The Environmental Kuznets Curve in a Multicountry Setting

(*Preliminary Version*)

Effrosyni Diamantoudi[*], Eleftherios Filippiadis[†]

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Abstract

The Environmental Kuznets Curve (EKC) hypothesis postulates an inverted U-shaped relationship between emissions and per capita income. The theoretical models developed to explain this stylized fact suggest that the EKC depends on various factors; the most prominent are high income elasticity for environmental quality, increasing returns to abatement, and pollution havens due to international trade. This study investigates the income-pollution path in a multicountry framework. We adopt a static model, wherein pollution is assumed global and social utilities are additively separable in consumption and total pollution. Individual countries compete in emissions as strategic substitutes. It is shown that an EKC can be obtained in this set up for a rich set of parameters, without any stringent conditions imposed on technology. Comparative static analysis is conducted to examine the role of such parameters.

[*]Department of Economics, Concordia University.
[†]Department of Economics, Concordia University.

1 Introduction

Ever since Malthus published his "Essay on the Principle of Population" (1798), the question of sustainable growth has been raised: Can human civilization grow without limits or are there obstacles on the way which will bring a stop to that growth? Malthus' pessimistic approach that "*...the power of population is indefinitely greater than the power in the earth to produce subsistence for man*" was refuted and disproved by the impending Industrial Revolution. Technological advancement and the use of machines in both agriculture and manufacturing led to a growth in production that exceeded population growth proving that economic growth and prosperity are strongly connected. The arguments, however, against the Malthusian theory were to be tested again in a different terrain. The Industrial Revolution soon brought up the problem of pollution and the exhaustion of natural resources. John Stuart Mill addressed this issue in his "Principles of Political Economy" (1848), recognizing the dangers of unlimited growth: destruction and exhaustion of natural resources, and depletion of the environment due to pollution.

The "Club of Rome," a group of macrotheorists in the 1970s, revived these early theories, claiming that growth is limited due to environmental constraints. Natural resources and clean environment cannot be sustained for ever, if the economies continue to grow without limit. This view is expressed by Georgescu-Roegen (1971) and Meadows *et al.* (1972). The latter provided general predictions about the exhaustion dates of resources like chromium, gold, and petroleum, under the assumption that the population and the use of natural resources increase exponentially while discovery and renewal of natural resources increase linearly. For the economists of the "Club of Rome," zero steady state growth is suggested as the only solution to the environmental problem. Responding to the "Club of Rome," other studies including Malenbaum (1978), Williams *et al.* (1987), and Tilton (1990) have shown a decrease in the intensity of use of some natural resources or, even more, an absolute decline in the use of some natural resources as economies grow. In the next decade, starting in early 1990s, the debate over the predictions of "The Limits to Growth" focused on the dynamic behavior of pollutants: a group of environmentalists claimed that growth generates pollution and that there is no adequate level of absorption and regeneration in the ecological system leading to a global natural disaster.[1] Concerns have also been raised about the globalization of economies: free trade increases output, thus stimulating growth, which leads to more pollution.

The inverse U-shaped relationship between pollution and per capita income was first pointed out in the empirical studies of Grossman and Krueger (1991), Shafik and Bandyopadhyay (1992), and Panayotou (1993). These studies use cross-country data of local air and water pollutants (such as CO, NO_x, SO_x, suspended particulate matter, municipal waste, lead) and for some of these pollutants the conclusion is that at early stages of economic development pollution rises until a turning point beyond which pollution steadily decreases as per capita income rises. Grossman and Krueger (1991) verify this relationship for SO_2 and smoke while for some of the pollutants the relationship between per capita income and pollution is monotonic (positive for municipal waste and negative for suspended particulate matter, for example). They are the first that decomposed pollution change into three effects: (1) the *scale effect*, where the higher the income the higher the production, thus the higher the pollution, (2) the *technological effect*, where the higher the income the more environmentally friendly the technology is, resulting in lower pollution, and (3) the *composition effect*, where higher income implies a change in the composition of production towards "greener" products. So, as income rises, the composition and technological effects may cancel out the scale effect resulting in an EKC. Using data from Mexico's maquiladora sector, they claim that trade liberalization can benefit the environment by enhancing the composition and the

[1] For references on this subject one can check Grosmman Krueger and (1991).

technological effect. Shafik and Bandyopadhyay (1992) for the World Bank and the "1992 World Development Report," and Panayotou (1993) for the International Labor Organization, have also reached similar conclusions about some local pollutants. The term Environmental Kuznets Curve is introduced by the latter.

Selden and Song (1994), and Grossman and Krueger (1995) use cross-country data and examine the relationship between per capita income and many different air and water pollutants. Their findings seem to support the aforementioned earlier studies. Recognizing that the process of improving environmental conditions with economic growth is not automatic but rather requires government regulation, they suggest three intuitive explanations for the downward-sloping part of the EKC: first, as income rises "*... citizens demand that more attention be paid to the noneconomic aspects of their living conditions,*" thus increasing the demand for cleaner environment; second, richer countries tend to lower the production of pollution-intensive industries and import these goods from less developed countries, thus changing the composition of production and consumption; third, as economies progress, cleaner technologies of production and more efficient abatement technologies are available. The explanation that the income elasticity of environmental quality is more than one (or just positive) seems to be the dominant one in the related literature while the environmental "dumping" - or the pollution haven hypothesis - constitutes the most persuasive counterargument to the EKC theory.

Among those who discuss the role of a positive income elasticity for environmental quality one can find Arrow *et al.* (1995) and Carson *et al.* (1997). Arrow *et al.* (1995) claim that if people spend on average more when their income increases, they will be willing to spend more for the environment as well. Carson *et al.* (1997) uses data from 50 U.S. states and finds that per capita emissions of seven major air pollutants (including among others CO_x, NO_x, and air toxics) decrease with increasing per capita income. The underlying explanation is that the income elasticity for environmental quality is greater than one.

However, the positive income elasticity of pollution is a result of different primary forces and, therefore, it cannot be used to explain the EKC. McConnell (1997), for example, shows that environmental quality being a luxury or even a necessary good is neither necessary nor sufficient condition for the existence of EKC. It is shown that preferences consistent with a positive income elasticity can coexist with lower willingness to pay for abatement, resulting in monotonically increasing pollution with income. Lopez (1994) shows that the downward-sloping part of the EKC can be explained if the production sector fully internalizes the "stock feedback effects on production." In the absence of such internalization, the inverse relationship between income and pollution can be explained by the high elasticity of substitution between pollution and conventional inputs and the high degree of relative risk aversion of the utility function. Stokey (1998) provides a theoretical model that derives an inverted V-shaped EKC. In the spirit of Lopez (1994), the primary explanation for the EKC is the high elasticity of consumption: when the marginal utility of consumption changes slowly, pollution as a production factor and conventional inputs (represented by national income) are substitutes, thus an increase in one decreases the other. An important implication of this model is that all types of pollutants must exhibit the EKC property, though the turning points might be different. Lieb (2002) extends the work of McConnell and shows that when preferences over consumption are satiated there will be a turning point in the pollution-per capita income relationship. This result holds partially even if the preferences over consumption are asymptotically satiated. It also shows that previous works like those of McConnell (1997) and Stokey (1998) are actually a special case of this model.

Others, like Roca (2003), deviate even farther from the positive income elasticity of pollution as an explanation for the EKC. Roca (2003) reviews theoretical models of EKC introduced earlier in the literature, and claims that the income elasticity alone cannot explain the observed patterns of pollution as a function of income. Since pollution causes external effects, income and power distribution can affect the curvature, the turning point, and even the shape of an EKC. Along these lines, one finds Torras and Boyce (1998) and Magnani (2000) who claim that a more equitable distribution of power and income can benefit the environment, thus explaining the downward-sloping part of an EKC. Finally, Andreoni and Levinson (2001) proposed a model where the EKC is a result of increasing returns (IRS) to abatement. The existence of IRS in the abatement process of some pollutants is verified empirically in their study.

In their seminal paper, Grossman and Krueger (1991), as well as in Grossman and Krueger (1995), provide evidence that, although there is a displacement of polluting industries from developed to developing countries, the magnitude of this shift is insignificant. Arrow *et al.* (1995) and Ekins (1997) examine the role of international trade and the EKC: "cleaner" production in the developed world does not coincide with higher demand for "greener" products. Therefore, in the presence of international trade, the demand for pollution-intensive goods on behalf of developed countries is satisfied by the production of these goods in the developing world. Despite deriving an EKC, these models are pessimistic: higher income does not imply lower pollution but rather a "transfer" of pollution from the developed to the developing world (environmental dumping). Empirical studies examining the role of international trade in the derivation of an EKC offer a blurred image. Some researchers, like Liddle (1996) tend to agree with the claim of Grossman and Krueger that the role of international trade and specialization is not significant in producing an EKC, while others, like Suri and Chapman (1997) are more sceptical and tend to disagree.

Transboundary and global pollutants are subject to analysis in the more recent EKC literature.

Local pollutants like SO_2, NO_x, and heavy metals tend to have immediate effects on the environment and, consequently, on public health. As a result, these pollutants are more likely to exhibit the inverse-U pattern in the income-pollution relationship. Transboundary pollutants, on the other hand, such as CO_2 and other greenhouse gases, do not seem to follow the path expected by the EKC theory. Instead of an inverted U-shaped curve, global pollutants often exhibit a strictly monotonic relationship between per capita income and emissions with the two being positively related. The reason is that transboundary pollutants do not seem to have an immediate and easily recognized effect on people's well-being. Moreover, these emissions are tied to energy consumption, when current technology cannot support the exponentially increasing demand for energy with solely environmentally friendly production processes. As a result, even if transboundary pollutants exhibit the EKC property, the turning point is expected at higher per capita income compared to the case of local pollutants. Due to the reasons presented above, emissions of transboundary pollutants remained unregulated for longer compared to local pollutants. Even when scientists presented their findings about the damage that global pollutants can cause (e.g. ozone layer depletion, global warming), regulating these emissions required action to be taken at an international level. International environmental agreements are slow processes, they usually fail to involve all the major polluters, and their targets are often missed by far (*e.g.* the Kyoto Protocol).

However, many of these problems have been mitigated in recent years. Scientific research on the results of global pollution has improved public awareness, international negotiations have resulted in many bilateral and multilateral environmental agreements, and innovative green production processes are becoming cost-efficient leading to more and more countries adopting them. In the recent literature about the CO_2 emissions, the existence of an EKC path cannot be confirmed or rejected with certainty. Galeotti *et al.* (2006) estimated an EKC with a reasonable turning point for the developed countries (OECD members) but not for the developing countries. The robustness of their findings was checked with the use of different data sets, and with the use of different functional forms of the regressions. Yaguchi *et al.* (2007) used data from China and Japan to investigate the dynamics of SO_2 and CO_2 with the EKC hypothesis to be clearly rejected for the case of CO_2 emissions. Wagner and Muller-Furstenberger (2007) question the results of the empirical research on CO_2 emissions claiming that the econometric techniques used are often inappropriate.

The review of the EKC's literature shows that the theoretical models proposed are inadequate to describe the cases of transboundary and/or global pollutants. It is a common characteristic of these models that are single-country models with two immediate implications: (i) countries that share the same technological and demand parameters should follow the same income-emission path independently of their initial income, and (ii) the pollution externality is restricted within the borders of a single country.

The present study introduces a mutlicountry model for the analysis of global pollutants. By doing so, we are allowing for interdependence in the countries' pollution decisions, thus changing the pollution problem from that of optimal control to a pollution game. We propose a fairly general set up where emissions can be considered a production factor and under relatively weak assumptions on preferences and technology we show that the pollution game has a unique solution. Comparative static analysis shows that an inverted U-shaped can be derived for the case of symmetric countries. However, when the symmetry is not extended to the initial income of the countries, various forms of income-pollution paths can be derived. These paths are country specific, thus being in accordance with the empirical literature on CO_2.

In what follows, the basic set up of the model is presented in Section 2. Section 3 shows that our model can be described as a Potential Game. Using the theory of Potential Games, existence and uniqueness of the model's solution is proven. Comparative statics for the symmetric case of n countries and the 2-country asymmetric case are presented in Section 4. Section 5 presents a special case with specific functional forms and Section 6 concludes.

2 The model

2.1 Basic setup

We are assuming an environment of n countries, each one producing a single good with pollution, $x_i \in X_i \subset \mathbb{R}_+$, being a by-product of the production process, where $i \in N = \{1, 2, ..., n\}$. Pollution is assumed global. For each country $i \in N$ a social utility function is given by

$$V_i = u_i(c_i) - h(\sum_{i \in N} x_i), \tag{1}$$

where u_i is a country-specific utility function, twice continuously differentiable, that is increasing and concave in consumption, c_i, and h is a damage function, also twice continuously differentiable and common for all countries, that is strictly increasing and convex in total pollution $\mathbf{x} = \sum_{i \in N} x_i$.

Individual pollution is bounded from above at every level of income. This upper bound is an increasing function of income, *i.e.*,

$$\overline{x}_i = \phi(y_i), \tag{2}$$

with $\partial\phi/\partial y_i \geq 0$, $\partial^2\phi/\partial y_i^2 \leq 0$, $\lim_{y \to \infty} \phi(\cdot) = \infty$. Under this formulation, improvements in abatement technology will reduce the maximum level of pollution at any given level of income. However, in our model there is no cost associated with improvements in abatement technology, thus abatement is considered exogenous.

Finally, assuming no trade, consumption possibilities are fully defined by the production process. The production function[2] is expressed as

$$c_i = y_i \sigma(x_i), \tag{3}$$

where $y_i \in \mathbb{R}_{++}$ is the potential income, that is the income when the dirtiest technology is used, and $\sigma(x_i)$ is a technology index that converts potential output into actual consumption with

$$\sigma(x_i) = \frac{x_i}{\overline{x}_i} = \frac{x_i}{\phi(y_i)}. \tag{4}$$

It is obvious that $\sigma(x_i) \in [0,1]$, $\sigma_x > 0$, and $\sigma_{xx} = 0$. Using this definition of technology index, the consumption possibilities are now defined as

$$c_i = y_i \frac{x_i}{\phi(y_i)}. \tag{5}$$

We can now define the following:

Definition 1 *Let* $G = \left\langle N, X, \{V_i\}_{i \in N} \right\rangle$ *the pollution game, where the set of players is* $N = \{1, 2, ..., n\}$*, the strategy space is* $X = X_1 \times X_2 \times ... \times X_n$*, and* $V_i : X \rightarrow R$ *is the payoff function of player* i*.*

[2] Note that the term production function is abused here. This function does not describe the production process, but rather it expresses the consumption possibilities relative to the intensity of pollution. A similar function is used by Stokey (1995).

In this setup, the maximization problem for a social planner in country i is given by

$$\max_{x_i \in [0,\phi(y_i)]} V_i = u_i \left(y_i \frac{x_i}{\phi(y_i)} \right) - h(x_i + \sum_{k \in N/\{i\}} x_k), \tag{6}$$

with first order and Kuhn-Tucker conditions being

$$\begin{cases} \dfrac{\partial u_i}{\partial c_i} \dfrac{y_i}{\phi(y_i)} - \dfrac{\partial h}{\partial x_i} - \lambda_i = 0 \\ \phi\left(y_i\right) - x_i \geq 0 \\ \lambda_i \geq 0 \\ \lambda_i \left(\phi\left(y_i\right) - x_i\right) = 0 \end{cases}, \tag{7}$$

where λ_i is the Langrangean multiplier for country i. Country i's reaction function is fully characterized by the above conditions: at the interior, the reaction function is given implicitly by the equation $(\partial u_i/\partial c_i)\left(y_i/\phi(y_i)\right) - (\partial h/\partial x_i) = 0$ with $\partial x_i/\partial x_j < 0$ implying strategic substitutability; at the corner, the reaction function is simply $x_i = \phi\left(y_i\right)$ suggesting that a dominant strategy may exist for country i. The reaction functions of all n countries constitute an $n \times n$ system of equations to be solved.

3 The competition in pollutants as a Potential Game

Solving an $n \times n$ system of non-linear reaction functions can be complicated. Most importantly, neither the existence nor the uniqueness of a solution can be guaranteed. As a result the pollution-income path might not be tractable. In what follows, we show that when the competition in pollutants takes the form defined in our model, it can be thought of as a Potential Game.

Definition 2 (***Monderer and Shapley, 1996***) *Let* $\Gamma = \langle N, X, \{V_i\}_{i \in N} \rangle$ *be a strategic form game with a finite number of players. The set of players is* $N = \{1, 2, ..., n\}$*, the strategy space is* $X = X_1 \times X_2 \times ... \times X_n$*, and* $V_i : X \to R$ *is the payoff function of player* i*. A function* $P : X \to R$ *is an ordinal potential for* Γ*, if for every* $i \in N$ *and for every* $x_{-i} \in X_{-i}$*,*

$$V_i(x, x_{-i}) - V_i(z, x_{-i}) > 0 \Leftrightarrow P(x, x_{-i}) - P(z, x_{-i}) > 0,$$

for every x*,* $z \in A_i$*. A game* Γ *is an ordinal potential game if it admits an ordinal potential.*

We apply the concept of Potential Game to prove existence and uniqueness of the solution to this problem. A simple inspection of the specifics of the model described in section 1.1 shows that the competition in pollutants satisfies the following conditions:

(i) individual strategy spaces are compact as intervals of real numbers, *i.e.* $X_i = [0, \phi(y_i)] \subset \mathbb{R}$,

(ii) the payoff functions are continuously differentiable, and

(iii) the cross-partial derivatives of any two payoff functions are equal, *i.e.*

$$\frac{\partial^2 V_i}{\partial x_i \partial x_j} = \frac{\partial^2 V_j}{\partial x_j \partial x_i} = -\frac{\partial^2 h(x_i + \sum_{k \neq 1}^{n} x_k)}{\partial x_i \partial x_j}.$$

Therefore, according to *Theorem 4.5* in Monderer and Shapley (1996) the Pollution Game G is a Potential Game.

Finding a Potential Function is not always immediate. In our setup, differentiability of individual social utilities is the key element for the algorithm used to find the Potential Function. The following Lemma identifies a potential function for the competition in pollutants game that is strictly concave, and thus it possesses a global maximum, in the strategy space X. The proof of this lemma is straightforward and it is presented in the appendix.

Lemma 3 *The following is a potential function of* G*:*

$$P(\mathbf{x}) = \sum_{i=1}^{n} u_i \left(y_i \frac{x_i}{\phi(y_i)} \right) - h(x_i + \textstyle\sum_{k \neq 1}^{n} x_k). \tag{8}$$

Moreover, this potential function is strictly concave.

The proof of Lemma 3 is provided in the Appendix. Note that, by definition, the maximization of the Potential Function has the same solution as the maximization problems expressed by (6) since both maximization problems yield the same first order and Kuhn-Tucker conditions. The maximization of the potential function is formally expressed as

$$\max_{\mathbf{x} \in \bigcup_{i=1}^{n} [0, \phi(y_i)]} P(\mathbf{x}) = \sum_{i=1}^{n} u_i \left(y_i \frac{x_i}{\phi(y_i)} \right) - h(x_i + \textstyle\sum_{k \neq 1}^{n} x_k), \tag{9}$$

where $\mathbf{x} \in \bigcup_{i=1}^{n} [0, \phi(y_i)]$ is the union of the individual constraints and it is a compact set. The first order and Kuhn-Tucker conditions are

$$\left\{ \begin{array}{l} \dfrac{\partial u_i}{\partial c_i} \dfrac{y_i}{\phi(y_i)} - \dfrac{\partial h}{\partial x_i} - \lambda_i = 0 \\ \phi\left(y_i\right) - x_i \geq 0 \\ \lambda_i \geq 0 \\ \lambda_i \left(\phi\left(y_i\right) - x_i \right) = 0 \end{array} \right. , \text{ for all } i \in N \tag{10}$$

Under the strict concavity of the objective function and the compactness of the constraints' set, the maximization of the Potential Function with respect to every individual country's pollution level yields a global maximum. Note that, in general, a global maximum of the Potential Function does not guarantee a unique solution for the Potential Game. However, in our case we get the following lemma:

Proposition 4 *Strict concavity of the Potential Function (8) implies a unique Nash equilibrium in the pollution game G.*

Proof. The Potential Function is continuous and it is defined over a compact set. According to Weierstrass Theorem it possesses at least one maximum. Strict concavity of the Potential function guarantees the uniqueness of a maximum. It suffices to show that the first order conditions (7) that correspond to the pollution game, and the first order conditions (10) of the Potential Function's maximization are the same. Therefore, if the maximization of the Potential Function has a unique solution, so does the system of reaction functions defined by (7). ∎

This result is of great significance since it excludes multiple equilibria and/or indeterminacy in the case of global pollutants.

4 Comparative statics

Assuming that the unique solution is interior, the system of reaction functions is given by

$$\frac{\partial u_i}{\partial c_i}\frac{y_i}{\phi(y_i)} - \frac{\partial h}{\partial x_i} = 0, \quad i = 1, 2, ..., n.$$

To simplify notation we have,

$$\frac{\partial u_i}{\partial c_i} = u'$$

and

$$\frac{y_i}{\phi(y_i)} = g(y_i),$$

where by assumption $g' \geq 0$. Moreover note that $\forall i \neq j \in N$ we have

$$\frac{\partial^2 h}{\partial x_i^2} = \frac{\partial^2 h}{\partial x_j^2} = \frac{\partial^2 h}{\partial x_i \partial x_j} = h''.$$

Therefore, the system of reaction functions that solves the pollution game can be written as

$$u_i' g(y_i) - \frac{\partial h}{\partial x_i} = 0, \quad i = 1, 2, ..., n.$$

Taking total derivatives yields

$$\left[u_i'' g(y_i)^2 - h''\right] dx_i + \left[u_i' g'(y_i) + u_i'' g(y_i) x_i\right] dy_i - \left[\sum_{j \neq i}^{n} h''\right] dx_j = 0.$$

The first term in brackets is the slope of country i's reaction function and it is negative. The second term in brackets shows how changes in own income affect the optimal choice of country i. This term depends on the degree of relative risk aversion (RRA).[3] The last term in the brackets shows the interdependency of country i's pollution decision with the choices of the rest of the world. To simplify the expression above denote

$$u_i'' g(y_i)^2 - h'' = R_i' < 0,$$
$$u_i' g'(y_i) + u_i'' g(y_i) x_i = u_i' g'(y_i)(1 - RRA_i).$$

[3] We are using the Arrow-Pratt measurement of relative risk aversion, that is $RRA = -\frac{c_i u_i''}{u_i'}$.

Therefore, we get

$$R_i' dx_i + u_i' g'(y_i)(1 - RRA_i) dy_i - \left[\sum_{j \neq i}^{n} h''\right] dx_j = 0 \Rightarrow$$

$$R_i' \frac{dx_i}{dy_i} + u_i' g'(y_i)(1 - RRA_i) - \left[\sum_{j \neq i}^{n} h''\right] \frac{dx_j}{dy_i} = 0 \Rightarrow$$

$$R_i' \frac{dx_i}{dy_i} + u_i' g'(y_i)(1 - RRA_i) - \left[\sum_{j \neq i}^{n} h''\right] \frac{dx_j}{dx_i}\frac{dx_i}{dy_i} = 0.$$

Note that the expression above contains the terms (dx_i/dy_i) and (dx_j/dx_i). The former describes the behavior of country i's pollution with income, while the latter describes the strategic effect of i's pollution on the choice of country j. In the remainder of this section we examine the sign (dx_i/dy_i) of the above expression under different setups, namely the n-country symmetric case and the two-country asymmetric case.

4.1 The n-country symmetric case

Assume that all n countries share the same utility function, they have the same initial income, and they all grow at the same rate, *i.e.* $\forall i,\ j \in N$ we have $dy_i = dy_j$. Therefore, in equilibrium all countries should choose the same pollution level, *i.e.*, $x_i^* = x_j^*$. If the solution is interior the following is true:

Lemma 5 *Assume n symmetric countries, where $n \in \mathbb{N}$, competing in pollutants. Then*

$$\frac{dx^*}{dy} \gtreqless 0 \ \textit{if}\ RRA \lesseqgtr 1$$

Moreover, x^* can be expressed as a function of y, i.e., $x^* = F(y)$ where $F : Y \to X$ is continuous.

Proof. For the proof of the above Lemma recall that

$$R_i' \frac{dx_i}{dy_i} + u_i' g'(y_i)(1 - RRA_i) - \left[\sum_{j \neq i}^{n} h''\right] \frac{dx_j}{dx_i} \frac{dx_i}{dy_i} = 0$$

Dropping the subscripts, the above equation becomes

$$R' \frac{dx}{dy} + u' g'(y)(1 - RRA) - (n-1) h'' \frac{dx}{dx} \frac{dx}{dy} = 0 \Rightarrow$$

$$\frac{dx^*}{dy} = -\frac{u' g'(y)(1 - RRA)}{R' - (n-1) h''}$$

Note that $R' - (n-1) h'' < 0$, thus this derivative is well defined over Y. Therefore, x^* can be expressed as a continuous function of y. Moreover, by assumption we have $u' \geq 0$, $g' \geq 0$. Thus, $\forall n \in \mathbb{N}$, $RRA \lesseqgtr 1 \Rightarrow \frac{dx}{dy} \gtreqless 0$. ■

This shows that, at the interior, pollution increases with income when the degree of relative risk aversion is low, while pollution decreases with income otherwise. Moreover, the optimal pollution level is a continuous function of the level of income eliminating the possibility for big changes in pollution for infinitesimal changes in income. However, the solution need not be interior. We can generalize the result of Lopez (1994) to include n symmetric countries and an upper bound in pollution at each level of income, with the following proposition:

Proposition 6 *Assume n countries, where $n \in \{1, 2, ..., N\}$, competing in pollutants. Assume that*

(i) u'' is bounded from below

(ii) either $\forall x > 0$, $\frac{\partial RRA}{\partial c} > 0$, or $\frac{\partial RRA}{\partial c} = 0$ and $RRA > 1$

Then pollution increases at first, but eventually decreases with income, thus generating an EKC.

Figure 1.1 provides a sketch of the proof for the case of increasing relative risk aversion. Curve F represents the interior solution that, according to Lemma 2, is continuous. Under increasing RRA, F has a global maximum at y_0. Pollution's upper bound, $\phi(y)$, is by assumption unbounded from above, increasing, and concave. Then the pollution-income path is the lower envelope of F and ϕ generating an EKC. A full proof of proposition 2 is presented in the appendix.

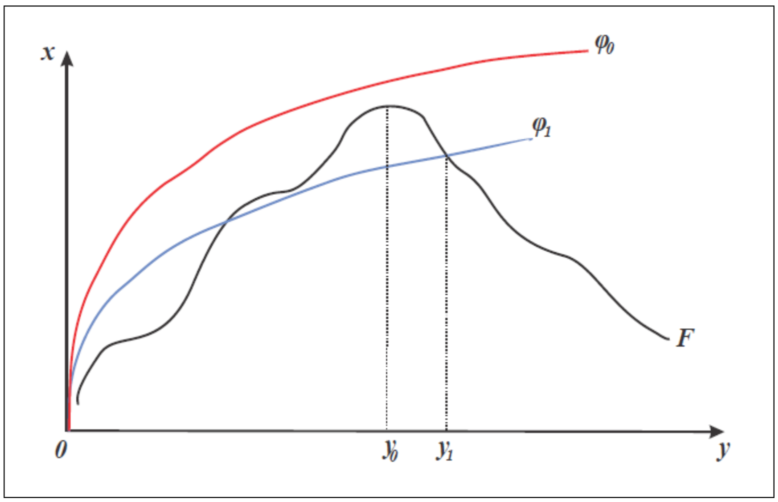

Figure 1.3: Proof of Proposition 2.

4.2 The 2-country asymmetric case

In **the** case of two asymmetric countries we get (from the perspective of country 1),

$$R_1' \frac{dx_1}{dy_1} + u_1' g'(y_1)(1 - RRA_1) - h'' \frac{dx_2}{dx_1} \frac{dy_1}{dy_2} = 0,$$

where from the first order conditions of country 2 we get,

$$\frac{dx_2}{dx_1} = \frac{h''}{R_2'} - \frac{u'g'(y)(1-RRA)}{R_2'}\frac{dy_2}{dx_1}.$$

Substituting in the above expression and solving yields

$$\frac{dx_1}{dy_1} = \left[R_1' - \frac{(h'')^2}{R_2'}\right]^{-1}\left[u_1'g'(y_1)(1-RRA_1) + \frac{h''}{R_2'}u_2'g'(y_2)(1-RRA_2)\frac{dy_2}{dy_1}\right].$$

Note that the fist term in brackets is negative, thus

$$sign\left[\frac{dx_1}{dy_1}\right] = sign\left[u_1'g'(y_1)(1-RRA_1) + \frac{h''}{R_2'}u_2'g'(y_2)(1-RRA_2)\frac{dy_2}{dy_1}\right].$$

It is worth noticing that the pollution-income relationship for country 1 depends not only on its own income, but also on country 2's income as well as on the income distribution. We can distinguish three cases, assuming that in each case country 1 is wealthier than country 2, and that both countries experience income growth and increasing relative risk aversion. In the first case, both countries have relatively low income so that $RRA_i < 1$, $\forall i = 1, 2$. The first term of the RHS is positive while the second term is negative. Depending on the size of the derivative dy_2/dy_1, the latter exceeds the former the higher this derivative is, resulting in a decreasing pollution-income path. Intuitively, when low-income countries converge in income, the wealthier countries among them have a lower turning point income compared to the symmetric case. In the second case, country 1 has relatively high income so that $RRA_1 > 1$, while country 2 has relatively low income so that $RRA_2 < 1$. Both terms of the RHS are negative and country 1 definitely decreases pollution with income. Intuitively, richer countries reduce pollution a lot faster when facing pollution-aggressive poor countries. Finally, in the third case, both countries have relatively high income so that $RRA_i > 1$, $\forall i = 1, 2$. The first term of the RHS is negative while the second term is positive. The latter exceeds the former the bigger the derivative dy_2/dy_1 is. Intuitively, when high-income countries converge in income, the wealthier countries among them reduce pollution at a slower rate compared to the symmetric case. In extreme cases, wealthier countries may even start increasing pollution with income (resulting in an N-shaped pollution-income path).

5 A special case

In order to investigate the subject further we adopt specific functional forms. The functional forms adopted satisfy all the assumptions made in Section 1.1. More specifically, the utility received from consumption is represented by a truncated quadratic function, *i.e.*

$$u_i(c_i) = \begin{cases} c_i - \frac{1}{2}\beta c_i^2, & \text{if } c_i \leq 1/\beta \\ 1/2\beta, & \text{otherwise} \end{cases},$$

with $\beta \in (0, 1)$. One can interpret β as a risk aversion indicator, with higher values of β corresponding to a higher degree of relative risk aversion[4]. The production function is assumed to be

$$c_i = y_i \sigma(x_i) = y_i \left(\frac{x_i}{\overline{x}_i} \right),$$

[4] Using the Arrow-Pratt measurement of relative risk aversion for this utility function we get $RRA = -\frac{c_i u_i''}{u_i'} \Rightarrow RRA = \frac{c_i \beta}{1 - c_i \beta}$ and $\partial RRA / \partial \beta = c_i / (1 - c_i \beta)^2 > 0$.

where $\sigma(x_i) = (x_i/\overline{x}_i)$. Substituting the production function in the utility function yields

$$u_i(c_i) = y_i \left(\frac{x_i}{\overline{x}_i}\right) - \frac{1}{2}\beta y_i^2 \left(\frac{x_i}{\overline{x}_i}\right)^2.$$

The social damage function, h, is quadratic in total pollution, *i.e.*

$$h_i = \frac{\rho}{2}\left(x_i + \sum_{j\neq i}^{n} x_j\right)^2,$$

where $\rho \in R_+$ is a scale parameter that shows how pollution is perceived by country i. Finally, we assume that pollution is bounded from above at every level of income. This upper bound is an increasing function of income, *i.e.*

$$\overline{x}_i = \phi(y_i) = y_i^{\alpha},$$

where $\alpha \in (0, 1)$. This parameter, α, incorporates technological improvement in the abatement process implicitly into our model. Note that, although $\alpha < 1$, it is still positive, meaning that any given degree of technological advancement in the abatement process is not enough to generate an EKC.

Under this setup, the maximization problem for a social planner in country i is given by

$$\max_{x_i\in[0,\overline{x}_i]} V_i = y_i \left(\frac{x_i}{\overline{x}_i}\right) - \frac{1}{2}\beta y_i^2 \left(\frac{x_i}{\overline{x}_i}\right)^2 - \frac{\rho}{2}\left(x_i + \sum_{j\neq i}^{n} x_j\right)^2 \Rightarrow$$

$$\max_{x_i\in[0,y_i^{\alpha}]} V_i = y_i^{1-\alpha} x_i - \frac{1}{2}\beta y_i^{2-2\alpha} x_i^2 - \frac{\rho}{2}\left(x_i + \sum_{j\neq i}^{n} x_j\right)^2. \tag{11}$$

Solving this model requires, first, finding the best responses for each country, and second, identifying the possible Nash Equilibria.[5] Given the definitions of the objective functions, country i's best response is a function of all other countries' choices of pollutants. Therefore, the solution of this model can be found by solving the system of the best response functions (*i.e.* reaction functions).

Taking the first order condition of the maximization problem defined by (11), yields

$$y_i^{1-\alpha} - \beta y_i^{2-2\alpha} x_i - \rho \left(x_i + \sum_{j \neq i}^{n} x_j \right) \geq 0,$$

with the equality to hold $\forall\ x_i \in [0, y_i^{\alpha}]$. The reaction function of a country is then given by

$$x_i = \begin{cases} \dfrac{y_i^{1-\alpha}}{\rho + \beta y_i^{2-2\alpha}} - \dfrac{\rho \sum_{j \neq i}^{n} x_j}{\rho + \beta y_i^{2-2\alpha}}, & \text{if } \sum_{j \neq i}^{n} x_j \geq \dfrac{(\rho + \beta y_i^{2-2\alpha} - y_i^{1-2\alpha}) y_i^{\alpha}}{\rho} \\ y_i^{\alpha}, & \text{otherwise} \end{cases}.$$

Note that, at the interior, the competition in pollutants is competition in strategic substitutes yielding downward-sloping reaction functions, *i.e.*

$$\frac{\partial x_i}{\partial x_j} = -\frac{\rho}{\rho + \beta y_i^{2-2\alpha}} < 0, \forall j \neq i.$$

In what follows, we solve this model for the case of (i) n symmetric countries, and (ii) two asymmetric countries. Comparative static analysis is conducted for the parameters of the model regarding the pollution behavior and the turning point income.

[5] Starting from a completely symmetric case with n countries, it is natural to assume that their decisions on pollution are taken simultaneously.

5.1 Solution and comparative statics

5.1.1 The symmetric case

We first examine the totally symmetric case where all countries share the same income**, i.e.,** $y_i = y$**,** and the same parameter values α, β, and ρ. Given these assumptions and for $n \in \{1, 2, 3, ...\}$ the optimal pollution is

$$x^* = \frac{y^{1-\alpha}}{\rho n + \beta y^{2-2\alpha}},$$

where $x^* = x_i^*$. Note that, given $x^* \leq \overline{x}$, the pollution-income path follows the EKC pattern, with

$$\begin{cases} \dfrac{\partial x^*}{\partial y} \geq 0, & \text{if } y \leq \left(\dfrac{\beta}{\rho n}\right)^{-\left(\frac{1}{2-2\alpha}\right)} \\ \dfrac{\partial x^*}{\partial y} < 0, & \text{otherwise.} \end{cases}$$

It is also interesting to see that $\partial x^*/\partial\beta < 0$, $\partial x^*/\partial\rho < 0$, and $\partial x^*/\partial n < 0$. That is, any increase in the degree of RRA, the degree of pollution dispersion and perception, or the number of countries, is associated *ceteris paribus* with a lower level of individual pollution. The explanation lies in the nature of the game: pollutants are strategic substitutes. For any given level of income, factors that make a country less aggressive in the pollution game will result in lower emissions. No clear conclusion can be drawn for the relationship between pollution and the abatement technology.

The optimal pollution level has a single peak (turning point) since

$$\frac{\partial x}{\partial y} = \frac{(1-\alpha)\left(\rho n - \beta y^{2-2\alpha}\right)}{y^{\alpha}\left(\rho n + \beta y^{2-2\alpha}\right)^2} = 0 \Rightarrow \rho n - \beta y^{2-2\alpha} = 0,$$

with

$$y_{TP} = \left(\frac{\beta}{\rho n}\right)^{-\left(\frac{1}{2-2\alpha}\right)},$$

where y_{TP} is the turning point income. Furthermore, $\partial y_{TP}/\partial \beta < 0$, $\partial y_{TP}/\partial \rho > 0$, and $\partial y_{TP}/\partial n > 0$.

Depending on specific values of α, β, ρ, and n, we get $\left[\rho n + \beta y^{2-2\alpha} - y^{1-2\alpha} \leq 0\right]$, and pollution reaches its upper bound, i.e. $x_i = \overline{x}$. At the corner, the pollution-income relationship follows an increasing and concave path since,

$$\begin{cases} \frac{\partial x^*}{\partial y} > 0 \Leftrightarrow \dfrac{\partial \left[y^{\alpha}\right]}{\partial y} = \alpha y^{\alpha-1} > 0, \text{ and} \\ \\ \frac{\partial^2 x^*}{\partial y^2} > 0 \Leftrightarrow \dfrac{\partial \left[\alpha y^{\alpha-1}\right]}{\partial y} = \alpha\left(\alpha-1\right) y^{\alpha-2} < 0. \end{cases}$$

At the corner, pollution does not depend on β, ρ, or n, while it increases monotonically with α.

Graphically, the comparative static analysis at the interior is represented by the following graphs. Figure 1.2 shows the relationship between EKC and the number of competing countries, while Figure 1.3 shows the relationship between EKC and environmental awareness. Figure 1.4 describes the relationship between RRA and the EKC. We observe that the EKC becomes smoother when the number of countries increases, when there is a hike in environmental awareness, and/or when the RRA increases.

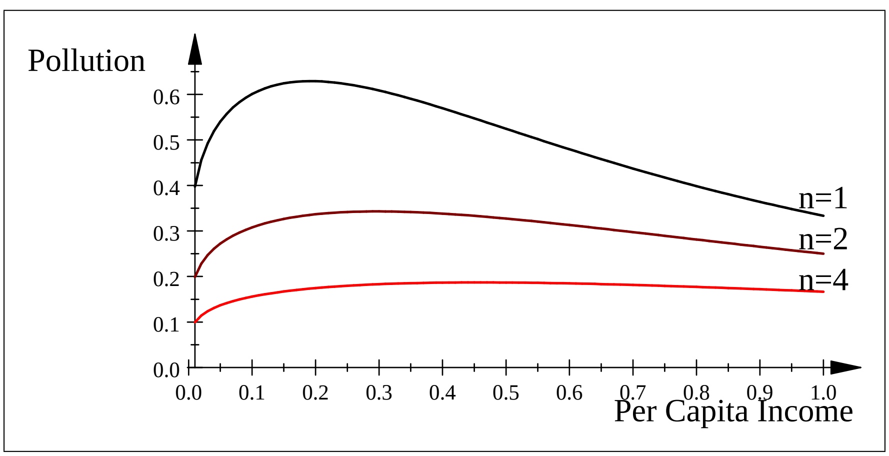

Figure 1.2: Number of countries and EKC.

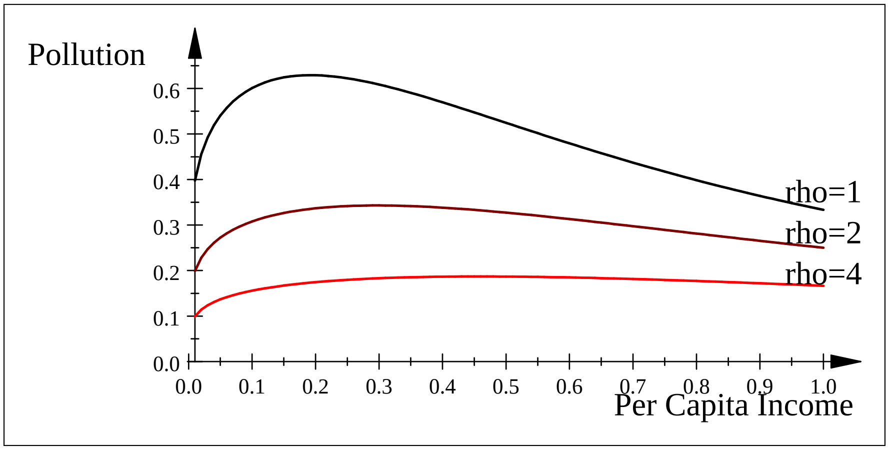

Figure 1.3: Environmental awareness and EKC.

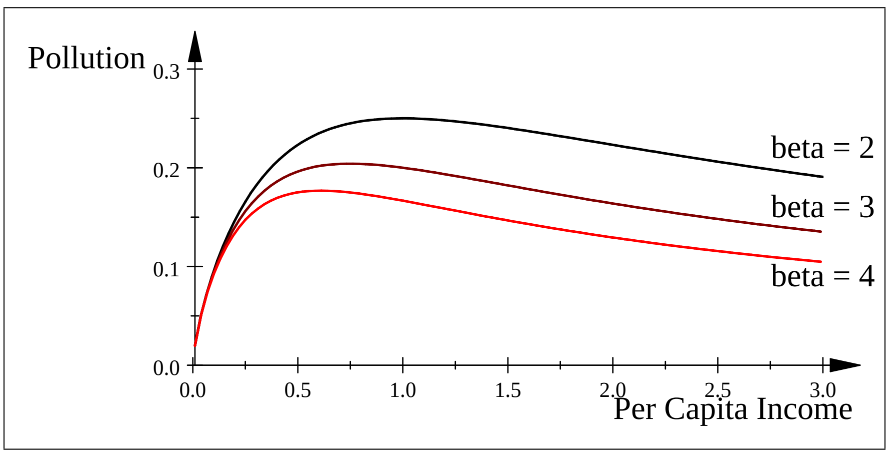

Figure 1.4: RRA and EKC.

5.1.2 The asymmetric case

For $n = 2$ we get

$$x_i^* = \begin{cases} 0, & \text{if } y_i < l\left(y_i\right) \\ \dfrac{\rho y_i^{1-\alpha} - \rho y_j^{1-\alpha} + \beta y_i^{1-\alpha} y_j^{2(1-\alpha)}}{\rho\beta y_i^{2(1-\alpha)} + \rho\beta y_j^{2(1-\alpha)} + \beta^2 y_i^{2(1-\alpha)} y_j^{2(1-\alpha)}}, & \text{if } y_i \geq l\left(y_j\right), \text{ and } y_j \geq l\left(y_i\right) \\ \dfrac{y_i^{1-\alpha}}{\rho + \beta y_i^{2-2\alpha}}, & \text{if } y_j < l\left(y_i\right) \end{cases}$$

where

$$l\left(y_k\right) = \left(\frac{\rho}{\rho + \beta y_k^{2(1-\alpha)}}\right)^{\frac{1}{1-\alpha}} y_k.$$

Under extreme income inequalities the poor country does not pollute, *i.e.,* $y_i << y_j \Rightarrow x_i^* = 0$. This situation is referred to as environmental poverty trap. At the interior, i.e. $x_i^* < \overline{x}$, the pollution-income path follows the EKC pattern for various values of α, β, and ρ. But most importantly, (i) the closer the incomes, y_i and y_j, are, and/or (ii) the greater the values of α and ρ are, and/or (iii) the lower the value of β is, the greater the value interval that yields EKC patterns:

$$\begin{cases} \dfrac{\partial x_i^*}{\partial y_i} \geq 0, & \text{if } 2\rho y_i^{1+\alpha} y_j^{1+\alpha} - \rho y_i^2 y_j^{2\alpha} + y_i^{2\alpha} y_j^2 - \beta y_i^2 y_j^2 \geq 0 \\ \dfrac{\partial x_i^*}{\partial y_i} < 0, & \text{otherwise.} \end{cases}$$

In Figure 1.7 one can see the EKC income-pollution path for country i (domestic country), for different values of country j's income (foreign country).

For specific values of α and β, *i.e.,* $\left(\beta y_j^{2(1-\alpha)} + 1\right) y_i^{1-\alpha} - \beta \left(\beta y_j^{2(1-\alpha)} + 1\right) y_i^{2-\alpha} + \left(\beta y_j^{2(1-\alpha)}\right) y_i^{\alpha} -$

$y_j^{1-\alpha} \leq 0$, x_i^* reaches the upper bound $\overline{x}_i$. If $x_i^* = \overline{x}_i$ then the pollution-income path follows an increasing and concave path:

$$\begin{cases} \frac{\partial x_i^*}{\partial y_i} > 0 \Leftrightarrow \frac{\partial [y_i^\alpha]}{\partial y_i} = \alpha y_i^{\alpha-1} > 0 \\ \\ \frac{\partial^2 x_i^*}{\partial y_i^2} < 0 \Leftrightarrow \frac{\partial \left[\alpha y_i^{\alpha-1}\right]}{\partial y_i} = \alpha \left(a-1\right) y_i^{\alpha-2} < 0. \end{cases}$$

If x_i^* is interior then the poorer country pollutes more if both countries' incomes are already high. Whereas if both countries are poor, then the richer one pollutes more. This observation is consistent with (cross-sectional) EKC patterns:

$$x_i > x_j \Leftrightarrow y_i < y_j \text{ and } y_i y_j > \left(\frac{2}{\beta}\right)^{\frac{1}{1-a}}$$

Income distribution and pollution Denote total income $y = y_1 + y_2$. Then, for any given share $t \in [0,1]$[6] of world income for country 1 we get $y_1 = ty$ and $y_2 = (1-t)\,y$. Country 1's pollution can be written as

$$x_1^* = \frac{(ty)^{1-\alpha} - (1-t)^{1-\alpha} y^{1-\alpha} + \beta\,(ty)^{1-\alpha}\,[(1-t)\,y]^{2(1-\alpha)}}{\beta\,(ty)^{2(1-\alpha)} + \beta\,[(1-t)\,y]^{2(1-\alpha)} + \beta^2\,(ty)^{2(1-\alpha)}\,[(1-t)\,y]^{2(1-\alpha)}}$$

and total pollution becomes

$$x_{TOTAL}^* = x_1^* + x_2^* = \frac{t\,(1-t)\left[t\,(1-t)^\alpha + t^\alpha\,(1-t)\right] y^{1-\alpha}}{t^2\,(1-t)^{2\alpha} + t^{2\alpha}\,(1-t)^2 + \beta t^2\,(1-t)^2\,y^{2(1-\alpha)}}.$$

[6] Note that we actually require $t \in \left[\underline{t}, \overline{t}\right]$ where $\underline{t} > 0$, $\overline{t} < 1$ such that interior solution is guaranteed.

In Figure 1.8 we can see the behavior of total pollution as world income increases for different shares t of country 1. Note that if the share t of a country remains relatively constant, an EKC is more likely to be obtained. However, this observation cannot be extended to n countries. When $n = 2$, fixing one country's income share also sets the share of its rival. For $n > 2$, this is no longer true: even if a country grows steadily over time, variations on the relative shares of the rest of the world might have a great impact on that country's income-pollution relationship.

It is worth noticing that for low levels of global income, equal income distribution seems to hurt the environment, while as global income rises, income equality is optimal from a global perspective. Intuitively, when the world is very poor, the marginal benefit from pollution is relatively high. An unequal income distribution decreases the marginal benefit from pollution of the relatively wealthier countries, leading them to decrease the rate at which they increase pollution. At the same time, the relatively poorer countries are experiencing a lower marginal benefit from pollution compared to the symmetric case. Their reaction is to decrease pollution at an increasing rate. The latter exceeds the former, thus unequal distribution is beneficial for the environment. For relatively higher global income the opposite is true. In Figure 1.9, we see the total pollution for different shares of global income for country 1. The red line represents high global income, while the black line represents low global income.

6 Conclusions

In this paper, we show that the competition in pollutants has a unique solution, thus giving rise to a tractable income-pollution path. Moreover, a hump shaped pollution-income path is generated for multiple symmetric countries under relatively weak assumptions and without any stringent conditions on technology. The setup of the model is fairly general and it can be modified to include production where pollution can be thought of as a factor of production. It is a generalization of Lopez (1994) in that it considers multiple countries and it assumes an upper bound on maximum pollution. It also generalizes Stokey (1998) in that it applies to a larger family of utility functions. Our model, in the presence of income asymmetries alone, can generate an N-shaped pollution income path, consistent with recent empirical studies. Finally, always for the asymmetric cases, it can give rise to different EKCs for different countries. This is solely a result of the externality caused from pollution and it is not related to environmental dumping.

7 Appendix

Proof of Lemma 3. First, note that $\forall i \in N$, the welfare functions are defined as $V_i : X_i \to \mathbb{R}$, where $X_i \subset \mathbb{R}$ and they are continuously differentiable. Function $P : X_i \to \mathbb{R}$ is also continuously differentiable and $\forall i \in N$ we have

$$\frac{\partial P}{\partial x_i} = \frac{\partial V_i}{\partial x_i}.$$

Therefore, according to *Lemma 4.4* in Monderer and Shapley (1996), (8) is a potential function of G. Furthermore, note that $\forall i \in N$

$$\frac{\partial^2 P}{\partial x_i^2} = \frac{\partial^2 V_i}{\partial x_i^2} = \left[\frac{\partial^2 u_i}{\partial c_i^2}\left(\frac{y_i}{\phi\left(y_i\right)}\right)^2\right] - \frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_i^2} = A_i - \frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_i^2},$$

where $A_i = \left[\frac{\partial^2 u_i}{\partial c_i^2}\left(\frac{y_i}{\phi(y_i)}\right)^2\right] < 0$. *Therefore, by definition* $P(x)$ *is a potential function. Moreover, the Hessian of* $P(x)$ *is given by*

$$H = \begin{bmatrix} \frac{\partial^2 P}{\partial x_1^2} & \frac{\partial^2 P}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 P}{\partial x_1 \partial x_n} \\ \frac{\partial^2 P}{\partial x_1 \partial x_2} & \frac{\partial^2 P}{\partial x_2^2} & \cdots & \frac{\partial^2 P}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 P}{\partial x_1 \partial x_n} & \frac{\partial^2 P}{\partial x_2 \partial x_n} & \cdots & \frac{\partial^2 P}{\partial x_n^2} \end{bmatrix} =$$

$$= \begin{bmatrix} A_1 & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_1 \partial x_2} & \cdots & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_1 \partial x_n} \\ -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_2 \partial x_1} & A_2 & \cdots & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_n \partial x_1} & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^{n} x_k\right)}{\partial x_n \partial x_2} & \cdots & A_n \end{bmatrix}$$

It worth noticing that the second derivatives of the social damage function - own and cross partial derivatives - are the same, *i.e.* for any k, l, m, $n \in N$ we have

$$\frac{\partial^2 h\left(\sum_{i=1}^n x_i\right)}{\partial x_k \partial x_l} = \frac{\partial^2 h\left(\sum_{i=1}^n x_i\right)}{\partial x_m \partial x_n} = B\left(\sum_{i=1}^n x_i\right) \geq 0$$

Consider $\mathbf{z} \in \mathbb{R}^n \diagup \{\mathbf{0}_{n\times 1}\}$ *and construct the quadratic form*

$$\mathbf{z}^T H \mathbf{z} =$$

$$= \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}^T \begin{bmatrix} A_1 & -B\left(\sum_{i=1}^n x_i\right) & \cdots & -B\left(\sum_{i=1}^n x_i\right) \\ -B\left(\sum_{i=1}^n x_i\right) & A_2 & \cdots & -B\left(\sum_{i=1}^n x_i\right) \\ \vdots & \vdots & \ddots & \vdots \\ -B\left(\sum_{i=1}^n x_i\right) & -B\left(\sum_{i=1}^n x_i\right) & \cdots & A_n \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} =$$

$$= \sum_{i=1}^n \left(z_i^2 A_i\right) - B\left(\sum_{i=1}^n x_i\right) \left(\sum_{i=1}^n z_i\right)^2 < 0$$

Thus the Hessian matrix of the potential function is negative definite and, therefore, the potential function $P(\mathbf{x})$ *is strictly concave.* ■

Proof of Proposition 6. Denote the interior solution as $x^* = F(y)$, while the corner solution is expressed by $x^* = \phi(y)$. Given the uniqueness of solution the relationship between income and pollution can be expressed as follows:

$$x^* = \min\{F(y), \phi(y)\}$$

This equation is by construction continuous since its arguments are continuous functions, function $\phi(y)$ by assumption and function $F(y)$ due to Lemma 5.

We first check the case of increasing RRA. Note that under the first part of assumption (ii) we get $\dfrac{\partial RRA}{\partial c} > 0 \Rightarrow \dfrac{\partial RRA}{\partial y} > 0$, while due to assumption (i) we get

$$\lim_{y\to 0} RRA = 0, \text{ and}$$

$$\lim_{y\to\infty} RRA = \infty$$

Therefore, and since RRA is obviously continuous in y, $\exists$ some $y_0 \in R_{++}$ such that $RRA = 1$, $\forall x \in R_{++}$. Due to Proposition 2 we know that $y_0 = \arg\max F(y)$. Moreover, $F(y_0) < \infty$, since $\forall y < \infty$, $\lim_{x\to\infty}[u'(y/\phi(y))] = 0 < \lim_{x\to\infty} h' = \infty$. At $y = y_0$ we distinguish two cases:

(a) $F(y_0) < \phi(y_0)$. Therefore, $\forall y > y_0$, $x^* = \min\{F(y), \phi(y)\} = F(y)$ and due to Proposition 2, $dx^*/dy < 0$.

(b) $F(y_0) \geq \phi(y_0)$. Recall that $\phi(y)$ is continuous and note that $\lim_{y\to\infty}\phi(y) = \infty$. Therefore, according to the intermediate value theorem, $\exists$ some $y_1 \in [y_1, \infty)$ such that $\phi(y_1) = F(y_0)$. Since $\phi(\cdot)$ is strictly increasing and unbounded form above, $\forall y > y_1 \Rightarrow \phi(y) > \phi(y_1) = F(y_0) \Rightarrow \phi(y) > F(y) \Rightarrow x^* = \min\{F(y), \phi(y)\} = F(y)$ and due to Proposition 2, $dx^*/dy < 0$.

We now turn to the case where RRA is constant and greater than unit. As a result $F(y)$ is strictly decreasing and since $\phi(y)$ is strictly increasing and unbounded from above, $\exists$ some $y_1 \in [y_1, \infty)$ such that $\phi(y_1) = F(y_0)$. Therefore, $\forall y > y_1 \Rightarrow \phi(y) > \phi(y_1) = F(y_1) \Rightarrow x^* = \min\{F(y), \phi(y)\} = F(y) \Rightarrow dx^*/dy < 0$. ■

References

[1] Andreoni, J., Levinson, A., 2001. The simple analytics of the Environmental Kuznets Curve. *Journal of Public Economics,* 80 (2), 269– 286.

[2] Antweiler, W., Copeland, B.R., Taylor, M.S., 2001. Is free trade good for the environment? *American Economic Review,* 91 (4), 877–908.

[3] Arrow, K., Bolin, B., Costanza, R., Folke, C., Holling, C.S., Janson, B., Levin, S., Maler, K., Perrings, C., Pimental, D., 1995. Economic growth, carrying capacity, and the environment. *Science, 15*, 91– 95.

[4] Carson, R.T., Jeon, Y., McCubbin, D.R., 1997. The relationship between air pollution emissions and income : US data. *Environment and Development Economics,* 2, 433– 450.

[5] Copeland, B.R., Taylor, M.S., 1995. Trade and Transboundary Pollution. *American Economic Review*, 85, 716-737.

[6] Ekins, P., 1997. The Kuznets Curve for the environment and economic growth: examining the evidence. *Environment and Planning*, A 29, 805– 830.

[7] Galeoti, M.,Lanza, A., Pauli, F., 2006. Reassessing the environmental Kuznets curve for CO2 emissions: A robustness exercise. *Ecological Economics*, 57, 152– 163.

[8] Georgescu-Roegen, N., 1971. The Entropy Law and the Economic Process. Harvard University Press, Cambridge, Massachusetts.

[9] Grossman, G.M., Krueger, A.B., 1991. Environmental impacts of the North American Free Trade Agreement. NBER Working paper 3914.

[10] Grossman, G.M., Krueger, A.B., 1993. Environmental impacts of the North American Free Trade Agreement. In: Garber, P. (Ed.), The U.S. – Mexico Free Trade Agreement. MIT Press, Cambridge, pp. 13–56.

[11] Grossman, G.M., Krueger, A.B., 1995. Economic growth and the environment. *Quarterly Journal of Economics*, 110 (2), 353– 377.

[12] Hutchinson, E., Kennedy P. W., 2010. Income Growth and Equilibrium Dynamics for a Global Pollutant. Manuscript.

[13] John, A., Pecchenino, R., 1994. An overlapping generations Model of Growth and the Environment. *Economic Journal*, 104, 1393-1410.

[14] Kuznets, S., 1955. Economic growth and income inequality. *American Economic Review*, 45, 1– 28.

[15] Liddle, B., 2001. Free trade and the environment – development system. *Ecological Economics*, 39 (1), 21– 36.

[16] Lieb, C.M., 2002. The Environmental Kuznets Curve and satiation: A simple static model. *Environment and Development Economics*, 7, 429– 448.

[17] Lopez, R., 1994. The environment as a factor of production: the effects of economic growth and trade liberalization. *Journal of Environmental Economics and management*, 27, 163–184.

[18] Malenbaum, W., 1978. World demand for raw materials in 1985 and 2000. McGraw-Hill, New York.

[19] McConnell, K.E., 1997. Income and the demand for environmental quality. *Environment and Development Economics*, 2, 383–399.

[20] Meadows, D. H., Meadows, D. L., Randers, J., Behrens, W. W., 1971. The Limits to Growth. Universe Books, New York.

[21] Monderer, D., Shapley, L., 1996. Potential games, *Games and Economic Behaviour*, 14, 124-143.

[22] Panayotou, T., 1993. Empirical tests and policy analysis of environmental degradation at different stages of economic development, ILO, Technology and Employment Programme, Geneva.

[23] Roca, J., 2003. Do individual preferences explain Environmental Kuznets Curve? *Ecological Economics*, 45 (1), 3– 10.

[24] Selden, T., Song, D., 1994. Environmental quality and development: is there a Kuznets Curve for air pollution emissions? *Journal of Environmental Economics and Management*, 27, 147–162.

[25] Stokey, N.L., 1998. Are there limits to growth? *International Economic Review*, 39 (1), 1– 31.

[26] Suri, V., Chapman, D., 1998. Economic growth, trade and the environment: implications for the environmental Kuznets curve. *Ecological Economics*, 25, 195– 208.

[27] Tilton, J.E., 1990. The OECD countries: Demand trend setters. In: Tilton, J.E. (ed.), World Metal Demand: Trend and Prospects. Resource for the Future, Washington, DC, 35-76.

[28] Torras, M., Boyce, J.K., 1998, "Income, inequality, and pollution: a reassessment of the Environmental Kuznets Curve," Ecological Economics 25, 147– 160.

[29] Wagner, M., Müller-Fürstenberger, G., 2008. The Carbon Kuznets Curve: A cloudy picture emitted by bad econometrics? *Resource and Energy Economics,* 30 (3), 388-408.

[30] WIlliams, R.H., Larson, E.D., Ross, M.H., 1987. Materials affluence and industrial energy use. *Annual Review of Energy*, 12, 99-144.

[31] World Bank, 1992. World Development Report. Oxford Univ. Press, New York.

[32] Yaguchi, Y., Sonobe, T., Otsuka, K., 2007. Beyond the Environmental Kuznets Curve: a comparative study of SO2 and CO2 emissions between Japan and China. *Environment and Development Economics*, 12, 445–470.