we conduct, and that changes in output are essentially driven by changes in human capital (quality of labor).

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1 Introduction

There are stark differences in schooling and retirement decisions across time and space. In the United States, 60 percent of men aged 65 or older were in the labor force in 1900, and the average worker had 5.4 years of schooling. By 2000, only 20 percent of men aged 65 or older were in the labor force, while the average worker had 12.4 years of schooling. Also in 2000, the labor force participation rate for men aged 65 or older was only 5 percent for a group of European countries. For these countries, the average years of schooling was 9.4 years.

In this paper we study a model of effective lifetime labor supply that is rich enough to understand these differences. We analyze the effects of changes in wages and taxes on the *quantity* dimension of lifetime labor supply (i.e. career length) as well as the *quality* dimension which includes the quantity and quality of schooling as well as investments in on-the-job training. We use this framework to explain the patterns of schooling and retirement across time and space, and to explore the effect of changes in retirement tax/benefit schemes.

The model that we study blends elements of a standard life-cycle model which are essential to understand retirement decisions, with a dynastic preference structure that guarantees that the long run behavior is easily characterized. We assume that individuals choose the quantity and quality of schooling and that, when in the labor force, decide how to allocate their time between producing and increasing their human capital (on-the-job training). We study a model with indivisible labor and we assume that, once the worker retires, he can enjoy leisure. It is this extra utility associated with not working that drives retirement. We model retirement benefits as a non-linear function of income that includes a fixed payment.

We use the model to explore, both theoretically and quantitatively, the impact of some policy changes (e.g. changes in the nature of the retirement system), demographic shocks (e.g. changes in population growth rate or life expectancy) and total factor productivity, which allows us to model variation in space (e.g. productivity differences between Europe and the U.S.). In general, a given change, say, a change in retirement benefits financed by changes in taxes, affects several margins. Higher taxes —holding retirement decisions constant— discourage individuals from acquiring human capital and, at the same time, can induce workers to retire earlier. Thus, it is possible that a policy that does not have a large impact on the length of a career (individuals join the labor force earlier and retire earlier as well) can have a significant impact on effective labor supply through their effect on human capital.

Theoretically, we show that in the long run, reducing the progressivity of the tax code and changing the composition of government expenditures (net of retirement benefits) in the direction of more goods and fewer transfers, increases the retirement age. We also explore the effects of unanticipated shocks. We show that wealth shocks, e.g. a drop in the stock market, increases the retirement age and that the effect is stronger for older workers. A permanent decrease in the wage rate (a TFP shock) has non-monotonic effects on the equilibrium age-earnings profile: on impact, effective labor supply increases, and over time it decreases to fall below the pre-shock level at retirement. We show that lower wage rates unambiguously induce older workers to retire earlier. Finally, we study the effect of an unanticipated change in the stock of human capital (e.g. a loss associated with reallocation in the presence of firm or sector specific human capital). We find that older workers respond by supplying less effective labor and retiring earlier.

We calibrate model to data from the United States and use the model to run a number of experiments. Our paper makes three key points. First, the model economy is able to capture the variation in retirement ages across several European countries as well as Japan and Mexico. The model is able to capture the higher retirement age in Mexico as well as the lower retirement age in France relative to the United States. Variations in the payroll tax play a first order role in understanding these differences. We also ask whether the model economy can capture the change in schooling and retirement in the U.S. between 1900 and 2000. We find that it can - while individuals in 2000 retire at age 64, their counterparts 100 years earlier retired at 52 (which was also their average lifespan). Hence a lower lifespan as well as the absence of a social security system can help our understanding of why the average household worked until death circa 1900.

Second, we analyze the effect on output per worker of a switch to a U.S. style retirement regime which, compared to the prevailing regimes in European countries, is less generous and less redistributive. We find that, for Denmark, France and Spain, the increase in output per worker is very large; of the order of an additional 1 percent extra growth for a period of 40 years. This significant response is driven almost exclusively by changes in human capital per worker. Perhaps surprisingly, we find that even though the retirement age increases so does schooling; consequently, lifetime labor supply is essentially unchanged. Our findings contribute to the recent debate on social security reform in Europe. We find that, in agreement with Prescott's view (see Prescott, 2006), that the impact of a reform that lowers taxes can be significant. However, consistent with the findings of Ljungqvist and Sargent (2010), our model suggests that lifetime labor supply does not change much and, in our setting, that the major source of increase in output is the accumulation of human capital. Overall, we find that a calibrated version of our model predicts significant responses of output to changes in the pension system but that these are mostly driven by the associated changes in taxes. In all cases, the predicted response of lifetime labor supply is essentially zero as growth is driven by increases in the quality and not the quantity of labor supply.

Finally, we use the model to evaluate the effect of demographic and policy shocks on U.S. output per worker. We find that increasing the normal retirement age by two years increases output per worker in the long run by 7 percent, but that eliminating the implicit progressivity in the benefit function while keeping the average replacement rate constant, does not have an appreciable impact. Eliminating the social security regime, and lowering taxes to match the lower levels of expenditures, has a dramatic impact: output per worker increases by over 45 percent. Increasing life expectancy by two years and halving the population growth rate result in decreases in output per worker of 4 and 7 percent. This is driven by the effect of the higher taxes needed to finance a larger retired population. In all cases the impact on lifetime labor supply is close to zero and the effects of policy changes are driven by changes in the quality of labor supply, measured as the amount of human capital per year of schooling.

2 Brief Literature Review

There is a large literature interested in the aggregate effects of endogenous retirement decisions. There several recent papers that are related to our research. Wallenius (2009) studies a life-cycle model and assumes that human capital is accumulated as a by-product of working with exogenously declining “ability to learn.” She ignores the effect of the retirement regime on schooling decisions but allows for endogenous work week length. She finds that increases in benefits consistent with European levels of benefits would reduce hours somewhere from 5% to 14% depending on the details of the experiment. She also finds that a switch to a system in which benefits are more dependent on income increases hours less than 1%

Laitner and Silverman (2008) analyze a life cycle model. They assume that age earnings profiles are exogenous (all human capital accumulation is exogenous) but they consider the effect of disability. Their main interest is to study the effect of changes in social security regimes that encourage individuals to retire later. In particular, they consider a reform that sets the social security tax rate to zero for individuals whose working career exceed a certain length and that collection of benefits —computed using the income data from the “taxable” income years— can be postponed with their value being adjusted in an actuarially fair manner. They find that a reform that vests individuals at age 54 or when they have worked for 34 years can increase retirement age by one over one year. This estimate does not take into account changes in wages associated with the policy changes.

Heckman and Jacobs (2009) also study a life cycle model. They assume that the human capital accumulation technology is such that the level of schooling affects the ability of individuals to accumulate human capital on the job. Their theoretical model is very stylized with a reduced form value of retirement that depends on the length of the retirement period (i.e. no income effects during retirement) and a social security regime in which benefits are independent of both the retirement age and the level of earnings during the active period. Heckman and Jacobs (2009) do not explore the full implications of their model. Rather they consider two subcases, one in which they do not allow for on-the-job training (but schooling is endogenously chosen) and another in which individuals are allowed to invest in human capital while on the job but schooling is taken as given. For the first model they find that higher taxes in Europe account for some of the lower labor supply and that the tax wedge at retirement —the difference between what a worker would earn if he stays active relative to the retirement benefits— is a major determinant of early retirement.

Ljungqvist and Sargent (2010) study how earnings shocks and retirement benefits affect career length. They vary the age earnings profile by choosing alternative learning by doing technologies. Their results show that changes in retirement benefits financed by income taxes can result in significant changes in the career lengths.

Other than the Heckman and Jacobs (2009) paper, the reviewed literature limits the number of margins that individuals have access to when confronted with policy or real shocks (Wallenius (2009) allows for hours to adjust as well) and this might be one of the reason that accounts for our different conclusions. Since we let schooling (both quantity and quality) and investments in human capital while working change our workers have more options when it comes to adjusting. Our numerical results suggest that these other dimensions are quantitatively more relevant.

3 A Model with Indivisible Labor

In this section we study a simple version of the model that assumes that leisure is indivisible. Each individual is endowed with one unit of time in each instant, which can be used for leisure, $\ell(a)$, human capital accumulation, $n(a)$, or working in the market at wage w, $1 - \ell(a) - n(a)$. Following Rogerson (1988) we concentrate on the extensive margin and assume that feasible values for the choice of leisure are $\{0, 1\}$. We derive the retirement decision using two forces: an increase in utility associated with higher consumption of leisure and access to retirement benefits.

We adapt the basic model from Manuelli and Seshadri (2009) to allow for endogenous retirement. The basic setting is one in which an individual lives for a finite horizon, has children who, after some time, leave the parent household to form their own household. Following Barro and Becker (1989) parents care about the utility of their children albeit in an "imperfect" manner.

The representative household is formed at age I (age of independence). At age B, e^f children are born.[1] The period of 'early childhood' (defined by the assumption that children

[1] f equals the natural logarithm of one-half the total fertility rate (TFR). It is natural to think of B in the model as corresponding to average age of child birth. The correlation between median age of child birth (data from World Fertility Report, 2003) and output per worker is very small —for our sample, it is 0.09 in 2000. Thus, a uniform median age at birth when we consider different countries is not an unreasonable approximation.

are not productive during this period) corresponds to the (parent) ages from B to $B+6$. The children remain with the household (and as such make no decisions of their own) until they become independent at (parent) age $B+I$. The parent retires at the endogenously chosen age R, and dies at (exogenous) age T.

Let a denote an individual's age. Each parent chooses his own consumption, $c(a)$, as well as consumption of each of his children, $c_k(a)$, during the years that they are part of his household, $a \in [0, I)$, to maximize his utility. We adopt the standard Barro-Becker approach, and we specify that parent's utility depends on his own consumption, as well as the utility of his children. In addition to consumption, the parent chooses the amount of market goods to be used in the production of new human capital, $x(a)$, and the fraction of the time allocated to the formation of human capital, $n(a)$ (and, consequently, what fraction of the available time to allocate to working in the market, $1-n(a)$) for him and each of his children while they are still attached to his household and before retirement. The parent also decides to make investments in early childhood, which we denote by x_E (e.g. medical care, nutrition and development of learning skills), that determine the level of each child's human capital at age 6, $h_k(6)$, or h_E for short. Finally, the parent chooses how much to bequeath to each children at the time they leave the household, q_k. We assume that each parent has unrestricted access to capital markets.

The utility of a parent who has h units of human capital, who received a bequest of q and a government transfer (other than social security) u at age I, is given by

$$V^P(h,q,u) = \max \int_I^T e^{-\rho(a-I)} U(c(a), \ell(a)) da +$$
$$e^{-\alpha_0+\alpha_1 f} \int_0^I e^{-\rho(a+B-I)} U(c_k(a), \ell_k(a)) da + e^{-\alpha_0+\alpha_1 f} e^{-\rho B} V^k(h_k(I), q_k, u_k).$$

The utility of the parent is comprised of three terms. The first summarizes the utility — until death— associated with his own consumption and leisure choices. The second represents the utility that the parent derives from his children's consumption while they are still part of the household, properly discounted. Finally, the third term corresponds to the "continuation utility" after the children leave the household.

The second component includes the integrals of terms like $e^{-\alpha_0+\alpha_1 f} e^{-\rho(a+B-I)} u(c_k(a))$, which measures the contribution to the parent's utility of an a year old child still attached to him. The time discount factor is $e^{-\rho(a+B-I)}$ since at that time the parent is $a+B$ years old. In this formulation, $e^{-\alpha_0+\alpha_1 f}$ captures the degree of altruism. If $\alpha_0=0$, and $\alpha_1=1$, it corresponds to a standard infinitively-lived agent model. Positive values of α_0, and values of α_1 less than 1 capture the degree of imperfect altruism.

Each parent maximizes $V^P(h,q,u)$ subject to two types of constraints: the budget constraint, and the production function of human capital. Before describing the budget constraint it is useful to discuss some of the different phases of the income process as well as the tax system. Net income at age a be given by[2]

$$y(a,R)=\begin{cases} 0 & 0\le a<6 \\ -x_E & a=6 \\ -x(a) & 6<a\le 6+s \\ (1-\tau)[wh(a)(1-n(a))-px(a)] & 6+s<a\le R \\ b(a,R) & R<a\le T. \end{cases}$$

The interpretation is straightforward: during early childhood (until age 6) individuals earn no income and we summarize the necessary expenditures on health and nutrition in a lump sum payment at age 6, x_E. From age 6 to age $6+s$, net income is equal to the negative of the value of all market goods used in schooling, $-x(a)$. The length of the schooling period, s, is endogenously determined. The active job market period is from ages $6+s$ to R, the retirement age (also endogenously chosen). During this period net after tax income is

$$(1-\tau)[wh(a)(1-n(a))-px(a)],$$

where we view p as either the market price of market resources used in on-the-job training which depends on the tax regime. To be precise, we allow for the possibility that a fraction $1-\xi$ of training expenditures are deductible. Thus, given the income tax rate is τ, p is given by

$$p=(1-\tau(1-\xi))/(1-\tau).$$

[2]To be more precise, we should note that there are other factors, e.g. time allocation, investments and human capital, that affect the function $y(a,R)$. We ommit listing those other factors to keep the notation simple.

In this formulation, $\tau = \tau_I + \tau_S$, where τ_I is the tax rate on both capital and labor income minus the tax rate on social security, and τ_S is the effective (not necessarily equal to the statutory) social security tax rate. Finally, $b(a, R)$ denotes social security payments which depend on age as well as earnings. We assume that the tax code is non-linear in the sense that individuals are not taxed during their schooling years —identified as the period in which 100% of their available time is allocated to human capital accumulation— but they are taxed as soon as they enter the labor force. In our analysis, we assume $\xi = 1$; that is, expenditures on goods used to produce human capital are not tax deductible. This assumption ensures that the tax treatment of goods input is the same in and out of school.

With this notation, the budget constraint is given by

$$\int_I^T e^{-r(a-I)}c(a)da + e^f \int_0^I e^{-r(a+B-I)}c_k(a)da + e^f e^{-rB} q_k$$
$$\leq \int_I^T e^{-r(a-I)}y(a,R)da + e^f \int_6^I e^{-r(a+B-I)}y_k(a,R_k)da + q + u,$$

where subscript k denotes a child's variable and r corresponds to the after tax real interest rate, $r \equiv (1-\tau_I)\hat{r}$.

We adopt Ben-Porath's (1967) formulation of the human capital production technology, augmented with an early childhood period. To be precise we assume that

$$\dot{h}(a) = z_h(n(a)h(a))^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [6, R) \tag{1}$$

$$h_k(6) = h_B x_E^{\upsilon}, \qquad 0 < \gamma_i, \upsilon < 1, \qquad \gamma = \gamma_1 + \gamma_2 < 1, \tag{2}$$

The technology to produce each child's human capital at the beginning of the potential school years, $h_k(6)$ or h_E is given by (2). This specification captures the idea that nutrition and health care are important determinants of early levels of human capital, and those inputs are, basically, market goods. Equation (1) corresponds to the standard human capital accumulation model initially developed by Ben-Porath (1967). For a parent who starts independent life at age I, the initial human capital (chosen by his parent), $h(I)$, is given.[3]

[3] Our formulation allows for the early childhood production function to be different. This permits a better match between model and data, but it is not essential for the main results of the paper. Similarly, the assumption that the production function for educational services is the same as that for consumption is not critical: We have studied the impact of assuming a more human capital intensive technology (labor share of 80%), and the results are similar.

We study two versions of the social security regime distinguished by the features that they capture. The simplest is one in which the benefit is given by

$$b(a,R) = \begin{cases} 0 & a < \max\{R, R_n\} \\ b_m + b_y Y(R) & a \geq \max\{R, R_n\} \end{cases}$$

Here, b_m is a fixed component and $Y(R)$ is the present discounted value of net income given by

$$Y(R) = \int_0^R e^{-ra} y(a,R) da$$

In an extension of the basic model we study a more general case in which different benefits become available at certain ages.

As indicated before, we assume that during the working years $\ell(a) = 0$ while in retirement the individual does not work, $\ell(a) = 1$. In future work we plan to relax this specification to allow for endogenous determination of hours worked during the active period as well as non zero labor supply during retirement. We assume that

$$U(c,\ell) = \frac{(c(1+\zeta\ell))^{1-\theta}}{1-\theta},$$

and the parameters are such that

$$\frac{(c(1+\zeta))^{1-\theta}}{1-\theta} > \frac{c^{1-\theta}}{1-\theta}.$$

This specification can accommodate both the view—see for example French (2005) and Laitner and Silverman (2008)—that consumption drops at retirement because of the substitutability between home and market goods or leisure and the view—see Aguiar and Hurst (2009)—that in order to supply labor to the market it is necessary to purchase some goods (e.g., transportation and some restaurant meals outside the home). See the Appendix for the latter specification.

For a given retirement age R, which we will endogeneize later, we proceed to study optimal consumption and human capital accumulation. Consumption expenditures when in school or working (in both cases leisure is zero), and consumption in retirement are given by

$$c(a) = c(I)e^{(r-\rho)(a-I)/\theta}, \ a \in [I, R)$$
$$c(a) = c(I)e^{(r-\rho)(a-I)/\theta}(1+\zeta)^{1/\theta-1}, \ a \in [R, T]$$

To guarantee that $\lim_{a \nearrow R} c(a) > c(R)$ it is necessary that $\theta > 1$, which we assume from now on. In addition, the attached children's consumption is

$$c_k(a) = c(I) e^{\frac{(r-\rho)(a+B-I)-\alpha_0-(1-\alpha_1)f}{\theta}}, \; a \in [0, I)$$

Since we do not restrict bequests, the standard separation result obtains: any allocation that maximizes utility should also maximize income. An intuitive (and heuristic) argument that shows the correspondence between the utility maximization and the income maximization problems is as follows: Suppose that parents (who make human capital accumulation decisions for their children until age I) do not choose investments in human capital to maximize the present value of income of their children (only part of which they keep). In this case the parent could increase the utility of each child by adopting the income maximizing human capital policy and adjusting the transfer to finance this change. It follows that the cost to the parent is the same and the child is made better off. It follows that, to determine the equilibrium level of human capital, it suffices to study the problem of maximizing the present discounted value of income taking into account that, potentially, the level of retirement benefits may depend on earnings. The details are in the appendix

It turns out to be convenient to simplify the problem by looking at a measure of utility that accrues to the parent while he is alive. This includes the utility of his own consumption as well as the utility of his children while still in the household. Let the interest rate, r, be fixed

Simple calculations show that,

$$\int_I^T e^{-\rho(a-I)} U(c(a), \ell(a)) da + e^{-\alpha_0+\alpha_1 f} \int_0^I e^{-\rho(a+B-I)} U(c_k(a), \ell_k(a)) da = \frac{c(I)^{1-\theta}}{1-\theta} G(I, R),$$

where

$$G(I, R) = e^{\upsilon(r)I} [(\Delta(R) - \Delta(I)) + (1+\zeta)^{\frac{1-\theta}{\theta}} (\Delta(T) - \Delta(R)) + e^{f-rB} e^{-\mu/\theta} \Delta(I)],$$

with

$$\Delta(x) = \int_0^x e^{-\upsilon(r)a} da = \frac{1 - e^{-\upsilon(r)x}}{\upsilon(r)},$$

$$\upsilon(r) = \frac{\rho - (1-\theta)r}{\theta} > 0,$$

and the term

$$\mu = \alpha_0 + (1 - \alpha_1)f + \rho B - rB$$

which captures the difference between the current interest rate, r, and the effective discount factor between the utility of different generations, $(\alpha_0 + (1 - \alpha_1)f)/B + \rho$.

The cost of consumption to the parent (including the cost of the attached children) is given by

$$\int_I^T e^{-r(a-I)} c(a) da + e^f \int_0^I e^{-r(a+B-I)} c_k(a) da = c(I)G(I, R)$$

With this notation, the utility of a parent of generation t is

$$V(h_t, q_t, u_t) = \frac{c_t(I)^{1-\theta}}{1-\theta} G(I, R_t) + e^{f-rB} e^{-\mu} V(h_{t+1}, q_{t+1}, u_{t+1}),$$

or

$$\bar{U} = \sum_{t=0}^{\infty} e^{(f-rB)t} e^{-\mu t} \frac{c_t(I)^{1-\theta}}{1-\theta} G(I, R_t). \tag{3}$$

The budget constraint is simply

$$c_t(I)G(I, R_t) + e^{f-rB} q_{t+1} \leq W(I, T, R_t) + e^{f-rB+rI} W(0, I, R_{t+1}) + q_t + u_t$$

where

$$W(z, b, R) = \max_{x,n} \int_z^b e^{-r(a-z)} y(a, R) da,$$

is the maximized —over the allocation of time and goods inputs used in producing human capital— present discounted value of labor income. This value function takes as given the retirement age, R.

Iterating forward the budget constraint we obtain

$$\begin{aligned} \sum_{t=0}^{\infty} e^{(f-rB)t} c_t(I) G(I, R_t) \leq & \sum_{t=0}^{\infty} e^{(f-rB)t} [W(I, T, R_t) + e^{f-rB+rI} W(0, I, R_{t+1})] \\ & + \sum_{t=0}^{\infty} e^{(f-rB)t} u_t + q_0. \end{aligned} \tag{4}$$

The problem faced by the household is to maximize equation (3) subject to equation (4). The first order conditions for an interior maximum assuming that $W(\cdot, \cdot, R)$ is differentiable with respect to its third argument (which it is not always the case due to policy-induced kinks in the retirement benefit function) are

$$e^{-\mu t} c_t(I)^{-\theta} = \Phi,$$

$$e^{-\mu t} \frac{c_t(I)^{1-\theta}}{1-\theta} G_R(I, R_t) - \Phi c_t(I) G_R(I, R_t) = -\Phi W_R(I, T, R_t),$$

which can be reduced to

$$W_R(I, T, R_t) = \frac{\theta}{\theta - 1} G_R(I, R_t) c_t(I), \tag{5}$$

since $W_R(0, I, R) = 0$ by the envelope theorem.[4]

There are two cases to consider. If $R \geq R_n$, then

$$W_R(I, T, R_t) = e^{-r(R_t - I)} \left[(1 + b_y D(R_t))(1 - \tau) w h(R_t) - [b_m + b_y \int_0^{R_t} e^{-ra} y(a, R_t) da] \right],$$

while in the case in which retirement occurs prior to the normal retirement age the relevant value is

$$W_R(I, T, R_t) = e^{-r(R_t - I)} (1 + b_y D(R_t))(1 - \tau) w h(R_t),$$

where $D(R) = \frac{e^{-rR} - e^{-rT}}{r}$. In both cases, the second order conditions require that the function $W_R(I, T, R_t)$ intersect the function $\frac{\theta}{\theta - 1} G_R(I, R_t) c_t(I)$ from above.

3.1 Properties of the Solution

In this section we describe some properties of the model and we distinguish between the long run (steady state) response of labor supply to changes in variables as well as the effect of unanticipated changes in some exogenous variables on effective labor supplies and retirement decisions as a function of a worker's age.

3.1.1 The Long Run

In this model the optimal choice of human capital, conditional on the retirement age, is independent of the discount factor: individuals maximize the present discounted value of

[4] Note that $W(0, T, R) = W(0, I, R) + W(I, T, R)$ and since $W(0, T, R)$ is a value function it follows that its derivative with respect to the retirement age is given by the appropriate marginal values in the interior case. This implies that $W(0, I, R)$ is independent of R.

income and since the relevant discount factor is the interest rate, investments in human capital are independent of μ. However, outside the steady state, i.e. when $\mu \neq 0$, the equilibrium retirement is not constant. To see this consider the case in which the interest rate exceeds the generational discount factor, i.e. $\mu < 0$. In this situation generational consumption increases over time. This shifts the right hand side of equation (5) upward and the equilibrium level of retirement, R_t, must be decreasing. Thus, this model delivers the result that whenever interest rates exceed their long run values —which happens during the transition to a steady state— there is a downward trend in retirement. In the case of this model this is driven by income effects as retirement allows individuals to enjoy more leisure.

In the rest of this section we restrict attention to the implications of the model when $\mu = 0$, which corresponds to the steady state.

It is straightforward to compute the steady state value of consumption. It is given by

$$c(I)G(I,R) = W(I,T,R) + e^{f-rB+rI}W(0,I,R) + (1-e^{f-rB})q + u.$$

The relevant first order condition for the equilibrium choice of retirement age (except at the kink) is

$$W_R(I,T,R) = \frac{\theta}{\theta-1}\frac{G_R(I,R)}{G(I,R)}[W(I,T,R) + e^{f-rB+rI}W(0,I,R) + (1-e^{f-rB})q + u] \quad (6)$$

Hereafter, we denote by $B(R)$ the right-hand side of equation (6). This expression summarizes the implications of the model for the choice of retirement. Since $G_R(I,R) > 0$ and the function $W_R(I,T,R)$ must intersect $B(R)$ from above, equation (6) implies that the net marginal benefit of working one more year exceeds the marginal monetary cost at the equilibrium retirement age (which would correspond to the condition $W_R(I,T,R) = 0$). The reason is simple: the opportunity cost of working is the additional utility associated with full time leisure. It is this second concept that is captured by the function $B(R)$.

If $R > R_n$ equation (6) is equivalent to

$$e^{-r(R-I)}\left[(1+b_y D(R))(1-\tau)wh(R) - [b_m + b_y\int_0^R e^{-ra}y(a,R)da]\right] = B(R) \quad (7)$$

Changing the Progressivity of the Retirement System The previous expression can be used to understand how the level and progressivity of the retirement system affect the retirement age. An increase in the fixed component, b_m, decreases retirement age as it shifts down the left hand side of equation (7) at the same time that it moves $B(R)$ up. It follows that equilibrium retirement increases.

To determine the impact of changing the progressivity of the retirement regime we study a change in (b_m, b_y) that *keeps the benefit level constant at the initial retirement age.* Let the benefit level be denoted

$$\bar{b} = b_m + b_y \int_0^R e^{-ra} y(a, R) da.$$

Equation (7) is now given by

$$e^{-r(R-I)} \left[(1 + b_y D(R))(1 - \tau) w h(R) - \bar{b} \right] = B(R)$$

and it follows that increases in b_y (which of course requires a decrease in b_m) shift the left hand side upward with no change in the function $B(R)$. In this case the equilibrium retirement age increases. The intuition is clear: returns to work associated with this change increase as the system becomes less redistributive and, hence, the pure tax component of the retirement system decreases. This shows that it is not only the level of benefits but the relative weight of the two components that influence retirement.

Changing the Composition of Government Expenditures A change in the composition of government spending has an impact on retirement. Consider a government that keeps tax rates constant and increases government consumption while at the same time decreasing transfers. In this case, holding the retirement age constant, the equilibrium level of consumption must decrease (see equation (11) in the next section). Since the function $B(R)$ satisfies

$$B(R) = \frac{\theta}{\theta - 1} G_R(I, R) c(I)$$

it follows that it must shifts down. Then, equation (5) implies that the retirement age must increase.

If individuals choose to retire before the normal retirement age the analog of equation (7) is

$$e^{-r(R-I)} (1 + b_y D(R_n))(1 - \tau) w h(R) = B(R) \tag{8}$$

but the effects of changes in the retirement regime are qualitatively the same.

Finally, it is trivial to show that increases in R holding taxes constant result in higher levels of schooling and more human capital. The intuition for this result is simple: as the rate of utilization of human capital increases its return becomes higher and individuals respond by increasing their investment.

3.1.2 The Short Run

In this section we describe the response of labor supply, both in terms of career length and quality, to three types of shocks: an unanticipated drop in the value of accumulated assets, a decrease in an individual's stock of human capital, and an unanticipated (and permanent) decrease in the wage rate.

We find it convenient to study the impact of a change that occurs when individuals are of age $a' > B+I$. This means that children are independent (and the parent does not adjust the bequests) and, in this case, the response captures the standard life cycle effects.[5] For simplicity, we consider the case in which individuals choose $R < R_n$, and they remain in this range after the shock. If this was not the case, the qualitative results are similar but the expressions need to be adjusted.

A Wealth Shock The relevant version of equation (6) is

$$W_R(a', T, R) = \frac{\theta}{\theta - 1} \frac{\tilde{G}_R(a', R)}{\tilde{G}(a', R)} [W(a', T, R) \tag{9}$$
$$+ e^{ra'} D(R_n)(b_m + b_y W(0, a', R)) + A(a')],$$

where $A(a')$ is the value of assets accumulated up until age $a' < R$, and

$$\tilde{G}(a', R) = e^{\upsilon(r)a'} [(\Delta(R) - \Delta(a')) + (1 + \zeta)^{\frac{1-\theta}{\theta}} (\Delta(T) - \Delta(R))]$$

As before, the second order conditions require that the left hand side of equation (9) crosses the right hand side from above.

Consider first the effect of an unanticipated decrease in the value of non-human wealth , $A(a')$. This shock shifts the benefits from retiring —the right hand side of equation (9)—

[5] The results are similar for all ages but the expressions are algebraically more cumbersome and income effects must include the adjustment of bequests associated with shocks.

down. Since $W_R(a', T, R)$ is independent of wealth the left hand side of equation (9) does not change and, therefore, the retirement age increases.

Moreover, since $\tilde{G}_R(a', R)/\tilde{G}(a', R)$ is increasing in a', the impact of a given change in $A(a')$ on the right hand side of equation (9) is increasing in age. This implies that the impact of a wealth shock on retirement depends on the age of the worker: older individuals will react to a loss of wealth postponing their planned retirement by more than younger workers do.

A Wage Shock What is the effect of an unanticipated permanent decrease in the wage rate, w, when a worker is a' years old? Abstracting from taxes and benefits here, we use equation (21) in the appendix to obtain

$$\frac{\partial h^e(a)}{\partial w} = \frac{\gamma_2}{1-\gamma} C_h \left(\frac{w}{p}\right)^{\frac{\gamma_2}{1-\gamma}} \frac{1}{w} \pi(a', a), \tag{10}$$

where $h^e(a)$ is the effective supply of labor —defined as $h^e(a) = h(a)(1-n(a))$— by a worker of age a,and

$$\begin{aligned} \pi(a', a) &\equiv (r+\delta_h) \int_{a'}^{a} e^{-\delta_h(a-u)} m(u)^{\frac{\gamma}{1-\gamma}} du - \gamma_1 m(a)^{\frac{1}{1-\gamma}}, \\ m(a) &= 1 - e^{-(r+\delta_h)(R-a)}, \\ C_h &= \left(\frac{z_h}{r+\delta_h} \gamma_1^{\gamma_1} \gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}}. \end{aligned}$$

The sign of the impact on effective labor of a change in the wage rate depends on the sign of $\pi(a', a)$. To understand the impact on retirement, let us fix R and $a' < R$. Simple calculations (omitted) show that exists $\varphi(a') \in (a', R)$ such that

$$\pi(a', a) = \begin{cases} < 0 & \text{if } a' \leq a < \varphi(a') \\ = 0 & \text{if } a = \varphi(a') \\ > 0 & \text{if } \varphi(a') < a \leq R \end{cases}$$

This result shows that, immediately following a permanent decrease in the wage rate, effective labor supply increases. The size of the increase depends on the worker's age: young workers increase their effective hours more than older workers.[6].

[6]To be precise. if $a' = R$ there is no change in effective labor supply.

Over time, the effect of the change in the wage rate reverses itself. For a given age a' at the time of the shock, there exist another age, $\varphi(a') > a'$, that is such that the post-shock supply of effective labor coincides with the pre-shock value. Finally, for a given a', if $a > \varphi(a')$, the post-shock effective supply of labor, $h^e(a)$, drops below its pre-shock value.

The intuition for this result is as follows: when faced with a permanent decrease in the wage rate the worker chooses to lower investment in human capital. In the short run, this results in more human capital supplied to the market as he changes his allocation of time and, hence, the increase in effective labor. Eventually, the lower level of investment in skills is such that the stock of human capital drops below its pre-shock level, and effective labor supply is lower.

For a fixed retirement age, the endogenous adjustment in the quality dimension implies that the response of the age-earnings profile to a drop in the wage rate is non-monotonic. Initially, workers supply more labor, but eventually the level of effective hours drops. As shown in the Appendix, the qualitative response of earnings mirrors that of effective labor.

What is the impact on the retirement age? In general, it is not possible to determine the effect of a wage change because the impact of a decrease in w on $W_R(a', T, R)$ is ambiguous. However, it is possible to show that for a' close to R (i.e. for older workers), a lower wage rate lowers $W_R(a', T, R)$. Thus, the left hand side of equation (9) decreases. The right hand side decreases as well. However, for older workers (i.e. as $a' \to R$), the change is arbitrarily small. Thus, for older workers the model predicts that a decrease in wages results in earlier retirement.

The situation for younger workers is more complicated since there are two forces at play. On the one hand even if it is the case that a lower wage rate decreases the marginal benefit of working, $W_R(a', T, R)$, the negative income effect lowers the right hand side of equation (9) and this tends to push retirement in the opposite direction.

A Shock to Human Capital Finally, consider the effect of an exogenous decrease in the stock of human capital at age a'. This could be due, for example, to a shock that requires reallocating workers across sectors and/or firms when human capital is (at least partially) sector or firm specific. In this case, the effect on effective labor supply diminishes over time.

To be precise, the effect of an exogenous change in $h(a')$, is given by

$$\frac{\partial h^e(a)}{\partial h(a')} = e^{-\delta_h(a-a')}, \text{ for } a \geq a'.$$

Unlike the case of a negative wage shock, the younger the worker the smaller the long run impact of a human capital loss, conditional on the retirement age. The intuition for this result is simple: a shock to the stock of human capital does not alter the payoff from investing. Since younger workers have a longer horizon, they accumulate more human capital than older workers and, hence, the impact of a loss of part of their human capital on their effective supply labor is lower.

As in the case of a wage shock, income and substitution effects push the retirement age in opposite directions. A sufficient condition for the substitution effect to dominate, and hence, for the worker to retire earlier than planned is

$$\frac{r+\delta_h}{e^{(r+\delta_h)(R-a')}-1} \geq \frac{\theta}{\theta-1}\frac{\tilde{G}_2(a',R)}{\tilde{G}(a',R)}.$$

It is straightforward to check that this condition is satisfied for older workers.

To summarize, we find that the impact of shocks on retirement depend both on the standard tension between income and substitution effects as well as the specifics of the determinants of the age-effective supply schedule. Unlike the majority of the models used to study career length, the shape of the age earnings profile and, consequently, the retirement decision, is not independent of the nature of the shock as the age-effective supply schedule is endogenously determined. Moreover, in the case of wage shocks, the new age earnings profile is not a simple translation of the old one as it shifts in non-monotonic fashion.

3.2 Equilibrium

In order to close the model, let $\eta = f/B$ be the growth rate of population. In this case, the fraction of the population of age a is

$$\phi(a) = \eta\frac{e^{-\eta a}}{1-e^{-\eta T}}.$$

The equilibrium (in the steady state) requires that

$$\int_0^T [c(a)+x(a)]\phi(a) + g = [zF(\kappa,1) - (\delta_k+\eta)\kappa]\int_{6+s}^R h^e(a)\phi(a)da. \tag{11}$$

This is simply the GDP identity. Since we assume competitive factor markets firms equal the marginal product of capital to the cost of capital and labor,

$$r = (1 - \tau_I)(zF_k(\kappa, 1) - \delta_k), \tag{12}$$

and

$$w = zF_n(\kappa, 1). \tag{13}$$

We assume that the government runs a balanced budget and hence

$$g + u = \tau_I[zF(\kappa, 1) - \delta_k \kappa] \int_{6+s}^{R} h^e(a)\phi(a)da.$$

Finally, output per worker is

$$y = zF(\kappa, 1) \frac{\int_{6+s}^{R} h^e(a)\phi(a)da}{\int_{6+s}^{R} \phi(a)da},$$

while output per capita is

$$y_C = zF(\kappa, 1) \int_{6+s}^{R} h^e(a)\phi(a)da.$$

3.3 Calibration

We calibrate the model to match recent U.S. data. We use the parameters of the human capital production function and the altruism function from Manuelli and Seshadri (2009). We calibrate the benefit function to approximate the U.S. benefits schedule as explained below, and pick preference parameters to match the retirement age and the drop in consumption.

Retirement benefit function We use OECD data on replacement rates at different levels of income. We consider individuals who earn 50 percent and 150 percent of the average wage per worker and assume that the benefit function is of the form

$$b_0 + b_1 y.$$

Let $\bar{r}_j, j \in \{1/2, 3/2\}$ be the replacement rates. Then if y denotes average income we require the simple specification to be consistent with the observed values.

$$b_0 + b_1 \times \frac{y}{2} = \bar{r}_{1/2} \times \frac{y}{2}$$
$$b_0 + b_1 \times \frac{3}{2}y = \bar{r}_{3/2} \times \frac{3}{2}y$$

Given data on $\bar{r}_j, j \in \{1/2, 3/2\}$, this system of equations can be solved for b_0/y and b_1. To calibrate the model we require that

$$b_m = b_0,$$
$$b_y Y(R) = b_1 y$$

In the model the social security regime is balanced. In the data it is not. As a first pass, we fix expenditures (benefits) according to the above calculations and choose a portion of total income taxes to finance the system. Note that, from an individual point of view, all that matters is the total tax rate (τ) and not necessarily the portion that is labeled social security contribution (τ_S). We find that for the U.S. $b_0/y = 0.12$—which corresponds to 12 percent of average labor income—and $b_1 = 0.26$.

In our calibration, we find that $b_m = 35$ which matches the U.S. value, and $b_y = 0.093$. We also find that the tax needed to finance this expenditure is $\tau_S = 0.076$. We use the values of the total tax wedge (OECD(2009a)) as our estimate of τ. For the U.S. OECD (2009a) reports that $\tau = 0.301$ which coincides with the estimate from Wallenius (2009).

Other Parameters The base calibration sets the parameters of the human capital accumulation function as follows, $\nu = 0.55$, $z_h = 0.3525$, $\gamma_1 = 0.63$, and $\gamma_2 = 0.3$. We set $\rho = 0.04$ and pick the imperfect altruism parameters equal to $\alpha_0 = .24, \alpha_1 = .65$. Since the U.S. population growth rate is about 1% this implies that the after tax interest rate in the calibration is 5.31%.

For the model to match a capital output ratio of 2.5 given a capital coefficient, α, equal to 0.33, we choose $\delta_k = 0.071$

Simple calculations show that

$$w = z^{1/(1-\alpha)}(1-\alpha)\left(\frac{\alpha(1-\tau_I)}{r + \delta_k(1-\tau_I)}\right)^{\alpha/(1-\alpha)}.$$

Thus, with knowledge of r (assumed to be 0.0531), the depreciation rate ($\delta_k = 0.071$), and the tax rate on ordinary income we can infer z. We find that $z = 0.4932$ and $w = 0.381$

We set $h_B = 6.3$ to match the average schooling in the U.S., and $\theta = 1.07$ to match the average retirement age. We find that $\zeta = 30$ matches the 20 percent drop in consumption at retirement. Finally, in this version, we only consider the case in which on the job training expenditures are not deductible at the individual level and set $\xi = 1$.

Laitner and Silverman (2008) present estimates of θ that range from 1.4 to 1.7 and they find that plausible values of ζ cluster in the interval [1.0,1.5]. The drop in utility at retirement is given by $(1+\zeta)^{\frac{1-\theta}{\theta}}$. Since we find that θ is closer to 1 than Laitner and Silverman (2008) we find a much larger value of ζ. The curvature parameter in our model plays a different role than in Laitner and Silverman (2008) given that in our paper the age-earnings profile is endogenous.

4 Results

We used the calibrated parameters for the U.S. to predict the performance of a small sample of European countries. As we move across countries we change their demographic characteristics (population growth rate η and life expectancy T), tax rates and benefit functions to match their replacement ratios, and we adjust the productivity parameter, z, to match actual output per worker. Since we do not require the model to match schooling for countries other than the U.S., it is good test of how well it performs. The predictions of the model and the data (from Barro and Lee (2010)) appear in Table 1.

Table 1: Years of Schooling

Country	Data	Model
Denmark	10.20	10.95
France	10.43	10.03
Japan	11.40	11.40
Mexico	9.20	9.32
Spain	10.35	10.88
U.S.	12.40	12.40

In Table 2 we compare the predictions of the model with the "average effective retirement age" (OECD(2009b)) and display the effective tax rates required by the retirement system.

Table 2: Retirement Age and Tax Rate				
Country	Official	Avg. Effective	Model	τ_S (percent)
Denmark	65	63.5	62.74	18.5
France	60	58.7	60.01	18.5
Japan	63	69.5	63.94	12.0
Mexico	65	73.0	69.98	3.0
Spain	65	61.4	64.78	21.0
U.S.	65	64.6	64.6	7.6

There are several interesting features. First the model does reasonable well in tracking retirement patterns. It clearly misses the high retirement age in Japan and this suggests that this simple model is not rich enough to account for the behavior of Japanese workers.

Second, even though the model underpredicts the retirement age in Mexico it shows that there are no inconsistencies between lower life expectancy and a high retirement age: Mexico's life expectancy is lower than the U.S. and nevertheless the model predicts that Mexican workers will retire five years after U.S. workers. There are several issues to consider that might account for the discrepancy between the predictions of the model and the Mexican evidence. Informality probably plays a role as some workers are not covered by the pension system. Taking into account the forces that appear in the model there are several that stand out: the tax rate is about half of the U.S. (15% vs. 30%) and this works in the direction of higher investment in human capital and a longer career length. Lower life expectancy and lower real wages push retirement in the opposite direction. Finally, the level of implicit transfers, defined as the difference between non-social security tax revenue and government spending is much smaller than in other countries. Thus, the smaller income effect has the effect of increasing the retirement age.

In the model, the more generous benefit function in Denmark results in a much earlier retirement. The Spanish system is almost proportional as individuals receive about 80 percent of their own earnings as retirement benefits. Overall, except in the case of Mexico, the model predicts that retirement occurs relatively close to, but not at, the age of eligibility.

Not surprisingly we find that to support some of the generous pension benefits in European countries high (relative to the U.S.) effective tax rates are needed.

4.0.1 Counterfactual Experiment I

In this section we study the effect of imposing a U.S. style retirement benefit system in each of the countries. The results are in Tables 3 through 5. In this exercise we adjust the effective tax rate on social security to balance the budget and we keep the tax rate on other income, τ_I, and the level of government spending relative to output unchanged. Thus, the total tax burden (τ) decreases by the magnitude of the drop in τ_S. We identify the post-change values with asterisks.

Table 3: Years of Schooling and Taxes

Country	s	s^*	τ_S	τ_S^*	τ	τ^*
Denmark	10.95	13.84	18.5	8.2	41.2	30.9
France	10.03	12.98	18.5	9.8	49.3	40.6
Japan	11.20	11.44	12.0	12.3	29.5	29.8
Mexico	9.32	9.82	3.0	3.0	15.1	15.1
Spain	10.88	13.55	21.0	9.8	37.8	26.6
U.S.	12.40	12.40	7.6	7.6	30.1	30.1

In the high tax European countries the lower tax induces an increase in schooling close to three years. Lower taxes, according to the model result in significant increases in output per worker (see Table 4). For example, the model predicts that Denmark's output per worker can be 50 percent higher than the U.S. value in the steady state. This is driven by our estimate of TFP. In the base case, since the Danish economy (as well as the other European economies) has relative large distortions, the model assumes that their level of TFP is relatively high in order to match the observed level of output per worker. When the distortions are reduced this produces a dramatic increase in output.

To put this number in perspective, since convergence to a steady state in version of this model takes approximately forty years, the impact of the social security reform in Denmark is equivalent to an increase in the growth rate of about 1%.

Table 4: Per-Worker Output and Retirement				
Country	y	y^*	R	R^*
Denmark	0.82	1.52	62.74	64.89
France	0.80	1.50	60.01	62.45
Japan	0.69	0.72	63.94	65.24
Mexico	0.34	0.37	69.98	70.30
Spain	0.72	1.30	64.78	65.36
U.S.	1.00	1.00	64.60	64.60

A switch to a U.S. style retirement system induces increases in retirement ages in the European countries but hardly any effects in Japan and Mexico since those countries are, in some sense, close to the U.S. in terms of their retirement systems.

Table 5 presents the predictions of the model for two measures of labor supply: lifetime hours and effective human capital per year of schooling relative to U.S. values.

Table 5: Labor Supply				
Country	L	L^*	$\bar{h}^e/s$	$\left(\bar{h}^e/s\right)^*$
Denmark	45.79	45.05	0.84	1.25
France	43.98	43.47	0.78	1.13
Japan	46.74	47.80	0.85	0.86
Mexico	54.66	54.48	0.69	0.72
Spain	47.90	45.81	0.82	1.17
U.S.	46.20	46.20	1.00	1.00

The effect on the two measures —quantity and quality of effective labor— of switching to a U.S. style retirement system is very different. The model predicts hardly any changes in lifetime labor supply. In the European countries the increase in the retirement age roughly matches the increase in schooling, leaving career lengths approximately unchanged. In Spain lifetime hours decrease. The big impact comes from the changes in the quality, or human capital, of the labor force. Relative to the U.S., European countries increase the effective human capital per year of schooling from a level that is about 15 percent below the U.S.

to values that are somewhere between 13 and 25 percent higher than the U.S. It is at this intensive margin where the changes in social security regimes have an impact.

4.0.2 Counterfactual Experiment II

In this section we explore the effect on the U.S. of demographic and policy changes. We study the predictions of the model for six experiments:

- A two year increase in expected lifetime (ΔT).
- A two year increase in the normal retirement age (ΔR_n).
- Eliminating the progressivity of the pension system holding the average replacement rate constant ($b_m = 0$).
- A decrease in the population growth rate from 1 percent to 0.5 percent ($\Delta\eta$)
- An income tax cut of 1 percentage point ($\Delta\tau$)
- Eliminating social security (P)

The results are in Tables 6 and 7

Table 6: Demographic Shocks

	Baseline	ΔT	$\Delta\eta$
y	1.00	0.96	0.93
s	12.40	12.26	12.04
R	64.60	64.99	63.39
τ_S	7.55	8.60	9.00
L	46.20	46.73	45.35
$\bar{h}^e/s$	1.00	0.98	0.95

Demographic changes of the magnitude that we study can have non-negligible effects on output but small effects on the retirement age and the length of careers. Higher life expectancy and lower population growth both have the same impact: they increase the size of the retired population and this necessitates an increase in taxes. This, in turn discourages investment in human capital and this drives the decline in output.

Table 7: Policy Shocks					
	Baseline	ΔR_n	$b_m = 0$	P	$\Delta\tau$
y	1.00	1.07	1.01	1.47	1.06
s	12.40	12.73	12.48	14.10	12.66
R	64.60	64.88	64.92	65.55	64.78
τ_S	7.55	6.30	7.55	0.00	7.50
L	46.20	46.15	46.44	45.45	46.12
$\bar{h}^e/s$	1.00	1.05	1.01	1.30	1.04

Somewhat surprisingly eliminating the progressivity built into the retirement system has very little effect on output or any other variable. Increasing the normal retirement age to 67 results in a long run increase in output of about 7 percent with hardly any changes in lifetime labor supply. Formal schooling changes by a little over a month but its quality (and the amount of on-the-job training) increases so that the effectiveness of formal education —as measured by $\bar{h}^e/s$— increases by 5 percent. The effects of an across the board (including taxes on capital income) 1 percentage point tax cut are very similar to those of the increase in the normal retirement age.

Eliminating the social security system has a dramatic long-run impact on output per worker, which is estimated to increase by 47 percent. This is attained with no change in lifetime labor supply and less than 16 percent increase in schooling. The largest factor is the increase in the amount of human capital per year of schooling (approximately 30 percent).

4.0.3 The U.S. in 1900

As a final experiment we used the model to simulate labor supply and retirement patterns for an economy with the same parameter values but with a much lower life expectancy ($T = 52$), no taxes (or government spending), no social security system, and a level of TFP that results in output per worker being very similar to the level of the U.S. in 1900 relative to 2000, about 19 percent. The results are in Table 8.

Table 8: U.S. 1900

	Baseline	1900
y	1.00	0.19
s	12.4	4.8
R	64.6	52
τ_S	7.6	0.0
L	46.2	41.2
$\bar{h}^e/s$	1.00	0.50

There are two interesting features of the results. First, the model predicts that people worked until the day they died, which is not that far from reality. Second, the model also does reasonably well predicting schooling for the U.S. economy in 1900. The prediction (4.8 years) is not that far from the actual value, 5.4 years of schooling.

5 A More General Model of Social Security Wealth

In this section we describe an extended version of the model defining the benefit function that adds two features: an age (and income) independent component and age-dependent eligibility criteria. In this version, the social security regime consists of three components:

1. *Medicare*: We model Medicare as a monetary payment from the system to a retired individual that accrues whenever he retires. Thus, we ignore health shocks and we take Medicare as a monetary transfer. Since our model has no health shocks, the monetary value of Medicare is, conceptually, the certainty equivalence value of Medicare payments. Alternatively, it can be approximated by the cost of purchasing insurance in an open market (e.g. the rate charged by the private sector for similar health insurance of a 64 year old individual is a lower bound). Denote this payment by b_m.

2. *Income based old age support*: We assume that an individual gets a fraction b_y of his present value of income after he retires. Note that we can compute the value of contributions and make this a "fair" return. However, for now, we take b_y as a small

number. Thus, the flow of income from this source in present value is $b_y Y(R)$, where $Y(R)$ is fully described in the Appendix and corresponds to the present discounted value of income of an individual who retires at age R.

3. *Redistributive old age support*: We assume that after they retire, all individuals receive payments—which depend on the retirement age—so that their present value is constant. We denote that value by b_r. This, we make actuarially fair, that is, the flow payment is adjusted —using the market interest rate— to take into account life expectancy.

In this case one an individual reaches a certain age he becomes eligible to receive different components of the benefit. There are two important ages, R_e, the earliest age at which an individual can retire and still receive social security benefits (but no health insurance through Medicare), and R_n, the "normal" retirement age (the age at which individuals are eligible for Medicare)

The flow of payments depending on the retirement age are given by the following table:

1. If an individual retires at age $R < R_e$ then he receives

$$b(a,R) = \begin{cases} 0 & \text{if } R \leq a < R_e \\ \frac{rb_r}{e^{-rR_e} - e^{-rT}} + b_y Y(R) & \text{if } R_e \leq a < R_n \\ \frac{rb_r}{e^{-rR_e} - e^{-rT}} + b_y Y(R) + b_m & \text{if } R_n \leq a \end{cases}$$

2. If an individual retires at age $R_e \leq R < R_n$

$$b(a,R) = \begin{cases} \frac{rb_r}{e^{-rR} - e^{-rT}} + b_y Y(R) & \text{if } R \leq a < R_n \\ \frac{rb_r}{e^{-rR} - e^{-rT}} + b_y Y(R) + b_m & \text{if } R_n \leq a \end{cases}$$

3. If an individual retires at age $R_n \leq R$

$$b(a,R) = \frac{rb_r}{e^{-rR} - e^{-rT}} + b_y Y(R) + b_m, \text{ for all } a.$$

Given this specification the present discounted value of Social Security wealth in each case, i.e. depending on the retirement age satisfies: (Need to change this)

1. If the individual retires at age $R < R_e$ the present value of social security benefits is

$$B_o(R) = b_r + \frac{e^{-rR_e} - e^{-rT}}{r} b_y Y(R) + \frac{e^{-rR_n} - e^{-rT}}{r} b_m.$$

2. For an individual who retires between the ages of R_e and R_n the present value is

$$B_e(R) = b_r + \frac{e^{-rR} - e^{-rT}}{r} b_y Y(R) + \frac{e^{-rR_n} - e^{-rT}}{r} b_m.$$

3. Social Security wealth for an individual who retires at the normal retirement age ($R_n \leq R$) is

$$B_n(R) = b_r + \frac{e^{-rR} - e^{-rT}}{r} [b_y Y(R, s) + b_m].$$

In this more realistic formulation the benefits function is continuous (in R) but not differentiable at points R_e and R_m. At those ages, the function has a kink. Let's denote, for any function B, the left and right derivatives by

$$\frac{\partial B_L^-}{\partial R} = \lim_{R \nearrow R_m} \frac{\partial B_L}{\partial R},$$

and, similarly, let

$$\frac{\partial B_D^+}{\partial R} = \lim_{R \searrow R_m} \frac{\partial B_D}{\partial R}.$$

Simple calculations show that

$$\frac{\partial B_n^-}{\partial R} |_{R=R_m} = \frac{\partial B_e^+}{\partial R} |_{R=R_m} - e^{-rR_n} b_m,$$
$$\frac{\partial B_e^-}{\partial R} |_{R=R_e} = \frac{\partial B_o^+}{\partial R} |_{R=R_e} - e^{-rR_n} b_y Y(R).$$

Thus, the derivative of the benefit function is discontinuous and, at the two statutory ages, it jumps down. In the version of the model that adds heterogeneity we expect these thresholds to account for the spikes seen in the data.

6 Future Work

In the next version we hope to add heterogeneity and to impose a non-negative bequest restriction. We view heterogeneity as being driven by differences in ability to learn, z_h, and we will pick the distribution of ability to match the U.S. distribution of schooling.

The non-negative bequest restriction was not binding in our exercises but we expect it to be binding when we add heterogeneity and when we study transitions, since policy changes that increase output in the future will probably result in parents borrowing from their (richer children).

7 Appendix

In this appendix we present some more detailed results about the equilibrium choice of human capital and a description of preferences that result in the same reduced form utility that we are using

Equilibrium Choice of Schooling and Human Capital In this section we characterize the functions $W_j(R)$. Since the choice of investments in human capital for a given retirement age is not distorted, it is possible to characterize the objects that we are interested in — $W_j(R)$ functions— by using the solution to a standard income maximization problem.

It is straightforward to show that the before tax net present discounted value of income of an a year old individual who is in the labor force, has human capital h, and who plans to retire at age R, $V(h,a,R)$ is given by

$$\tilde{W}(h,a,R) = (1-\tau)w\left[\frac{m(a)h}{r+\delta_h} + (1-\gamma)C_h\left(\frac{w}{p}\right)^{\frac{\gamma_2}{1-\gamma}}\int_a^R e^{-r(u-a)}m(u)^{\frac{1}{1-\gamma}}du\right], \tag{14}$$

where

$$m(a) = 1 - e^{-(r+\delta_h)(R-a)}$$

$$C_h = \left(\frac{z_h}{r+\delta_h}\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma}}$$

Thus, the problem faced by a "hypothetical" individual who makes optimal decision since birth, conditional on the retirement age, is

$$Y(R) = \max -x_E - \int_6^{6+s} e^{-ra}x(a) + e^{-r(6+s)}(1-\tau)\tilde{W}(h(6+s),6+s,R),$$

subject to

$$h(6) = h_E = h_B x_E^{v},$$
$$\dot{h}(a) = z_h h(a)^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \quad \text{for } 6 \leq a \leq 6+s.$$

The Hamiltonian for this problem (before age $6+s$) is simply

$$H = -x + \hat{q}(z_h h^{\gamma_1} x^{\gamma_2} - \delta_h h).$$

where the costate variable satisfies

$$\frac{d\hat{q}(a)}{da} = r\hat{q}(a) - \hat{q}(a)[\gamma_1 z_h h^{\gamma_1} x^{\gamma_2} h^{-1} - \delta_h].$$

The boundary conditions are

$$\hat{q}(6+s) = \frac{(1-\tau)w}{r+\delta} m(6+s),$$

and that the value of the Hamiltonian at the boundary point (that is, at age $6+s$) must equal the negative of the impact on the continuation value $V(h(6+s), 6+s, R)$. Formally, it is given by

$$-x(6+s) + \hat{q}(6+s)[z_h h(6+s)^{\gamma_1} x(6+s)^{\gamma_2} - \delta_h h(6+s)]$$
$$= -\frac{\partial[e^{-r(6+s)} V(h(6+s), 6+s, R)]}{\partial s}.$$

Since the optimal choice of investment in school quality requires that

$$x(a) = \hat{q}(a)\gamma_2 z_h h(a)^{\gamma_1} x(a)^{\gamma_2}, \ 6 \leq a \leq 6+s,$$

the choice of schooling must satisfy,

$$\frac{m(6+s)}{r+\delta_h} \Bigg[(1-\gamma_2) \left(\gamma_2 \frac{(1-\tau)w}{r+\delta_h} m(6+s)\right)^{\frac{\gamma_2}{1-\gamma_2}} (z_h h(6+s)^{\gamma_1})^{\frac{1}{1-\gamma_2}} - \delta_h h(6+s)\Bigg] \tag{15}$$
$$= \frac{h(6+s)}{r+\delta_h}[rm(6+s) + (r+\delta_h)(1-m(6+s))]$$
$$+ (1-\gamma)C_h \left(\frac{w}{p}\right)^{\frac{\gamma_2}{1-\gamma}} m(6+s)^{\frac{1}{1-\gamma}}.$$

The previous condition gives a relationship between years of schooling and human capital at the end of the schooling period, $h(6+s)$. It is possible to show (see Manuelli and Seshadri (2009)) that the optimal (from an individual point of view) level of human capital at age a (for $a \leq 6+s$) is given by

$$h(a) = h_E e^{-\delta_h(a-6)} \left[1 + \left(h_E^{-(1-\gamma)} \hat{q}_E^{\gamma_2} \gamma_2^{\gamma_2} z_h\right)^{\frac{1}{1-\gamma_2}} \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + \delta_h(1-\gamma_1)} \left(e^{\frac{\gamma_2 r + \delta_h(1-\gamma_1)}{(1-\gamma_2)}(a-6)} - 1\right)\right]^{\frac{1}{1-\gamma_1}}, \ a \in [6, 6+s]. \tag{16}$$

Thus, in particular, $h(6+s)$, satisfies

$$h(6+s) = h_E e^{-\delta_h s} \left[1 + \left(h_E^{-(1-\gamma)} \hat{q}_E^{\gamma_2} \gamma_2^{\gamma_2} z_h\right)^{\frac{1}{1-\gamma_2}} \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + \delta_h(1-\gamma_1)} \left(e^{\frac{\gamma_2 r + \delta_h(1-\gamma_1)}{(1-\gamma_2)}s} - 1\right)\right]^{\frac{1}{1-\gamma_1}}. \tag{17}$$

Given the shadow price of capital $\hat{q}$, the optimal investment in early childhood capital solves

$$\max \hat{q}_E h_E - x_E,$$

subject to

$$h_E \leq h_B x_E^{\upsilon},$$

which implies that

$$h_E = (\upsilon \hat{q}_E)^{\frac{\upsilon}{1-\upsilon}} h_B^{\frac{1}{1-\upsilon}}.$$

Since we know that $M(a) = \hat{q}(a) h^{\gamma_1}(a)$ satisfies

$$\dot{M}(a) = M(a) \left[\frac{d\hat{q}(a)}{da} \frac{1}{\hat{q}(a)} + \gamma_1 \frac{\dot{h}(a)}{h(a)}\right],$$

simple manipulations show that

$$\frac{d\hat{q}(a)}{da} \frac{1}{\hat{q}(a)} + \gamma_1 \frac{\dot{h}(a)}{h(a)} = r + \delta_h(1-\gamma_1).$$

Thus, the function $M(a)$ satisfies the first order ordinary differential equation

$$\dot{M}(a) = M(a)[r + \delta_h(1-\gamma_1)]$$

whose solution is

$$M(a) = M(6)e^{[r+\delta_h(1-\gamma_1)](a-6)}.$$

Thus,

$$\hat{q}(6+s)h(6+s)^{\gamma_1} = \hat{q}_E h_E^{\gamma_1} e^{(r+\delta_h(1-\gamma_1))s},$$

or

$$(1-\tau)\frac{w}{r+\delta}m(6+s)h(6+s)^{\gamma_1} = (\upsilon^{\upsilon}h_B)^{\frac{\gamma_1}{1-\upsilon}}\hat{q}_E^{\frac{1-\upsilon(1-\gamma_1)}{1-\upsilon}}e^{(r+\delta_h(1-\gamma_1))s}. \tag{18}$$

To summarize, the equilibrium values of $(h(6+s), s, \hat{q}_E)$ are the solutions to the system formed by equations (15) and (18) and the equilibrium level of human capital at the end of the schooling period given by

$$\begin{aligned} h(6+s) = {} & (\upsilon\hat{q}_E)^{\frac{\upsilon}{1-\upsilon}} h_B^{\frac{1}{1-\upsilon}} e^{-\delta_h s}\Bigg[1 \\ & + \left((\upsilon\hat{q}_E)^{-\frac{(1-\gamma)\upsilon}{1-\upsilon}} h_B^{-\frac{(1-\gamma)}{1-\upsilon}} \hat{q}_E^{\gamma_2}\gamma_2^{\gamma_2} z_h\right)^{\frac{1}{1-\gamma_2}} \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r+\delta_h(1-\gamma_1)} \\ & \left(e^{\frac{\gamma_2 r+\delta_h(1-\gamma_1)}{(1-\gamma_2)}s}-1\right)\Bigg]^{\frac{1}{1-\gamma_1}}, \end{aligned} \tag{19}$$

The solution is conditional on R. The human capital of a working age individual is given by

$$h(a) = e^{-\delta_h(a-6-s)}h(6+s) + (r+\delta_h)C_h\left(\frac{w}{p}\right)^{\frac{\gamma_2}{1-\gamma}}\int_{6+s}^{a} e^{-\delta_h(a-u)}m(u)^{\frac{\gamma}{1-\gamma}}du, \tag{20}$$

Since effective labor supply is $h^e(a) = h(a)(1-n(a))$, it follows that the amount of human capital supplied to the market by individuals of age a is

$$h^e(a) = \begin{cases} h(a) - \left(\frac{z_h}{r+\delta_h}\gamma_1^{1-\gamma_2}\gamma_2^{\gamma_2}\left(\frac{w}{p}\right)^{\gamma_2}\right)^{\frac{1}{1-\gamma}} m(a)^{\frac{1}{1-\gamma}}, & a \in [6+s, R] \\ 0 & a \notin [6+s, R] \end{cases} \tag{21}$$

Alternative Specification of Preferences As an example of a utility function that yields the specification in the text as a reduced form but that explicitly considers the presence of some component of consumption expenditures that are job related consider a function $\hat{u}(c_\ell, c_w, \ell)$. In this formulation c_ℓ denotes consumption not related to supplying labor, c_w

denotes consumption necessary to supply labor, ℓ denotes leisure. Given that total consumption expenditures are given by $c_\ell + c_w = c$, the indirect utility function is simply

$$u(c,l) = \max_{c_\ell} \hat{u}(c_\ell, c - c_\ell, \ell).$$

As an example of a functional form consistent with our specification let

$$\hat{u}(c_\ell, c_w, \ell) = \left(\frac{c_\ell^{\phi(1-\ell)+\ell} c_w^{(1-\phi)(1-\ell)}}{\phi^\phi (1-\phi)^{1-\phi}} \right)^{1-\theta} \frac{1}{1-\theta},$$

the indirect utility function over consumption expenditures corresponds to our specification with

$$1 + \zeta = (\phi^\phi (1-\phi)^{1-\phi})^{-1}.$$

Data on Taxes and Pension Systems We use the data on *replacement rates* from **Pensions at a Glance 2009: Retirement-Income Systems in OECD Countries** to estimate the parameters of the benefit function. Then we set

$$b_1 y = b_y Y(R).$$

and we pick b_m so that b_m/y from the model equals b_0/y in the data. The values for the benefit function for the different countries in the sample are:

Benefit Function Parameters			**Replacement Rates**	
Country	b_0/y	b_1	50%	150%
Denmark	.42	.39	124.0	67.5
France	.10	.39	61.7	48.5
Germany	.03	.42	43.0	42.6
Japan	.13	.21	47.1	29.4
Mexico	.16	.24	55.3	34.5
Spain	0.0	.81	81.2	81.2
U.S.	.12	.26	50.3	34.1

Source: OECD (2009a)

The data about the social security regime comes from OECD.

Social Security

Country	$OASI(\%GDP)$	$R = 0.5$	$R = 1.5$
Denmark	7.2	124	67.5
France	12.7	61.7	48.5
Japan	8.6	47.1	29.4
Mexico	1.3	55.3	34.5
Spain	8.4	81.2	81.2
United States	6.1	50.3	34.1

Source: OECD (2010)(col 1), OECD (2009b) (col 2-3)

Notes: $R = 0.5$ is the pension income replacement ratio for a worker who earns 50% of the average wage while $R = 1.5$ is the replacement ratio corresponding to an individual who earns 150% of the average wage

Our estimate of the tax rate is taken from what OECD labels Total Tax Wedge which is defined as the "wedge between total labour costs to the employer and the corresponding net take-home pay for single workers without children at average earning levels in 2008" OECD (2009a). The data comes from Table 0.1.

OECD Tax Rates

Country	Tax Wedge (2008)
Denmark	41.2
France	49.3
Japan	29.5
Mexico	15.1
Spain	37.8
United States	30.1

References

[1] Aguiar, M. and E. Hurst, 2009, "Deconstructing Life Cycle Expenditure," working paper.

[2] Au, A., O. Mitchell, and J. W. Phillips, (2004), “Modeling Lifetime Earnings Paths: Hypothetical versus Actual Workers,” Michigan Retirement Research Center working paper 2004-074.

[3] Barro, R. and G. Becker, (1989), “Fertility Choice in Model of Economic Growth,” *Econometrica*, Vol. 57 No. 2, pp: 481-501.

[4] Barro, R. and J. Lee, (2010), “,” NBER working paper

[5] Boskin, M. and M.D. Hurd, (1978), “The Effect of Social Security on Retirement,” *Journal of Public Economics*, Vol. 10, pp: 361—377.

[6] Cavalcanti Ferreira, P. and M. Rodrigues dos Santos (2008?), “The Effect of Social Security, Demography and Technology on Retirement Behavior,” working paper, Graduate School of Economics, Fundacao Getulio Vargas.

[7] Feldstein, M., (1974), “Social Security, Induced Retirement, and Aggregate Capital Accumulation,” *Journal of Political Economy*, Vol. 82, No. 5, pp: 905-926.

[8] French, E., (2005), “The Effects of Health, Wealth, and Wages on Labour Supply and Retirement Behavior,” *Review of Economic Studies*, Vol. 72, No. 2, pp: 395-427

[9] Gruber, J. and D. A. Wise, (1999), **Social Security Programs and Retirement Around the World**, University of Chicago Press, Chicago.

[10] _____ and _____, (2004), **Social Security Programs and Retirement Around the World: Micro Estimation**, University of Chicago Press, Chicago.

[11] _____ and _____, (2007), **Social Security Programs and Retirement Around the World: Fiscal Implications of Reform**, University of Chicago Press, Chicago.

[12] Heckman, J. and B. Jacobs, (2009), “Policies to Create and Destroy Human Capital in Europe,” IZA Discussion Paper No. 468.

[13] HHS, (2009) Data on Medicare expenditures. From NHE Web Tables, accessed at http://www.cms.hhs.gov/NationalHealthExpendData/.

[14] Laitner, J. and D. Silverman, (2008), "Consumption, Retirement and Social Security: Evaluating the Efficiency of Reform that Encourages Longer Careers," working paper, University of Michigan.

[15] Ljungqvist, L. and T. Sargent, (2010), "Career Length: Effects of Curvature of Earnings Profiles, Earnings Shocks, and Social Security," working paper.

[16] Mitchell, O. S. and J. W. Phillips, (2006), "Social Security Replacement Rates for Alternative Earnings Benchmarks," Pension Research Working Paper 2006-6.

[17] OECD, (2009a), **Taxing Wages 2007/2008: 2008 Edition**, OECD: Paris

[18] OECD, (2009b), **Pensions at a Glance 2009: Retirement Income Systems in OECD Countries**, OECD: Paris.

[19] OECD, (2010), **Tax Database**, OECD:Paris.

[20] Rogerson, R., (1988), "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21, pp: 3-16

[21] Wallenius, J., (2009), "Life Cycle Labor Supply and Retirement in a Model with Endogenous Human Capital Accumulation," working paper