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Abstract

We investigate the dynamic effect that the Tandroy's unsustainable practices have on the forest. The Tandroy people lives in Androy, a region located in the southern part of Madagascar. They are mainly an agricultural and cattle hearding society whose subsistence relies on the slash-and-burn farming agriculture (*hatsake)* and the burning of cactus which are given as fodder to the livestock (*ororaketa*). These activities generate ecological pressure on the surrounding dry forest and socioeconomic risks related to the lack of sustainability of these practices in the long run. In this paper we address the notion of sustainability and confront it with the Tandroy's current productive and economic system. By means of viability theory, we characterize the actions and scenarios that are compatible with a sustainable use of the forest in the region.

Key Words: Sustainability; Viability Theory; Deforestation; Madagascar.

1 Introduction

Androy is a region located in the southern part of Madagascar and is home to one of the most unique and biologically rich dryland areas on Earth (Olson and Dinerstein, 2002; USAID, 2008). It contains a relatively thick vegetation cover dominated by several species of *Didiereaceae*, *Euphorbia* and *Adansonia za* and hosts locally endemic succulent plants that have evolved under arid and poor-soil conditions (Elmqvist et al., 2007).

In Madagascar, 85% of the population are farmers whose main economic activities are farming and cattle raising. The Androy region is no exception and its population, the Tandroy, are characterized by their reliance on slash-and-burn farming agriculture (*hatsake*) and the burning of cactus for livestock (*ororaketa*). These two activities, along with fuelwood and charcoal production, are directly related to land degradation and forest depletion. Forest trees are seen as a common-property resource, and the Tandroy cut and sell them as an additional source of revenue. Deforested land is used to grow maize, manioc, sweet potatoes and a variety of legumes (e.g., *antake*, catjang, beans and lentils) which are essential to people's livelihoods, particularly in the south.

The Tandroy population has more than doubled in the last 30 years and the combined effect of an increasing population and of the reluctance to change their traditional lifestyle and production system has led to great ecological pressure on the dry forest. As both human and cattle populations increase, *hatsake* and *ororaketa* activities require ever-increasing land space. This has resulted in great environmental degradation. Forest cover in the region has declined: It occupied only 23% of Androy's total area in 2005 compared to more than 30% in 1960 (Humbert and Cours-Darne, 1965; RM, 2008; USAID, 2007). Rapid forest depletion imperils Tandroy's whole production system and has negative impacts on soil productivity and the hydrological cycle, causing recurrent famines, increased aridity, drought, migration and cultural changes (Fanokoa, 2007).

Our main research question is whether there is something that the Tandroy can do to stop or slow down this relentless process, and ultimately, whether sustainability can be achieved. To formalize the notion of sustainability, we look at a number of socioeconomic indicators, such as consumption, capital accumulation and timber demand for heating, and define minimum thresholds. We build an economic model to explain the evolution of three key variables (i.e., the Tandroy population, forest cover in the Androy region and physical capital) and we define sustainability thresholds with respect to all three.

We make use of viability theory (Aubin, 1991) to assess the sustainability of the Tandroy economic system. Viability theory is particularly well-suited to model environmental problems and applications (e.g., Aubin et al., 2011; Aubin et al., 2005; Béné and Doyen, 2008; Doyen et al., 2007; Martinet and Blanchard, 2009; Martinet and Doyen, 2007) and has already been used to model deforestation problems (Andrés-Domenech et al., 2011) and to assess the sustainability in other regions of Madagascar (Bernard, 2011).

Using viability theory, we find that the Tandroy's current production system is clearly not sustainable. We show that neither the human nor the cattle population can continue to increase at the current levels. We compute the system's maximum carrying capacity for human and cattle populations in the long run. We also show that there is a trade-off between forest area and human and cattle carrying capacities. Our results also acknowledge the importance of reducing the current net deforestation rate in order to achieve sustainable outcomes.

Social aspects, such as open access to the resource, may make it impossible to slow deforestation. To tackle this problem, we propose an alternative solution: increased afforestation on abandoned agricultural and farming land. We show that the current deforestation rate, though very high, may be part of a sustainable solution, provided that the afforestation rate increases greatly.

Total current population is too high and the current state of the system is not sustainable. To achieve sustainability, either the population has to decrease or the Tandroy have to greatly reduce their current consumption rate. Deforestation in Androy is mainly caused by poverty; hence, asking the Tandroy to reduce their consumption will probably exacerbate the problem of forest depletion.

We analyze the role that monetary transfers can play in easing the economic-environmental problem. We determine the amount with which the Tandroy should be compensated so that they are able to (i) guarantee

their current consumption levels, and (ii) ensure that the forest may be preserved. According to our results, it will take roughly 65 billion ariary[1] per year (approximately 31M 2011$US) to preserve the dry forest in Androy.

The remainder of this paper is organized as follows: In Section 2, we model Tandroy economic system and Androy forest evolution. In Section 3, we specify the economic and environmental objectives and in Section 4, we explicitly define the states, the set of controls and their bounds. Our main findings are presented in Section 5. Section 6 summarizes and concludes.

2 Model

2.1 State Dynamics

Time is discrete and denoted by t. The Tandroy economic and environmental system is described by three state variables, namely, the Tandroy population N, the forest area in the Androy region F, and the physical capital K.

Population dynamics: The Tandroy population evolves according to the following difference equation:

$$N_{t+1} - N_t = N_t \cdot (\alpha_t - \beta + \gamma_t)\,, \tag{1}$$

where α_t denotes the birth rate at time t, β the death rate and γ_t is the net migration rate at time t.

We model the birth rate as a linear process formed of a given fixed component $\underline{\alpha}$ and a controllable one a_t, that is,

$$\alpha_t = \underline{\alpha} + a_t. \tag{2}$$

One can think of a_t as the increment or reduction in the fertility rate due to some governmental policy (e.g., birth-control campaigns, tax cuts to support larger families). Because the Tandroy's lifestyle remains traditional and seems not to change significantly with higher levels of wealth or income, we assume that the mortality rate β is constant over time. The sign of the migration rate γ_t depends on the difference between the number of immigrants and the number of emigrants. We measure the Androy region's attraction level at time t by the difference between the per capita income in the region and the corresponding figure in the rest of Madagascar. More specifically, we denote by R_t the total revenue in the Androy region and by M the per capita income in Madagascar. Then, the migration rate is given by

$$\gamma_t = \theta \cdot \left(\frac{R_t}{N_t} - M\right), \tag{3}$$

where θ is a positive parameter measuring the propensity to migrate. Admittedly, our specification is not the most general. Indeed, the migration rate may also depend on sociological and political factors. Our implicit assumption is that these factors are somehow embedded in parameter θ. In the same vein, we could have considered the magnitude of the propensity to migrate to vary with the sign of the difference $\frac{R_t}{N_t} - M$. Further, our assumptions that M is constant and that it corresponds to the per capita income in the whole country, as opposed to in Madagascar excluding Androy, are simplifying assumptions, which are mainly dictated by data availability. We refer the reader to the Appendix for more details on all of the model's parameters.

Forest dynamics: The forest area in the Androy region is measured in hectares. The evolution of its size depends on decisions about forestation[2] and deforestation. Denote by b_t the afforested area at t and by d_t the size of deforested area. Forests also grow and spread naturally. According to Bennett (1983), Chen (1988), Sugita and Tsukada (1982) and Tsukada (1981, 1982) tree expansion is non-linear and the area occupied by a given taxon can be well approximated by exponential or logistic functions. We follow this line of thought and assume the expansion of forest area follows a logistic growth function with a maximum carrying capacity

[1] The national currency.

[2] In this paper we use the term afforestation and forestation interchangably to denote "[...] planting of trees on land that was not previously forested" (FAO, 2010).

$F_{\max}$ to which the forest naturally converges. The value of $F_{\max}$ is given by the maximum size of land suitable for forest colonization and is set equal to the forest area in 1960 (Humbert and Cours-Darne, 1965). The evolution of the forest area is then governed by the following difference equation:

$$F_{t+1} - F_t = b_t - d_t + \eta \cdot \left(1 - \frac{F_t}{F_{\max}}\right) \cdot F_t, \tag{4}$$

where η is a positive parameter. The above specification was also adopted in Andrés-Domenech et al. (2011).

Note that there exist several reasons for deforestation; namely, *hatsake*, *ororaketa*, and demand for fuelwood and timber. Hence, the total deforested area d_t, in equation (4) is allocated to three different uses. We denote the *hatsake* area by da_t; the deforested area for wood production by dw_t; and, the *ororaketa* area by dz_t. The three together give the identity below:

$$d_t = da_t + dw_t + dz_t. \tag{5}$$

Capital dynamics: One of the most unique characteristics of the Tandroy traditional society is the dual role played by zebus, as both productive capital and stock of value. Zebu is a type of domestic cattle of the bovid family that is widespread across Madagascar. Zebus are used in Androy to harvest, as a means of transport, as source of milk and food, as stock of wealth and as source of social status. Because they are central to Tandroy life and have all the characteristics of physical capital, we have considered the number of zebus in the region to be a good proxy for the stock of capital in Tandroy society.[3] Denote by K the number of heads of zebu, and by μ the difference between the birth and mortality rate of the zebu population. A first source of variation in the stock of zebus is captured by the term μK. A second source is trade and consumption. Denote by c_t the per capita consumption at time t in monetary units. The economic surplus (or deficit) in the Androy region is then given by $R_t - c_t \cdot N_t$. Denote by P_z the constant price of a zebu in an adjacent region. This assumption of constant price is reasonable because the Androy region can be considered an atomistic player in the zebu trade.[4] Assuming that the surplus is entirely devoted to the acquisition of zebus, then the capital dynamics are given by

$$K_{t+1} - K_t = \mu \cdot K_t + (R_t - c_t \cdot N_t)\frac{1}{P_z}. \tag{6}$$

Note that if the Tandroy society is witnessing a deficit, that is, if $R_t - c_t \cdot N_t$ is negative, this would mean that the Tandroy either kill or sell off part of their cattle in order to feed themselves.

2.2 Revenues

As we have seen, Tandroy revenues have an impact on the evolution of both the population (through γ_t) and the capital stock. The total revenues can be decomposed into four main categories or sources: First are the revenues from agricultural activities (e.g., growth of corn, manioc, chickpeas, potatoes). These revenues are related to *hatsake* and are denoted by RA_t. Second, the revenues from selling wood and charcoal are denoted by RW_t. Third, revenues from cattle breeding and *ororaketa*-related activities (e.g., milk, meat) are denoted by RZ_t. Fourth, we have revenues from monetary transfers T_t. These transfers are almost negligible nowadays, but could play a crucial role in the future to ensure sustainability, as we will see in Section 5. Total revenues R_t can then be written as

$$R_t = RA_t + RW_t + RZ_t + T_t. \tag{7}$$

[3] Other types of cattle in Androy (e.g., goats or sheep) have minor importance compared to the zebu.

[4] The Tandroy population is roughly 550 000 people, whereas the Malgache population is over 20 million. Extending this specification to the national level, however, would necessarily require general-equilibrium considerations on the price and availability of zebus in the market.

Revenues from agriculture: For simplicity, we suppose that agricultural revenues RA_t come from the growth of a single (or composite) good whose price is given by P_A. The total yield depends on the area available for agriculture, that is, $F_{\max} - F_t - \delta K_t$, and on the productivity of this land, denoted X_t. Therefore, we have

$$RA_t = P_A \cdot X_t \cdot [F_{\max} - F_t - \delta K_t]\,. \tag{8}$$

Note that in our specification, we have made the assumption that the total available area for either forest, agriculture or pasture is given by $F_{\max}$. The extension of agricultural land is then defined as the land that is not occupied by neither forest, F_t, nor zebu pastures, δK_t. Parameter δ captures the area of pasture land needed per zebu.

The productivity of the agricultural land, X_t, depends on the fertility of the soil, which is affected by *hatsake* activities. Indeed, the Tandroy slash and burn the forest and grow mostly corn, manioc and potatoes on the fertile ashes that remain. The productivity of agriculture land is approximated as follows:

$$X_t = \overline{X} + \lambda_t \cdot da_t - \varphi \cdot \frac{F_{\max} - F_t}{F_{\max}}, \tag{9}$$

where $\overline{X}$ is the average productivity of this land. The term $\lambda_t \cdot da_t$ captures the productivity gained by burning an area da_t, and the term $-\varphi \cdot \frac{F_{\max} - F_t}{F_{\max}}$ represents the reduction in productivity due to forest depletion, with φ being a positive parameter. The time-varying coefficient λ_t is given by

$$\lambda_t = \frac{\psi \cdot \overline{X}}{F_{\max} - F_t - \delta K_t}, \tag{10}$$

where ψ is a positive parameter. In words, λ_t measures the normalized increase in productivity due to burning land.

Revenues from wood: In Androy, people log trees from the forest and use them primarily for heating and cooking. Denote by q_t the quantity of wood retrieved at time t; by c_w the minimum acceptable per capita yearly consumption; and by P_w the price of timber. The revenues from selling wood are then given by

$$RW_t = p_w \cdot (q_t - c_w \cdot N_t)\,. \tag{11}$$

Note that whenever $q_t - c_w \cdot N_t < 0$ there is not enough wood to cover the Tandroy society's minimum needs for heating and cooking. Such a deficit is not realistic for as long as the forest exists, but it could happen in the future if the forest is depleted and/or the population continues to increase. If that were the case, then the Tandroy would need to import wood and, rather than a source of revenue, the term RW would become a cost.

Tandroy society obtains wood through logging, which is done by men, and the gathering of fallen branches by women. Denote by n the amount of wood per hectare of forest, measured in cubic metres. Then, the product $n \cdot dw_t$ gives the total timber yield from deforestation. Let τ be the forest patches where the Tandroy do not enter for religious or superstition reasons. These sacred parts are known as taboo forest (Ferguson, 2007). The total quantity of wood harvested at time t can then be approximated by

$$q_t = n \cdot dw_t + n \cdot \varepsilon \cdot (F_t - \tau)\,, \tag{12}$$

where ε is a *small* positive parameter capturing the fraction of trees and branches that fall every period.

Revenues from zebus: Denote by ω the estimated yearly revenues obtained from milk and meat. The revenues from zebus is simply given by

$$RZ_t = \omega \cdot K_t. \tag{13}$$

To summarize, we have a model with three state variables, namely population N_t, forest area F_t, and capital stock K_t, and seven control variables: a control on the birth rate a_t, afforestation b_t, per capita consumption c_t, the deforestation rate for agriculture da_t, the deforestation rate for wood dw_t, the deforestation

rate for cattle breeding dz_t, and total transfers T_t. Note that the deforestation rate d_t is controlled through its components, not directly. Also, it is interesting to point out that these control variables do not belong to a single economic actor but rather to a number of them. We are concerned with the possibility or impossibility to commonly drive the system to a desirable (i.e. sustainable) set of states.

3 Sustainability objectives

Tandroy society heavily relies on the dry forest. However, its increasing population, traditional lifestyle and extensive production system (characterized by *hatsake* and *ororaketa* activities) has great destructive impact. Consequently, forest area continues to decrease alarmingly. It appears natural that their economic system will be sustainable only if it encompasses a sustainable use of the forest. This being said, it is quite arbitrary to define a threshold (in terms of forest area) beyond which Tandroy development stops being sustainable. Due to the difficulty of defining a minimum sustainable level of forest area, we instead enumerate a number of desirable properties that any so-called sustainable development must meet.

According to the definition given by the Brundtland Commission (WCED, 1987), a development is said to be sustainable if it "meet(s) the needs of the present generations without compromising the ability of future generations to meet their own needs." We shall put this definition in practice through a number of desirable economic requirements that, in our view, reflect the needs (the basic needs at least) of the Tandroy. We analyze the extent to which these requirements can be satisfied in time. In particular, following Ekins et al. (2003), we are interested in the underlying relation between compliance with these economic requirements and the state of Androy forest. As it turns out, we do not need to define explicit threshold levels for the stock of forest. Rather, the viability techniques used will tell us what initial levels of our system's three state variables (and not just the forest) are compatible with the perennial satisfaction of such needs.

Finding a set of initial states that allow the Tandroy to forever satisfy such needs is equivalent to saying that a sustainable development (both economic and environmental) is achievable for those initial states of the system. More precisely, what viability theory does is to give a clear-cut yes or no answer to the following two questions:

1. Can we find a subset of initial states such that the economic needs of the Tandroy can be satisfied forever?
2. Is the current situation (i.e., current population, forest area and capital stock) part of this set of initial states?

If the answer to the two questions is positive, then both economic and environmental sustainability are achievable. Else one has to identify the necessary policy changes that could lead to sustainable development.

We express the needs of Tandroy society in terms of consumption, production of timber and capital. In line with the definition of sustainability given above, we require the future levels of these variables to be at least as high as their current ones. More specifically, we adopt the following constraints:

$$c_t \geq \underline{c}, \tag{14}$$

$$q_t \geq c_w \cdot N_t, \tag{15}$$

$$K_t \geq K_{2005}, \tag{16}$$

$$\frac{K_t}{N_t} \geq \frac{K_{2005}}{N_{2005}}. \tag{17}$$

Constraint (14) states that the per capita level of consumption c_t at any period t must be at least equal to the current per capita consumption $\underline{c}$.[5] In (15), we impose that the production of wood covers at least the basic needs of the Tandroy population at any time period. Given the great importance of zebus in Tandroy society in terms of source of food, revenues, wealth and social status, the constraints in (16) and (17) require

[5]Current per capita consumption is measured as the per capita consumption in the region in 2005.

maintaining both the absolute and relative levels of capital, at least, at their 2005-levels. The year 2005 is chosen as a benchmark because it is the most recent year for which we have reliable data on many of our model's key variables.

4 State and control variables

4.1 State variables

Recalling that our model has three state variables, namely, population (N), forest area in Androy (F), and capital (K), our state space is defined as follows:

$$\mathbb{k} := \{x = (N, F, K) / N \in [N_{\min}, N_{\max}], F \in [F_{\min}, F_{\max}], K \in [K_{\min}, K_{\max}]\},$$

where x denotes a point in this set $\mathbb{k}$.

Forest area in Androy is bounded. Current forest area (453 561 hectares in 2005) is 37% less than it used to be before massive deforestation began in the 1950s-1960s (Fanokoa, 2007). According to Humbert and Cours-Darne (1965), forest area in 1960 was equal to 623 000 hectares. We use their estimation as a proxy for the upper bound of forest area. This upper bound, $F_{\max}$, is set as equal to the forest's carrying capacity.

To determine $F_{\min}$, recall that part of the forest is taboo and that the Tandroy do not enter it. Taboo forest covers an area approximately equal to 60 000 hectares (Ferguson, 2007). Therefore, we shall use $F \in [F_{\min}, F_{\max}] = [60\,000,\ 623\,000]$.

Once we have the bounds on F, we can endogenously determine the bounds for the other two state variables as a function of F itself. Recall from equation (8) that parameter δ denotes the area of pasture required by a head of cattle. Suppose that the whole available area in the model was turned into pasture land. Then, maximum cattle population would be given by the expression $\frac{1}{\delta} \cdot (F_{\max} - F_{\min}) = 1\,555\,248$ zebus. We thus have $K \in [K_{\min}, K_{\max}] = [0,\ 1\,555\,248]$.

We can determine the upper bound on population in a similar manner. Minimum per capita consumption is given by inequality (14) and revenues are given by (7). This means that $N_{\max} = \frac{1}{\underline{c}} \max[R]$. From (8), if all available land was dedicated to agriculture, then revenues would equal $P_A \cdot X_t \cdot (F_{\max} - F_{\min})$. Conversely, if all available land was dedicated to cattle herding, then, according to expression (13), cattle-herding revenues would be $\omega \cdot K_{\max}$. Finally, if all available land was used to obtain and sell wood, then timber revenues would equal $P_w \cdot n \cdot \varepsilon \cdot (F_{\max} - \tau)$. Given the fact that monetary transfers T are set equal to zero, the upper bound on population can be computed as follows:

$$N_{\max} = \frac{1}{\underline{c}} \max\left[P_A \cdot \left(\overline{X} - \varphi\right) \cdot (F_{\max} - \tau), \omega \cdot K_{\max}, P_w \cdot n \cdot \varepsilon \cdot (F_{\max} - \tau)\right]. \tag{18}$$

If we substitute all variables and parameters in (18) by their values, we obtain that agriculture is the most efficient way, per unit of land, of creating revenue and that: $N_{\max} = \frac{1}{\underline{c}} \cdot P_A \cdot \left(\overline{X} - \varphi\right) \cdot (F_{\max} - \tau) = 1\,836\,128$ people. We thus have that $N \in [N_{\min}, N_{\max}] = [0,\ 1\,836\,128]$.

The bounds on the three state variables are summarized in Table 1 below.

Table 1: State variables

State	Lower Bound	Current	Upper Bound
N	0	548 418 ps	1 836 128 ps
F	60 000 ha	453 561 ha	623 000 ha
K	0	290 000 zb	1 555 249 zb

4.2 Control variables

As stated above, our model includes seven control variables. Table 2 provides their current values as well as the adopted upper and lower bounds.

Table 2: Control variables

Control	Lower Bound	Current	Upper Bound
a_t	0 %	0 %	1 %
b_t	0	69 ha yr^{-1}	100 ha yr^{-1}
c_t	298 600 ar yr^{-1}	298 600 ar yr^{-1}	597 200 ar yr^{-1}
d_t	0	5 708 ha yr^{-1}	6 000 ha yr^{-1}
da_t	0	− − −	6 000 ha yr^{-1}
dw_t	0	− − −	6 000 ha yr^{-1}
dz_t	0	− − −	6 000 ha yr^{-1}
T_t	0	0	0 ar yr^{-1}

Control a_t denotes the variation in the fertility rate that can be induced by local authorities. To start with, we have kept a lower bound on a_t that is equal to zero. This means that the birth rate cannot be reduced. We shall analyze later the impact of letting a_t take negative values (i.e., $a_{\min} < 0$). Control b_t denotes the afforestation rate. We have chosen an upper bound on the afforestation rate that is quite low or pessimistic. Again, we will assess later how the solutions change when we allow for a larger afforestation rate, i.e., higher $b_{\max}$. With respect to the per capita consumption variable, c_t, we have set the lower bound exactly equal to current per capita consumption in order to be consistent with the definition of sustainable development put forward above. The deforestation rate d_t has been included in the table even if it is not a true control variable. We have no current observation for da, dw and dz. However we do have an estimate of the current value of d_t. Recall from (5) that $d = da + dw + dz$; and, hence, that the upper bound is shared for all four variables even if da, dw and dz cannot simultaneously have the maximum value of 6000 hectares per year. Although it has not been made explicit in Table 2, deforestation is limited by the availability of forest land, i.e., $d \in [0, \min(F - \tau,\ 6000)]$. Finally, we set the monetary transfers T_t equal to zero and, in the sequel, check the impact of having positive values for T_t. All monetary values are stated in ariary, where "ar" denotes the local currency of Madagascar. Note that for all of our control and state variables, the current observation lies somewhere between these two bounds.

To wrap up, our three-dimensional dynamic system, to which we shall refer to as $\mathcal{S}$, consists of

$$(\mathcal{S}) \quad \begin{cases} N_{t+1} - N_t &= N_t \cdot (\alpha_t - \beta + \gamma_t) \\ F_{t+1} - F_t &= b(t) + \eta \cdot \left(1 - \frac{F(t)}{F_{\max}}\right) \cdot F(t) - d(t), \\ K_{t+1} - K_t &= \mu \cdot K_t + (R_t - c_t \cdot N_t)\frac{1}{P_z}, \\ \text{with } u &:= (a, b, c, da, dw, dz, T) \in U, \end{cases} \tag{19}$$

where U is the feasible set of controls

$$\begin{aligned} U \quad = \quad & [a_{\min}, a_{\max}] \times [b_{\min}, b_{\max}] \times [c_{\min}, c_{\max}] \times [da_{\min}, da_{\max}] \times \\ & [dw_{\min}, dw_{\max}] \times [dz_{\min}, dz_{\max}] \times [T_{\min}, T_{\max}]\,. \end{aligned}$$

For future reference, we generically write the three first lines of $\mathcal{S}$ as $x_{t+1} = g(x_t, u_t)$ with $x_t = (N_t, F_t, K_t)$, $t \in \mathbb{N}$.

5 Results

5.1 Feasibility Set

In the previous section, we introduced the set $\mathbb{k}$, which is our computational area. Denote now by $\mathbb{k}_C$ the feasibility set, that is, the subset of $\mathbb{k}$ for which our constraints (14)-(17) can be met.

The first constraint (14) refers to the minimum per capita consumption and was used to obtain the boundary $N_{\max}$.

The second constraint (15) refers to the minimum wood consumption level necessary to satisfy the Tandroy demand for heating and cooking. If we plug (12) in (15), we obtain the following expression: $n \cdot dw_t + n \cdot \varepsilon \cdot (F_t - \tau) \geq c_w \cdot N_t$. From this last expression, we can obtain the minimum forest area required to meet our demand as a function of the population N and dw:

$$\underline{F} \geq \frac{c_w \cdot N - n \cdot dw + n \cdot \varepsilon \cdot \tau}{n \cdot \varepsilon}. \tag{20}$$

Substitute N and dw by their maximum in expression (20) bearing in mind that $dw \in [0, \min(F - \tau, 6000)]$ to obtain the minimum forest area required to guarantee the heating and cooking needs of the Tandroy, that is,

$$\underline{F} \geq \frac{c_w \cdot N_{\max} - n \cdot \min(F - \tau, 6000) + n \cdot \varepsilon \cdot \tau}{n \cdot \varepsilon}. \tag{21}$$

We can compute the value of $\underline{F}$ by inserting the values of our parameters into expression (21). The minimum is active, i.e., $\underline{F} \geq \frac{c_w \cdot N_{\max} + n \cdot \tau \cdot (\varepsilon+1)}{n \cdot (\varepsilon+1)}$ and $\underline{F}$ equals 62 319 ha. Now define $\Bbbk_q$ as the subset of $\Bbbk$ for which condition (21) can be satisfied in the absence of timber commerce: $\Bbbk_q := \{(N, F, K) / F \geq \underline{F}\}$. Figure 1 is a plot of set $\Bbbk_q$. The set comprises all the states above the horizontal plane at the bottom of the figure. As it appears, this constraint is only binding when forest area is very low.

The third constraint (16) refers to the minimum level of capital K. We require the absolute level of wealth to be at least as high as it was in 2005. Denote by $\Bbbk_K$ the subset of $\Bbbk$ for which condition (16) can be satisfied: $\Bbbk_K := \{(N, F, K) \ / \ K \geq K_{2005}\}$. Figure 2 is a plot of set $\Bbbk_K$. This set includes all the states to the right of K_{2005}.

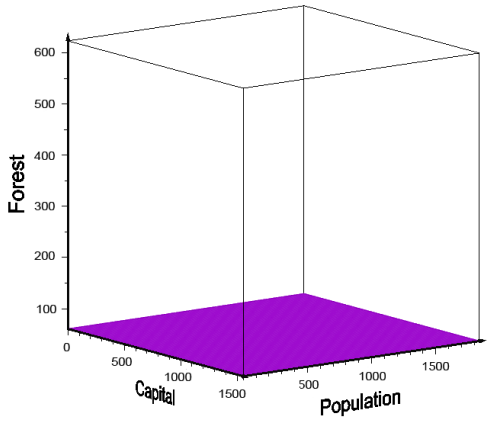

Figure 1: Set $\Bbbk_q$ is above the horizontal plane

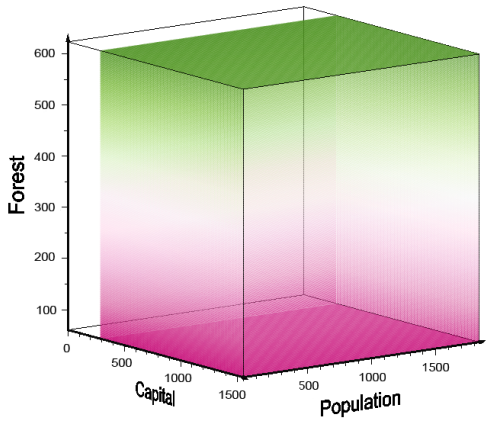

Figure 2: Constraint on capital: Set $\Bbbk_K$

Our fourth constraint refers to the minimum per capita level of capital. We require that wealth in per capita terms always be at least as high as in year 2005. Denote now by $\Bbbk_{K_{pc}}$ the subset of $\Bbbk$ for which condition (17) can be satisfied: $\Bbbk_{K_{pc}} := \{(N, F, K) \ / \ \frac{K}{N} \geq \frac{K_{2005}}{N_{2005}}\}$. Figure 3 is a plot of set $\Bbbk_{K_{pc}}$.

By merging all our constraints together, we obtain our feasibility set $\Bbbk_C$. We can now formally define $\Bbbk_C$ in the following manner:

$$\Bbbk_C := \{x = (N, F, K) \ / \ \Bbbk_q \cap \Bbbk_K \cap \Bbbk_{K_{pc}}\}. \tag{22}$$

Figure 4 is a plot of set $\Bbbk_C$.

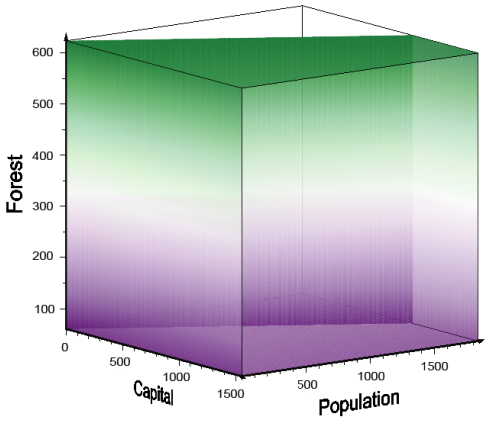

Figure 3: Constraint on capital per capita: Set $\mathbb{k}_{K_{pc}}$

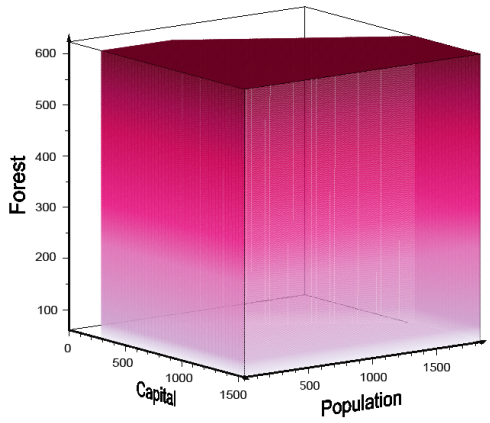

Figure 4: Feasibility set: $\mathbb{k}_C$

5.2 Benchmark case: $Viab_{\mathcal{S}}(\mathbb{k}_C)$

Set $\mathbb{k}_C$ does not involve the system dynamics; it is a static set. Our three state variables N, F, K are not constant and evolve in time. To determine whether or not Tandroy's productive system is sustainable we refer to viability theory. Viability theory provides us with the concept of the viability kernel. Denote by $Viab_{\mathcal{S}}(\mathbb{k}_C)$ the viability kernel of set $\mathbb{k}_C$ for system S. The viability kernel $Viab_{\mathcal{S}}(\mathbb{k}_C)$ is the largest subset of $\mathbb{k}_C$ for which one can find at least one control rule such that its resulting trajectory remains in $\mathbb{k}_C$ forever (such a trajectory is then called viable).

For our application, set $Viab_{\mathcal{S}}(\mathbb{k}_C)$ can be understood as the set of initial states for which sustainable development is possible, in the sense that an evolution that always remains in $\mathbb{k}_C$ is an evolution for which our constraints can always be satisfied. If the set $Viab_{\mathcal{S}}(\mathbb{k}_C)$ is empty, then this would mean that there is no initial state in $\mathbb{k}_C$ from which a viable evolution starts. In such case Tandroy productive system is said unsustainable.

More formally, denote by $x^0 = (N, F, K)$ any point in $\mathbb{k}_C$. Denote now by $S_{\mathcal{S}}(x^0)$ the set of all evolutions $x(\cdot)$ for the dynamic system (19) which emanate from x^0, that is,

$$S_{\mathcal{S}}(x^0) := \left\{(x_n)_{n\in\mathbb{N}} / \exists\, (u_n)_{n\in\mathbb{N}} \in U^{\mathbb{N}} \text{ such that } x_{n+1} = g\,(x_n, u_n)\,,\ x_0 = x^0\right\}.$$

Then, the viability kernel is given by

$$Viab_{\mathcal{S}}(\mathbb{k}_C) := \{x \in \mathbb{k}_C \ / \ \exists x(\cdot) \in S_{\mathcal{S}}(x), \forall n \in \mathbb{N},\ x_n \in \mathbb{k}_C\}\,.$$

Using the viability algorithm proposed in Saint-Pierre (1994),[6] we obtain that $Viab_{\mathcal{S}}(\mathbb{k}_C)$ is an empty set. This means that there are no states in $\mathbb{k}_C$ for which all our constraints can be satisfied forever.

For this particular case, it is easy to identify the reason why sustainability fails, i.e., $Viab_{\mathcal{S}}(\mathbb{k}_C)$ is empty. Substituting for α_t and γ_t, from (2) and (3), in the population dynamics (1) we obtain

$$\frac{N_{t+1} - N_t}{N_t} = \underline{\alpha} + a_t - \beta + \theta \cdot \left(\frac{R_t}{N_t} - M\right).$$

[6]The viability algorithm is a set-valued algorithm to numerically approximate the viability kernel. VIMADES Inc. developed this algorithm and allows us to use a version that is well adapted to environmental problems. All the viability kernels appearing in this paper were computed with this type of algorithm.

Given that R_t is positive, and that $a_t \in [a_{\min}, a_{\max}]$, then we have the following lower bound on the population's variation rate:

$$\frac{N_{t+1} - N_t}{N_t} \geq \underline{\alpha} + a_{\min} - \beta - \theta M. \tag{23}$$

Substituting for the parameters values in (23), we get $\frac{N_{t+1}-N_t}{N_t} \geq 0.01$, i.e., total population will increase by at least 1% every year. As the set $\mathbb{k}_C$ is bounded and the population can only increase, then, clearly, all evolutions will leave $\mathbb{k}_C$ in finite time. Consequently, the set $Viab_{\mathcal{S}}(\mathbb{k}_C)$ is empty and this means that Tandroy's productive system is not sustainable.

Faced with this negative result, the relevant question becomes the following: Can the Tandroy do something to turn their system into a sustainable one? Our previous results show that the current birth rate is too high, so we begin by exploring what changes in the birth rate are required in order to have a non-empty viability kernel.

5.3 Reduction in the birth rate

Recall that the net birth rate in (2) is given by two terms: a fixed one $\underline{\alpha} = 0.026$ and a variable one a_t. A necessary, though not sufficient, condition for the viability kernel to be non-empty is that $\frac{N_{t+1}-N_t}{N_t} \leq 0$ is feasible. This is possible if $a_{\min} \leq -0.010$. Denote by $\mathcal{S}_1$ the dynamic system that results when we let $a_{\min} = -0.015$. The viability kernel of $\mathbb{k}_C$ for the dynamic system $\mathcal{S}_1$ can be denoted by $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$. In Table 3, we specify the bounds for all of our control and state variables for system $\mathcal{S}_1$. We also include the specification used for all the different scenarios computed throughout this paper:

Table 3: Bounds on control variables for all scenarios studied

System	a_t	b_t	c_t	d_t	T_t
$\mathcal{S}$	$[\mathbf{0}, 0.01]$	$[0, 100]$	$[298\,600, 597\,200]$	$[0, 6000]$	$N.A.$
$\mathcal{S}_1$	$[-0.015, 0.01]$	$[0, 100]$	$[298\,600, 597\,200]$	$[0, 6000]$	$N.A.$
$\mathcal{S}_2$	$[-0.015, 0.01]$	$[0, 100]$	$[298\,600, 597\,200]$	$[\mathbf{5708}, \mathbf{5708}]$	$N.A.$
$\mathcal{S}_3$	$[-0.015, 0.01]$	$[0, \mathbf{3000}]$	$[298\,600, 597\,200]$	$[\mathbf{5708}, \mathbf{5708}]$	$N.A.$
$\mathcal{S}_4$	$[-0.015, 0.01]$	$[0, 100]$	$[298\,600, 597\,200]$	$[0, 6000]$	$N.A.$
$\mathcal{S}_{5.5}$	$[-0.015, 0.01]$	$[0, 100]$	$[\mathbf{180\,000}, 597\,200]$	$[0, 6000]$	$N.A.$
$\mathcal{S}_6$	$[-0.015, 0.01]$	$[0, 100]$	$[298\,600, 597\,200]$	$[0, 6000]$	$[0, \mathbf{6.5 \cdot 10^{10}}]$

The units for all variables in Table 3 have been omitted for simplicity. The values in bold indicate a change with respect to $\mathcal{S}_1$. Monetary transfers are constant and equal to zero for most of the scenarios considered. The label $N.A.$ is used here to denote that they are not applicable. Scenarios $\mathcal{S}_4$-$\mathcal{S}_6$ were computed requiring an extra constraint on the total forest area ($F \geq F_{2005}$).

Figure 5 below is a plot of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$.

Figure 5 shows two views of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ from two different perspectives.[7] Set $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ is made of all the points contained within the coloured volume. There are at least three distinct regions in the cube that do not belong to $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$. We have labelled these regions with letters A, B and C in the figure. Region A is determined by the constraint on minimum capital (16). Region B is related to the existence of constraint (17) on the minimum *per capita* capital. Finally, region C is also determined by constraint (17), but here, the relation is more subtle. Recall that if the population increases, capital must also increase in order to satisfy (17). Recall also that every head of cattle requires δ hectares of pasture land. The inclined surface that we observe relates these three facts. Take any point in the boundary of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ in region C. For a variation in population equal to ΔN, capital must increase at least $\Delta N \frac{K_{2005}}{N_{2005}}$ units, i.e., $\Delta K \geq \Delta N \frac{K_{2005}}{N_{2005}}$. Now, if capital increases ΔK units, then pasture land must increase (i.e., forest area must decrease) at least δ hectares for every additional head of cattle: $\Delta F \leq -\delta \cdot \Delta K$. We thus have that $\Delta F \leq -\delta \cdot \Delta N \frac{K_{2005}}{N_{2005}}$. This explains the shape of the frontier of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ in region C.

[7] Most of the figures representing the viability kernels plotted in this paper consist of two views of the same set from two different perspectives. The sets are complex and this double representation allows the reader to better seize the geometry of the set. It is also worth noting that the axes representing capital and population are inverted in each of these views.

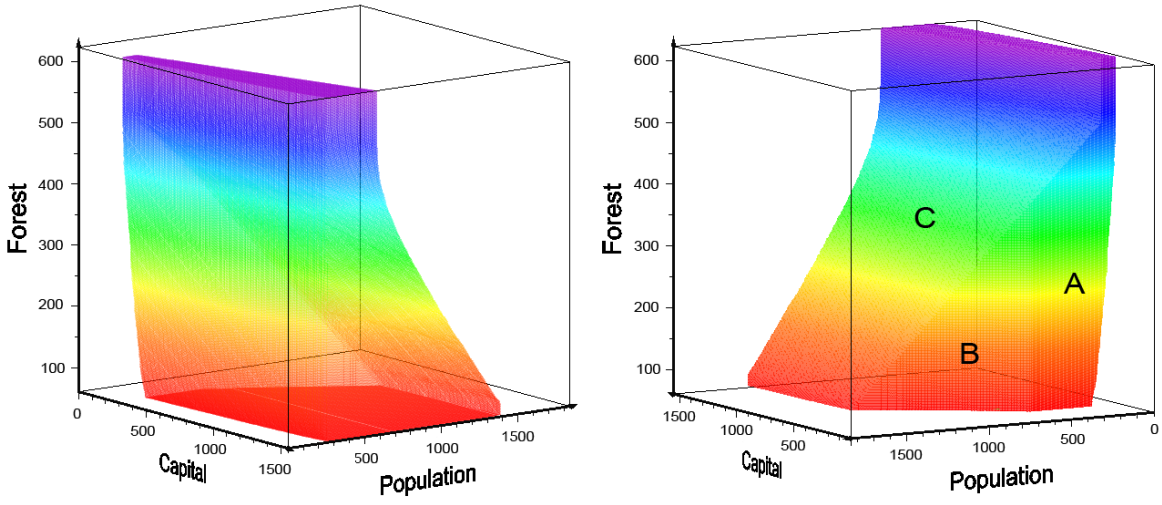

Figure 5: Two views of set $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$

5.4 Forest ownership and afforestation

Until now, we have supposed that the deforestation rate in Androy can be controlled. One of the reasons why $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ was not empty is that low deforestation rates are feasible. Figure 6 is a plot of the highest possible deforestation rate, d, along the boundary of the previously computed $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$.

Figure 6 shows that the Tandroy can afford to have a high deforestation rate for as long as they are not close to the lower boundary of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$.

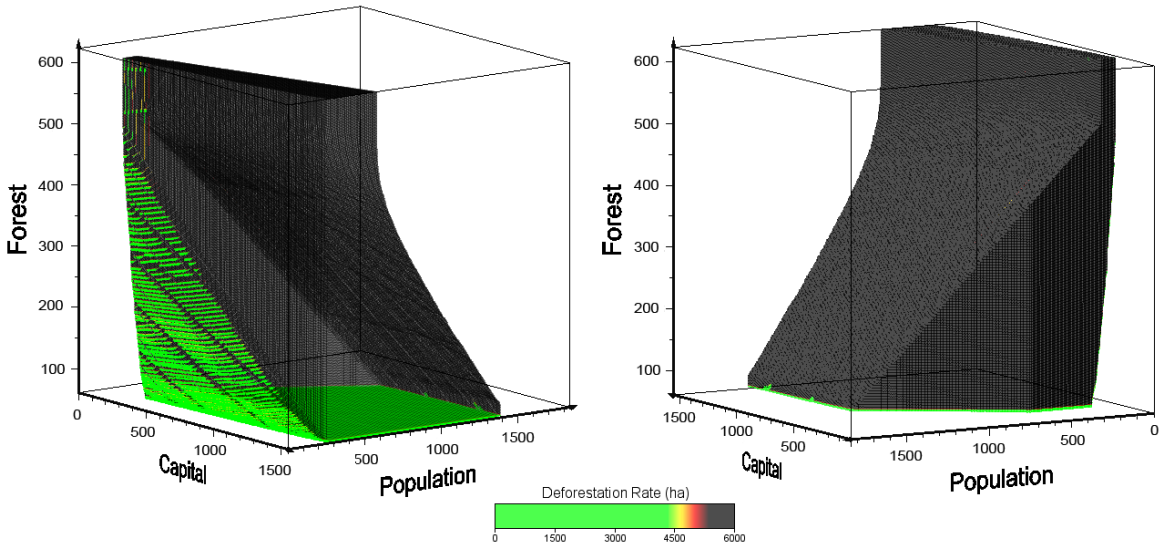

Figure 6: Highest viable deforestation rate along the boundary of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$

Forest in Androy is, in practice, an open-access resource. Whoever clears and subsequently occupies a piece of land is identified as its *de facto* owner. This type of behaviour poses a great problem and makes the task of reducing deforestation quite hard -if at all possible- to achieve.

Now suppose that it was not possible to reduce the deforestation rate. That is, suppose d is constant and equal to its current rate (i.e., $d = d_{current} = 5708$ ha yr^{-1}). This raises an interesting question: Can Tandroy's economic system be sustainable if deforestation cannot be reduced? Put differently: Would the viability kernel ($Viab_{\mathcal{S}_2}(\mathbb{k}_C)$) be non-empty if d were fixed and equal to the current deforestation rate?

Our results show that $Viab_{\mathcal{S}_2}(\mathbb{k}_C)$ is empty. In fact, it is easy to prove analytically that the current deforestation rate is not sustainable. Consider equation (4) and substitute d_t by its current value in (4) to obtain

$$F_{t+1} - F_t = b_t + \eta \cdot \left(1 - \frac{F_t}{F_{\max}}\right) \cdot F_t - d_{current}. \tag{24}$$

The left-hand side of (24) is maximized when $F = \frac{F_{\max}}{2}$ and $b = b_{\max}$ and can be expressed as follows:

$$F_{t+1} - F_t \leq b_{\max} + \frac{\eta \cdot F_{\max}}{4} - d_{current}. \tag{25}$$

Substituting for the parameters' values, we obtain that $F_{t+1} - F_t \leq -2390$ ha yr^{-1}. Hence, if the deforestation rate is set constant and equal to its current level, the forest area will decrease by at least 2390 ha every year. The current afforestation rate in Androy ($b_{current}$) is equal to 69 ha yr^{-1}(RM, 2003). Up to now, the upper bound used for the afforestation rate, $b_{\max}$, was equal to 100 ha yr^{-1}. Now suppose that the yearly afforestation rate could be as high as 3000 ha and denote by $\mathcal{S}_3$ the dynamic system that results when we allow for $b \in [0, 3000]$. Figure 7 below is a plot of $Viab_{\mathcal{S}_3}(\mathbb{k}_C)$.

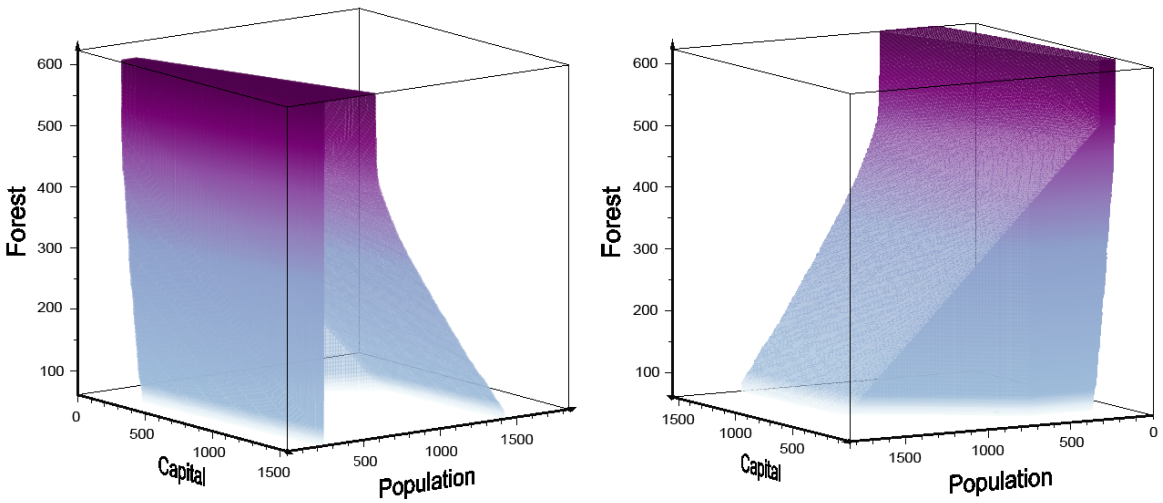

Figure 7: $Viab_{\mathcal{S}_3}(\mathbb{k}_C)$: Impact of increasing the upper bound on reforestation

There is not a great difference between Figures 5 and 7. In fact the two sets almost coincide. This means that if we are able to fully control deforestation but that forestation is low ($Viab_{\mathcal{S}_1}(\mathbb{k}_C)$), we obtain a similar result as when we are not able to reduce current deforestation levels but the forestation rate is high ($Viab_{\mathcal{S}_3}(\mathbb{k}_C)$). This is a positive message for local forestry authorities: Implementing a high forestation rate yields similar results as controlling the deforestation rate, and it is probably less difficult and costly to monitor.

5.5 Keeping the forest intact

Figure 7 provides an optimistic message: Sustainability is achievable even if the current deforestation rate remains high and unchanged. This same figure, however, also conveys a negative message: Sustainability can be achieved even when the forest area is quite low. As we saw in Figure 7, forest area can converge to $F_{\min}$ and still be part of a sustainable solution. In other words, it is sustainable to virtually deplete the forest.

Therefore, we ask ourselves if it would be feasible to maintain the forest untouched provided it were possible to monitor and control the deforestation rate. Once more, we can translate this question into the language of viability theory. Denote by $\mathbb{k}_{C_F}$ the following set: $\mathbb{k}_{C_F} := \{\mathbb{k}_C \cap \{(N, F, K) \ / \ F \geq F_{2005}\}\}$. Now denote by $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$ the viability kernel for set $\mathbb{k}_{C_F}$ where $\mathcal{S}_4 \equiv \mathcal{S}_1$. We want to determine whether or not $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$ is empty. Note that, by construction, $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F}) \subseteq Viab_{\mathcal{S}_4}(\mathbb{k}_C) \cap \mathbb{k}_{C_F}$. Figure 8 is a plot of $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$.

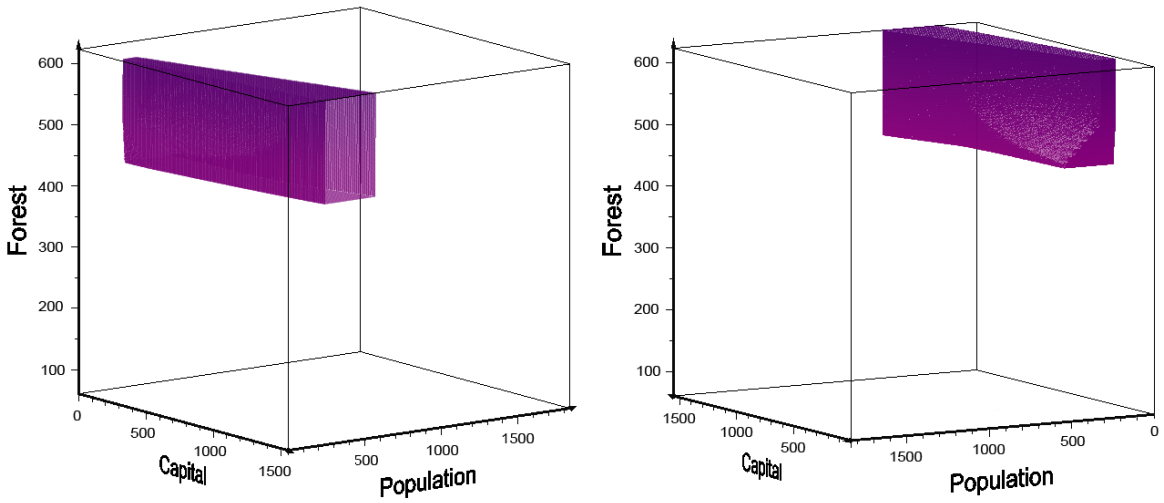

Figure 8: Two views of $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$. Maintaining current forest area

One can observe that $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$ is not empty. In fact it is equal to the intersection between $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ and $\mathbb{k}_{C_F}$.[8] The main message that we get is that if one can control the deforestation rate then it is possible to satisfy all our constraints without harming the forest.

5.6 Current state of the Tandroy economy

Up to now we have computed a number of viability kernels. Some of them were empty, while others were not. However, we have not yet addressed our second research question, namely, is the Tandroy economy's current state sustainable? Or put differently, does $x_{2005} = (N_{2005}, F_{2005}, K_{2005})$ belong to any of the kernels that we have computed so far?

The answer to this question is negative. Figures 9 and 10 are a plot of $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ and $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$ respectively. The initial state x_{2005} does not belong to either.

Figure 9 is a plot of the previously computed $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$ from a different perspective. One can observe that $x_{2005} \notin Viab_{\mathcal{S}_1}(\mathbb{k}_C)$. Figure 10 shows the same set as in Figure 8. We have now added x_{2005} and again show that $x_{2005} \notin Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$. What these two figures tell us is that, even if Tandroy's economic system is

[8] We cannot guarantee that the sets are exactly equal since we are numerically approximating them. However, the numerical approximation of both sets coincides.

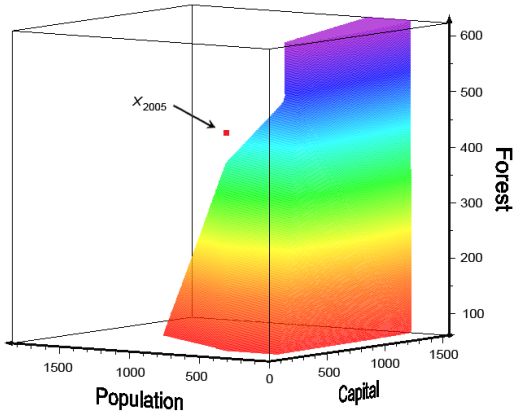

Figure 9: State x_{2005} does not belong to $Viab_{\mathcal{S}_1}(\mathbb{k}_C)$

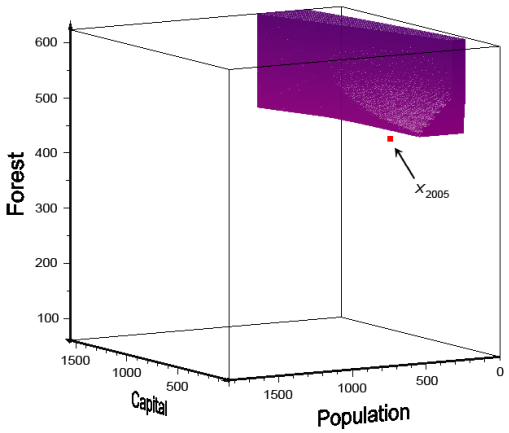

Figure 10: State x_{2005} does not belong to $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$

sustainable for some initial states, this is not the case for the current state. The main reason why x_{2005} is not sustainable in any of the scenarios studied is that overall Tandroy consumption, $\underline{c} \cdot N_{2005}$, is not sustainable in time. Recall that parameter $c_{\min} = \underline{c} = 298\,600$ ar yr^{-1}. Figure 11 is a plot of the highest feasible (i.e., affordable) consumption along the boundary of our previously computed $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$. We show two views of this set from different perspectives.

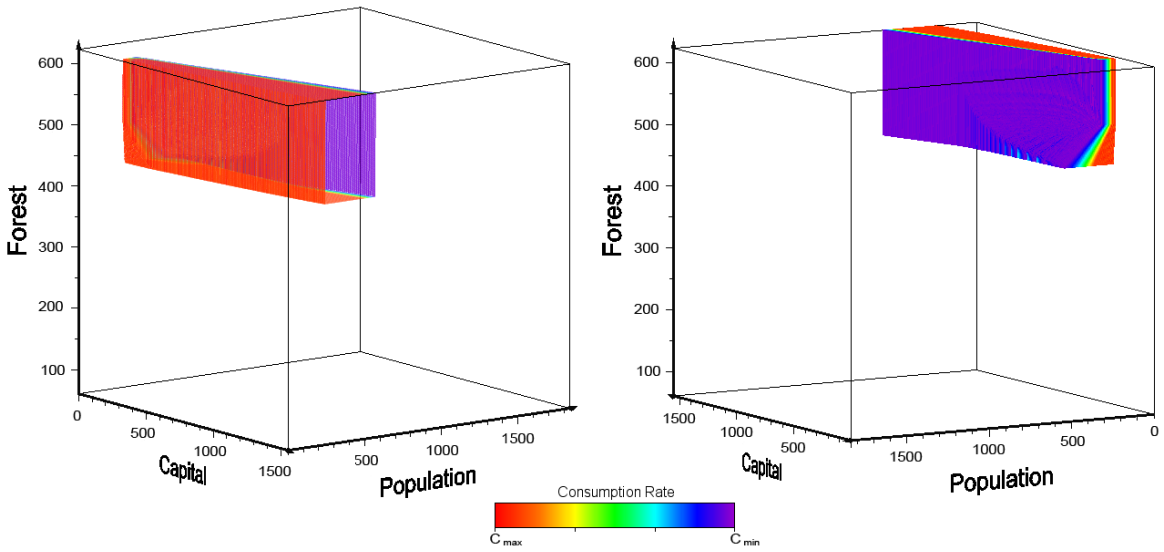

Figure 11: Highest affordable consumption along the boundary of $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$

Figure 11 shows the obvious fact that as population increases the highest affordable consumption must decrease. It also suggests that if the lower bound in consumption, $c_{\min}$, decreases (for a constant level of capital and forest area) then the boundary of the viability kernel would shift and include states with greater population.

To clarify this last point, we have computed the viability kernel for several scenarios. Each scenario has a different lower bound on the per capita consumption variable, as specified in Table 4:

Table 4: Different lower bounds in consumption per capita

Scenario	C1	C2	C3	C4	C5
$c_{\min}$	298 600	270 000	240 000	210 000	180 000

Denote by $Viab_{\mathcal{S}_{5.1}}(\mathbb{k}_{C_F})$ the viability kernel of scenario C1. The minimum consumption rate in scenario C1 is exactly the same one that we used to compute $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$. In fact, $Viab_{\mathcal{S}_{5.1}}(\mathbb{k}_{C_F})$ is exactly the same set as the previously computed $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$. Now, denote by $Viab_{\mathcal{S}_{5.2}}(\mathbb{k}_{C_F})$-$Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$ the viability kernel corresponding to scenarios C2 to C5. The viability kernel for C5 is the only one of the of the five proposed scenarios that includes x_{2005} inside its boundaries. In this scenario, $c_{\min} = 180\,000$ ar yr^{-1}.

Figure 12 is a plot of the kernel for scenario C5. The size of the viability kernel increases when $c_{\min}$ decreases. Since x_{2005} lies at the boundary of $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$, we can write that, *ceteris paribus*, $c_{\min} = 180\,000$ is the largest minimum consumption rate for which the current state of the system is viable. In other words, the current per capita consumption rate in Androy must be reduced by 40% in order for x_{2005} to be viable.

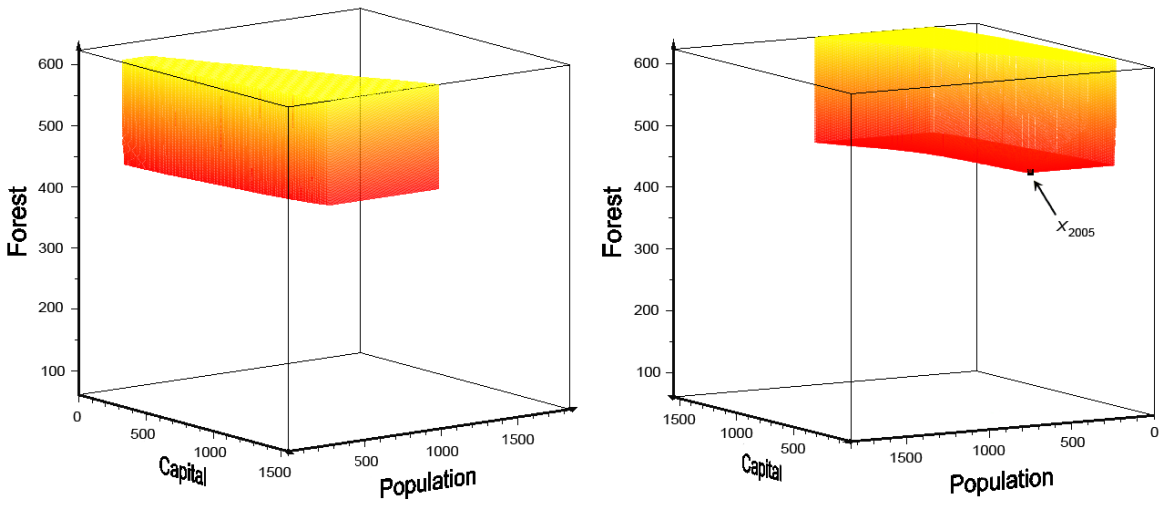

Figure 12: State x_{2005} belongs to $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$

5.7 Monetary transfers

The viability kernel computed in Figure 12 somehow presupposes that reducing consumption is feasible. The current consumption rate is already very low and the Tandroy perceive the forest as something to be exploited if needed. Therefore, it is not clear that the Tandroy would be willing to sacrifice 40% of their consumption in order to be viable and/or preserve the forest. It is important to mention that reducing consumption is not the only way to ensure that x_{2005} is included in the viability kernel. Another possibility would be using monetary transfers as a means to ease Tandroy's problems and reduce the deforestation rate.

So far we have considered monetary transfers as being equal to zero. It is possible, however, that national authorities or international organizations or countries would be willing to compensate the Tandroy for their keeping the forest intact. The question then becomes how much money would have to be transferred in order that there are no decreases in either the forest area or the current consumption rate.

To answer this question, we need to review our previous findings: $Viab_{\mathcal{S}_4}(\mathbb{k}_{C_F})$ is not empty; therefore it is possible to preserve the current forest area and current consumption levels, although not at the current population levels. Analogously $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$ contains x_{2005}, which means that it is possible to preserve the current forest area and the current population, but not the current consumption rate. The only difference between these two sets is the lower bound in consumption, $c_{\min}$, used to obtain them.

Denote by $\widetilde{T}$ the minimum value of transfers that guarantees that current consumption is sustainable for the current Tandroy population. One can compute $\widetilde{T}$ analytically as the product between current population and the difference in the $c_{\min}$ used for scenarios S_4 and $S_{5.5}$:

$$\widetilde{T} = N_{2005}\,(\underline{c} - c_{C5}) = 6.5 \cdot 10^{10}\ ar,$$

where $\underline{c}$ and c_{C5} are the minimum per capita consumption rates allowed in scenarios S_4 and $S_{5.5}$, respectively. Now denote by S_6 the dynamic system that results when we use $T \in [0, \widetilde{T}]$. Figure 13 below is a plot of $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$.

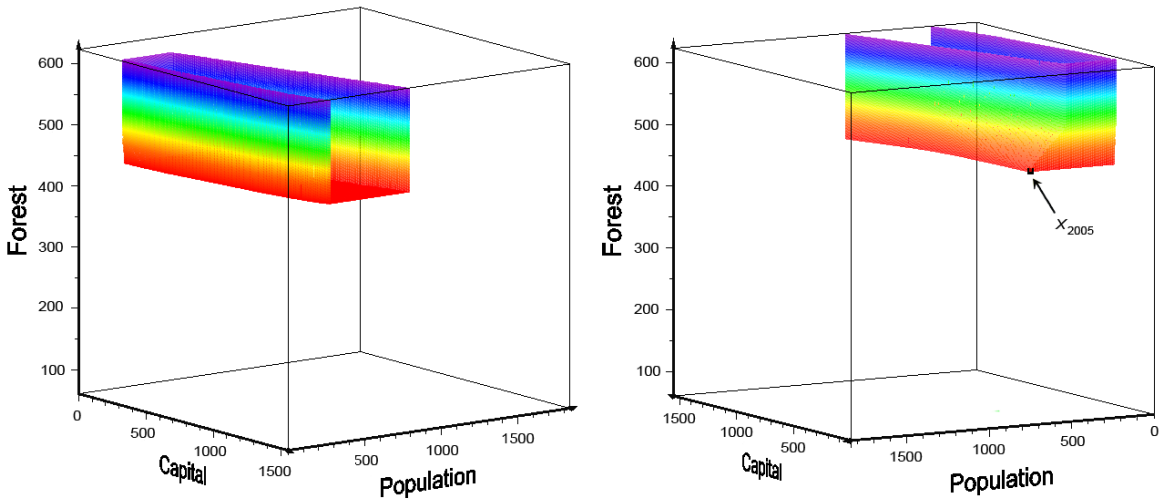

Figure 13: Monetary transfers can make x_{2005} viable: $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$

State x_{2005} is included in the boundary of set $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$ by construction. Hence, for transfers as high as $6.5 \cdot 10^{10}$ ar yr^{-1} it is possible for the Tandroy to maintain their current per capita consumption, and for their valuable forest to be preserved. It is also worth noting that $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$ and $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$ do not strictly coincide. We have that for $N > N_{2005}\ /\ N \cdot (\underline{c} - c_{C5}) > \widetilde{T}$. Hence $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$ includes some states with high population levels that do not belong to $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$. Conversely, for $N < N_{2005}\ /\ N \cdot (\underline{c} - c_{C5}) < \widetilde{T}$. Hence $Viab_{\mathcal{S}_6}(\mathbb{k}_{C_F})$ includes some states with low population levels that do not belong to $Viab_{\mathcal{S}_{5.5}}(\mathbb{k}_{C_F})$.

6 Conclusions

The dry forest in Southern Madagascar has great ecological importance and much of its biodiversity is unique. In this paper we modelled the economic and productive system of the Tandroy people and showed that it is far from being sustainable. We proposed a number of policy changes that are necessary to recover sustainability. The Tandroy need to reduce their birth rate, else population will continue to grow and put pressure on the forest resource. Reducing natality is necessary, but not sufficient.

The current deforestation rate is also far from being sustainable. Forests in Androy are an open-access resource and it is not clear that deforestation can be reduced. If reducing deforestation is beyond reach, or if

it is too costly to monitor and control deforestation, then it may be desirable to boost afforestation by means of public programs to offset part of the deforestation damage and help stabilize the area of forest.

We have found a number of requirements (e.g., low birth rates, lower deforestation rates than the current levels, low consumption rate) that make the Tandroy production system sustainable for a small set of states. However, the current population in Androy is high and is not part of the set of initial states for which sustainability can be guaranteed. One way to ensure a sustainable development for the current level of population (more than 550 000 people) is to reduce per capita consumption in the region by roughly 40% with respect to 2005 levels.

Reducing consumption can have devastating consequences, though. Deforestation is mainly caused by the mix of poverty and ill-defined property rights over the forest. It is thus very unlikely that the forest will be spared if the Tandroy's consumption rate is reduced. On the contrary, the deforestation problem may worsen if consumption decreases or if population increases persist over time.

Malgache national authorities, developed countries and NGOs are showing increasing interest in preserving the dry forest in Androy. We have computed the amount of money that would be required from them on a yearly basis to preserve the forest as it currently is. According to our estimates, as much as 65 billion ariary (roughly US$ 31M), are required to maintain the current level of consumption for the region's population.

Appendix

State Variables

N: Tandroy population

Tandroy population in the Androy region. The data on the Tandroy population that we used was forecasted by RM (2003). According to their forecasts Tandroy population in 2005 totalled 548 418 individuals. State variable N can take values within two bounds $N \in [N_{\min}, N_{\max}]$. $N_{\min} = 0$ is exogenously determined and $N_{\max}$ is determined endogenously as the largest population that can consume their subsistence level $\underline{c}$:

$$N_{\max} = \frac{1}{\underline{c}} \max\left[RA, RW, RZ\right] = \frac{1}{\underline{c}} P_A \overline{X} \left(F_{\max} - \tau\right) = 1\,836\,128,$$

where the terms inside the brackets represent the maximum agricultural yield, the maximum yield from cattle farming, and the maximum timber yield, respectively, when the whole available area is allocated to either of the three activities.

F: Forest area in Androy

The forest stock in our model is measured as the total area of dry forest in the Androy region given in hectares. RM (2008) and USAID (2007) estimate that the forest in 2005 covered 453 561 ha. Forest area can take values in the range $F \in [F_{\min}, F_{\max}]$ where $F_{\min}$ is set equal to the size of the taboo forest (see estimate of parameter τ below) and parameter $F_{\max}$ denotes the maximum extent of the Androy forest measured in hectares. $F_{\max}$ was set equal to the original pristine size of the forest. The oldest and most accurate estimate we have access to is the one by Humbert and Cours-Darne (1965) who estimated that the forest in Androy covered 623 000 ha in 1960.

K: Capital stock in Tandroy society

The stock of capital is proxied by the zebu population. The current zebu population in Androy for 2005 is 290 000 heads (Fanokoa, 2007). The capital stock is bounded in our model and belongs to the set $K \in [K_{\min}, K_{\max}]$. $K_{\min}$ is set equal to zero and represents a physical constraint (we do not allow for banking or negative capital assets) and $K_{\max}$ is the maximum achievable cattle stock that is determined endogenously according to the region's maximum carrying capacity. A zebu needs, on average, an area of δ ha for pasture.[9] If we suppose that all available land, $F_{\max} - \tau$, is used for pastures, we then obtain $K_{\max} = 1\,555\,994$ zebus.

[9] The value of δ is estimated below.

Control Variables

a: Control on the birth rate

Ability that the governmental institutions have to influence the base birth rate with respect to its natural value $\underline{\alpha}$. A positive value indicates a policy that aims at increasing natality (e.g., tax cuts for larger families) and *vice versa* (e.g., advertising and educational campaigns for the use of contraceptive measures). For our benchmark case, we consider $a \in [a_{\min}, a_{\max}] = [0\ ,\ 0.01]$.

b: Afforestation rate

Total afforested area measured in hectares. Total afforestation in the region is very small. RM (2003) estimates the afforestation for 2005 to be 69 *ha*. The afforestation rate in our model belongs to the interval $b \in [b_{\min}, b_{\max}] = [0\ , 100]$.

c: Consumption per capita

Yearly per capita consumption in the region. We have allowed consumption to fluctuate around the following interval: $c \in [c_{\min}, c_{\max}] = [\underline{c}\ ,\ 2\underline{c}]$, where $\underline{c}$ denotes current consumption (estimated below). The upper bound chosen for maximum consumption, $c_{\max}$, is completely arbitrary but has little or no impact on the solution. We simply chose a value that doubles the current per capita consumption. This upper bound is just required for mathematical tractability.

d: Deforestation rate

Total human deforestation measured in hectares. We do not have direct measures or estimations of current deforestation in the region. Nevertheless, we have access to observations of the yearly variation in total forest area in Androy (USAID, 2007). According to USAID, the total forest area decreased at a rate of 3098 ha yr^{-1} during the period 2000-2005, meaning that average forest loss in equation (4) equals 3098 ha yr^{-1}. We know all the other terms in equation (4). We can thus compute current deforestation as a residual. Substituting for parameters and current values of the afforestation rate, we obtain an estimate of current deforestation: $d_{2005} = 5708.4$ *ha*. In our paper d is modelled as a bounded pseudo-control variable that the Tandroy can adjust, i.e., $d \in [0\ ,\ d_{\max}]$ where we rounded up and set $d_{\max}$ equal to 6000 *ha* so that the value of the upper bound is consistent with current deforestation figures, that is, we set the current deforestation as a worst-case scenario.

da: Area deforested and burnt for agricultural purposes

Total area burnt and subsequently used to grow agricultural goods. Typically, the productivity of this type of land is much higher.

dw: Area deforested to obtain wood

Forest area deforested to obtain wood primarily used for heating and cooking. The wood obtained can also be sold to obtain economic revenues.

dz: Area deforested for pasture use

The Tandroy burn the cacti and use them to feed cattle. This practice is known as *ororaketa*.

T: Monetary transfers

Total monetary transfers expressed in ariary. These are monetary transfers coming from the national authorities or foreign aid agencies. These can be seen as transfers or subsidies to the Tandroy people with the objective of compensating them for reducing deforestation or applying a given forest or environmental policy.

Parameters

The parameters are written in order of appearance in the text.

$\underline{\alpha}$: Average birth rate

The birth rate in Androy is measured in child births per person and year. We compute $\underline{\alpha}$ using the estimates from RM (2003) for the four main districts in the region in 2005. The numbers inside parentheses correspond to the birth rates while the other figures correspond to the population in each of these four districts.

$$\underline{\alpha} = \frac{146078\cdot(0.020) + 51592\cdot(0.023) + 50652\cdot(0.028) + 99194\cdot(0.032)}{146078 + 51592 + 50652 + 99194} = 0.026.$$

β: Average mortality rate

Average mortality rate was computed using the same source (RM, 2003) and in the same manner as $\underline{\alpha}$.

$$\beta = \frac{146078\cdot(0.006) + 51\,592\cdot(0.003) + 50652\cdot(0.009) + 99194\cdot(0.006)}{146078 + 51592 + 50652 + 99194} = 0.006.$$

θ: Population willingness to migrate in terms of revenue

Current migration in the region is $\gamma = -0.005$ (Fanokoa, 2007). We have calibrated θ using equation (3) to fit current migration. This gives us an estimate of $\theta = 5.956 \cdot 10^{-8}$.

M: Per capita income for Madagascar

Average per capita income for Madagascar measured in ariary per year for 2005. IMF (2007) estimates per capita income at US$ 309. The average exchange rate between the ariary and the US dollar for the period from 2003 to 2006 is 1238.33 ar per dollar. This gives a value of $M = 382\,644$ ariary.

η: Natural growth rate of the forest

Elmqvist et al. (2007) report that the Androy forest cover grew 4% in abandoned rural areas from 1993 to 2000. The authors show that these findings are also consistent with data for the period between 1984 and 1993. This change in forest area can be expressed on an annual basis to obtain a mean annual increase of 0.5619%. Considering that total area of Androy forest in 2005 was 453 561 ha, we estimate the annual increase in forest area to be 2548.6 ha. If forest grows following expression (4), substituting, we obtain $\eta = 0.021$.

μ: Natural net growth rate of livestock population per annum

This parameter denotes the yearly net growth rate of zebu population when taking into account natality and mortality rates for natural causes. Note that the Tandroy often kill zebu in times of famine in order to eat. This has to be taken into account when computing μ or else the parameter could be underestimated. We used data from RM (2007) to estimate μ at 0.029.

P_z: Price per zebu

The average price of a zebu has been proxied by the price of the meat in an adult animal or rather what is called a tropical cattle unit (TCU). Barral (1974) and Kirsch-Jung and Soeftestad (2006) introduce this synthetic unit to compare across zebus of different ages and different types. An adult zebu weights approximately 250 kg on the hoof, while the average one is slightly smaller (approximately 25 % smaller). The price of meat being $P_{meat} = 2900\ ar/kg$ we estimate the price per zebu at 543 750 ariary.

δ: Area needed for pasture per head of cattle

This parameter is estimated by computing the ratio of current pasture land, here denoted SZ_{2005}, with respect to the current stock of cattle, i.e., $\delta = \frac{SZ_{2005}}{K_{2005}}$. To obtain SZ_{2005}, we subtract the surface area dedicated to agriculture, SA_{2005}, from the already deforested area ($F_{\max} - F_{2005}$). The area dedicated to agriculture SA_{2005} equaled 65 000 ha in 2005 (INSTAT, 2006) and $F_{\max} - F_{2005} = 170\,000\ ha$, hence $SZ_{2005} = 105\,000$ ha. Parameter $\delta = 0.362\ ha/zebu$.

$\overline{X}$: Average land productivity

Average land productivity is measured in kilograms of agricultural good per hectare. We use a single agricultural good in our model even if the Tandroy grow a variety of goods. To compensate for this shortcoming we computed $\overline{X}$ taking into account the three most important agricultural products in the region (i.e. corn, manioc and sweet potato). RM (2006a and 2006b) provide us with estimations of the average productivity per hectare for each of these goods, namely $\overline{X}_{corn} = 1000\ kg/ha$, $\overline{X}_{manioc} = 6300\ kg/ha$ and $\overline{X}_{potato} = 4500\ kg/ha$. Note that in one year one may grow corn and sweet potato on the same land since both growing calenders are compatible; however, manioc cannot be mixed with either of them. We compute the average yield in tons of crop per hectare by aggregating together corn and sweet potato on the one hand and manioc on the other: $\overline{X} = \frac{\overline{X}_{corn}+\overline{X}_{potato}+\overline{X}_{manioc}}{2} = 5900\ kg/ha$. We can do this type of aggregation as long as we are careful in computing the average price of the representative agricultural product P_A.

P_A: Price of the representative agricultural good

The price of the representative agricultural good in ariary per kilogram of crop. We aggregate the information on the three most important crops, to compute both the price and the average land productivity of our representative good. RM (2006a and 2006b) and WFP (2006) provide us with the price, in ariary, per kilogram of crop of corn, manioc, and sweet potato: $P_{corn} = 191\ ar/kg$, $P_{manioc} = 178.8\ ar/kg$ and $P_{potato} = 178.8\ ar/kg$. To determine the average price, P_A, we aggregate the economic value of these three products as follows:

$$P_A = \frac{P_{corn} \cdot \overline{X}_{corn} + P_{potato} \cdot \overline{X}_{potato} + P_{manioc} \cdot \overline{X}_{manioc}}{2\overline{X}} = 179.8 ar/kg.$$

φ: Reduced agricultural productivity due to forest depletion

Eswaran et al. (2001) estimate the loss in productivity as a consequence of land degradation, erosion and desertification. They report that the productivity loss due to such processes in Africa ranges between 2% and 40% and they provide an average estimate for the whole continent: 8.2%. We use their estimates to compute φ. Average productivity in our model is $\overline{X}$. Hence $\varphi = 0.082 \cdot \overline{X} = 483.8 kg/ha$.

$\underline{c}$: Per capita consumption per annum

WB (2007) estimates that per capita consumption in Androy in 2005 is 298 600 ariary.

ψ: Increased agricultural productivity per ha deforested and burned

This parameter is computed as a ratio with respect to average land productivity. Blanc-Pamard and Rebara (2001) estimate this increase between 50% and 100% for the first two years after the land is burnt. We have set $\psi = 0.75$.

P_w: Average price of wood

Price of wood per cubic meter. This parameter is determined by the ratio between the total economic value of wood (industrial round wood and fuelwood) in Madagascar and its total volume. FAO (2006) estimates the economic value of this volume at US$ 75.34 million and the total volume of wood in year 2005 at 7.03 million m^3. We compute the price as $P_w = \frac{75.3}{7.03} = 10.7\$$ and translate it into the local currency. This gives $P_w = 13250\ ar/m^3$.

c_w: Per capita consumption of wood

This parameter is measured as per capita consumption of wood per annum. In Madagascar, one person consumes 0.037 m^3/person of fuelwood (FAO, 1983).

n: Per-hectare timber yield

The timber yield is measured in cubic metres of wood per hectare of deforested forest. Timber yield is estimated by DEF-ME (1996) at 29 m^3/ha for Eastern and Southern Africa, particularly for the *Euphorbiaceae, Acanthaceae*, etc.

ε: Free collection of wood ratio

This is the ratio of free collection of wood per hectare with respect to the average timber yield of deforested land. The ratio has been estimated in the literature to range between $[0.01, 0.025]$. We follow Andrés-Domenech et al. (2011) and adopt $\varepsilon = 0.01$.

τ: Total area of taboo forest

Parameter τ denotes the part of the forest that is taboo for the Tandroy for various reasons, namely, religion and superstition. The Tandroy do not enter and do not collect wood in these parts of the forest. According to Ferguson (2007), the total area of taboo or sacred forest in Androy is 60 000 ha.

ω: Yearly revenue per cattle unit

This parameter measures annual economic revenues in ariary coming from cattle. Zebus give milk and provide the Tandroy with meat, two revenue sources that are far from being negligible. Parameter ω can be computed as follows:

$$\omega = p_{milk} \cdot q_{milk} \cdot \frac{K_L}{K} + \frac{p_{meat} \cdot q_{meat}}{\text{life expectancy}},$$

where p_{milk} is the price of a litre of milk; q_{milk} is the number of litres of milk produced yearly by a *cow* in lactation. The ratio $\frac{K_L}{K}$ denotes the proportion of zebus in lactation. With respect to meat revenues, p_{meat} stands for the price per kilogram of meat, q_{meat} equals the amount of meat per average zebu (its weight is 75% of that of a TCU). We divide by the life expectancy to obtain an average quantity of meat on an annual basis. The values estimated for these parameters by RM (2006c) are $p_{milk} = 400$ ar/litre, $q_{milk} = 300$ litres/year, $\frac{K_L}{K} = 0.1875$, $p_{meat} = 2900$ ar/kg, $q_{meat} = 0.75 \cdot 250$ kg, and their life expectancy is 8 years. Merging these figures together we obtain an estimation of $\omega = 90469$ ar zb^{-1} yr^{-1}.

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