Limit-Pricing Oil Monopolies when Demand is Inelastic, Resource Taxation and Substitutes Subsidies*

Saraly Andrade de Sá
Professorship of Environmental Policy and Economics, ETH Zurich

Ujjayant Chakravorty
Department of Economics, University of Alberta

Julien Daubanes
Center of Economic Research, ETH Zurich

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Abstract

When demand for a resource is relatively inelastic and there exist substitutes for the resource, a monopolistic extractor may maximize her profits by supplying the minimum quantity that deters some substitutes' production. This limit-pricing behavior, although mentioned in some prior studies, is ruled out by the assumptions of the traditional treatments of market power in exhaustible resource supply. In this note, we extend the limit-pricing theory in several respects and draw its implications in terms of optimal taxation policies. The price inelasticity of oil demand, the existence of drastic substitutes, the rate at which the monopoly discounts future profits, increasing returns to scale in short-term extraction and limited effects of the remaining stock on extraction costs, are all empirically-relevant factors of limit pricing on the part of a resource monopoly. Under standard assumptions borrowed from the literature on optimal taxation of monopolistic extractors, we compare traditional optimal policies with optimal policies when limit pricing arises. We show that resource taxes/subsidies are strongly neutral, having no effect on the equilibrium extraction flows. However, well-designed taxes/subsidies on drastic substitutes can implement any optimal extraction path, but have not-so-intuitive effects on current extraction. These results apply even when the equilibrium also features an environmental distortion.

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1 Introduction

Assume that a monopolistic supplier faces a demand with price elasticity below unity. Assume furthermore that a perfect substitute for the monopoly's output is available at some positive constant marginal cost and is competitively supplied. In this context, the monopoly adopts a "limit pricing" at the price level of the competing substitute: above the substitute's price, the monopoly does not sell any unit; below, due to demand inelasticity, its marginal revenue is negative and its profits are thus increased with higher prices. This reasoning holds in particular if the monopoly under study supplies flows of a non-renewable resource.[1] Such limit pricing sharply differs from the conventional treatment of market power on resource markets. Although substantial support can be found for this theory (more on this shortly below), the economics literature has only paid poor attention to it. With this note, we want to dwell upon this neglected theory by extending it further and by drawing its main implications as regards fossil fuels' market regulation policies.

A major feature of the above example is the discontinuity of the monopoly's marginal revenue at the level of the substitute's unit cost. It stems from the joint properties that the residual demand is perfectly elastic at this price level and relatively inelastic below that price, implying marginal revenue to be negative. In traditional treatments of market power in the supply of exhaustible resources (Hotelling, 1931; Stiglitz, 1976; Lewis *et al.*, 1979; and others) either the marginal revenue's discontinuity or its negative sign are assumed away.[2] Following the well-known result that a static monopoly always operates in price regions where demand elasticity is above unity, those studies restrict demand elasticity to always be so. As Stiglitz (1976, p. 656, footnote 2) explained, this is to avoid that "one can obtain larger profits by reducing [the quantity]", and thus to guarantee that there exists a solution to the monopoly's problem. Some followers set up more general models and did not specify the form of the demand function. Instead,

[1] This simple example is developed in Section 2.

[2] In their analyses, a monopoly extracts the resource such that the marginal revenue net of marginal cost of extraction – instead of the price net of marginal cost under competition – rises at the rate of interest. Their assumptions allow the application of dynamic-optimization tools, in the tradition of the classical Hotelling literature. This is not so in our introductory example as well as in the general model of the paper where the monopoly's revenue is not differentiable at the equilibrium quantity.

as for example in Lewis *et al.* (1979), the demand's reaction was implicitly featured in the monopoly's revenue function, assumed to be increasing and concave. More recent examples include Gerlagh and Liski (2011) where the assumption that seller's flow payoff is always differentiable excludes limit pricing.

Three exceptions must be mentioned. Salant (1979), Dasgupta and Heal (1979) and Newbery (1981) evoked the possibility of a dynamic limit-pricing behavior by a monopolistic extractor. Salant's (1979) paper deals with cases where the residual demand for an exhaustible resource is kinked due to the existence of a perfect substitute. Under the assumption that elasticity is greater than unity, he established that reserves need not be exhausted at the time when the price reaches the substitute's unit cost. Instead, there might be a period during which the monopoly chooses to price marginally below this cost.[3] At this occasion, he quickly remarked what would occur if demand elasticity were lower than unity[4], *i.e.* if marginal profit were negative for prices below the substitute's cost: "if the profit function has a linear segment with a positive slope to the left of [the kink] but a negative slope for all extraction rates to the right of [the kink], it is intuitively plausible that the monopolist" would always adopt a limit-pricing behavior (p. 8). Therefore, when demand elasticity is lower than unity, not only during a second phase, but at all dates of the extraction period, the monopoly chooses to price marginally below the backstop's cost. Dasgupta and Heal (1979), in their comprehensive book, were led to point out at the same phenomenon: "the phenomenon of limit-pricing appears in a sharp form for the case where the elasticity of demand (...) is less than unity (...) (*i.e.* demand is inelastic)" (p. 343). Newbery (1981) addressed the same issue and provided a similar remark. So far, overall, the academic literature has thus not dedicated more than a few lines to the case the present paper is focusing on, *i.e.* that of a lower-than-unity demand elasticity.

Is there any reason to give any particular attention to the combination of low-elasticity demand with the existence of a substitute? First, it is often remarked that energy demand, oil demand in particular, is relatively price inelastic. As Stiglitz (1976) noted, "some have

[3]Hoel (1978) also mentioned these two phases. This general limit-pricing theory has recently received renewed attention by Withagen and van der Ploeg (2010).

[4]Salant's setting then becomes equivalent to that of our starting example.

suggested the demand for oil in the short run has less than unitary elasticity" but correctly added that "whether it is optimal for the monopolist to raise its price (...) depends on the long-run demand elasticity as well." (p. 656, footnote 2). Estimates of the long-run oil-demand elasticity are generally lower than unity (Krichene, 2002; Killian and Murphy, 2010).[5] Hence assuming the price elasticity of oil demand to be lower than unity not only simplifies the analysis, but also enhances its empirical relevance.

Second, the existence of substitutes for an exhaustible resource is often taken into account in the resource economics literature. In this respect, the very classical assumption is that of a choke price where the demand for the resource gets nil. This follows Heal (1976) by assuming that a "backstop" technology exists that guarantees the possible production of a perfect substitute in infinite quantity at a given constant marginal cost and that this production is made in a context of competition. Hoel (1978), Salant (1979), Dasgupta and Heal (1979, ch. 11), Newbery (1981) and Withagen and van der Ploeg (2010) adopt this simple and meaningful way of representing substitution opportunities. We will do away with the assumptions that marginal costs are constant in the substitute's production as well as in extraction. We will moreover consider extraction costs to depend on the remaining reserves and will discuss non-stationary substitutes' production costs, decreased over time by technical progress. Our assumptions are not incompatible with limit pricing and will bring new insights on the factors favoring its manifestation.

In particular, the existence of a substitute is not in contradiction with the assumption that oil demand is relatively inelastic. First, substitutes may not deserve production for low price levels. However, the gradual availability of substitutes with higher oil prices should imply the elasticity of the residual demand for oil to increase with price. Marshall's (1920) remark that such property is expected to be satisfied is clearly justified by the existence of substitution opportunities for high prices. Second, substitutes' pro-

[5] Pindyck's (1979) estimate for liquid fuels' demand is close but greater than the unitary threshold. However, as Krichene (2002) clearly stated, this is in contrast with all prior studies as well as with most of recent ones whose estimates for long-run price elasticity of oil or energy demand are significantly below unity and sometimes very close to zero. Among the 12 studies he reviewed – including his own one – only one corroborates Pindyck's findings with a long-run elasticity of gasoline demand of 1.01. See also Hausman and Newey (1995) and Kilian and Murphy's (2010) discussion on the consistency of their short-run estimate with standard long-run estimates.

duction affects the residual demand for oil only to the extent of their price-elasticity of supply. A sufficiently acute kink of the residual oil demand thus corresponds to the entry price of a substitute with sufficiently high supply elasticity. Substitutes with low supply elasticity cannot cause a kink of the kind that triggers limit-pricing. Indeed, such substitutes cannot sufficiently affect the residual demand for oil so that this demand remains relatively inelastic. This will lead us to make a sharp distinction between drastic (high-supply-elasticity) and non-drastic (low-supply-elasticity) substitutes. An oil monopoly will find attractive to deter the production of drastic substitutes while tolerating non-drastic substitutes. Hence, unlike when substitution opportunities are modeled with a backstop technology, the observed production of some substitutes is not in contradiction with limit pricing and may be observed during the extraction period.

Since the marginal cost function is the inverse of the supply function, supply elasticity is also the reciprocal of the elasticity of marginal costs, making it a measure of returns to scale; drastic, high-supply-elasticity substitutes are produced under not-too-decreasing returns to scale and vice versa for non-drastic ones. Substitutes play a role according to their ability to meet demand at a large scale at reasonable costs. Heal (2009) suggests that the production of realistically available oil substitutes exhibits capacity constraints which limit substitution opportunities. Hence, a limit-pricing oil monopoly, while deterring the production of substitutes which provide large-scale substitution opportunities, tolerates the production of those only offering limited opportunities.

We will also discuss how the degree of concentration in an oligopolistic market structure affects the residual demand elasticity of oligopolists. It will turn out from this discussion that the limit-pricing theory is more adapted to the case of coordinated cartels like the OPEC than to non-coordinated oligopolistic extractors. At this stage, a legitimate question would be whether there is any reason why we should believe that some resource producers act or intend to act following such a limit-pricing fashion. The OPEC cartel is a natural example. In a famous 1974 interview, Iran's Minister of the Interior and the Shah's right-hand oil expert, Jamshid Amuzegar, when explaining that OPEC's strategy is to have the oil price following the industrialized countries' inflation, had these revealing words: "The first of our (...) principles is that the price of oil should be equivalent to the cost of alternative sources of energy.".[6] Another interesting feature of this simple theory is its prediction that, under stationary conditions, the paths of oil prices and of exhausted quantities would be flat.[7]

Limit pricing is the simplest form of market power exercise. The literature on optimal policies in exhaustible resource markets in the presence of market power were all based on Hotelling theories which ruled out the possibility of limit pricing. The paper also aims at filling this gap. It will turn out that optimal policies under limit pricing sharply differ from those the classical literature has advocated so far.

Bergstrom *et al.* (1981) and Karp and Livernois (1992) characterized optimal resource-taxation policies that induce a resource monopoly to extract efficiently. Unlike in those analyses, we find that resource taxes are neutral, not affecting the supply behavior of the monopoly. Bergstrom *et al.*'s and Karp and Livernois' optimal taxes are determined up to a neutral term rising at the rate of interest. This is because, as is well known from Dasgupta, Heal and Stiglitz (1981), any unit tax rising at the rate of discount is generally neutral since it does not affect suppliers' incentives to revise their plan to extract inelastic reserves. Unlike this standard form of dynamic neutrality, taxes under limit pricing are strongly neutral. They stem from the perfect inelasticity of instantaneous supply resulting from the monopoly's limit pricing. The commitment issue raised by Karp and Livernois (1992) does not arise here. One the other hand, we show that the regulator can induce efficiency by taxing/subsidizing the substitute instead of the monopoly's output. However, it is sufficient that taxes/subsidies are credible threats that do not need to be levied/paid. Moreover, the commitment issue does not arise here neither. The standard trade-off between inducing efficiency and raising tax revenues under market power is generally alleviated when large agents are subject to the exhaustibility of their input (Daubanes, 2011). When limit pricing manifests itself, this trade-off does not arise. In this context, the neutrality of resource taxes provides the regulator with new possibilities to raise

[6] Time Magazine, October 14, 1974, p. 36.

[7] See Livernois' (2009) synthesis on why incorporating elements of market power that provide rationale for non-increasing resource real-value prices has the interesting potential of improving Hotelling theory's explanatory power.

revenues.

The paper is organized as follows. Section 2 presents the dynamic limit-pricing resource monopoly theory in the simplest case. Following Salant (1979), Dasgupta and Heal (1979) and Newbery (1981), we will assume the substitute to be perfect and produced at a given stationary unit cost, the resource to be extracted at a constant marginal cost that is independent of past extraction, and the total demand for energy to be relatively inelastic. Then, in Section 3, we will investigate how robust the predictions of the simplest model are to the introduction of stock-dependent, non-linear extraction costs and of several decreasing-returns-to-scale substitutes. We will also discuss technical progress in the substitutes' production and oligopolistic structures. The setting inherited from Section 3 is similar to that of Bergstrom *et al.* (1981) and Karp and Livernois (1992). The rest of the paper focuses on optimal regulation of a resource market where limit pricing manifests itself. Section 4 aims at extending the classical literature to cases of limit pricing. Section 5 discusses the introduction of environmental objectives and concludes.

2 A simple introductory setting: constant marginal cost of extraction and backstop technology

In this short section, we formally develop the starting example of the introduction, following previous contributions by Dasgupta and Heal (1979, ch. 11, p. 343), Salant (1979, p. 8) and Newberry (1981, p. 625).

At each date $t \in [0, +\infty)$, a monopoly can extract oil at a constant marginal cost $c > 0$ from an initial stock $Q_0 > 0$. There is no storage possibility; the resource has to be supplied as it is extracted. Let us denote the flow of oil extracted and supplied at date t by q_t.

Let us define $\overline{D}(p)$ as the total demand for oil, function of the oil price p; it is continuously differentiable and strictly decreasing. Assume the price elasticity of oil demand to be lower than unity all along the demand curve: $\xi_{\overline{D}}(p) = \frac{-\overline{D}'(p)p}{\overline{D}(p)} < 1$. There is a backstop technology allowing the production of a perfect oil substitute at a constant marginal cost

$b > c$. This substitute is supplied by a competitive sector and meets the same demand as oil. Therefore, the residual demand $D(p)$ the monopoly faces is zero for all prices $p > b$ while $D(p) = \overline{D}(p)$ for all prices $p < b$. In other words, as illustrated in Figure 1, the residual demand is kinked at level $p = b$.

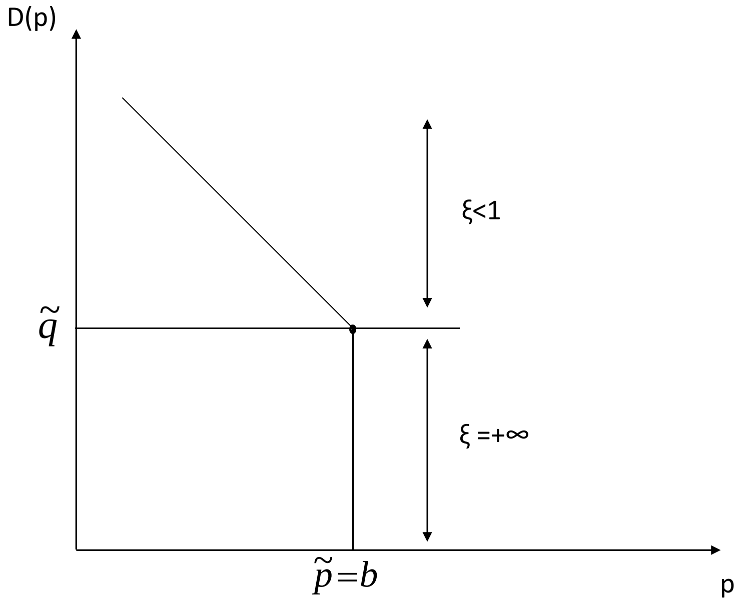

Figure 1: Residual demand with a backstop substitute

When the monopoly supplies an amount q_t that is lower than the threshold quantity

$$\widetilde{q} \equiv \overline{D}(b), \tag{1}$$

then $p_t = b$; its spot profit is $(b - c)q_t$, which is strictly increasing. However, with excess supply $q_t > \widetilde{q}$, the monopoly's supply depresses the price below b; its spot profit is thus $\big(\overline{D}^{-1}(q_t) - c\big)q_t$, which is strictly decreasing because of the assumed demand's relative inelasticity. Indeed, marginal profit is $-p_t\big(\frac{1}{\xi_{\overline{D}}(p)} - 1\big) - c$, where $\xi_{\overline{D}}(p) < 1$ implies the term into parenthesis to be positive.

If the monopoly discounts its profits at rate $r > 0$, it is then optimal for her to constantly extract the quantity $q_t^m = \widetilde{q}$ for all dates $t \leq T^m \equiv \frac{S_0}{\widetilde{q}}$. The resource is exhausted from that date on. Along the extraction period, the price is constant: $p_t = b = \overline{D}^{-1}(\widetilde{q})$ (Figure 2).

To show this, let us consider deviations from this extraction path. We shall see that

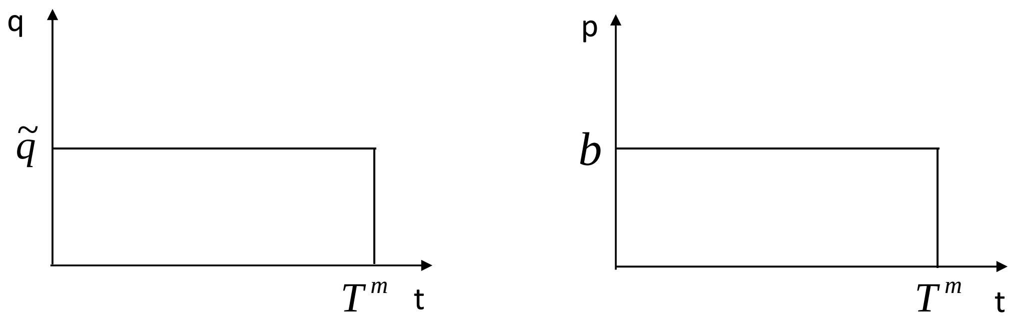

Figure 2: Extraction and price paths

these cannot increase the sum of discounted profits. First, consider reallocations of an infinitesimal quantity $\Delta > 0$ of resource from any date t to any date $t' \neq t$ such that $t, t' < T^m$. Reducing extraction by Δ at date t decreases present-value profits by $(b - c)\Delta e^{-rt}$ while increasing extraction at date t' decreases profits as well, since profits are decreasing for quantities exceeding $\widetilde{q}$.

Second, consider deviations which consist in a reallocation of an infinitesimal quantity $\Delta > 0$ of resource from any date t to any date t' such that $t \leq T^m < t'$. Again, reducing extraction by Δ at date t decreases present-value profits by $(b - c)\Delta e^{-rt}$. Increasing extraction at date t', from zero, by Δ, increases present-value profits $(b - c)\Delta e^{-rt'}$. Since $r > 0$, the overall effect on the discounted stream of profits is negative.

In this simple context, demand's relative inelasticity makes it optimal for the monopoly to choose a flat extraction path, with a constant extraction quantity that prevents the substitute's production from being profitable. Unlike in conventional treatments of monopoly power on resource markets, the oil price p is constant over time and present-value, net-of-cost marginal revenue $(b - c)e^{-rt}$ is strictly decreasing over time.

3 A more general, synthetic setting: stock-dependent, non-linear extraction cost and decreasing-return substitute(s)

In this section, we extend the simple setting presented above. First, following Karp and Livernois (1992) and Withagen and van der Ploeg (2010), we depart from the assumption that marginal costs of extraction/production are constant and introduce stock dependent extraction costs. The latter is a traditional way to distinguish units of reserves according to their depth and thus to the difficulty to extract them. As is well known, this introduces incentives to conserve the resource as extracting more at some date decreases the remaining stock to be exploited at following dates and thus increases future extraction costs.

Second, we do away with the single substitute assumption. As Hoel (1984) pointed out, there are several substitutes, each corresponding to some uses. Therefore, they generally differ according to the degree of substitutability they offer. We will show that limit pricing is not incompatible with some substitutes being deterred while some others are produced along the extraction path. Refining the assumptions in this way will serve two purposes. On the one hand, we shall see that, under these more general assumptions, the resource monopoly does not always choose to adopt a limit-pricing behavior. Hence, we will derive conditions under which limit pricing manifests itself. This exercise will bring relevant insights on the characteristics of the oil sector that render such behavior plausible. On the other hand, these refinements yield a setting that is similar to traditional models which have been used in the literature on the optimal taxation of a resource monopoly (*e.g.* Bergstrom *et al.*, 1981; Karp and Livernois, 1992). Characterizing the conditions for limit pricing in their model will thus be useful to draw the implications of the theory under study in terms of optimal taxation.

We shall discuss other relevant refinements later on, without including them in the baseline model we use to study optimal taxation policies.

3.1 The model

The residual demand the monopoly faces is relatively inelastic for low prices and kinked. In other words, elasticity does not need to get infinite at a certain price. As will be clear, it is sufficient that elasticity discontinuously jumps to a sufficiently high level at a given price. We make the following assumption, that will be given theoretical grounds further below.

Assumption 1 *The residual demand $D(p)$ the oil monopoly faces is continuous everywhere and continuously differentiable almost everywhere but, at least, at one price level. There exists $\widetilde{p} > 0$ such that*

$$\begin{cases} \xi_D(p) < 1, \ \forall p < \widetilde{p} \\ \xi_D(p) > \xi_D^m > 1, \ \forall p > \widetilde{p} \end{cases},$$

where $\xi_D = \frac{-D'(p)p}{D(p)}$ and $\xi_D^m > 1$ will be defined further below, in Expression (6).

The elasticity of the residual demand is often interpreted as the extent of substitution opportunities (*e.g.* Lewis *et al.*, 1979). Marshall (1920) argued that, ordinarily, demand curves can be expected to have the property that the price elasticity is increasing with price. Assumption 1 is consistent with such demand curves, although further assuming that elasticity increases discontinuously. In particular, it is consistent with an infinite elasticity of demand at level $\widetilde{p}$, as would be the case if there were a backstop technology available at this cost level, as in Section 2.

Let us develop an example where the presence of substitutes for oil causes such a discontinuity in the residual demand's price elasticity. We will consider two substitutes, although only one of them causes the residual demand's elasticity to turn above unity. This will lead us to make the distinction between two classes of substitutes, that we shall term "drastic" and "non-drastic" substitutes.

Assume for simplicity the total demand for oil and for its substitutes $\overline{D}(p)$ to be continuously differentiable and relatively inelastic everywhere: $\xi_{\overline{D}}(p) < 1$, for all $p > 0$. The supply of oil substitutes does not need to be perfectly elastic. As a matter of fact, Heal

(2009) suggests capacity constraints to limit substitution possibilities.[8] The finiteness of a substitute's price elasticity of supply represents its ability to meet demand at a large scale, without implying costs to increase too much. Put another way, supply price elasticity is a measure of returns to scale. Indeed, since supply under competition is the inverse of marginal cost, supply elasticity is also the reciprocal of the elasticity of marginal cost.

Assume that there exist two substitutes for oil. The first substitute has a supply function $S^d(p)$ which is increasing and non-perfectly elastic – as would be the case if its production schedule was subject to decreasing returns to scale (increasing marginal cost of production) – and which is strictly positive for and only for prices p that strictly exceed p^d, as would be the case if its minimum marginal cost was non zero; thus not deserving production for prices lower than this cost. To be complete, $S^d(p)$ is assumed continuously differentiable, except, maybe, at level p^d. This substitute will be said to be drastic in the sense that its production drastically increases the elasticity of the residual demand the monopoly faces. Let us moreover consider an additional substitute that is non drastic in the sense that its sole production does not reverse the relative inelasticity of the residual demand. It is supplied according to the function $S^{nd}(p)$, increasing, non-perfectly elastic and strictly positive for and only for prices that strictly exceed p^{nd}. Except, maybe, at p^{nd}, $S^{nd}(p)$ is continuously differentiable. To avoid uninteresting situations where the non-drastic substitute is only supplied when the drastic substitute is supplied, let us assume $p^{nd} < p^d$. Hence, the residual demand the oil monopoly faces is

$$\begin{cases} D(p) = \overline{D}(p), \ \forall p \leq p^{nd} \\ D(p) = \overline{D}(p) - S^{nd}(p), \ \forall p^{nd} < p \leq p^d \\ D(p) = \overline{D}(p) - S^{nd}(p) - S^d(p), \ \forall p > p^d \end{cases} ,$$

and is represented in Figure 3.

Denoting by $x^d = S^d(p)$ and $x^{nd} = S^{nd}(p)$ respectively the equilibrium quantities of the drastic and non-drastic substitutes, by $q = D(p)$ the equilibrium oil quantity and defining

[8]Substitutability may also be partial because substitutes of oil are only used for some of oil uses (Hoel, 1984). We will not investigate imperfect substitution between oil and its substitutes. However we will see, as Gerlagh (2011) remarked, that decreasing returns in substitutes' production are not unlike their imperfect substitutability with oil.

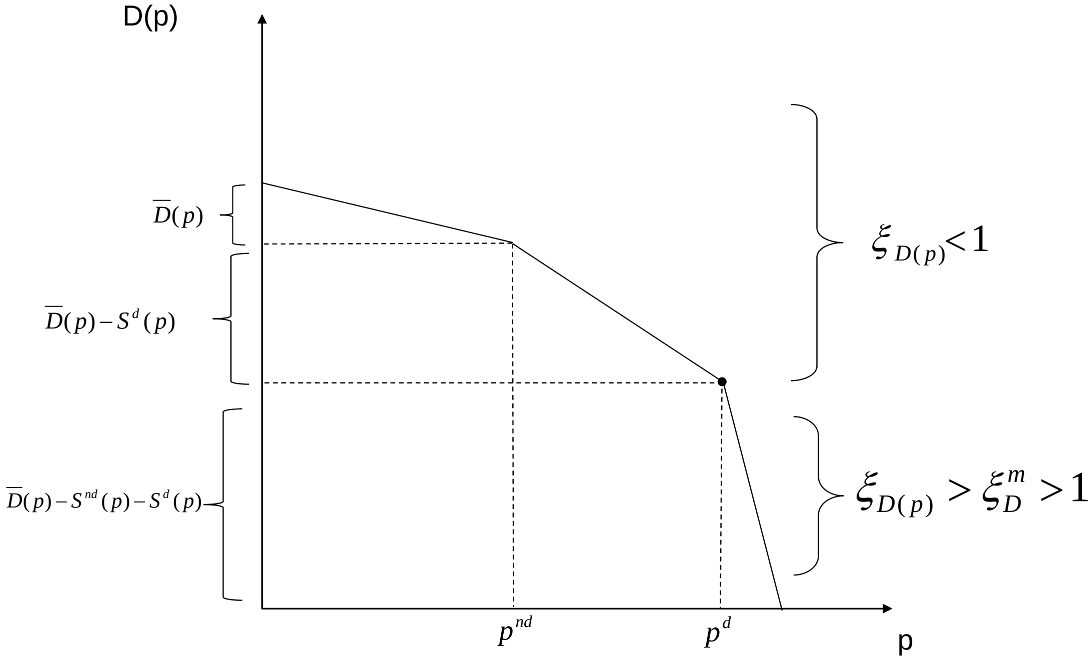

Figure 3: Residual demand with drastic and non-drastic substitutes

$d \equiv q + x^d + x^{nd} = \overline{D}(p)$, we have the following expression of the residual demand's price elasticity, in terms of the price elasticities of both the total demand and the substitutes' supplies:

$$\xi_D(p) = \frac{d}{q}\xi_{\overline{D}}(p) + \frac{x^{nd}}{q}\xi_{S^{nd}}(p) + \frac{x^d}{q}\xi_{S^d}(p),$$

where $\xi_{S^d}(p) \equiv \frac{S^{d\prime}(p)p}{S(p)}$ is the price elasticity of the drastic substitute's supply and $\xi_{S^{nd}}(p) \equiv \frac{S^{nd\prime}(p)p}{S(p)}$ the one of the non-drastic substitute. Additionally, note that $x^{nd} > 0$ only when $p > p^{nd}$. Therefore, $\xi_D(p) < 1$, for $p \leq p^d$ is consistent with sufficiently low price elasticity $\xi_{S^{nd}}(p)$ and market share x^{nd} of the non-drastic substitute. Finally, for $p > p^d$, it is sufficient that the supply elasticity $\xi_{S^d}(p)$ and the market share x^d of the drastic substitute are large enough to warrant $\xi_D(p) > \xi_D^m > 1$, as *per* Assumption 1.

In this example, defining the price level at the kink $\widetilde{p}$ as the price above which the drastic substitute becomes profitable, *i.e.*

$$\widetilde{p} \equiv p^d, \tag{2}$$

justifies Assumption 1. Note that the latter allows the production of a non-drastic substitute at and below the kink level $\widetilde{p}$. This example naturally extends to the case of several drastic and non-drastic substitutes. However, only a drastic substitute could be a backstop technology, subject to constant returns to scale, with unit marginal cost of production of level p^d. If so, $\xi_{S^d}(p^d)$ is infinite and so is the elasticity of the residual demand at this level. Because the supply elasticity is also the reciprocal of the elasticity of marginal cost, it is clear that drastic substitutes, as we have defined them, have not-too-decreasing returns to scale, while non-drastic ones have sufficiently-decreasing returns to scale. Since the resource is non-renewable, it must be that

$$\int_0^{+\infty} q_t \, dt \leq Q_0, \tag{3}$$

where Q_0 is the initial stock to be exploited.

Following Karp and Livernois (1992), the total cost of extracting q_t at date $t \geq 0$ is $C(q_t, Q_t) \geq 0$, where Q_t is the remaining stock to be exploited at this date: $Q_t = Q_0 - \int_0^t q_t \, dt$. This cost function is twice differentiable in its two arguments, increasing in q_t, decreasing in Q_t and such that $\dfrac{\partial^2 C(q_t, Q_t)}{\partial q_t \partial Q_t} \leq 0$ and that $C(0, Q_t) = 0$ for any $Q_t \geq 0$. However, unlike Bergstrom *et al.* (1981) and Karp and Livernois (1992), we do not restrict extraction cost to be convex in the extracted quantity.

The monopoly's gross revenue is $R(q_t) = D^{-1}(q_t)q_t$, where $D(.)$ satisfies Assumption 1. Therefore, the monopoly's current spot profit at date t is

$$\pi(q_t, Q_t) = R(q_t) - C(q_t, Q_t), \tag{4}$$

such that $\pi(0, Q_t) = 0$ for any $Q_t \geq 0$.

Marginal profit thus writes $\dfrac{\partial \pi(q_t, Q_t)}{\partial q_t} = R'(q_t) - \dfrac{\partial C(q_t, Q_t)}{\partial q_t}$ wherever $R'(q_t)$ exists. For $q_t > \widetilde{q} \equiv D^{-1}(\widetilde{p})$, $p_t < \widetilde{p}$ and Assumption 1 implies that $R'(q_t) < 0$. Hence, $\dfrac{\partial \pi(q_t, Q_t)}{\partial q_t} < 0$. For quantities $q_t < \widetilde{q}$, $p_t > \widetilde{p}$ and, by Assumption 1, the elasticity of residual demand is greater than $\xi_D^m > 1$. On this range of quantities, as shown in Appendix 1, marginal profit is strictly positive if and only if

$$\xi_D(q_t) > \frac{D^{-1}(q_t)}{D^{-1}(q_t) - \dfrac{\partial C(q_t, Q_t)}{\partial q_t}}. \tag{5}$$

Thus, the following definition of ξ_D^m ensures that, by Assumption 1, marginal profit is positive below $\widetilde{q}$ at all dates:

$$\xi_D^m \equiv \sup_{q_t \in [0,\widetilde{q}], Q_t \in [0,Q_0]} \frac{D^{-1}(q_t)}{D^{-1}(q_t) - \dfrac{\partial C(q_t, Q_t)}{\partial q_t}}. \tag{6}$$

In (5), the right-hand side is the reciprocal of the Lerner index evaluated at (q_t, Q_t). At the optimum of a static monopoly, this index should generally be equal to the price elasticity of the residual demand the monopoly faces. First, the Lerner index being lower than unity by definition, we know that such an optimum can occur only at points of the demand curve where elasticity is greater than unity. Second, this makes clear that the elasticity's discontinuity in Assumption 1, and our definition of ξ_D^m as *per* (6) eliminate such static-monopoly optima. This ensures profit to be increasing for all $q_t < \widetilde{q}$.

Overall, Assumption 1 with Definition (6) imply the following sign for marginal profit:

$$\begin{cases} \dfrac{\partial \pi(q_t, Q_t)}{\partial q_t} > 0, \ \forall q_t < \widetilde{q} \\ \dfrac{\partial \pi(q_t, Q_t)}{\partial q_t} < 0, \ \forall q_t > \widetilde{q} \end{cases}, \ \forall Q_t \in [0, Q_0]. \tag{7}$$

The objective of the oil monopoly is to maximize the discounted stream of her spot profits

$$\int_0^{+\infty} \pi(q_t, Q_t) e^{-rt} \, dt, \tag{8}$$

where $r > 0$ is the constant rate at which profits are discounted, subject to the exhaustibility constraint (3).

In this context, further conditions are required for the monopoly to adopt a limit-pricing behavior, constantly supplying a quantity $q_t^m = \widetilde{q}$ over the extraction period, *i.e.* until the exhaustion date $T^m = \frac{Q_0}{\widetilde{q}}$. To derive these conditions, we proceed as in the previous section, by considering deviations from this path that are reallocations of the extraction.

First, the monopoly cannot increase her profits by extracting the resource more rapidly than along the limit-pricing extraction path. Since, by (7), profit is decreasing in the extraction quantity above $\widetilde{q}$ and increasing below, increasing extraction in excess of $\widetilde{q}$ at some date $t < T^m$ and decreasing extraction below $\widetilde{q}$ at any posterior date $t' \leq T^m$, $t' > t$ deteriorate the spot profit at both dates. Moreover, increased extraction at date t further deteriorates profits between dates t and t' as it increases costs via the stock effect. The monopoly thus never benefits from a more rapid depletion than along the limit-pricing path. In fact, deviations from limit pricing can only be attractive when entailing a reduction of extraction costs via the stock effect, *i.e.* when deviations consist in conserving the resource by adopting a less rapid depletion. In Appendix 2, we show that the two following conditions rule out potential benefit of the monopoly from deviations that consist of shifting extraction from any date of the extraction period to any posterior date, be it during the extraction period or at a subsequent date.

Condition 1 *The path $q_t^m = \widetilde{q}$ for any $0 \leq t \leq T^m = \frac{Q_0}{\widetilde{q}}$, $q_t^m = 0$ for any $t > T^m$ satisfies*[9]

$$\frac{\partial \pi(\widetilde{q}^-, Q_t)}{\partial q_t} - \frac{\partial \pi(\widetilde{q}^+, Q_{t'})}{\partial q_t} e^{-r(t'-t)} \geq \int_t^{t'} \frac{\partial \pi(\widetilde{q}, Q_s)}{\partial Q_t} e^{-r(s-t)}\, ds, \ \forall t < t' \leq T^m \tag{9}$$

and

$$\frac{\partial \pi(\widetilde{q}^-, Q_t)}{\partial q_t} - \frac{\partial \pi(0,0)}{\partial q_t} e^{-r(t'-t)} \geq \int_t^{T^m} \frac{\partial \pi(\widetilde{q}, Q_s)}{\partial Q_t} e^{-r(s-t)}\, ds, \ \forall t \leq T^m < t'. \tag{10}$$

Conditions (9) and (10) are reminiscent of deviations from the standard no-arbitrage condition applying in conventional models with stock effects, and must be interpreted as

[9]In Condition 1 and in the rest of the paper, a superscript $^-$ ($^+$) on the value taken by a function's variable means that the function is measured at the limit taken from below (above) the value of this variable.

such.[10]

Condition 1 tells us which factors provide the monopoly with incentives to adopt a limit-pricing behavior. In the sequel, we identify three factors and discuss their relevance.

First, both (9) and (10) are satisfied for sufficiently high discount rates. The rate at which profits are discounted does not need to be the rate of interest. From the producer's point of view, future risks on profits are not unlike an increase of the rate at which profits are discounted. In the resource economics literature, one can refer to Long's (1975) analysis on the risk of future expropriation. The similarity between risk and discount rate arises in a sharp fashion when the probability of expropriation is expected to be constant over time; in that case, they are formally equivalent (*e.g.* Sinn, 2008, p. 370). Other risks, as for instance the anticipation of a gradual introduction of future environmental demand-reducing policies might also have such effects[11].

As anticipated, the introduction of a stock effect on the extraction cost provides the monopoly with incentives to conserve the resource. Condition (9) is always satisfied in the absence of such effect, *i.e.* when $\frac{\partial \pi(q_t,Q_t)}{\partial Q_t} = 0$ as in Section 2. In general, the weaker the stock effect, the more likely the condition is satisfied. The same remark applies to condition (10). However, it is not sufficient that the stock effect vanishes for this second

[10]In Appendix C.3, we show that if the profit function were differentiable, the condition that producers are indifferent between extracting a marginal unit at two different dates t and t' is

$$\frac{\partial \pi(q_t, Q_t)}{\partial q_t} - \frac{\partial \pi(q_{t'}, Q_{t'})}{\partial q_t} e^{-r(t'-t)} = \int_t^{t'} \frac{\partial \pi(q_s, Q_s)}{\partial Q_t} e^{-r(s-t)}\, ds. \tag{11}$$

Were the left-hand side greater than the right-hand side, as *per* the inequalities of Condition 1, producers would decrease their profits by leaving a marginal unit in situ at date t and shifting its extraction to date $t' > t$.

It is from the indifference condition (11) that the well-known Hotelling rule in presence of stock effects is derived:

$$\frac{\partial \dot{\pi}(q_t, Q_t)}{\partial q_t} = r\frac{\partial \pi(q_t, Q_t)}{\partial q_t} - \frac{\partial \pi(q_t, Q_t)}{\partial Q_t}, \tag{12}$$

where the dot on top of a variable or function denotes its derivative with respect to time (See *e.g.* Dasgupta and Heal (1979), ch. 6). Condition (12) rules out profitable deviations that consist in shifting extraction between two dates that are very close to each other.

Interestingly, such a condition cannot exist in the setting of this paper. Appendix C shows that taking the limit as t' approaches t, Condition (9) is always satisfied. The discontinuity of marginal profits makes the left-hand side negatively infinite while by continuity of stock effects, the left-hand side remains finite. Hence, a standard indifference condition such as (11) cannot be satisfied when marginal profits are discontinuous as *per* Assumption 1.

[11]On this, see also Sinn (2008), who showed that the anticipated gradual greening of environmental policies are not unlike credible threats of future expropriation.

condition to be satisfied.

Condition (10) further involves the difference between marginal profit at a zero extraction level and below the limit-pricing extraction quantity. Absent any stock effect, *i.e.* when $\frac{\partial \pi(q_t,Q_t)}{\partial Q_t} = 0$, the condition is always satisfied if $\frac{\partial \pi(\widetilde{q}^-,Q_t)}{\partial q_t} \geq \frac{\partial \pi(0,0)}{\partial q_t}$ for any $Q_t \leq Q_0$. This is clearly the case in Section 2 where marginal extraction costs are constant. In general, the greater the gap between both marginal profits, the more likely the condition is satisfied. Increasing marginal costs introduce a gap between marginal profits at a zero extraction level and at higher extraction levels, thus enhancing the benefit of conserving some of the reserves beyond T^m. If marginal costs are increasing in extraction so that profits are concave, (10) imposes that the marginal profit is not too elastic below $\widetilde{q}$. If costs were concave (decreasing marginal extraction costs) in the extracted quantity q_t, profits would be convex. Such a case, that we have not ruled out, clearly favors the possibility of limit pricing. Increasing returns to scale in short-term extraction of a mine have not received remarkable attention. One interesting exception is Hartwick *et al.* (1986) who addressed the presence of set-up costs. Although not equivalent to the presence of set-up costs, concave extraction costs have been interpreted as reflecting increasing returns to scale in short-run extraction (*e.g.* Neher, 1999, p. 68). Our results on the manifestation of limit pricing on the part of a resource monopoly in our general model are summarized in the following proposition.

Proposition 1 *Let the residual demand the oil monopoly faces satisfy Assumption 1. Let Condition 1 be satisfied. Then, the monopoly chooses a constant extraction* $q_t^m = \widetilde{q}$ *at all dates* $t \leq T^m = \frac{Q_0}{\widetilde{q}}$.

Proof. See Appendix C ■

In Section 4, we will work under the conditions of Proposition 1, *i.e.* Assumption 1 and Condition 1, in order to draw the implications of limit pricing in terms of optimal taxation of a resource monopoly.

The following subsection discusses several features of the oil market that are either relevant or have interesting implications for the limit-pricing theory. We have not introduced them in our general model in order to build a framework that is as close as possible to that used in the literature on the optimal taxation of exhaustible resource monopolies.

3.2 Remarks

3.2.1 Technical progress in the substitute's sector

For simplicity, we have assumed stationary functional forms. This is why the limit-pricing path consists in the supply of constant quantities. It is worth noting that technical progress in the substitute's production, if it leads to decreased costs of production over time, would imply increasing extraction $\widetilde{q}$ over time and thus decreasing equilibrium oil price.

The monopoly would then be more and more pressured by the substitute as its production would become profitable for lower and lower prices. To deter its production, the monopoly would then choose to supply more and more, inducing a lower and lower resource price.

Although not equivalent and rather simplistic here, it is not unreminiscent of the situation described by Gerlagh and Liski (2011) where the oil monopoly supplies more over time since consumers have stronger incentives, as depletion goes, to invest in a perfect substitute.

3.2.2 Oligopolistic market structure

We have assumed total residual demand for oil, net of the demand for its substitutes, to have a lower than unity price elasticity. When several suppliers form a single coordinated cartel, this demand is met by the monopoly alone. As we have seen, demand relative inelasticity thus implies the monopoly's marginal profit to be negative beyond the limit-pricing quantity.

If suppliers formed an oligopoly, the demand each member would face is its residual oil demand, net of the supply of all other members. The residual demand $D(p)$ of each member would then be more elastic than the total residual demand. This is because an increase in one member's supply, the quantities supplied by the others being given, affects less the market price. In fact, in the symmetric case, the individual residual demand price elasticity is the total residual demand price elasticity multiplied by the number of members. Hence, even if the total demand for oil is inelastic, it is all the less likely that individual demands are inelastic as there are many oligopolists. Limit-pricing theory is therefore more relevant to the case of a coordinated oil cartel than to the case of an non-coordinated oligopoly.

As remarked by Salant (1976, p. 1085), the existence of a competitive fringe also implies that the residual demand that is relevant to oligopolists differs from the total demand. The elasticity of the former is that of the latter, divided by the cartel's market share. The same conclusion thus applies to the case where there is a competitive fringe. For the limit-pricing theory to apply to such cases, the total oil demand must be very inelastic, or the cartel must have a relatively large market share.

4 Optimal taxation of a dynamic limit-pricing monopoly

From Section 3, we inherit from a model that is close to the traditional model of the literature on the optimal taxation of non-renewable monopolies. In particular, it follows Bergstrom *et al.* (1981) and Karp and Livernois (1992) in including costs of extraction non-linearly dependent on current extraction flows and on the currently remaining stock underground.

Our model essentially differs from theirs because it allows for the possibility of limit pricing on the part of the monopoly to be regulated. Such a possibility was implicitly ruled out in Bergstrom *et al.* (1981) and Karp and Livernois (1992) by their assumption that the sales' revenue and the spot profit of the monopoly are continuously differentiable in extracted quantities. Unlike them, we assume the profit to be kinked due to the existence of threshold quantities below which drastic substitutes are profitable. Precisely, we will work under the conditions of Proposition 1, following which the monopoly adopts a limit-pricing behavior.

The question we address here is the same as Bergstrom *et al.* (1981) and Karp and Livernois (1992): Which paths of taxes/subsidies induce the monopoly to extract the resource efficiently? Like these previous studies, and as is standard in the optimal taxation literature, we will focus on unit taxes, added to the market producer price, that are possibly negative (subsidies). The regulator's single objective is the restoration of efficiency. The split of the surplus induced by his policy is not an issue here.[12]

Karp and Livernois (1992) do not assume the regulator to be able to *ex ante* commit to a path of taxes/subsidies. In this context, the regulator should rely on subgame-perfect policies and taxes/subsidies are restricted to Markovian rules, functions of the remaining stock. As we shall see, the regulator's commitment ability is never an issue when the monopoly adopts a limit-pricing behavior.

Our model assumes the presence of a substitute, as reflected by the property that profits are kinked, that may offer an alternative opportunity to influence the monopoly's supply. Such a substitute is absent in Bergstrom *et al.* (1981) and in Karp and Livernois (1992), although substitution possibilities might be implicitly recognized by the form of the demand the monopoly faces, that is also of her profits. We have seen that its explicit modeling implies some discontinuity strongly influencing the monopoly's choice. Under conditions of limit pricing, this influence arises in a sharp fashion as the monopoly exactly supplies in such a way as to deter the substitute's production. This raises the question of whether taxes/subsidies applied on the substitute allow the regulator to implement some optimal extraction path. We will deal with this question in a subsequent section.

4.1 Taxation of the resource

Let us assume that the regulator imposes a specific tax on top of the producer resource price[13], θ_t, positive or negative, at each date $t \geq 0$. Let us furthermore assume that this tax is not too high:

$$\theta_t \leq \overline{\theta}_t, \tag{13}$$

where $\overline{\theta_t}$ is to be defined later in Expression (17).

In the presence of the resource tax, the final (tax inclusive) resource price above which the substitute is produced is no longer given by (2). Although the tax does not affect

[12]On this, see Benchekroun and Long (2008) and Daubanes (2011).

[13]This is a consumer tax. As shown by Bergstrom *et al.* (1981, p. 30), the reasoning and results survive the case where the tax is a producer tax.

the price at which the substitute enters, a distinction should now be made between the producer, before-tax price $\widetilde{p}_t$ and the final, after-tax price $\widetilde{p}_t + \theta_t$ consumers pay for the resource. Moreover, we shall see that in the presence of a time-varying tax, the producer price will depend on time. Hence, the limit before-tax price that deters the substitute's production is now $\widetilde{p}_t$ such that

$$\widetilde{p}_t + \theta_t = p^d. \tag{14}$$

Proposition 1 still holds, to the extent that profits are modified to take account of the tax. $R(q_t)$ should be replaced by the function of the tax $R(q_t, \theta_t) = D^{-1}(q_t)q_t - \theta_t q_t = R(q_t, 0) - \theta_t q_t$, where $R(q_t, 0) = R(q_t)$ is the gross revenue in the absence of resource taxation. Spot profits thus not only depend on the extraction rate and on the remaining stock, but also on the current tax; (4) should be modified to become

$$\pi(q_t, Q_t, \theta_t) = \pi(q_t, Q_t, 0) - \theta_t q_t, \tag{15}$$

where $\pi(q_t, Q_t, 0) = \pi(q_t, Q_t)$ is as defined in (4).

When the monopoly limit prices as *per* Proposition 1, *i.e.* when Assumption 1 and Condition 1 are satisfied, (14) is the consumer price she will choose to induce at each date of the extraction period. The supplied amount that induces this price is

$$q_t = D(p^d) \equiv \widetilde{q}, \tag{16}$$

which is the same quantity as in the absence of taxes. Hence, such resource taxes do not affect the equilibrium quantities at any date. However, as is clear by (14), the tax at each date reduces the producer price that accrues to the monopoly: $\widetilde{p}_t = p^d - \theta_t$; in the presence of a resource tax, this implies that $\widetilde{p}_t$ is lower than its zero-tax counterpart $\widetilde{p}$ of Section 3.

The neutrality of resource taxes in a limit-pricing context is an extreme form of neutrality. The former studies based on traditional Hotelling models found that there exists a family of optimal resource tax/subsidy paths. This family is indexed by the magnitude a tax component which rises at the rate of interest of the form Ke^{rt}, where K is some scalar.

As shown by Dasgupta *et al.* (1981), such a component is neutral as it does not affect the Hotelling rule applying to the producer price and thus the intertemporal arbitrage of extractors. As Karp and Livernois (1992) highlighted it: "If the amount Ke^{rt} is added to [the optimal unit tax], the monopolist will still want to extract at the efficient rate, provided that the dynamics rationality constraint is satisfied (...).".

Here, not only taxes/subsidies rising at the rate of interest are neutral, but any tax/subsidy paths. The neutrality of Dasgupta *et al.*'s (1981) taxes is dynamic in the sense that they do not affect the intertemporal arbitrage of extractors, thus not affecting the allocation of a given stock to several dates. Here, the form of neutrality that applies is of another form which is simpler. At each date, any tax does not affect the minimum quantity that deters the substitute's production and that the monopoly will choose. This implies a static supply inelasticity. This results in the well-known static form of tax neutrality, when supply (or demand), is perfectly inelastic.

To which extent the resource can be taxed? Like in Bersgtrom *et al.* (1981) and Karp and Livernois (1992), the amount that the regulator can raise is bounded by a "dynamics rationality constraint". This constraint is the positivity of the marginal profits of the monopoly at each date. Indeed, if marginal profit is negative at some date because of too high a tax, the monopoly deviates from the limit-pricing path. In our context, what matters is the positivity of the marginal profit derived from the infra-marginal unit of the resource. Along the limit-pricing path, infra-marginal profit at date $0 \leq t \leq T^m = \dfrac{Q_0}{\widetilde{q}}$ is $\dfrac{\partial \pi(\widetilde{q}^-, Q_0 - t\widetilde{q}, \theta_t)}{\partial q_t} = \dfrac{\partial \pi(\widetilde{q}^-, Q_0 - t\widetilde{q})}{\partial q_t} - \theta_t$. Therefore, at any date t, all values of θ_t lower than

$$\overline{\theta}_t \equiv \frac{\partial \pi(\widetilde{q}, Q_0 - t\widetilde{q})}{\partial q_t} \tag{17}$$

guarantee the incentives of the monopoly to extract.

It follows from Assumption 1 that $\overline{\theta}_t > 0$. Therefore, not only all resource subsidies, but also some taxes satisfy the rationality constraint that infra-marginal profits are positive. Moreover, as the stock decreases over time, thus reducing marginal profits, the maximum neutral unit tax $\overline{\theta}_t$ decreases over time.

The following proposition summarizes the findings of this subsection.

Proposition 2 *Let the residual demand the oil monopoly faces satisfy Assumption 1. Let Condition 1 be satisfied, where profits are now* $\pi(q_t, Q_t, \theta_t)$ *as defined by (15). Then, any path of unit resource taxes/subsidies* $\theta_t \leq \overline{\theta}_t$ *does not affect the limit-pricing extraction path and Proposition 1 still applies:* $q_t^m = \widetilde{q}$ *at all dates* $t \leq T^m$.

Proof. See Appendix D. ■

Taxes greater than $\overline{\theta}_t$ at some date render the extraction of $\widetilde{q}$ unattractive as the monopoly would increase her profit by reducing her extraction at this date. Such taxes may even deteriorate net marginal profits to such an extent that the monopoly prefers not to extract at all at this date. When profits are convex in extraction, this is always the case. When profits are concave in extraction, the monopoly reduces her extraction continuously with the tax. If the tax exceeds the value of marginal profit in the absence of taxation as extraction approaches zero, the monopoly does not extract at all.[14]

From the monopoly's perspective, whatever the tax, provided it is not extreme, the minimum quantity that induces the maximum price deterring the drastic substitute's production is always $\widetilde{q}$. This is because the resource tax does not affect the price p^d at which the substitute becomes profitable.

On the contrary, we will see in the next subsection that taxes/subsidies on this substitute affect the monopoly's quantities by modifying the substitute's entry price.

4.2 Optimal substitute's taxation

Alternatively, let us assume that there is a unit tax γ_t on top of the substitute's producer price, which is also its marginal cost. When γ_t is negative, it is a subsidy. However, let us assume that it cannot be too high a subsidy. Precisely, let us rule out situations where the subsidy is greater than the minimum marginal cost of the substitute:

$$\gamma_t > -p^d. \tag{18}$$

[14]However, even when profits are concave, such extreme taxes cannot optimally regulate the monopoly when the only market failure is her monopoly power. Indeed, restoring efficiency implies the monopoly to supply less at some date and more at some others than in the absence of taxation. High taxes in the sense that they make the marginal profit negative at the limit-pricing quantity can only reduce the quantity compared to the equilibrium without intervention.

In fact, as will be shown later on, the regulator will never set a greater subsidy.

At any date $t \geq 0$, the final price beyond which the substitute's production is profitable now depends on the substitute's tax/subsidy:

$$\widetilde{p}(\gamma_t) = p^d + \gamma_t. \tag{19}$$

When the monopoly limit prices in order to deter the substitute's production, this is the resource price she will choose to induce. Such a price is induced by the resource supply

$$\widetilde{q}(\gamma_t) = D(\widetilde{p}_t) = D(p^d + \gamma_t). \tag{20}$$

Let us still denote by $\widetilde{q}$ the quantity the monopoly supplies in absence of any tax/subsidy policy. As $D(.)$ is decreasing, it follows that a tax on the substitute $\gamma_t > 0$ allows the monopoly to decrease her supply $\widetilde{q}(\gamma_t) < \widetilde{q}$ to induce a larger price. On the contrary, a subsidy $\gamma_t < 0$ pressures the monopoly to increase her supply; thus $\widetilde{q}(\gamma_t) > \widetilde{q}$. Changes entailed by the introduction of the subsidy to the drastic substitute are represented in Figure 4 (thick line).

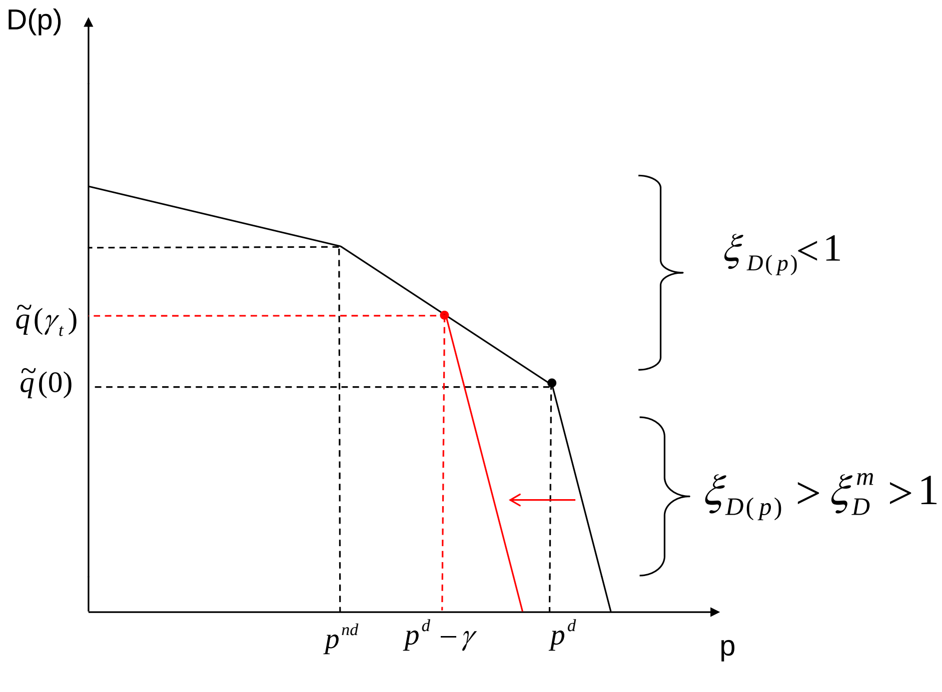

Figure 4: Effects of a subsidy to the drastic substitute

Hence, taxes or subsidies on the substitute modify the kink of the residual demand at which the monopoly may limit-price. Unlike in Section 3, the kink of the residual demand curve and the associated resource price may vary over time if the tax/subsidy γ_t is time-dependent. However, the same reasoning applies here. When Assumption 1 and Condition 1 are satisfied along the path $(\widetilde{q}(\gamma_t), \widetilde{p}(\gamma_t))$ defined by (19) and (20), the monopoly maximizes her profits by adopting a limit-pricing behavior at this level.

As (20) makes clear, the regulator can induce any extraction quantity between 0 and $D(0)$ with an appropriate γ_t above $-p^d$. The optimal tax/subsidy is such that $q_t^* = \widetilde{q}(\gamma_t^*) = D(p^d + \gamma_t^*)$, *i.e.*

$$\gamma_t^* = D^{-1}(q_t^*) - p^d, \tag{21}$$

which cannot be lower than $-p^d$, thus justifying (18).

The result on the possibility to induce efficiency in the extraction of the resource by taxing/subsidizing the drastic substitute is summarized in the following proposition.

Proposition 3 *Let Assumption 1 and Condition 1 hold along any optimal extraction path $(q_t^*)_{t\geq 0}$. Then, the regulator can implement this optimal path by setting a unit tax/subsidy γ_t^* on the drastic substitute, as given by (21). At any date t during the extraction period, a tax on the substitute reduces resource supply, while a subsidy increases it.*

Proof. See Appendix E. ■

The monopoly chooses the quantity that deters the substitute's production; under the optimal tax, this quantity is optimal at each date. Since limit-pricing manifests itself by the exclusion of the substitute's use, to which the instrument is applied, it has an important distributional implication. The substitute is not used at any date of the extraction period, *i.e.* at any date when a positive amount of the resource is produced. Hence, during the extraction period, the tax (subsidy) on the substitute is not levied (paid). Actually, the optimal policy consists in credibly announcing at each date the appropriate tax/subsidy on the substitute that will prevail at that date and only at that date. The ability to commit over one date is a necessary minimum requirement, and is satisfied here as in any, even static, model of optimal taxation. The ability of the regulator to commit for a longer interval is not an issue here.[15]

Generally, depending on whether it is optimal to induce a greater or a lower extraction compared with the quantity the monopoly would have chosen in absence of intervention, it will be optimal to either set a tax or a subsidy. Above, we have characterized the effect of a tax on the substitute (inducing the monopoly to reduce her quantity) and of a subsidy (inducing her to increase her quantity). If the monopoly tends to supply too large an amount of resource at any date t, $q_t^* < \widetilde{q}$, then the substitute should be taxed, *i.e.* $\gamma_t^* > 0$. Otherwise, when the monopoly does not sufficiently extract, $q_t^* > \widetilde{q}$, then the substitute should be subsidized, *i.e.* $\gamma_t^* < 0$.[16]

5 Final discussion

In the above section, we have not specified the optimal extraction path $(q_t^*)_{t\geq 0}$.

The equilibrium extraction path might only be distorted to the extent that the monopoly exercises her market power. It might as well be that the limit-pricing equilibrium features an environmental distortion. If so, the results of the above section apply. In particular, the taxation of the resource offers poor opportunities to implement the first-best extraction path. In contrast, taxes/subsidies to drastic subsidies allow to regulate the monopoly in a way that induces an optimal extraction, even when account is taken of environmental concerns.

Still in that case, if the optimal policy implies to reduce resource extraction at some dates, the substitute should be taxed so as to increase the price p^d at which it becomes

[15] Only is necessary that the monopoly anticipates conditions of limit-pricing to apply. Indeed, under these conditions, her problem is static at each date of the extraction period. Therefore, the regulator's setting of the optimal tax/subsidy date-by-date is sufficient for the monopoly not to strategically deviate from the optimal path.

[16] In general, when the cost of extraction depends on the remaining reserves and when the substitute's cost of production is non linear, it is difficult to analytically compare the optimal path with the monopoly path without intervention. A standard particular case is that of a backstop technology (constant marginal cost of production of the substitute). In that case, it is well known that the resource should be optimally depleted before its price reaches the level of the unit cost of production of the backstop substitute. It implies that during the exploitation period, the optimal extracted resource quantity is always greater than the quantity that induces the price of the resource to be equal to the substitute's unit cost. The latter quantity is the limit-pricing quantity $\widetilde{q}$. Hence, in the backstop case, $q_t^* > \widetilde{q}$ at all dates before the optimal date of exhaustion. Therefore, the optimal taxation policy consists in subsidizing the substitute at all dates. However, the substitute's production is deterred by the monopoly until the date of exhaustion, and the subsidy is not effectively paid by the regulator.

profitable. Consequently, given that marginal profits are negative below p^d – since demand elasticity is lower than unity – the limit-pricing monopoly has incentives to supply less of the resource at these dates so as to increase the price up to p^d. In the same fashion, subsidizing the substitute at some dates, and therefore decreasing p^d, would have undesired environmental effects as it results in more of the resource being supplied at these dates. Since, in this context, such substitute's subsidies could seem attractive at first sight, the increase in extraction they provoke is a kind of green paradox.

The stock of reserves does not generally become available without some prior developments. Following the simple representation of Gaudet and Lasserre (1988), Fisher and Laxminaryan (2005) and Daubanes and Lasserre (2011), the *ex ante* supply of exploitable reserves should positively react to the monopoly's valuation of an extra reserves' unit. Resource taxes and substitutes' subsidies may affect this valuation and thus the cumulative stock that is ultimately extracted. In order to use a setting comparable with the main literature on Hotelling-resource-monopoly taxation, we have assumed away this possibility. This aspect of resource taxation is thus out of the scope of the present paper. However, it may be an important consideration in the design of environmental policies.

A Proof of Expression (5)

From (4), the monopoly's marginal profit is $\dfrac{\partial \pi(q_t, Q_t)}{\partial q_t} = R'(q_t) - \dfrac{\partial C(q_t, Q_t)}{\partial q_t}$, where $R'(q_t) = D^{-1'}(q_t)q_t + D^{-1}(q_t) = D^{-1}(q_t)\big(1 + \frac{D^{-1'}(q_t)q_t}{D^{-1}(q_t)}\big)$.

In this expression, one could recognize $\dfrac{D^{-1'}(q_t)q_t}{D^{-1}(q_t)}$ to be the reciprocal of $\xi_D(q_t) \equiv \dfrac{-D^{-1}(q_t)}{D^{-1'}(q_t)q_t}$. Hence, $\dfrac{\partial \pi(q_t, Q_t)}{\partial q_t} = D^{-1}(q_t)\big(1 - \dfrac{1}{\xi_D(q_t)}\big) - \dfrac{\partial C(q_t, Q_t)}{\partial q_t}$, from which we obtain, after rearranging, that $\dfrac{\partial \pi(q_t, Q_t)}{\partial q_t} > 0$ is equivalent to (5).

B Proof of Proposition 1

To prove Proposition 1, it is sufficient to show that the limit-pricing path $q_t^m = \widetilde{q}$, $\forall 0 \leq t \leq T^m = \frac{Q_0}{\widetilde{q}}$, $q_t^m = 0$, $\forall t > T^m$, maximizes the monopoly's discounted profits under Assumption 1 and the two inequalities of Condition 1.

$\imath$) First, let us consider deviations from this path that consist in shifting extraction by $\Delta > 0$ units from any date $t \leq T^m$ to any date t' such that $t < t' \leq T^m$.

The net discounted benefit from such a deviation is

$$-\frac{\partial \pi(\widetilde{q}^-, Q_t)}{\partial q_t}\Delta e^{-rt} + \int_t^{t'} \frac{\partial \pi(\widetilde{q}, Q_s)}{\partial Q_t}\Delta e^{-rs}\, ds + \frac{\partial \pi(\widetilde{q}^+, Q_{t'})}{\partial q_t}\Delta e^{-rt'}. \tag{B.1}$$

Any such deviations do not increase profits if and only if this net discounted benefit is non-positive $\forall t < t' \leq T^m$. Setting (B.1) negative, dividing by Δe^{-rt} and rearranging immediately give the first inequality (9) of Condition 1.

As stated in Footnote 10, the above deviations are never profitable when dates t and t' are very close to each other. In that case, the right-hand side of (9) can be approximated as $(t'-t)\frac{\partial \pi(\widetilde{q}, Q_t)}{\partial Q_t}$. Substituting, dividing by $(t'-t)$, and multiplying by e^{-rt}, (9) thus yields

$$\frac{\frac{\partial \pi(\widetilde{q}^-, Q_t)}{\partial q_t}e^{-rt} - \frac{\partial \pi(\widetilde{q}^+, Q_{t'})}{\partial q_t}e^{-rt'}}{t'-t} \geq \frac{\partial \pi(\widetilde{q}, Q_t)}{\partial Q_t},$$

for t and t' sufficiently close to each other. Using the fact that $\frac{\partial \pi(\widetilde{q}^-, Q_t)}{\partial q_t}e^{-rt} - \frac{\partial \pi(\widetilde{q}^+, Q_{t'})}{\partial q_t}e^{-rt'} = \big(\frac{\partial \pi(\widetilde{q}^-, Q_t)}{\partial q_t}e^{-rt} - \frac{\partial \pi(\widetilde{q}^-, Q_{t'})}{\partial q_t}e^{-rt'}\big) + \big(\frac{\partial \pi(\widetilde{q}^+, Q_t)}{\partial q_t}e^{-rt} - \frac{\partial \pi(\widetilde{q}^+, Q_{t'})}{\partial q_t}e^{-rt'}\big) + \big(\frac{\partial \pi(\widetilde{q}^-, Q_{t'})}{\partial q_t}e^{-rt'} - \frac{\partial \pi(\widetilde{q}^+, Q_t)}{\partial q_t}e^{-rt}\big)$, we have the equivalent condition

$$\frac{\frac{\partial\pi(\widetilde{q}^-,Q_t)}{\partial q_t}e^{-rt} - \frac{\partial\pi(\widetilde{q}^-,Q_{t'})}{\partial q_t}e^{-rt'}}{t'-t} + \frac{\frac{\partial\pi(\widetilde{q}^+,Q_t)}{\partial q_t}e^{-rt} - \frac{\partial\pi(\widetilde{q}^+,Q_{t'})}{\partial q_t}e^{-rt'}}{t'-t} + \frac{\frac{\partial\pi(\widetilde{q}^-,Q_{t'})}{\partial q_t}e^{-rt'} - \frac{\partial\pi(\widetilde{q}^+,Q_t)}{\partial q_t}e^{-rt}}{t'-t} \geq \frac{\partial\pi(\widetilde{q},Q_t)}{\partial Q_t}e^{-rt}.$$

Taking the limit as t' approaches t, the first two terms on the left-hand side respectively become $-\big(\frac{\partial\pi(\widetilde{q}^+,\dot{Q}_t)}{\partial q_t}e^{-rt}\big)$ and $-\big(\frac{\partial\pi(\widetilde{q}^-,\dot{Q}_t)}{\partial q_t}e^{-rt}\big)$, while the third term tends to be positively infinite. The right-hand side being finite by continuity of $\pi(q_t, Q_t)$ in Q_t, this implies that condition (9) is always satisfied for deviations involving dates sufficiently close to each other.

ii) Second, let us consider deviations that consist in shifting $\Delta > 0$ units from any date $t \leq T^m$ to any date $t' > T^m$. Since the stock effect on the extraction cost is irrelevant at dates when there is no extraction, *i.e.* for all dates greater than T^m, the net discounted benefit from such a deviation is

$$-\frac{\partial\pi(\widetilde{q}^-,Q_t)}{\partial q_t}\Delta e^{-rt} + \int_t^{T^m}\frac{\partial\pi(\widetilde{q},Q_s)}{\partial Q_t}\Delta e^{-rs}\,ds + \frac{\partial\pi(0,0)}{\partial q_t}\Delta e^{-rt'}. \tag{B.2}$$

Setting it negative and dividing all terms by Δe^{-rt} immediately give the second inequality (10) of Condition 1.

C Proof of Expressions (11) and (12)

This appendix assumes that the profit function is differentiable and concave. The monopoly's problem consists in maximizing (8) with respect to $(q_t)_{t\geq 0}$, subject to $\dot{Q}_t = -q_t$, Q_0 being given.

Condition (12) can easily be derived from the dynamic optimization problem of the monopoly.

Its heuristic derivation starts from the condition that the extractor's discounted benefit from shifting extraction of an infinitesimal amount of resource $\Delta > 0$ from any date t to any subsequent date t'

$$-\frac{\partial\pi(q_t,Q_t)}{\partial q_t}\Delta e^{-rt} + \int_t^{t'}\frac{\partial\pi(q_s,Q_s)}{\partial Q_t}\Delta e^{-rs}\,ds + \frac{\partial\pi(q_{t'},Q_{t'})}{\partial q_t}\Delta e^{-rt'}$$

is nil.

Dividing by Δe^{-rt} and rearranging give the indifference condition (11).

When t and t' are sufficiently close to each other, the right-hand side of (11) can be approximated by $(t'-t)\frac{\partial\pi(q_t,Q_t)}{\partial Q_t}$. Dividing all terms by $(t'-t)$, multiplying by e^{-rt} and taking the limit as t' approaches t yields (12).

D Proof of Proposition 2

Shown in the main text.

E Proof of Proposition 3

Shown in the main text.

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