International environmental agreements under endogenous uncertainty

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Abstract

This paper explores the implications of the possibility of a shift in environmental damages on the participation in environmental treaties. Using a two-period model where the probability of a regime shift increases with the first-period emissions, we examine the issue of coalition formation under both fixed and dynamic membership. Our analysis suggests that endogenous uncertainty may increase participation. Moreover, we find that full cooperation may be sustained, but only in the presence of endogenous uncertainty. Interestingly, when the shift in the environmental damage is large enough, the model provides a way to solve the "puzzle of small coalitions" found in the literature related to International Environmental Agreements (IEAs).

Keywords: International Environmental Agreements; Endogenous Uncertainty; Emissions; Shift in Damage.
JEL classification: C73; D81; F53; Q54

1 Introduction

There is a growing scientific consensus that excessive greenhouse gas emissions accumulation can trigger abrupt changes in global climatic patterns. These sudden changes can potentially result in large environmental damages.[1] A number of authors have examined this issue. Clarke and Reed (1994), Tsur and Zemel (1998), Haurie and Moresino (2006), de Zeeuw and Zemel (2011) analyze the issue of environmental disruption in the case of a single polluter. Nkuiya (2011) extends their analysis to the case where an arbitrary number of countries whose production and consumption activities entail pollution. All the papers in this literature analyze the problem either within the fully cooperative or the fully non-cooperative framework, or both. The cooperative setting assumes that countries cooperate at any point in time. However, due to the lack of supra-national authority, such a solution may be viewed as extreme. On the other hand, the non-cooperative solution may not be appropriate for transboundary pollution problems; analyzing such environmental changes within the framework of international negotiations may thus be an interesting alternative.

The possibility of an endogenous regime change (in environmental damages), which has been used as a stylized fact within the scientific community, has not been modeled yet in the literature on dynamic environmental games. The purpose of this paper is to study how the prospect of such a sudden shift in environmental damages affects the participation in climate treaties and the emissions of the countries. We consider a world constituted of an arbitrary number of countries involved in polluting production activities. Pollution is considered as a global, pure public bad which harms all the countries. In our model, current emissions can exacerbate future environmental damages. More precisely, an upward shift in the environmental damages (suffered by the countries) may occur in the future with a positive probability (which is increasing in current total emissions). This raises a number of research questions. How does this type of uncertainty affect the participation in climate treaties? How does it affect the countries' emission policies? How does the magnitude of

[1] For further details, see the for instance IPCC (2007).

such a shift in environmental damages affect the incentives to ratify environmental treaties?

In order to address these questions we consider the concept of International Environmental Agreements (IEAs) introduced by Carraro and Siniscalco (1993) and Barrett (1994). An IEA is a two-stage game. In the first stage (membership game), each country decides whether or not to ratify the treaty. In the second stage (emissions game), the signatories choose their emission strategies cooperatively, while the non-signatories act unilaterally. This basic framework has been used in both the static and the dynamic membership games (see for example Rubio and Ulph, 2006, 2007).

We consider a two-period dynamic game within a Cournot framework. We first examine the effects of a possible shift in environmental damages within the fully non-cooperative setting. Our results indicate that the possibility of an upward shift in the damage function leads to lower emissions today and higher emissions tomorrow. That is in comparison with the cases where there is no uncertainty about the future damages of pollution and when there is certainty about the shift in the damages. We then examine the issue of coalition formation under fixed membership, when countries commit to their membership decision for both periods. We find that the possibility of an upward shift in the damages may increase the size of the stable coalition. We also find that, under the possibility of a damage shift, the grand coalition may emerge as the stable coalition in period 1. In order to prove this possible result, we resort to a series of simulations. The results are maintained under dynamic membership, when countries negotiate an IEA at the outset of each period.[2] Interestingly enough, our findings suggest that considering the possibility of a shift in environmental damages is one route to solve the "puzzle of small coalitions" found in the literature on IEAs.[3]

Our model also builds on the IEA literature with uncertainty and learning. Ulph (2004) studies how the prospect of getting better information about the future can affect the ability

[2] Ulph (2004) and Rubio and Ulph (2002) use the dynamic and the fixed membership frameworks to address different questions.

[3] For a detailed discussion see Finus (2003).

of countries to ratify an IEA. To analyze this issue, he incorporates uncertainty about damages in each of the two periods in the model of Rubio and Ulph (2002). He then compares the resulting outcome to that of the model where there is no uncertainty in period 2 in order to measure the effect of learning (which results in larger stable IEAs). Our approach differs on two counts. First, uncertainty has always been considered to be exogenous in that literature. Nevertheless, human activities can influence the occurrence of a dangerous event, thus making the uncertainty endogenous. This feature is taken into account in the present paper. Second, we consider a particular type of uncertainty. The period-1 damage function is known with certainty. However, unlike what has been assumed in the prevailing IEA literature, the global emissions of period 1 affect the probability of an upward shift in the environmental damage of period 2.

There exists also an interesting literature that studies IEAs with risk averse countries (see for instance Boucher and Bramoullé, 2010; Finus et al., 2010). However, that strand of literature considers exogenous uncertainty and makes use of static models. We use instead a dynamic framework and we argue that the level of uncertainty is endogenously determined by the emission strategies of the countries.

The paper unfolds as follows. The upcoming Section 2 presents the model. The non-cooperative equilibrium is derived in Section 3. Assuming a quadratic damage function, Section 4 and Section 5 examine the membership game under fixed membership and dynamic membership, respectively (the case of a linear damage function is discussed in Appendix B). Finally, Section 6 makes concluding comments.

2 The model

We consider a world constituted of N countries (with $N \geq 3$), with two periods $t = 1, 2$. We use $i = 1, 2, ..., N$ to refer to an arbitrary country. As the result of their production and consumption activities, countries emit global pollution. We assume for simplicity that, in each period, one unit of production in any of the countries generates one unit of pollution.

Country i's benefit function is given by $U(e_{it})$, $t = 1, 2$, where U is an increasing and concave utility function, and e_{it} is i's emission level at date t. Following Ulph (2004) and Rubio and Ulph (2002), we assume that the total emissions accumulate according to the law of motion:

$$E_t = \sum_{i=1}^{N} e_{it} + \rho E_{t-1},\ E_0 = 0, \tag{1}$$

where E_t denotes the total emissions of period t and $1-\rho$ is the natural decay of accumulated emissions. Emissions are a global public bad and they inflict an environmental damage on each of the countries. We consider two states of the damage function, the business as usual state where the damage function is D, and the dangerous state, which is characterized by the damage function $D + \bar{\theta}$.

In period 1 (the present), the damage function is perfectly known and is in the business as usual state. However, the period-1 emissions (E_1) increase the likelihood of the occurrence of the dangerous state of damages in period 2 (the future). To capture these features, we assume that in period 1, θ_1 is known and is equal to zero, whereas θ_2 is a binomial random variable which can take the values $\bar{\theta}$ [with probability $P(E_1)$] or 0 [with probability $1 - P(E_1)$]. The parameter $\bar{\theta}$ is a positive number, which represents the level of loss inflicted by the upward shift in the damages. Specifically, the damage function in period t is

$$D_t(E_t) = D(E_t) + \theta_t,\ t = 1, 2,$$

where D is an increasing and convex function, which satisfies

$$D'(0) = 0 \text{ and } \lim_{E \to +\infty} D'(E) = +\infty.$$

We assume that the probability $P(E_1)$ is an increasing, piecewise twice differentiable and convex function of the period-1 emissions, with

$$P(0) = P'(0) = 0. \tag{2}$$

The net benefit function of country i in period t is given by

$$\pi_{it}(e_{it}, E_t) = U(e_{it}) - D(E_t) - \theta_t.$$

Notice that the case where the function P is identically equal to zero corresponds to the model with no shift in damages, whereas the situation where P is identically equal to one corresponds to the model where there is certainty about the shift in the damage function. We now turn to the fully non-cooperative solution, which provides the Nash equilibrium outcome.

3 The non-cooperative equilibrium

The game is solved using backward induction, starting from the second period. In period 2, each country decides unilaterally the emission level that maximizes its expected net benefit, considering the emission levels of other countries as given. Formally, country i solves

$$\max_{e_{i2}} \mathbb{E}\pi_{i2}(e_{i2}, E_2) = [U(e_{i2}) - D(E_2) - \bar{\theta}P(E_1)],$$

$$\text{subject to: } E_2 = \sum_{i=1}^{N} e_{i2} + \rho E_1.$$

The first-order conditions for this maximization problem are

$$U'(e_{i2}) = D'(\sum_{i=1}^{N} e_{i2} + \rho E_1), i = 1, ..., N. \tag{3}$$

These conditions indicate that each country chooses the period-2 emission level that equates its marginal benefit of emissions and its marginal damage (given the emission strategies of the others). We restrict attention to symmetric equilibria. Letting e_{2n} denote the emission level of each country in period 2, we can rewrite (3) as:

$$U'(e_{2n}) = D'(Ne_{2n} + \rho E_1). \tag{4}$$

Differentiating the two sides of (4) with respect to E_1 and rearranging, we get

$$e'_{2n}(E_1) = \frac{\rho D''(Ne_{2n} + \rho E_1)}{U''(e_{2n}) - ND''(Ne_{2n} + \rho E_1)} < 0.$$

Notice that in (4), $e_{2n}(E_1)$ is the inter-temporal emission rule. Since it has a negative slope, if the total emissions of period 1 increase then each country will respond by reducing its emission level in the second period.

In period 1, anticipating on the emission rule of the second period given by (4), each country decides the emission rate that maximizes its expected discounted net benefit. Therefore, each country i solves the problem:

$$\max_{e_{i1}}\{[\pi_{i1}(e_{i1},E_1)+\delta\mathbb{E}\pi_{i2}(e_{2n},E_2)]=[U(e_{i1})-D(E_1)]+\delta[U(e_{2n})-D(Ne_{2n}+\rho E_1)-\bar{\theta}P(E_1)]\},$$

where $\delta\in(0,1)$ is the discount factor.

For a symmetric equilibrium, a necessary condition for an interior solution is

$$U'(e_{1n})-D'(Ne_{1n})+\delta[U'(e_{2n})e'_{2n}-D'(E_{2n})(Ne'_{2n}+\rho)-\bar{\theta}P'(Ne_{1n})]=0,$$

where $E_{2n}=Ne_{2n}+N\rho e_{1n}$.

Making use of (4), this condition can be rewritten as

$$U'(e_{1n})=D'(Ne_{1n})+\delta[(\rho+(N-1)e'_{2n})D'(Ne_{2n}+N\rho e_{1n})+\bar{\theta}P'(Ne_{1n})]. \qquad (5)$$

Equation (5) gives the period-1 optimality condition stating that each country's marginal benefit of emissions must be equal to its intertemporal marginal damage. The term $\bar{\theta}P'(Ne_{1n})$ captures the effects of a possible upward shift in the damages. Notice that this term would vanish if the uncertainty was exogenous ($P'=0$). We check in Appendix A that an interior solution always exists under our assumptions.

Proposition 1 *In the non-cooperative equilibrium, emissions under endogenous uncertainty are lower in period 1, but higher in period 2 (in comparison with the case without a shift in the damage function).*

Proof. See Appendix A.■

Proposition 1 shows that countries emit less in period 1 when faced with the possibility of an upward shift in the damages. In period 2, each country emits more than it would when a shift in the damages is not possible. The intuition behind these results can be explained by the fact that countries have incentives to emit less in period 1, given that their period-1 emissions increase the probability of an upward shift in the damages. In addition, the optimal emissions in period 2 are decreasing in the period-1 emissions. Since countries produce less in period 1, it then follows that the emissions in period 2 are higher (than they would be without a shift).

Notice that, given our specification, countries are risk neutral. By contrast, Bramoullé and Treich (2009) propose a static model with risk-averse polluters and exogenous uncertainty. In that context, one of their findings is that uncertainty always lowers emissions.

To better understand the pure effect of uncertainty, one can compare the results of Proposition 1 to the outcome of the model where the shift in period 2 is predictable ($P = 1$). Using (5), we can see that the marginal damages when the shift is certain are smaller than that obtained under endogenous uncertainty. An argument similar to the one used for Proposition 1 allows us to conclude that: (a) the period-1 emissions under certainty are greater and (b) the period-2 emissions are lower.

The above results show that the possibility of an upward shift in the damages leads to a reduction in global emissions if countries act non-cooperatively.[4] We next examine the effects of endogenous uncertainty on the size of stable international environmental agreements.

In the remainder of the paper, we assume that

$$U(e_{it}) = ae_{it} - \frac{b}{2}e_{it}^2, \;\; t = 1, 2, \tag{6}$$

where a and b are positive parameters. This specification is used by a number of papers in this literature.[5] Since U reaches its maximum at a/b, the total emissions (in equilibrium) will never exceed Na/b when the damages are taken into account.

We also assume that

$$D(E_t) = \frac{\gamma}{2}E_t^2, \;\; t = 1, 2,$$

where γ is a positive parameter. The case of a linear damage function is examined in Appendix B.

[4]We have shown that all the results of this section also hold qualitatively under full cooperation in the two periods.

[5]Among many others and in a deterministic framework, Dockner and Long (1993) use a quadratic benefit function in a dynamic game with two players. Rubio and Ulph (2006) use the same specification but in a static IEA game.

4 IEAs with fixed membership

In this section we incorporate endogenous uncertainty into the dynamic version of the canonical IEA membership game introduced by Carraro and Siniscalco (1993) and Barrett (1994). An IEA is a two-stage game where, in the first stage (membership game), each country decides whether or not to ratify the treaty. In the second stage, signatories jointly choose their emissions, while each non-signatory chooses its emissions non-cooperatively. Using backward induction, we solve the game starting by its second stage.

4.1 The emissions game

Assume that an IEA is negotiated at the beginning of period 1, which results in the set of signatories S, with $n \leq N$ being the cardinality of S. In this section, every country commits to their membership decision for the two periods (there is no renegotiation at the outset of period 2).[6]

In period 2, signatories solve

$$\max_{e_{i2}, i \in S} \sum_{i \in S} \mathbb{E}\pi_{i2}(e_{i2}, E_2) = \sum_{i \in S} [ae_{i2} - \frac{b}{2}e_{i2}^2 - \frac{\gamma}{2}(E_2)^2 - \bar{\theta}P(E_1)],$$
$$\text{subject to: } E_2 = \sum_{i=1}^{N} e_{i2} + \rho E_1.$$

The typical non-signatory $i \in N \setminus S$ solves

$$\max_{e_{i2}} \mathbb{E}\pi_{i2}(e_{i2}, E_2) = [ae_{i2} - \frac{b}{2}e_{i2}^2 - \frac{\gamma}{2}(E_2)^2 - \bar{\theta}P(E_1)],$$
$$\text{subject to: } E_2 = \sum_{i=1}^{N} e_{i2} + \rho E_1.$$

For the symmetric equilibria, the first-order conditions for the respective maximization problems above are

$$a - be_{2s} = n\gamma E_2, \tag{7}$$

$$a - be_{2ns} = \gamma E_2, \tag{8}$$

[6] This assumption will be relaxed further on.

where $E_2 = ne_{2s} + (N-n)e_{2ns} + \rho E_1$.[7] Solving (7) and (8), we get the inter-temporal decision rule for the emissions of signatories and non-signatories, respectively

$$e_{2s} = \frac{ab - a\gamma(N-n)(n-1) - bn\gamma\rho E_1}{b(b+\gamma(N-n+n^2))}, \tag{9}$$

$$e_{2ns} = \frac{ab + an(n-1)\gamma - b\gamma\rho E_1}{b(b+\gamma(N-n+n^2))}. \tag{10}$$

In period 1, signatories decides jointly the emission levels that maximize their expected discounted net welfare. More formally, they solve

$$\max_{e_{i1}, i\in S}\{\sum_{i\in S}[\pi_{i1}(e_{i1}, E_1) + \delta\mathbb{E}\pi_{i2}(e_{i2}, E_2)] = \sum_{i\in S}[ae_{i1} - \frac{b}{2}e_{i1}^2 - \frac{\gamma}{2}(E_1)^2] + n\delta[ae_{2s} - \frac{b}{2}e_{2s}^2 - \frac{\gamma}{2}(E_2)^2 - \bar{\theta}P(E_1)]\},$$

where $E_2 = ne_{2s} + (N-n)e_{2ns} + \rho E_1$.

A non-signatory $i \in N \setminus S$ solves

$$\max_{e_{i1}}\{[\pi_{i1}(e_{i1}, E_1) + \delta\mathbb{E}\pi_{i2}(e_{2ns}, E_2)] = [ae_{i1} - \frac{b}{2}e_{i1}^2 - \frac{\gamma}{2}(E_1)^2] + \delta[ae_{2ns} - \frac{b}{2}e_{2ns}^2 - \frac{\gamma}{2}(E_2)^2 - \bar{\theta}P(E_1)]\},$$

subject to: $E_2 = ne_{2s} + (N-n)e_{2ns} + \rho E_1$.

In a symmetric equilibrium, given (7) and (8), the first-order conditions for signatories and non-signatories are respectively:

$$a - be_{1s} = n\gamma E_1 - n\delta[n\gamma E_2 e'_{2s} - \gamma(ne'_{2s} + (N-n)e'_{2ns} + \rho)E_2 - \bar{\theta}P'(E_1)], \tag{11}$$

$$a - be_{1ns} = \gamma E_1 - \delta[\gamma E_2 e'_{2ns} - \gamma(ne'_{2ns} + (N-n)e'_{2ns} + \rho)E_2 - \bar{\theta}P'(E_1)]. \tag{12}$$

Using a similar reasoning as in the non-cooperative scenario, it can be shown that in period 1 and for any coalition of size n, the signatories' emissions as well as the non-signatories' emissions are lower under endogenous uncertainty in comparison with the certainty framework. But these two results do not hold in period 2.

[7] Condition (7) is in fact the Samuelson (1954) condition for the provision of a public good within the coalition S, which states that the marginal benefit of each coalition member is equal to the sum of the marginal damages of all the coalition members.

Multiplying (11) by n and (12) by $N-n$, we get

$$\begin{aligned} na - bE_{1s} &= n^2\gamma E_1 - n^2\delta[n\gamma E_2 e'_{2s} \\ &\quad - \gamma(ne'_{2s} + (N-n)e'_{2ns} + \rho)E_2 - \bar{\theta}P'(E_1)], \quad (13) \\ (N-n)a - bE_{1ns} &= (N-n)\gamma E_1 - (N-n)\delta[\gamma E_2 e'_{2ns} \\ &\quad - \gamma(ne'_{2ns} + (N-n)e'_{2ns} + \rho)E_2 - \bar{\theta}P'(E_1)], \quad (14) \end{aligned}$$

where E_{1s} and E_{1ns} are the total emissions of signatories and non-signatories (that is to say, $E_1 = E_{1s} + E_{1ns}$). Adding up side by side (13) and (14) yields

$$\begin{aligned} Na - bE_1 &= (N-n+n^2)\gamma E_1 + \delta[(N-n+n^2)(ne'_{2s} + (N-n)e'_{2ns} + \rho)\gamma E_2 \\ &\quad - (n^3 e'_{2s} + (N-n)e'_{2ns})\gamma E_2 + (N-n+n^2)\bar{\theta}P'(E_1)]. \end{aligned}$$

Using the fact that

$$E_2 = ne_{2s} + (N-n)e_{2ns} + \rho E_1 = \frac{Na + b\rho E_1}{b + \gamma(N-n+n^2)}, \quad (15)$$

this condition can be rewritten as

$$Na(1-\alpha(n)) = (d(n) + \alpha(n)b\rho)E_1 + \delta(N-n+n^2)\bar{\theta}P'(E_1), \quad (16)$$

where $d(n) = b + \gamma(N-n+n^2)$ and $\alpha(n) = \delta\gamma\rho[(N-n+n^2)b + (N-n+n^4)\gamma]/d(n)^2$.

Solving Condition 16 determines $E_1(n)$, which is the total emissions of period 1 given the coalition of size n. Substituting $E_1(n)$ into (9) and (10), we get the equilibrium emission levels $e_{2s}(n)$, $e_{2ns}(n)$ of period 2. Likewise, substituting $E_1(n)$ into (11) and (12) and rearranging, we obtain the emission equilibrium emission levels of period 1

$$\begin{aligned} e_{1s}(n) &= \{a - n\gamma E_1(n) + n\delta[n\gamma e'_{2s}E_2(n) \\ &\quad - \gamma(ne'_{2s} + (N-n)e'_{2ns} + \rho)E_2(n) - \bar{\theta}P'(E_1(n))]\}/b, \quad (17) \\ e_{1ns}(n) &= \{a - \gamma E_1(n) + \delta[\gamma e'_{2ns}E_2(n) \\ &\quad - \gamma(ne'_{2ns} + (N-n)e'_{2ns} + \rho)E_2(n) - \bar{\theta}P'(E_1(n))]\}/b, \quad (18) \end{aligned}$$

where $E_2(n) = ne_{2s} + (N-n)e_{2ns} + \rho E_1(n)$ denotes the accumulated emissions in period 2.

Substituting the equilibrium emissions in the objective functions above, we get the discounted payoffs of the typical signatory and the typical non-signatory, which are respectively given by

$$\begin{aligned} V_s(n) &= ae_{1s}(n) - \frac{b}{2}e_{1s}(n)^2 - \frac{\gamma}{2}E_1(n)^2 \\ &+ \delta[ae_{2s}(n) - \frac{b}{2}e_{2s}(n)^2 - \frac{\gamma}{2}E_2(n)^2 - \bar{\theta}P(E_1(n))], \end{aligned} \tag{19}$$

$$\begin{aligned} V_{ns}(n) &= ae_{1ns}(n) - \frac{b}{2}e_{1ns}(n)^2 - \frac{\gamma}{2}E_1(n)^2 \\ &+ \delta[ae_{2ns}(n) - \frac{b}{2}e_{2ns}(n)^2 - \frac{\gamma}{2}E_2(n)^2 - \bar{\theta}P(E_1(n))]. \end{aligned} \tag{20}$$

Proposition 2 *For any $n \in [2, N-1]$, (i) each non-member of a coalition of size n pollutes more than any individual member; (ii) each member of a coalition of size n gains less than each non-member.*

Proof. It is clear from (9) and (10) that $0 < e_{2s} < e_{2ns} < a/b$. Form (17) and (18), we have $0 < e_{1s}(n) < e_{1ns}(n) < a/b$. Since the function $U(x) = ax - bx^2/2$ is increasing over the interval [0, a/b), we have: $U(e_{1s}(n)) < U(e_{1ns}(n))$ and $U(e_{2s}(n)) < U(e_{2ns}(n))$. Using (19) and (20), we get

$V_{ns}(n) - V_s(n) = [U(e_{1ns}(n)) - U(e_{1s}(n))] + \delta[U(e_{2ns}(n)) - U(e_{2s}(n))] > 0.$■

Proposition 2 highlights the free riding incentive that affects the membership game. Indeed, the abatement effort of signatories is greater than that of non-signatories. However, the benefit of abatement goes to all players, while non-signatories gain more from that abatement. Notice that for $n = 1$ and $n = N$, this section retrieves the respective outcomes of the non-cooperative game and the cooperative game.

4.2 Stable agreements

In what follows, we use the stability concept of D'Aspremont et al. (1983) to determine the equilibrium of the membership game. It states that a coalition of size n is stable or self-enforcing if it is both internally stable and externally stable. A coalition of size n is internally stable if no member has incentives to leave the coalition, while it is externally stable if no non-member can increase its payoff by joining the coalition.

For convenience, we summarize the stability conditions using the stability function introduced by Hoel and Schneider (1997),

$$\phi_i(n) = V_s(n) - V_{ns}(n-1), \text{ for } n = 2, ..., N.$$

Clearly, $\phi_i(n) \geq 0$ for $i \in S$ means that the coalition S of size n is internally stable whereas $\phi_i(n+1) < 0$ for $i \in N \setminus S$ means that the coalition S of size n is externally stable. Notice that: (a) many stable coalitions may exist and (b) a coalition whose size is the greatest integer for which the internal stability holds is also externally stable. We therefore consider the largest coalition size for which internal stability is verified to be self-enforcing.

Proposition 3 *In the fixed membership framework, full cooperation may be sustained as a stable agreement.*

Proof. To prove that full cooperation may be sustained, it is sufficient to provide an example. We run a numerical analysis on a set of $N = 100$ countries to examine how endogenous uncertainty affects stable coalitions in period 1. We consider the probability function

$$P(E_1) = \frac{2b^2}{N^2a^2}\int_0^{E_1} u\,\mathbb{I}_{[0,aN/b]}(u)du = \begin{cases} \frac{b^2E_1^2}{N^2a^2}, & \text{if } E_1 < aN/b \\ 1, & \text{if } E_1 \geq aN/b \end{cases}, \tag{21}$$

where $\mathbb{I}_{[0,aN/b]}(u)$ is the function that takes the value: one if u lies in the interval $[0, aN/b]$, and zero otherwise. Note that this specification of P satisfies the requirements needed for our probability functions: it is increasing, piecewise twice differentiable and convex, with $P(0) = P'(0) = 0$. We use the parameter values $a = 5; b = 1; \gamma = 0.0001; \delta = 0.9$ and $\rho = 0.95$ (these values always result in nonnegative emissions when computing the stable coalitions). We obtain two cases depending on the magnitude of the shift in the damages function, and each case is associated with a distinct equilibrium. The first case corresponds to a small shift in the damage function *i.e.* $\bar{\theta} \in [0, 0.0065)$. Our simulations reveal that the higher $\bar{\theta}$ is, the lager are stable coalitions. Moreover, for $\bar{\theta} = 0$,[8] the model generates stable coalitions of two members. This is consistent with the result in Rubio and Casino (2005) who analyze an IEA under fixed membership using a differential game (with open-loop emission strategies). They find in their model, without uncertainty, that an IEA can sustain only a coalition of size two. In our analysis, we use a two-period model with a fixed membership strategy and we also find a two-member stable coalition in the case without uncertainty. The second case is for a large shift in the damage function *i.e.* $\bar{\theta} \geq 0.0065$, which always leads to full cooperation. These findings are illustrated in Figure 1.■

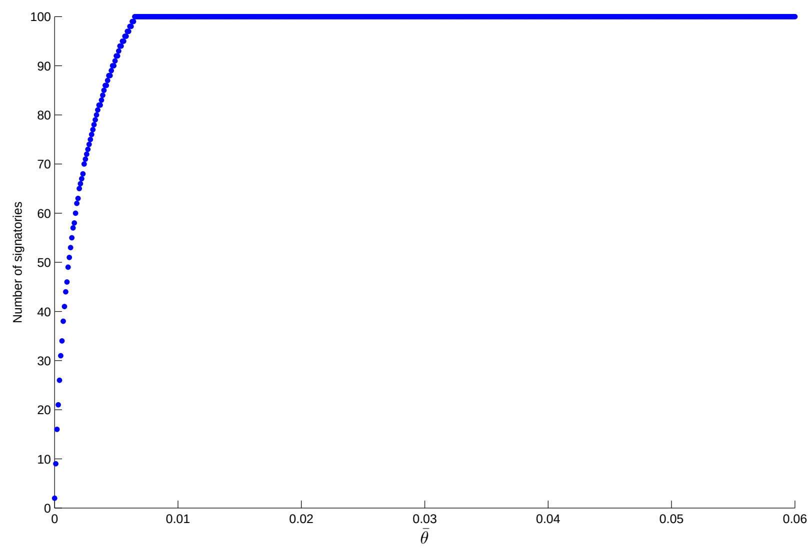

Figure 1: The effects of endogenous uncertainty on participation with fixed membership and quadratic damages.

5 IEAs with dynamic membership

So far we have assumed that an IEA is negotiated once and for all at the outset of period 1. In this section, we will examine the effects of endogenous uncertainty when an IEA is negotiated at the beginning of each period.

[8]Notice that the coalition size under $\bar{\theta} = 0$ is equal to the coalition size under both the certainty case ($P = 1$) and the no-shift in the damage case ($P = 0$).

In period 2, given the total emissions of the first period (E_1), we assume that an IEA is negotiated at the outset of period 2, which results in a coalition S of size n. The optimal emissions of each signatory $e_{2s}(n)$ and those of each non-signatory $e_{2ns}(n)$ are defined respectively by (9) and (10).

The associated expected payoff of a signatory and that of a non-signatory for the period are respectively

$$W_{2s}(n) = U(e_{2s}(n)) - \frac{\gamma}{2}E_2(n)^2 - \bar{\theta}P(E_1), \tag{22}$$

$$W_{2ns}(n) = U(e_{2ns}(n)) - \frac{\gamma}{2}E_2(n)^2 - \bar{\theta}P(E_1), \tag{23}$$

where $E_2(n)$ is given by (15).

In this setting, the stability function is

$$\phi_2(n) = W_{2s}(n) - W_{2ns}(n-1).$$

Proposition 4 *Under dynamic membership, any coalition of size n greater than 2 is not internally stable in period 2.*

Proof. See Appendix A.■

Proposition 4 states that no coalition of more than two members can be stable in period 2. This leaves us with only two-member coalitions as potential candidates for stability. Depending on the parameters, the coalitions of size 2 may or may not be stable. We show in Appendix A that, if the marginal damage parameter γ is sufficiently large ($\gamma > \bar{\gamma} \equiv b[4 - N + 2\sqrt{N^2 - 3N + 3}]/(3N+2)(N-2)$), then no stable coalition can be sustained by the model. However, if the marginal damage parameter γ is sufficiently small ($0 \leq \gamma \leq \bar{\gamma}$), the model generates stable coalitions of two signatories in period 2.

We now turn to the analysis of coalition formation in period 1. Let us first examine how membership decisions taken in period 1 affect the countries' payoffs in period 2. Following Rubio and Ulph (2007) and Ulph (2004), we use the Random Assignment Rule, which states that countries cannot commit to their membership across the two periods. Formally, we assume that there is a binomial random variable whose realization in period 1 determines whether or not a particular country will be among the members in period 2. For any stable IEA of size n^* in period 2, the *a priori* probability of any given country being a member of the coalition is n^*/N. In a symmetric equilibrium, this probability is the same for all countries. Therefore, in period 1, each country has the same period-2 expected payoff, which depends on the total emissions of period 1.

The common payoff that signatories as well as non-signatories expect to get in period 2 is

$$\psi(E_1) = \frac{n^*}{N} W_{2s}(n^*) + (1 - \frac{n^*}{N}) W_{2ns}(n^*),$$

where $n^* = 0$ if $\gamma > \bar{\gamma}$ and $n^* = 2$ if $0 \leq \gamma \leq \bar{\gamma}$.

This expression can be rewritten as:[9]

$$\psi(E_1) = U(e_{2s}(n^*)) \frac{n^*}{N} + (1 - \frac{n^*}{N}) U(e_{2ns}(n^*)) - \frac{\gamma}{2} E_2(n^*)^2 - \bar{\theta} P(E_1).$$

Assume that an IEA game is played at the outset of period 1, which results in n signatories.

Signatories solve

$$\max_{e_{i1}, i \in S} \{ \sum_{i \in S} [a e_{i1} - \frac{b}{2} e_{i1}^2 - \frac{\gamma}{2} (E_1)^2] + n \delta \psi(E_1) \}.$$

A non-signatory $i \in N \setminus S$ solves

$$\max_{e_{i1}} \{ [a e_{i1} - \frac{b}{2} e_{i1}^2 - \frac{\gamma}{2} (E_1)^2] + \delta \psi(E_1) \}.$$

The first-order conditions for optimality are

$$a - b e_{1s} = n \gamma E_1 + n \delta [\frac{\gamma \rho}{N d(n^*)^2} (Na + b \rho E_1)(Nb + \gamma(N + n^{*3} - n^*)) + \bar{\theta} P'(E_1)], \tag{24}$$

$$a - b e_{1ns} = \gamma E_1 + \delta [\frac{\gamma \rho}{N d(n^*)^2} (Na + b \rho E_1)(Nb + \gamma(N + n^{*3} - n^*)) + \bar{\theta} P'(E_1)]. \tag{25}$$

[9]Notice that in the expression of $\psi(E_1)$, the terms $e_{2s}(n^*)$, $e_{2ns}(n^*)$ and $E_2(n^*)$ all depend on E_1.

Multiplying (24) by n and (25) by $(N-n)$ and adding up side by side the two resulting equations, we obtain

$$Na - bE_1 = (N-n+n^2)\gamma E_1 + (N-n+n^2)\delta[\frac{\gamma\rho}{Nd(n^*)^2}(Na+b\rho E_1)(Nb + \gamma(N+n^{*3}-n^*)) + \bar{\theta}P'(E_1)],$$

which can be rewritten as

$$a[N-\beta(n,n^*)] = [b+\gamma(N-n+n^2) + \frac{b\rho}{N}\beta(n,n^*)]E_1 + (N-n+n^2)\bar{\theta}\delta P'(E_1), \qquad (26)$$

where $\beta(n,n^*) = (N-n+n^2)\gamma\rho\delta(Nb+\gamma(N+n^{*3}-n^*))/d(n^*)^2$.

The positive root of (26) is the equilibrium total emissions of period 1, $E_1(n)$. Substituting $E_1(n)$ in (24) and (25), we get the respective equilibrium emissions of a signatory and a non-signatory country

$$e_{1s}(n) = \{a - n\gamma E_1(n) - n\delta[\frac{\gamma\rho}{Nd(n^*)^2}(Na+b\rho E_1(n))(Nb+\gamma(N+n^{*3}-n^*)) + \bar{\theta}P'(E_1(n))]\}/b,$$
$$e_{1ns} = \{a - \gamma E_1(n) - \delta[\frac{\gamma\rho}{Nd(n^*)^2}(Na+b\rho E_1(n))(Nb+\gamma(N+n^{*3}-n^*)) + \bar{\theta}P'(E_1(n))]\}/b.$$

Replacing the equilibrium emissions into the objective functions, we obtain the respective payoff of a signatory and a non-signatory

$$W_{1s}(n) = U(e_{1s}(n)) - \frac{\gamma}{2}E_1(n)^2 + \delta\psi(E_1(n)), \qquad (27)$$
$$W_{1ns}(n) = U(e_{1ns}(n)) - \frac{\gamma}{2}E_1(n)^2 + \delta\psi(E_1(n)). \qquad (28)$$

The payoffs we have just derived are used to provide the following result on coalition formation in period 1.

Proposition 5 *In the dynamic membership framework, full cooperation may be sustained in period 1 (as a stable agreement), but only under endogenous uncertainty.*

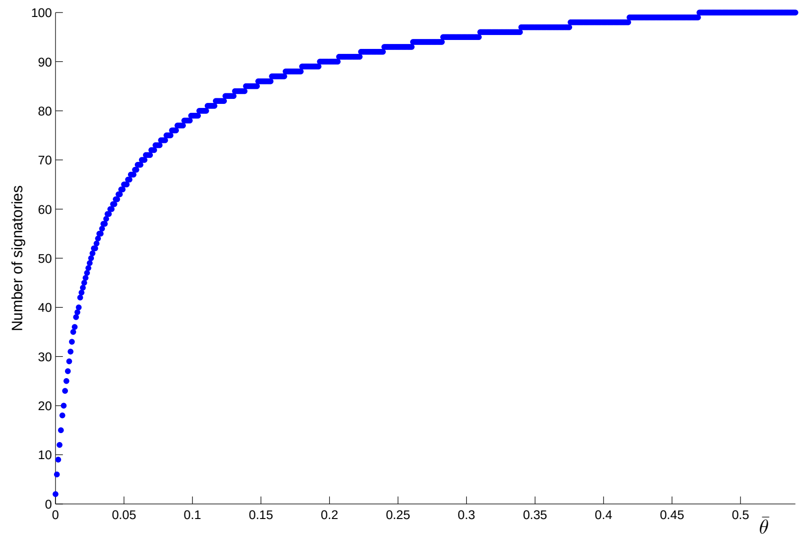

Figure 2: The effects of endogenous uncertainty on period-1 participation with dynamic membership and quadratic damages.

Proof. To prove that full cooperation may be sustained in period 1, it suffices to provide an example. We carry out a numerical analysis on a set of $N = 100$ countries to study how endogenous uncertainty can impact stable coalitions in period 1. We consider the probability function defined by (21) with the parameter values $a = 0.04; b = 1; \gamma = 0.0001; \delta = 0.9$ and $\rho = 0.95$ (which always result in nonnegative emissions levels). Our findings are illustrated in Figure 2. The number of signatories under either certainty ($P = 1$) or no-shift in the damages ($P = 0$) is equal to two. However, when endogenous uncertainty prevails, our analysis identifies two intervals of the magnitude of the shift in the damages function (captured by the parameter $\bar{\theta}$) across which the equilibrium patterns differ considerably. Below a given threshold for $\bar{\theta}$, we find a positive relationship between $\bar{\theta}$ and the number of signatories. However, above that threshold, the model generates the fully cooperative equilibrium. Notice that under exogenous uncertainty, we have $P' = 0$ and the probability disappears from the stability function. Using the same argument as in the proof of Proposition 4, it can be shown that the size of a stable coalition cannot exceed two.■

The results we have established show that in the first period, the number of signatories is greater than under the certainty case whether the IEAs are designed in the dynamic membership or in the fixed membership game. We explore the validity of these results under linear damages in Appendix B. Our results are maintained for period 2 where the puzzle of small coalitions still holds. In period 1, we find again that our model may sustain full cooperation, but only under endogenous uncertainty.

6 Conclusion

In this paper we have examined how the possibility of an upward shift in environmental damages can affect the willingness of countries to adhere to an IEA. To do this we have extended the traditional IEA membership model by including endogenous uncertainty within a two-period framework. This allows us to compare our results to the case where the shift in the damages is certain. Our findings show that this type of uncertainty has specific impacts on the equilibrium behavior. For a given membership size, endogenous uncertainty always lowers the emissions in period 1 as compared to the certainty case. But we get a reverse result in period 2. Moreover, endogenous uncertainty may increase the size of self-enforcing coalitions in period 1 (the grand coalition may even emerge as the stable coalition). The intuition behind our results is that the period-1 emissions not only will add to the environmental damage in both periods (this effect is traditionally present in the models of this literature), but also can trigger a regime change. As a consequence, the countries adopt a precautionary behavior by reducing their emissions and forming larger coalitions (in comparison with (a) the case without a shift in the damages and (b) the case with a certain shift in the damages).

Our results may have interesting ramifications within the IEA literature that uses the Cournot approach (see Finus, 2003, for an overview of this literature). When emission choices are simultaneous, the findings in the literature suggests that, the size of self-enforcing coalitions cannot be greater than three. Using the Cournot approach, however, we find that considering endogenous environmental risk provides a route to disentangle the puzzle of small coalitions.

Our results also shed new light on the effects of uncertainty on the incentives to ratify an IEA. A number of studies conclude that participation in IEAs is greater under uncertainty than under certainty (See for e.g. Young, 1994; Na and Shin, 1998). Our analysis suggests that it is not uncertainty per se, but rather the endogenous risk, that may increase participation in IEAs. In our model, uncertainty prevails in period 2, yet we have stable coalitions of at most three members (just as in the traditional models) in this period.

For elucidation purposes we have used a two-period model; considering more periods would not fundamentally change our results, however. In such a framework and under dynamic membership, the puzzle of small coalitions would still hold for the last period (in which no coalition of size greater than two would be stable as suggested by Proposition 4). Moreover, full cooperation would also emerge in some cases as a stable agreement in every non-terminal period. In the fixed-membership scenario too, full cooperation could possibly be generated with more than two periods.

Appendix A

Proof of Proposition 1

1. **Proof of the fact that "emissions under endogenous uncertainty are lower in period 1"**

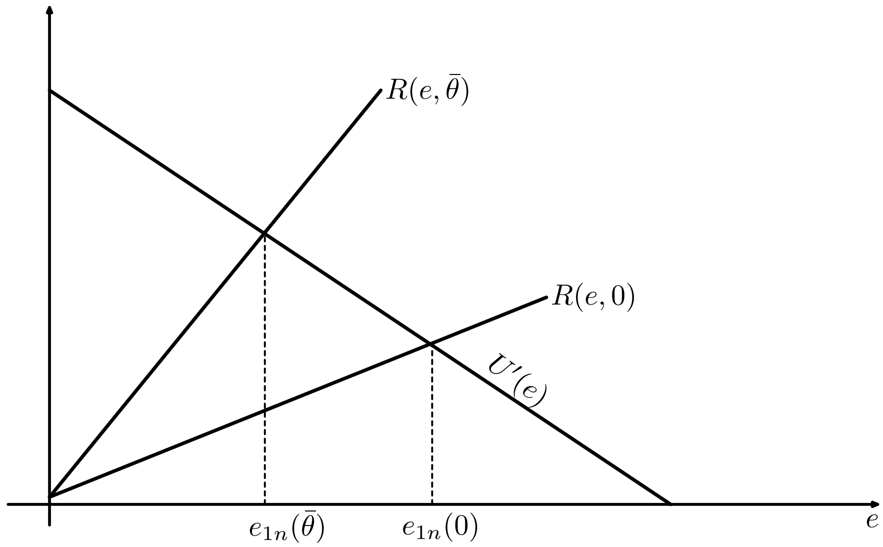

Figure 3: Effects of uncertainty on the period-1 optimal emissions

Denote by $R(e_{1n},\theta)$ the right-hand side of (5), that is

$$R(e_{1n},\theta) = D'(Ne_{1n}) + \delta[(\rho + (N-1)e'_{2n})D'(Ne_{2n} + N\rho e_{1n}) + \theta P'(Ne_{1n})],$$

for $\theta \in \{0,\bar{\theta}\}$ and $e_{1n} \geq 0$.

Since $D'(0) = P'(0) = 0$, $e'_{2n}(0) < 0$, $U'' < 0$ and $\delta, \rho \in [0,1]$ we have:

$$R(0,\theta) = \delta(\rho + (N-1)e'_{2n}(0))D'(Ne_{2n}(0)) < D'(Ne_{2n}(0)) = U'(e_{2n}(0)) < U'(0).$$

The existence of an interior solution in both cases ($e_{1n}(\bar{\theta})$, $e_{1n}(0)$) is guaranteed by the facts that U' is decreasing, $R(0,\theta)$ is lower than $U'(0)$, and $R(e_{1n},\theta)$ has an infinite limit as e_{1n} goes to infinity.

Notice that

$$R(e_{1n},\bar{\theta}) - R(e_{1n},0) = \delta\bar{\theta}P'(Ne_{1n}) > 0.$$

Thus, the marginal damage under the endogenous uncertainty is greater than the marginal damage under no uncertainty. As illustrated in Figure 3, the emission level is smaller under endogenous uncertainty.

2. **Proof of the fact that "emissions under endogenous uncertainty are greater in period 2"**

Since period-1 global emissions are lower under endogenous uncertainty and $e_{2n}(E_1)$ has a negative slope, the period-2 individual emission levels will be higher under the uncertainty case.

Proof of Proposition 4

Given E_1, we have shown that the stability function can be written as

$$\phi_2(n) = G(n)\frac{\gamma^2(n-1)(Na+b\rho E_1)^2}{2b(b+N\gamma-n\gamma+n^2\gamma)^2(b+2\gamma+N\gamma-3n\gamma+n^2\gamma)^2},$$

where $G(n) = -\gamma^2 n^5 + 5\gamma^2 n^4 + \gamma(-2b-7\gamma-2N\gamma)n^3 + \gamma(8b+3\gamma+4N\gamma)n^2 + (2N\gamma^2 - 2Nb\gamma - N^2\gamma^2 - b^2 - 6b\gamma)n + (2Nb\gamma - N^2\gamma^2 + 3b^2 + 4b\gamma)$. Notice that the sign of this expression is determined by $G(n)$.

Since $G(3) = -4\gamma(N^2\gamma + 3N\gamma + b(N-1)) < 0$ and $G(n+1) - G(n) = -((N-4)N + n^2)\gamma^2 - 2Nb\gamma - 2N(3n-1)n\gamma^2 - b^2 - 2bn(3n-5)\gamma - 5n^3(n-2)\gamma^2 < 0$ for all $n \geq 2$, any coalition of size $n \geq 3$ is not internally stable.

$G(2) = (2(2\gamma - b) - 3\gamma N)\gamma N + b^2 + 8b\gamma + 4\gamma^2$, which is positive if and only if $0 \leq \gamma \leq \bar{\gamma} \equiv b[4 - N + 2\sqrt{N^2 - 3N + 3}]/(3N+2)(N-2)$.

As consequence, if $\gamma > \bar{\gamma}$ no stable coalitions can be formed. However, for $0 \leq \gamma \leq \bar{\gamma}$ stable coalitions are constituted of two members.

Appendix B

In this appendix, we consider the following damage function

$$D(E_t) = \gamma E_t,\ t = 1, 2, \tag{29}$$

where γ is a positive parameter.

B.1 Fixed membership

We find that "*In the fixed membership framework, stable coalitions are constituted of three members under the certainty case* ($P = 1$) *or under the case where there is no shift in the damages* ($P = 0$)".

Proof

Assume for the moment that the coalition S of size n is the outcome of the membership game.

In period 2, signatories solve

$$\max_{e_{i2}, i\in S} \sum_{i\in S} \mathbb{E}\pi_{i2}(e_{i2}, E_2) = \sum_{i\in S} [a e_{i2} - \frac{b}{2} e_{i2}^2 - \gamma E_2 - \bar{\theta} P(E_1)],$$
$$\text{Subject to: } E_2 = \sum_{i=1}^{N} e_{i2} + \rho E_1.$$

A non-signatory $i \in N \setminus S$ solves

$$\max_{e_{i2}} \mathbb{E}\pi_{i2}(e_{i2}, E_2) = [a e_{i2} - \frac{b}{2} e_{i2}^2 - \gamma E_2 - \bar{\theta} P(E_1)],$$
$$\text{Subject to: } E_2 = \sum_{i=1}^{N} e_{i2} + \rho E_1.$$

In a symmetric equilibrium, for an interior solution, the optimal emissions for a signatory and a non-signatory are respectively

$$e_{2s}(n) = \frac{a - n\gamma}{b}; \;\; e_{2ns}(n) = \frac{a - \gamma}{b}. \tag{30}$$

In period 1, using a similar method as for the the quadratic damages presented in Section 4, the necessary and sufficient conditions for an interior solution are

$$a - b e_{1s} - n\gamma(1 + \delta\rho) - n\bar{\theta} P'(E_1) = 0, \tag{31}$$

$$a - b e_{1ns} - \gamma(1 + \delta\rho) - \bar{\theta} P'(E_1) = 0, \tag{32}$$

where $E_1 = n e_{1s} + (N - n) e_{1ns}$. Multiplying (31) by n and (32) by $N - n$ and adding up side by side the two resulting equations, we get

$$Na - (N - n + n^2)\gamma(1 + \rho\delta) = bE_1 + (N - n + n^2)\bar{\theta} P'(E_1).$$

The above equation gives $E_1(n)$, the total emissions of period 1 for a coalition of size n.

Solving (31) and (32), we obtain

$$e_{1s}(n) = [a - n\gamma(1+\delta\rho) - n\bar{\theta}P'(E_1(n))]/b, \tag{33}$$

$$e_{1ns}(n) = [a - \gamma(1+\delta\rho) - \bar{\theta}P'(E_1(n))]/b. \tag{34}$$

Hence the present payoff for a signatory and a non-signatory are respectively

$$\begin{aligned} W_s(n) &= ae_{1s}(n) - \frac{b}{2}e_{1s}(n)^2 - \gamma(1+\delta\rho)E_1(n) - \delta\bar{\theta}P(E_1(n)) \\ &\quad + \delta[ae_{2s}(n) - \frac{b}{2}e_{2s}(n)^2 - \gamma(ne_{2s}(n) + (N-n)e_{2ns})] \\ W_{ns}(n) &= ae_{1ns}(n) - \frac{b}{2}e_{1ns}(n)^2 - \gamma(1+\delta\rho)E_1(n) - \delta\bar{\theta}P(E_1(n)) \\ &\quad + \delta[ae_{2ns}(n) - \frac{b}{2}e_{2ns}(n)^2 - \gamma(ne_{2s}(n) + (N-n)e_{2ns})] \end{aligned}$$

The stability function is then $\phi(n) = W_s(n) - W_{ns}(n-1)$ for $n = 2, ..., N$. Notice in the no-uncertainty case ($P = 1$) and in the no-shift in the damages case ($P = 0$), the above expression simplifies and the stability function can be written as

$\phi(n) = A(-n^2 + 4n - 3)/2b$, where A is a positive constant, which depends on the model parameters. If $N \geq 3$, the largest value of n for which $\phi(n) \geq 0$ is $\bar{n} = 3$.■

Using (29), in the fixed membership game, we find that full cooperation may be sustained in period 1.

Proof

To prove this statement, it suffices to provide an example. We undertake a numerical analysis on a set of $N = 100$ countries. Again, we use the probability function (21) with the parameter values $a = 0.47; b = 1; \gamma = 0.0001; \delta = 0.9$ and $\rho = 0.95$. Our simulations identify three cases for the damage parameter $\bar{\theta}$ across which the equilibrium behavior differs considerably as illustrated in Figure 4.

The first case is for small values of the damage parameter *i.e* $\bar{\theta} \in [0, 0.03)$. There is a positive relation between $\bar{\theta}$ and the size of stable coalitions. In addition, the stable coalition associated to $\bar{\theta} = 0$, is constituted of three members, which confirms the analytical result that stable coalitions are of size three under the certain case. The second case corresponds to values of the damage parameter $\bar{\theta}$ in the interval $[0.03, 0.78)$, where the model always sustains full cooperation. The third case is for large values of the damage parameter *i.e* $\bar{\theta} \geq 0.78$. Our simulations identify a negative relation between $\bar{\theta}$ and the size of stable coalitions.■

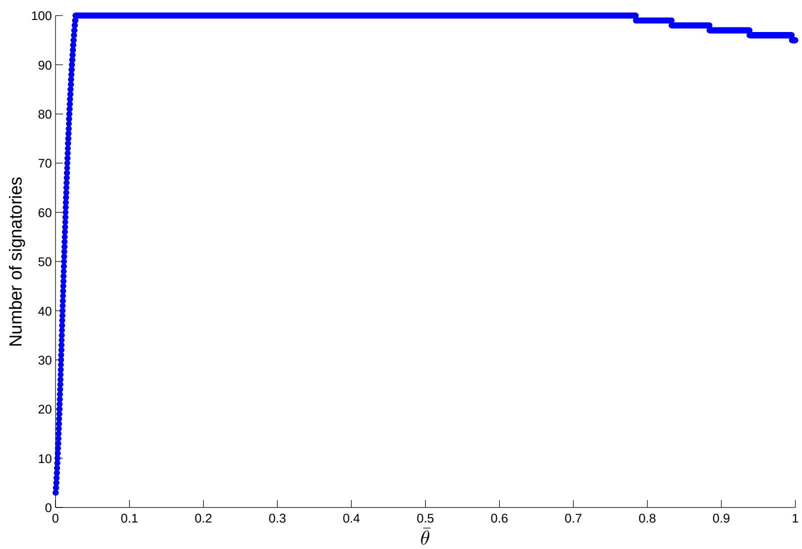

Figure 4: The effects of endogenous uncertainty on participation: fixed membership and linear damages.

B.2 Dynamic membership

We find that "*In the dynamic membership framework, stable coalitions are also of size three in period 2*".

Proof

Assume that an IEA is played at the beginning of period 2 and that a coalition S of size n is the outcome of the membership game. Given E_1, we use a similar method as in Section 5, to derive the optimal emissions of each signatory and that of each non-signatory, which are

$$e_{2s}(n) = \frac{a - n\gamma}{b}, \tag{35}$$

$$e_{2ns}(n) = \frac{a - \gamma}{b}. \tag{36}$$

The associated expected payoff for a signatory and a non-signatory is

$$V_{2s}(n) = U(e_{2s}(n)) - \gamma(ne_{2s}(n) + (N-n)e_{2ns}(n)) - \gamma\rho E_1 - \bar{\theta}P(E_1), \tag{37}$$

$$V_{2ns}(n) = U(e_{2ns}(n)) - \gamma(ne_{2s}(n) + (N-n)e_{2ns}(n)) - \gamma\rho E_1 - \bar{\theta}P(E_1). \tag{38}$$

In this situation, the stability function is

$$\phi_2(n) = V_{2s}(n) - V_{2ns}(n-1).$$

Simplifying, we get $\phi_2(n) = \gamma^2(-n^2+4n-3)/2b$. Since $N \geq 3$, the largest value of n for which the stability function is non-negative is $\bar{n} = 3$, which is the unique stable coalition size.■

We find that "*In the dynamic membership framework, stable coalitions are constituted of three members in period 1 under the certainty case* $(P = 1)$ *or under the no-shift in the damages case* $(P = 0)$*.*"

Proof

In order to derive the outcome of period 1. We use the random assignment rule defined in Section 5. The common payoff that signatories as well as non-signatories expect to get in period 2 is

$$\Psi(E_1) = \frac{\bar{n}}{N}V_{2s}(\bar{n}) + (1 - \frac{\bar{n}}{N})V_{2ns}(\bar{n}),$$

which can be rewritten as

$$\Psi(E_1) = -\gamma\rho E_1 - \bar{\theta}P(E_1) + \bar{C},$$

where

$$\bar{C} = \frac{\bar{n}}{N}U(e_{2s}(\bar{n})) + (1 - \frac{\bar{n}}{N})U(e_{2ns}(\bar{n})) - \gamma(\bar{n}e_{2s}(\bar{n}) + (N-\bar{n})e_{2ns}(\bar{n})).$$

In period 1, signatories and the typical non-signatory solve respectively

$$\max_{e_{i1}, i\in S} \sum_{i\in S}[ae_{i1} - \frac{b}{2}e_{i1}^2 - \gamma E_1] + n\delta\Psi(E_1),$$

$$\max_{e_{1i}}[ae_{i1} - \frac{b}{2}e_{i1}^2 - \gamma E_1] + \delta\Psi(E_1),$$

from which we derive the first-order conditions

$$a - be_{1s} = n\gamma(1 + \delta\rho) + n\bar{\theta}P'(E_1), \tag{39}$$

$$a - be_{1ns} = \gamma(1 + \delta\rho) + \bar{\theta}P'(E_1), \tag{40}$$

where $E_1 = ne_{1s} + (N - n)e_{1ns}$.

Notice that these conditions are similar to (31) and (32). Therefore, given coalitions of size n, the total emissions of period 1 $E_1(n)$ is the positive root of the equation

$$Na - (N - n + n^2)\gamma(1 + \rho\delta) = bE_1 + (N - n + n^2)\bar{\theta}P'(E_1). \tag{41}$$

The equilibrium emissions of each signatory and non-signatory are respectively

$$e_{1s}(n) = [a - n\gamma(1 + \delta\rho) - n\bar{\theta}P'(E_1(n))]/b,$$

$$e_{1ns}(n) = [a - \gamma(1 + \delta\rho) - \bar{\theta}P'(E_1(n))]/b,$$

and their associated discounted present payoffs are respectively

$$V_{1s}(n) = U(e_{1s}(n)) - \gamma E_1(n) + \delta\Psi(E_1(n)),$$

$$V_{1ns}(n) = U(e_{1ns}(n)) - \gamma E_1(n) + \delta\Psi(E_1(n)).$$

In this situation, the stability function is $\phi_1(n) = V_{1s}(n) - V_{1ns}(n-1)$. Notice that in the no-uncertainty case ($P = 1$) and in the case without a shift in the damages ($P = 0$), the above expression simplifies and the stability function can be written as $\phi_1(n) = k(-n^2 + 4n - 3)$, where k is a positive number depending on the parameters of the model. Therefore, in the no-uncertainty case ($P = 1$) and in the no-shift in the damages case, $\bar{n} = 3$ is the unique stable coalitions size for period 1.■

Using (29) in the dynamic membership game, we find that "*full cooperation may be sustained in period 1*".

Proof

To prove this statement, it suffices to provide an example. We carry out a numerical analysis on a set of 100 countries to study how endogenous uncertainty can impact stable coalitions in period 1. We use the probability function (21) and the parameter values $a = 0.54; b = 1; \gamma = 1/10^4; \delta = 0.9;\ \rho = 0.95$. Our results are illustrated in Figure 5.

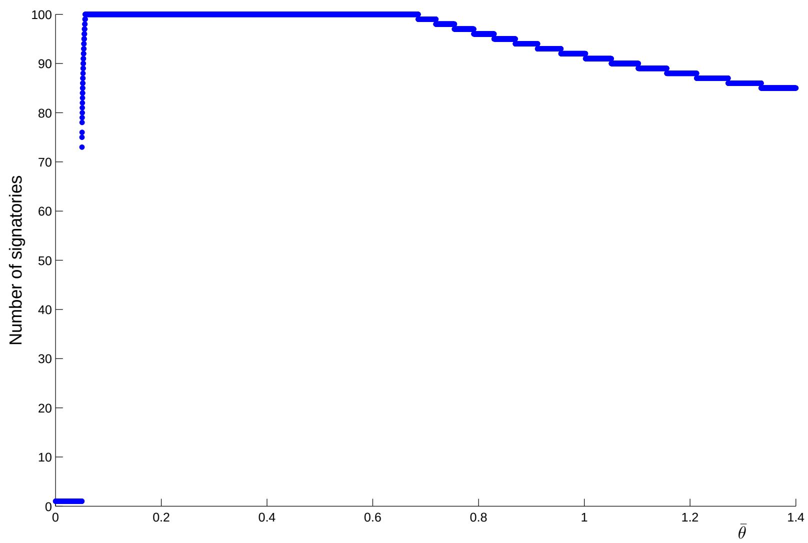

Figure 5: The effects of endogenous uncertainty on period-1 participation: dynamic membership and linear damages.

As in the fixed membership game, there are three cases (depending on the damage parameter $\bar{\theta}$) across which we have different equilibrium behaviors.

The first case corresponds to small values of the damage parameter, $i.e$ $\bar{\theta} \in [0, 0.06)$. We find a positive relation between $\bar{\theta}$ and the number of signatories. The second case is for intermediate values of the damage parameter, $i.e$ $\bar{\theta} \in [0.06, 0.685]$; in this case the model generates full cooperation. The third case corresponds to large values of the damage parameter, $i.e$ $\bar{\theta} > 0.685$. In this last case there is a negative relation between $\bar{\theta}$ and the size of stable coalitions.■

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