(Anti-)Coordination Problems with Sparse Water Resource*

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A VERY PRELIMINARY VERSION

Abstract

This paper extends the previous literature on differential games of groundwater pumping in two directions. First, we take up the often claimed necessity to model the physical interactions between groundwater and rainwater, instead of analyzing these water sources as non-connected systems, and confirm the crucial importance of this hydrological aspect. Next, considering strategic interaction between resource users, we show that there may exist multiple equilibria, which may occur simultaneously. Moreover, we prove that farmers may opt for opposite irrigation strategies, one group choosing groundwater extraction, the other rainwater storage. This is a typical feature of anti-coordination games. Open-loop and feedback equilibrium have been computed to characterize these equilibria.

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1 Introduction

Many papers in water resource economics have focused on one single aquifer and have dealt with the question whether policy intervention was necessary. While Gisser and Sanchez showed that the difference between the laisser-faire case and the case with policy intervention was possibly small (Gisser and Sanchez [6], see also Koundouri [8] for a literature overview), more recent papers pointed out the necessity of policy intervention in the presence of additional externalities, such as drought risk (see Provencher and Burt [14]) or pollution (see Roseta-Palma [15]) or when resource users adopt strategic behavior (see Negri [12] or Rubio and Casino [17],[18]). In this paper, we analyze another case of policy intervention which may be necessary when several long-term equilibria exist.

The need to deal with water as a system of different water bodies has been recognized early in the literature. Three strands of literature can be distinguished in water resource economics. The first deals with the problem of saltwater intrusion into coastal aquifers that are connected to the sea (see for example Cummings [3], Moreaux and Reynaud [10],[11]), the second deals with the optimal management of multiple groundwater resources (see Roumasset and Wada [16] or Zeitouni and Dinar [23]), the third studies the conjunctive use of ground and surface water (see Burt [1], Chakravorty and Umetsu [2], Gemma and Tsur [5], Knapp and Olson [7], Krulce, Roumasset and Wilon [9], Pongkijvorasin and Roumasset [13], Stahn and Tomini [19], [20] or Tsur and Graham-Tomasi [21]).

The way the physical link between different water bodies is modeled is crucial to understand the optimal management strategy. Consider first the case where resources are not linked. Roumasset and Wada [16] showed in the case of several non-linked groundwater resources that the optimal management depends on their marginal opportunity cost: only the resource with the lowest marginal opportunity cost is used in the first place, but all resources are used in the stationary Nash equilibrium. When resources are linked the optimal management has to consider the interactions among costs. Zeitouni and Dinar [23] studied the case of two interrelated aquifers: water flows from one aquifer to the other, depending on the relative height of the water tables. This may potentially contaminate the aquifer with the better water quality. Optimal water management is then defined by the joint management of these interrelated resources, the threat of contamination representing an additional externality that has to be taken into account. Stahn and Tomini [19] considered the joint use of groundwater and rainwater and showed that the introduction of rainwater harvesting may lead to a greater depletion of the groundwater aquifer in the long-run. This results from two different externalities: first the fact that the groundwater recharge rate is negatively affected by rainwater harvesting and second the fact that the efficiency of water use depends on the relative size of evapotranspiration in the storage reservoir and infiltration to the groundwater aquifer.

In this paper, we analyze a model in which rain (thereafter RW) and groundwater (thereafter GW) are physically linked: rainwater may either be harvested before it infiltrates into the soil or it may replenish the aquifer. All resource users, farmers in our case, have access to the same aquifer but can opt to use either rainwater or groundwater, or both. Whereas rainwater collection has a marginal constant cost, groundwater pumping depends on the height of the water table: the deeper the water height, the higher is marginal pumping cost. We can therefore expect that there exists a trade-off based on the difference in costs of groundwater extraction and rainwater storage. Furthermore, beyond this difference in costs, we introduce a difference in terms of productivity. For example, evaporation may be greater for rainwater than for groundwater and hence the consumptive use of groundwater may be greater. On the other hand, groundwater may be salty or charged with other toxic substances (e.g. chloride) and hence the consumptive use of rainwater may be greater. We will consider both possible cases.

We are interested in qualifying the management of this interrelated water resource system, given strategic interactions between N homogeneous resource users (Dockner et al. [4]). We suppose in the first place that resource users make consistent commitments. We focus on the role of the cost and productivity differences between rain and groundwater use (we do not consider other externalities such as the buffer value of groundwater use). Because resources are physically interlinked, we cannot merely compare marginal (extraction and user) costs of separate use, as proposed by Pongkijvorasin and Roumasset [13] but we compare all possible equilibria of joint and separate uses. The optimal choice of one or the other resource depends on the ratio of their cost and productivity parameters.

We show that there are multiple equilibria. In particular, there is an asymmetric equilibrium, similar to those in anti-coordination games, although our players are homogeneous in all their characteristics (see Vives [22] for conditions on the non-existence of asymmetric equilibria). This is linked to the pumping cost externality, i.e. the fact that pumping becomes more costly when the water-table has been reduced by another resource-user. Some farmers try to avoid this pumping cost externality by adopting rainwater harvesting (where this externality does not exist). As we show in a numerical example, they thereby generate higher gains than those players, who continue to use the groundwater resource. For all players, the overall gain of such a strategy can be greater or smaller than the gain from another equilibrium (for example the groundwater equilibrium), depending on the initial resource stock and the approach path chosen.

Furthermore, we show how the number of equilibria depends on the magnitude of the recharge rate of the resource. For large recharge rates, it is optimal to use both resources when the productivity of rainwater is high or the cost is low (relative to groundwater). On the other hand, when the productivity of rainwater is low or the cost is high (relative to groundwater), it is optimal to use only the groundwater resource. For small recharge rates, many equilibria coexist. For example, for intermediate values of groundwater productivity and costs, some users opt for rainwater-use and others for groundwater use, which is the asymmetric equilibrium. For low and high groundwater productivity (compared to rainwater), resource users may either opt for groundwater use or for conjunctive use of rain- and groundwater. Overall, the type of equilibrium depends on the ratio of marginal productivity and marginal costs of both resources and the strategic decisions of the other players.

In terms of policy implications, our results suggest that policy intervention may be necessary to make users switch from those equilibria that are socially sub-optimal (because joint welfare is lower or because the resource stock is smaller) to those equilibria that are better for all users. Our results also suggest that it may be difficult to define a policy for a resource with low recharge rates, because many different equilibria can occur. In addition, it might be impossible to infer the outcome of a situation with low recharge rates from the outcome of a situation with high recharge rates, as resulting equilibria are very different. In the context of climate change, where temperature may raise and precipitation may become more variable (that is mean recharge rate may decrease in many areas), the above analysis highlights all the difficulties of predicting future equilibria and designing sound policy interventions.

The rest of the paper is organized as follows. In section 2, we introduce the underlying model, in section 3, we provide necessary conditions for both, symmetric and asymmetric open-loop Nash equilibria, emphasizing the interactions between the two water sources. In section 4 we discuss the coexistence of these equilibria and the problem of anti-coordination. In section 5, we discuss the case of feedback Nash equilibria. Finally, in section 6, we conclude and discuss our results.

2 The Model

We consider a continuous time strategic interaction problem where a fixed number $N \geq 2$ of farmers needs water as an input and can use rainwater and groundwater.

2.1 Groundwater Dynamics

We consider a single-cell, unconfined and "bathtub" type aquifer with flat bottom and perpendicular sides in which the water table increases because some part of rain soaks into the soil and reaches the ground to replenish the aquifer and decreases because of farmers' withdrawals. We denote R the quantity of rain and $\rho \in [0, 1]$ the infiltration rate. In line with the wider part of literature (e.g. Gisser and Sanchez [6], Koundouri [8], Rubio and Casino [18]), the natural recharge is exogenously determined (i.e. not stock dependent). Moreover, for simplification, we do not take into account the local percolation and discharge.[1]

At time t farmer i pumps a quantity $w_g^i(t)$ directly in the aquifer and therefore the decline of the level of water table results from the total pumping : $\sum_{i=1}^{N} w_g^i(t)$. More-

[1] When the water table is near the ground surface, there will be little opportunity for recharge and shallow aquifers are thus recharged by local percolation of surface water and discharged by crops that use the water out of the ground. But the great aquifers run deep and are highly dependent on rain and melting snow.

over, they can also directly collect some part of rainwater, $w_r^i(t)$ before a drop of water could seep into the ground and collect this water quantity at the surface. Consequently, rainwater harvesting reduces the amount of water that can replenish the aquifer by the total quantity of rainwater that resource users have stored, that is $\sum_{i=1}^N w_r^i(t)$. Thus, when the farmers collect rainwater, the quantity of water that reaches the aquifer is $\rho\left(R - \sum_{i=1}^N w_r^i(t)\right)$.

Combining these assumptions all together, the change in the water table is defined by:

$$\dot{h} = \rho\left(R - \sum_{i=1}^{N} w_r^i(t)\right) - \sum_{i=1}^{N} w_g^i(t) \tag{1}$$

where h is the water table elevation.

2.2 Net Farmers' Benefits

A combination of water inputs, $w_g^i(t)$ and $w_r^i(t)$ at each period t, is used for production. We make the simplifying assumption that water is the sole input. However, we assume that each water source has various properties and impacts differently the output. Namely, the productivity of groundwater is denoted $\mu > 0$ and the productivity of rainwater is denoted $\theta > 0$. These two parameters represent the contribution of each water source to the production. We assume that both types of water are perfect substitutes:

$$W(t) = \mu w_g^i(t) + \theta w_r^i(t). \tag{2}$$

We then denote $F(W(t))$, a concave production function.[2]

The use of both sources of water is costly. The cost of extraction of groundwater is denoted C. Since we assume a "bathtub" type aquifer, the total cost of extraction of groundwater, $C(.,.)$ is the same at each point of the aquifer. It depends on the quantity of water withdrawn in the aquifer $w_g(t)$ and the head of water table $h(t) \in (0, c)$, thus $C(h(t), w_g(t))$.

The collection of rainwater is not affected by the head of the water table. The cost of rainwater collection (e.g the cost of transport from the point of the reservoir to the irrigation area) is assumed to be constant and exogenous: $K > 0$.

Finally, farmer i's net benefit at time t is then:

$$F\left(\mu w_g^i(t) + \theta w_r^i(t)\right) - C(h(t), w_g^i(t)) - K w_r^i(t).$$

We assume that agents behave non-cooperatively, they maximize the present value of their stream of profits given the extraction path of others with a common discount rate δ. The ith farmer faces the following dynamic optimization problem:

[2] An example is given in subsection 3.3

$$
\max_{\{w_g^i, w_r^i\}} \int_0^\infty \left(F\left(W(t)\right) - C(h(t), w_g^i(t)) - K w_r^i(t)\right) \exp^{-\delta t} dt
$$

$$
\text{w.r.t} \left| \begin{array}{l} \dot{h} = \rho\left(R - \sum_i^N w_r^i(t)\right) - \sum_i^N w_g^i(t) \\ w_g^i(t) \geq 0 \\ w_r^i(t) \geq 0 \\ \lim\limits_{t\to\infty} \exp^{-\delta t} p^i(t) \geq 0 \quad \lim\limits_{t\to\infty} \exp^{-\delta t} p^i(t)h(t) = 0 \\ h(0) \text{ given and } h(\infty) \text{ free} \end{array} \right. \tag{3}
$$

where $p^i(t)$ is the shadow price of groundwater for farmer i.

3 The Open-loop Stationary Nash Equilibria

The resource users solve the same dynamic problem simultaneously. They choose simultaneously their irrigation strategies at the beginning of the game and commit to their actions over the entire planning horizon. We can define N current-value Hamiltonian functions such as:

$$
H_i = F\left(W(t)\right) - C(h(t), w_g^i(t)) - K w_r^i(t) + p^i \left[\rho\left(R - \sum_{i=1}^N w_r^i(t)\right) - \sum_{i=1}^N w_g^i(t)\right] \tag{4}
$$

and the N corresponding Lagrangian functions[3]:

$$
L_i = H_i + \lambda_g^i w_g^i(t) + \lambda_r^i w_r^i(t) \tag{5}
$$

The first order conditions are:

$$
\begin{aligned}
\frac{\partial L_i}{\partial w_g^i} &= \mu F'\left(W(t)\right) - \frac{\partial C(h(t), w_g^i(t))}{\partial w_g^i} - p^i(t) + \lambda_g^i(t) = 0 && (6) \\
\lambda_g^i(t) &\geq 0 \ ; \ \lambda_g^i(t) w_g^i(t) = 0 && (7) \\
\frac{\partial L_i}{\partial w_r^i} &= \theta F'\left(W(t)\right) - K - \rho p^i(t) + \lambda_r^i(t) = 0 && (8) \\
\lambda_r^i(t) &\geq 0 \ ; \ \lambda_r^i(t) w_r^i(t) = 0 && (9) \\
\dot{p^i} &= \delta p^i(t) - \frac{\partial L_i}{\partial h} = \delta p^i(t) + \frac{\partial C(h(t), w_g^i(t))}{\partial h} && (10) \\
\dot{h} &= \rho R - \rho \sum_{i=1}^N w_r^i(t) - \sum_{i=1}^N w_g^i(t) && (11)
\end{aligned}
$$

[3] We are only interested in the study of equilibria in which the water table is strictly positive in order to compare the various steady states in different situations where the aquifer is not depleted. However, we consider the different cases in which farmers can use only one of the two water sources or a combination of both.

Equation (6) states that the marginal benefit for using one additional unit of GW in each period must be equal to the total marginal pumping cost (that is the sum of extraction costs with the opportunity cost of removing one unit of water from the ground), if GW extraction is used by farmers. Equation (8) states that the marginal benefit for using one additional unit of rainwater in each period must be equal to the total cost (that is the sum of the marginal RWH costs with the opportunity cost of removing one unit of rainwater that could have infiltrated into the soil), if farmers collect rainwater. The shadow price of player i, p^i represents the effect that the depletion of the water table in the current period has on future profits. Equation (10) characterizes the time variation of this price along the optimal extraction path of player i. It is positively affected by the discount rate, the current price and the marginal effect of the water table depletion on pumping cost. Equations (7) and (9) are the complementary slackness conditions.

To completely characterize the scope of the paper, we have to mention that our focus is on situations where the aquifer is not totally exhausted in the long-run, i.e. $h(\infty) > 0$. Nevertheless, along the optimal path, farmers may adopt different irrigation strategies. In particular, they can use only groundwater or only rainwater but they can also use simultaneously both water sources. In the following, we focus on the analysis of the stationary Nash equilibria (if they exist) in these various regimes.

It is convenient to notice that because all farmers are identical, we can easily show that when they use the same resource, they choose the same level of resource extraction/collection, i.e. they pump the same amount of groundwater and/or collect the same amount of rainwater. For instance, in an asymmetric stationary Nash equilibrium where a subset of the farmers uses groundwater only and the others use rainwater only (and we show this stationary Nash equilibrium exists), all the farmers who use groundwater only will use the same amount of groundwater and all the farmers who use rainwater only will use the same amount of rainwater.

We now show that there is no stationary Nash equilibrium such that all agents use rainwater and do not use groundwater and then turn to the characterization of the other possible stationary Nash equilibria.

3.1 Symmetric Stationary Nash Equilibria

This model may have various symmetric stationary Nash equilibria, that is stationary Nash equilibria in which all farmers choose the same irrigation strategies : (i) the pure RW harvesting stationary Nash equilibrium where all the farmers use RW only, (ii) the pure GW pumping stationary Nash equilibrium where all the farmers use groundwater only and (iii) the conjunctive use stationary Nash equilibrium where each farmer uses both water sources.

In a symmetric equilibrium we have $w_g^i = w_g^j = w_g$ and $w_r^i = w_r^j = w_r$. Consequently, the aggregate amount of GW is $\sum_{i=1}^N w_g^i = Nw_g$ and the total amount of RW harvested is $\sum_{i=1}^N w_r^i = Nw_r$. Therefore, the $5 \times N$ equations defined by (6) to (43) reduce to 5 equations. In the following we use the superscript RW, GW and c for denoting stationary Nash equilibria variables in the RW harvesting stationary Nash equilibrium, the GW pumping stationary Nash equilibrium and the conjunctive use stationary Nash equilibrium, respectively.

3.1.1 The Pure Rainwater Harvesting Stationary Nash Equilibrium

Let us first consider the stationary Nash equilibrium where farmers use rainwater only. This implies that the slackness condition is : $w_g = 0$, $\lambda_r = 0$ and $\lambda_g \geq 0$. Taking into account that at the stationary Nash equilibrium we must have $\dot{h} = \dot{p} = 0$, from equation (10) it is straightforward that:[4]

$$p^{RW} = 0. \tag{12}$$

Then, conditions (8) and (11) lead to two solutions for rainwater collection w_r^{RW}, which is impossible. We conclude that a stationary Nash equilibrium where only rainwater is used fails to exist.

3.1.2 The Pure Groundwater Stationary Nash Equilibrium

Let us now consider the state where all the farmers withdraw groundwater only. This implies that the variable w_r is equal to zero and, from the slackness condition (9), we have $\lambda_r \geq 0$ while the slackness condition (7) gives $\lambda_g = 0$.

Taking into account that $\dot{h} = \dot{p} = 0$, equations (10) and (11) can be used to find the characterization of the extraction rate and the shadow price in the stationary Nash equilibrium:[5]

$$\begin{aligned} w_g^{GW*} &= \frac{\rho R}{N} && (13) \\ p^{GW*} &= -\frac{1}{\delta} \cdot \frac{\partial C}{\partial h}(h^{GW*}, w_g^{GW*}) && (14) \end{aligned}$$

In the long-run, farmers use an identical share of the recharge of the aquifer, as we can see in condition (13), which is in line with the literature on groundwater management (see for example Negri [12].

By substituting these two conditions into equation (6), we obtain an implicit characterization of the water head in the stationary Nash equilibrium, h^{GW*} :

$$\mu F'\left(\mu w_g^{GW*}\right) = \frac{\partial C\left(h^{GW*}, w_g^{GW*}\right)}{\partial w_g} - \frac{1}{\delta} \frac{\partial C\left(h^{GW*}, w_g^{GW*}\right)}{\partial h}. \tag{15}$$

Then, combining these equations with equation (8), we obtain the stationary Nash equilibrium value for the Lagrangian multiplier,

$$\lambda_r^{GW*} = K - \frac{\rho}{\delta} \cdot \frac{\partial C(h^{GW*}, w_g^{GW*})}{\partial h} - \theta F'\left(\mu w_g^{GW*}\right). \tag{16}$$

[4]One can easily check that all the farmers face the same steady sate shadow price.
[5]One can easely check that all the farmers face the same steady sate shadow price.

Condition (16) and $\lambda_r^{GW*} \geq 0$ imply that the full marginal rainwater cost in the long-run must be greater than the long-run marginal productivity or rainwater.

$$K - \frac{\rho}{\delta}\frac{\partial C(h^{GW*}, w_g^{GW*})}{\partial h} > \theta F'\left(\mu w_g^{GW*}\right) \tag{17}$$

Finally, combining (15) and (17) enables us to characterize a necessary condition to have a pure GW stationary equilibrium that will help us to study the coexistence of several stationary Nash equilibrium in section 4. This result is summarized in the following proposition.

Proposition 1 *If there exists a pure GW stationary Nash equilibrium, then the ratio of the marginal productivity of the two water sources (the productive ratio) is higher than the ratio of the full marginal cost (cost ratio).*

$$\frac{\mu}{\theta} > \frac{MC^{GW}\left(h^{GW*}, w_g^{GW*}\right)}{MC^{RW}\left(h^{GW*}, w_g^{GW*}\right)},$$

with $MC^{GW} = \frac{\partial C(\cdot)}{\partial w_g} - \frac{1}{\delta}\frac{\partial C(\cdot)}{\partial h}$ *and* $MC^{RW} = K - \frac{\rho}{\delta}\frac{\partial C(\cdot)}{\partial h}$.

Proposition 1 shows that the pure GW equilibrium, if it exists, requires that the productivity ratio between GW and RW is higher than the cost ratio. This means the following: If the relative productivity gain of groundwater use, compared to rainwater use, exceeds the additonal relative costs of GW use, compared to rainwater use, farmers will use groundwater only. In simplifying terms, given the respective characteristics of the two water sources, rainwater remains "too costly" in the long-term, compared to groundwater. In contrast Roumasset and Wada [16], refering to the literature on backstop technologies, optimal extraction is not only driven by extraction costs, but also by differences in the water sources' productivity (μ andθ). Moreover, the physical interaction between GW and RW plays a very important role here in the sense that the choice of using one resource or the other does not only depend on the difference in costs, but on the *relative weight* of productivity and costs of the two water sources.

3.1.3 The Conjunctive Use Stationary Nash Equilibrium

Now, we turn to a second possible stationary Nash equilibrium in which each farmer uses both water sources. In this case, the complementary slackness conditions (7) and (9) require that the two Lagrangian multipliers are nil, $\lambda_r = \lambda_g = 0$.

Let us consider the stationary Nash equilibrium conditions, $\dot{p} = \dot{h} = 0$ and deduce that the rainwater harvesting level and the shadow price depend on the quantity of groundwater pumped:

$$p^{c*} = \frac{w_g^{c*}}{\delta}, \tag{18}$$

$$w_r^{c*} = \frac{\rho R - N w_g^{c*}}{N\rho}. \tag{19}$$

Then, by substituting these expressions into conditions (6) and (8), given that $\lambda_r = 0$ and condition (2), we get a system of two equations depending on the groundwater extraction rate and the water head :

$$\begin{cases} \mu F'(W^{c*}) - \frac{\partial C(h^{c*}, w_g^{c*})}{\partial w_g^c} + \frac{1}{\delta} \cdot \frac{\partial C(h^{c*}, w_g^{c*})}{\partial h^c} = 0 \\ \theta F'(W^{c*}) - K + \frac{\rho}{\delta} \cdot \frac{\partial C(h^{c*}, w_g^{c*})}{\partial h^c} = 0 \end{cases}$$

with $W^{c*} = w_g^{c*} \left(\frac{\rho N - \theta}{\rho} \right) + \frac{\theta R}{N}$.

This system enables us to fully characterize the stationary Nash equilibrium in the conjunctive use situation. Moreover, at this stationary Nash equilibrium, the productive ratio must be equal to the cost ratio as it is stated in the following proposition.

Proposition 2 *If there exists a conjunctive stationary Nash equilibrium, then the ratio of the marginal productivity of the two water sources is equal to the ratio of the full marginal cost.*

$$\frac{\mu}{\theta} = \frac{\frac{\partial C(h^{c*}, w_g^{c*})}{\partial w_g^c} - \frac{1}{\delta} \cdot \frac{\partial C(h^{c*}, w_g^{c*})}{\partial h^c}}{K - \frac{\rho}{\delta} \cdot \frac{\partial C(h^{c*}, w_g^{c*})}{\partial h^c}}. \tag{20}$$

This condition simply means that farmers are indifferent between the two water sources. They will use indifferently both water sources as long as the sum of these two quantity of water is equivalent to the available water in the long-term, ρR.

3.2 Asymmetric Stationary Nash Equilibria

We now consider the only possible form of asymmetric stationary Nash equilibrium, i.e. when farmers specialize and use different irrigation strategies. Indeed, as we have argued before, the farmers always use the same amount of a resource (groundwater or rainwater), provided that they use it. In other words, the only possible asymmetric stationary Nash equilibria are such that a subset of farmers use groundwater only and the others use rainwater only. This is thereby an illustration of a situation where players have an incentive to choose different strategies as it is the case in anti-coordination games. Here, given the existence of a negative externality through groundwater pumping costs, some farmers may choose to "escape" from this externality by directly harvesting rainwater and thus only face the exogenous rainwater collection cost.

The analysis of these asymmetric stationary Nash equilibria entails to distinguish the first order conditions for two groups of farmers. Let us therefore assume that $M \geq 1$ agents use GW only (group G) and $N - M$ farmers use RW only. Formally, for every given path for the other farmers, the choice of farmer $i \in G$ is characterized by the following necessary conditions (with the slackness conditions being $\lambda_g(t) = 0$):

$$\mu F'(W(t)) - \frac{\partial C(h(t), w_g(t))}{\partial w_g} - p_g(t) = 0 \tag{21}$$

$$\theta F'(W(t)) - K - \rho p_g(t) + \lambda_r(t) = 0 \tag{22}$$

$$\dot{p}_g = \delta p_g(t) + \frac{\partial C(h(t), w_g(t))}{\partial h} \tag{23}$$

where p_g denotes the shadow price of farmers who use GW only.[6]

Similarly, for every given path for the other farmers, farmer $i \notin G$ is characterized by the following necessary conditions (with the slackness condition being $\lambda_r(t) = 0$):

$$\mu F'(W(t)) - \frac{\partial C(h(t), 0)}{\partial w_g(t)} - p_r(t) + \lambda_g(t) = 0 \tag{24}$$

$$\theta F'(W(t)) - K - \rho p_r(t) = 0 \tag{25}$$

$$\dot{p}_r = \delta p_r + \frac{\partial C(h(t), 0)}{\partial h} \tag{26}$$

where p_r is the shadow price for the farmers who use RW only.[7]

Finally, the water table dynamics is affected by the resource use of all the farmers (11):

$$\dot{h} = \rho R - M w_g(t) - \rho(N - M) w_r(t). \tag{27}$$

Still considering the stationary Nash equilibrium, i.e. $\dot{p}_g = \dot{p}_r = \dot{h} = 0$, we directly deduce that $p_r = 0$ because $\frac{\partial C(h(t),0)}{\partial h} = 0$. In other words, farmers who use RW only do not care about the aquifer because it does not affect their profits. Hence, they do not give any value to the aquifer.

From equation (25), we obtain the following expression for the stationary Nash equilibrium rainwater collection :

$$w_r^{a*} = (F'(W))^{-1} \left(\frac{K}{\theta} \right). \tag{28}$$

Substituting into (27), with $\dot{h} = 0$, we obtain the stationary level of groundwater extraction :

$$w_g^{a*} = \rho \left(\frac{R - (N - M)(F'(W))^{-1} \left(\frac{K}{\theta}\right)}{M} \right) \tag{29}$$

Remark 1 The groundwater stationary Nash equilibrium extraction level is positive if and only if the recharge R is larger than a threshold R_{inf}.

$$w_g^{a*} > 0 \Leftrightarrow R > (N - M)(F'(W))^{-1} \left(\frac{K}{\theta} \right) \equiv R_{inf}$$

[6] One can easely check that all the farmers in group G face the same shadow price.

[7] One can easily check that all the farmers not in group G face the same shadow price.

Then the stationary Nash equilibrium shadow price for the farmers who use groundwater only is easily deduced from (23),

$$p_g^{a*} = \frac{\rho}{\delta}\left(\frac{R - (N-M)\,(F'(W^{a*}))^{-1}\left(\frac{K}{\theta}\right)}{M}\right)$$

Using equations (24) and (23) and the stationary Nash equilibria values for groundwater pumping and collection of rainwater, we obtain the following implicit characterization of the water table head :

$$\mu F'(W^{a*}) - \frac{\partial C(h^{a*}, w_g^{a*})}{\partial w_g} + \frac{1}{\delta}\cdot\frac{\partial C(h^{a*}, w_g^{a*})}{\partial h} = 0 \tag{30}$$

Now we combine these conditions and obtain necessary conditions that will enable us to study the coexistence of the stationary Nash equilibria. First consider farmers who use RW only. Condition (22) and $\lambda_g \geq 0$ implies that the long term rainwater marginal cost must be larger than the marginal productivity. Combining equation (22) with (21) and using $\lambda_g \geq 0$ leads to a necessary condition so that some farmers prefer choose to use GW only:

$$\frac{\mu}{\theta} \geq \frac{\frac{\partial C(h^{a*}, w_g^{a*})}{\partial w_g} - \frac{1}{\delta}\cdot\frac{\partial C(h^{a*}, w_g^{a*})}{\partial h}}{K - \frac{\rho}{\delta}\cdot\frac{\partial C(h^{a*}, w_g^{a*})}{\partial h}} \tag{31}$$

This condition states that the relative marginal productivity of groundwater, compared to rainwater, is larger than its relative marginal costs, compared to rainwater. This condition is similar to the condition for the pure GW stationary Nash, as resource users do not use rainwater in both cases. As we can see in condition 31, the cost ratio contains the effect of water harvesting today on the cost of water-use tomorrow. Likewise, it contains the effect of the depletion of the water-table, due to rainwater collection by the other group of resource users. In this sense, the choice of this group of groundwater users is driven by long-term considerations.

Second, we use equations (24) and (25) and obtain a necessary condition for the choice of the group who uses RW only. This condition is:

$$\frac{\mu}{\theta} \leq \frac{\frac{\partial C(h^{a*}, 0)}{\partial w_g}}{K}. \tag{32}$$

This condition states that the productivity ratio has to be smaller than the marginal cost ratio. The choice of this group is driven by short term effects only. Indeed, players who use rainwater do not take into account future costs associated to the exploitation of the aquifer, their shadow price is nil, because they do not pump groundwater.

Combining condition (31) and (32), we obtain that the asymmetric equilibrium may exists when the productive ratio gives an incentive to use GW when the dynamic effect of water use (on the aquifer) is taken into account and an incentive not to use GW when only short term effects are taken into account.

Proposition 3 *If an asymmetric equilibrium exists, then the productive ratio between GW and RW must be lower-bounded by the long run cost ratio and upper-bounded by the instantaneous cost ratio.*

$$\frac{\frac{\partial C(h^{a*},w_g^{a*})}{\partial w_g} - \frac{1}{\delta}\cdot\frac{\partial C(h^{a*},w_g^{a*})}{\partial h}}{K - \frac{\rho}{\delta}\cdot\frac{\partial C(h^{a*},w_g^{a*})}{\partial h}} \leq \frac{\mu}{\theta} \leq \frac{\frac{\partial C(h^{a*},0)}{\partial w_g}}{K}$$

3.3 An Example

3.3.1 Analytical Expressions

We now turn to an analytical example in order to derive a closed-form characterization for the three stationary Nash equilibria and to study wether they may coexist or not. Suppose that the production function is of the form

$$F\left(W(t)\right) = W(t) - \frac{1}{2}\left(W(t)\right)^2 \tag{33}$$

and the pumping cost function is given by:

$$C(h(t), w_g(t)) = (c - h(t))\, w_g^i(t), \text{ with } c > 0. \tag{34}$$

This cost is linear in the quantity of extracted groundwater and the marginal cost is linearly decreasing in the head of water table. To avoid unrealistic cases where the net benefit is always decreasing in the amount of rainwater used, we assume that $\theta > K$.

Let us first rewrite conditions (14), (15) and (16) for the *pure GW stationary Nash equilibrium* :

$$\begin{aligned}
p^{GW*} &= \frac{\rho R}{\delta N}\\
h^{GW*} &= c - \mu + \frac{\rho R}{N}\left(\frac{1+\delta\mu^2}{\delta}\right)\\
\lambda_r^{GW*} &= K - \theta + \frac{\rho R}{N}\left(\frac{\delta\theta\mu + \rho}{\delta}\right)
\end{aligned}$$

Hence, if a pure groundwater stationary Nash equilibrium exists, groundwater must be big enough, ie suitable enough for irrigation to ensure positive values for the Lagrangian multiplier, λ_r^{GW}:

Remark 2 If there exists a pure GW stationary Nash equilibrium, then :

$$\mu^{sep} \equiv \frac{N}{\rho R}\left(\frac{\theta - K}{\theta}\right) - \frac{\rho}{\delta\theta} < \mu \tag{35}$$

This condition states that the productivity of groundwater μ must be sufficiently large and/or the productivity of rainwater θ must be sufficiently small (because the LHS is increasing in θ).

Turning to the *conjunctive use stationary Nash equilibrium*, let us first remark that the groundwater withdrawals now writes :

$$w_g^{c*} = \underbrace{\left(\frac{\delta\rho}{\theta\mu\delta\rho - \theta^2\delta + \rho^2}\right)}_{A}\underbrace{\left(\frac{N(\theta - K) - \theta^2 R}{N}\right)}_{B} \tag{36}$$

This expression implies that there are two situations that ensure positive values of this extraction rate. In the first situation, the productivity of groundwater is sufficiently small (and/or the productivity of rainwater is sufficiently large) and the recharge is sufficiently large, in the second situation the productivity of groundwater is sufficiently large (and/or the productivity of rainwater is sufficiently small) while the recharge is sufficiently small (see equations 46 and 48 in the appendix).

Inserting equation (36) in equations (18) and (19), we can deduce the stationary Nash equilibrium for rainwater collection and the shadow price :

$$w_r^{c*} = \frac{R}{N} - \frac{1}{N}\left[\frac{\delta\left(N(\theta - K) - \theta^2 R\right)}{\theta\mu\delta\rho - \theta^2\delta + \rho^2}\right] \tag{37}$$

$$p^{c*} = \frac{\rho\left(N(\theta - K) - \theta^2 R\right)}{N\left(\theta\mu\delta\rho - \theta^2\delta + \rho^2\right)} \tag{38}$$

and then, substituting into (6), and given the pumping cost (34), we obtain the steady-state value of the water table

$$h^{c*} = c - \mu + \frac{\mu}{N}\left[\frac{\delta\rho\left(N(\theta - K) - \theta^2 R\right)}{\theta\mu\delta\rho - \theta^2\delta + \rho^2}\left(\frac{\rho\mu - \theta}{\rho}\right) + \theta R\right] + \frac{\rho\left(N(\theta - K) - \theta^2 R\right)}{N\left(\theta\mu\delta\rho - \theta^2\delta + \rho^2\right).} \tag{39}$$

Contrary to the case of the pure GW stationary Nash equilibrium, the level of precipitation plays a crucial role in the characterization of the conjunctive use stationary Nash equilibrium. The critical points $\{w_r^c; p^c; h^c\}$ also depend on the parameters of μ and R. We thus have to check the various ranges allowing to have positive values in the long run for rainwater collection, the shadow price and the water table.

Remark 3 If there exists a conjunctive use stationary Nash equilibrium, then :

$$\begin{array}{lll} \mu < \mu^{sep} & ; \ \breve{R} > R > \overline{R} & ; \rho^2 - \delta\theta^2 > 0 \\ \mu > \mu^{sep} & ; \ R < \overline{R} & \end{array}$$

with $\mu^{sep} \equiv \frac{N}{\rho R}\left(\frac{\theta - K}{\theta}\right) - \frac{\rho}{\delta\theta}$, $\overline{R} \equiv \frac{N(\theta-K)}{\theta^2}$ *and* $\breve{R} \equiv \frac{\delta N(\theta-K)}{\rho^2}$.

Remark 3 highlights that farmers may use the two water sources conjunctively only in two specific situations : the first one corresponds to a case where there is a high recharge but GW is not very productive and the second one corresponds to the reverse, i.e. a small recharge with a highly productive GW.

Finally, we can rewrite the *asymmetric stationary Nash equilibrium* characterization. Note that conditions (28), (29), (30), and the two Lagrangian multipliers can now be written as follows :

$$
\begin{aligned}
w_r^{a*} &= \frac{\theta - K}{\theta^2} \\
w_g^{a*} &= \frac{\rho}{M}\left[R - (N-M)\left(\frac{\theta - K}{\theta^2}\right)\right] \\
h^{a*} &= c - \mu + \left(\frac{1+\mu^2\delta}{\delta}\right)\frac{\rho}{M}\left[R - (N-M)\frac{\theta - K}{\theta^2}\right] \\
\lambda_r^{a*} &= \left(\frac{\rho}{\delta} + \mu\theta\right)\frac{\rho}{M}\left[R - (N-M)\frac{\theta - K}{\theta^2}\right] - (\theta - K) \\
\lambda_g^{a*} &= \mu\frac{\theta - K}{\theta} - \left(\frac{1+\mu^2\delta}{\delta}\right)\frac{\rho}{M}\left[R - (N-M)\frac{\theta - K}{\theta^2}\right]
\end{aligned}
$$

As previously, some supplementary necessary conditions have to be checked in order to ensure positive values for the two Lagrangian multipliers λ_r^{a*} and λ_g^{a*}. The investigation of these conditions (provided in the Appendix) outlines that the asymmetric stationary Nash equilibrium requires that the GW productive parameter belongs to a critical interval and the recharge has to be sufficiently small.

Remark 4 If there exists an asymmetric stationary Nash equilibrium then

(i) $\mu > \frac{\theta}{\rho}$. *In particular, if* $\mu = \theta$, *there is not asymmetric equilibrium because* $\rho \in [0, 1]$.

(ii)

$$
\begin{cases}
\max\left\{\underline{\mu}_a ; \widehat{\mu}_a\right\} < \mu < \overline{\mu}_a \\
\widehat{R} < R < \underline{R} \\
\textit{and} \quad \rho^2 - \delta\theta^2 > 0
\end{cases}
$$

where $\underline{\mu}_a$, $\widehat{\mu}_a$ *and* $\overline{\mu}_{asym}$ *are given in the appendix.*

These necessary conditions relate to proposition 3 and remark 1. For intermediate levels of GW productivity, the characteristics of the aquifer remain sufficiently good such that some farmers have incentives to use it; it is also sufficiently small such that some farmers prefer to use RW. Likewise, the recharge is sufficiently big to make groundwater use interesting (see section 4 for more detailed insights) and sufficiently small to allow also for other resource use. This drives the fact that a group of farmers will only use GW while another group will turn to RW.

Table 1: The equilibrium's necessary conditions

	GW Productivity parameter	Recharge level	Infiltration rate
Groundwater equilibrium	$\mu > \mu^{sep}$		
Conjunctive use equilibrium	$\mu > \mu^{sep}$	$R < \overline{R}$	
	$\mu < \mu^{sep}$	$\overline{R} < R < \breve{R}$	$\rho^2 - \delta\theta^2 > 0$
Asymmetric equilibrium	$\max\left\{\underline{\mu}_{asym}; \widehat{\mu}_{asym}\right\} < \mu < \overline{\mu}_{asym}$	$\widehat{R} < R < \underline{R}$	$\rho^2 - \delta\theta^2 > 0$

To conclude we can summarize the necessary conditions required by the various stationary Nash equilibria, if they exist, in table 1.

Before turning to the numerical examples, it is relevant to notice that the long run water table in the pure groundwater strategy is always greater than the stationary Nash equilibrium water table level in the asymmetric regime for the required level of recharge.

Remark 5 Since the asymmetric equilibrium exists when the recharge is small enough, i.e. $\widehat{R} < R < \underline{R}$ *with* ($\underline{R} < \overline{R}$), *we observe that the water table in the groundwater is higher than the long run water table in the asymmetric regime.*

$$h^{GW*} - h^{a*} > 0 \Leftrightarrow R < \overline{R}$$

3.3.2 A Numerical Case

Consider a numerical case where the three possible steady states exist. We choose the following hypothetical parameter values: $c = 50$, $\delta = 0.02$, $K = 2.2$, $M = 1$, $\mu = 27$, $N = 2$, $R = 0.1$, $\rho = 0.2$, $\theta = 2.7$. In the long-run asymmetric equilibrium, we have[8]:

$$h^{a*} = 27.894,\ \ w_g^{a*} = 0.006,\ \ w_r^{a*} = 0.069,\ \ p_g^{a*} = 0.314,\ \ \lambda_r^{a*} = 0.021,\ \ \lambda_g^{a*} = 0.106.$$

In the long-run groundwater equilibrium, we have:

$$h^{GW*} = 30.790,\ \ w_g^{GW*} = 0.001,\ \ p_g^{GW*} = 0.500,\ \ \lambda_r^{GW*} = 0.329.$$

In the long-run conjunctive-use equilibrium, we have:

$$h^{c*} = 27.854,\ \ w_g^{c*} = 0.003,\ \ w_r^{c*} = 0.035,\ \ p^{c*} = 0.146.$$

We can see that the stationary equilibrium water table in the pure groundwater regime is higher than the steady state water table in the asymmetric equilibrium. The lowest steady state water head is obtained in the conjunctive use equilibrium, i.e. when farmers use simultaneously the two water sources :

$$h^{GW*} > h^{a*} > h^{c*}.$$

[8] Values are rounded.

4 Anti-coordination and Coordination Problems

Up to now, we have identified three admissible stationary Nash equilibria where one corresponds to the interior solution, i.e. the conjunctive use of both water sources and the two other stationary Nash equilibria are at the boundary either with no RW or no GW (in the asymmetric regime). Now, we wonder if these stationary Nash equilibria are mutually exclusive or not. We use the necessary conditions on the groundwater productivity parameter, μ, and on the recharge R to check if there exists a non-empty set when we combine all the necessary conditions. We prove that these stationary Nash equilibria cannot exist simultaneously when the groundwater productivity parameter and the recharge fall in particular intervals but multiplicity of stationary Nash equilibria may also appear.

4.1 A Large Recharge

The previous discussion shows that there may exist two stationary Nash equilibria when the recharge is sufficiently large. Indeed, Proposition 1 states that the pure groundwater equilibrium may exist regardless of the level of the recharge but the groundwater productivity must be high enough, that is higher than the threshold μ^{sep}. Proposition 2 outlines that the conjunctive use equilibrium may exist when the recharge is high enough and the groundwater productivity is lower than the same threshold μ^{sep}. Therefore, according to the level of the productivity parameter μ, we could obtain a unique equilibrium.

Proposition 4 *When the recharge is large, (i) the pure groundwater equilibrium and the conjunctive use equilibrium do not coexist, and (ii) the asymmetric equilibrium fails to exist.*

Proposition 4 states that there is no (anti-)coordination problem when the recharge is large. In particular, (ii) states that there is no anti-coordination problem when the recharge is large. The intuition of this result deserves some comments. If there were an asymmetric equilibrium, some farmers would use rainwater only. These farmers makes their choices by comparing short-term marginal benefits and marginal costs of rainwater collection. In other words, as the head of the water table does not affect their costs, they do not take the dynamic of the aquifer into account ($p^{RW} = 0$). Hence, the quantity of rainwater they collect does not depend on the recharge. Moreover, at the stationary equilibrium, the "actual" recharge of the aquifer, ρR, is fully extracted (either with rainwater collection or with groundwater pumping). Any increase in the level of recharge must therefore be pumped from the aquifer (by the farmers who use groundwater only). This is consistent only if the head of the water table increases. Now for a sufficiently large increase in the recharge, the head of the water table would become sufficiently large so that no farmer would have incentives to use rainwater anymore. Consequently, there cannot be any asymmetric equilibrium when the level of recharge is sufficiently large.

4.2 A Small Recharge

Investigating the stationary Nash equilibria is somehow more difficult when the level of precipitation is limited. This case is nevertheless relevant in a context where water becomes scarce and adaptive solutions are required. Thus, in terms of public policy, the implications could be more ambiguous than expected.

4.2.1 Low Productivity of Groundwater

According to Proposition 1 and 2, we know that there is no symmetric equilibrium when the productive parameter of groundwater is sufficiently low. Similarly, Proposition 3 shows that the asymmetric equilibrium fails to exist when the productive parameter of groundwater is sufficiently low. Consequently, we can assert the following:

Proposition 5 *When the recharge R and the productive parameter of groundwater μ are small enough, i.e. $R < \overline{R}$ and $\mu < \mu^{sep}$, there is no equilibrium.*

Proposition 5 shows that there can exist no equilibrium when the recharge is low and GW is not productive enough ($\mu < \mu^{sep} = \frac{N}{\rho R}\left(\frac{\theta-K}{\theta}\right) - \frac{\rho}{\delta\theta}$). Obviously, this result comes from our specification and the fact that we claim that the shadow price of the aquifer is zero to ensure a stationary Nash equilibrium.[9] Beyond this technical observation, this proposition captures real situations in arid areas or in areas under severe drought. In fact, some parts of the world are characterized by little rain (e.g. the Sahel belt) and also suffer from a high concentration of salts in GW. That is why farmers have no incentives to extract GW in the long run. The only remaining possibility is that farmers use rainwater only, but this is not an equilibrium (because we excluded situations where the resource is exhausted in the long run).

4.2.2 High Productivity of Groundwater

We now turn to the situation in which the productive parameter is sufficiently high. From Proposition 1 and 2, we know that, if they exist, the pure GW equilibrium and the conjunctive equilibrium require that the value of this parameter falls in a critical interval and Proposition 3 requires values for μ that are higher than a threshold. Consequently, it is clear that the asymmetric equilibrium does exist simultaneously with the pure GW equilibrium and the conjunctive equilibrium as soon as $\mu > \mu^{sep}$. We can therefore claim the following :

Proposition 6 *When the recharge is smaller than the threshold $R < \overline{R}$ and the productive parameter of groundwater is such that $\mu \in \left]\mu^{sep}; \max\left\{\underline{\mu}_{asym}; \widehat{\mu}_{asym}\right\}\right[\bigcup \left]\overline{\mu}_{asym}; +\infty\right[$, the asymmetric equilibrium does not exist simultaneously with the pure GW equilibrium and the conjunctive use equilibrium.*

[9] Remember that we obtain the differential equation $\dot{p} = \delta p(t)$ when we analyze the RW regime and in the case of the asymmetric equilibrium when farmers do not use any GW. Because we study the steady state $\dot{p} = 0$, we conclude that $p = 0$.

Proposition 6 shows that the three stationary Nash equilibria may coexist for intermediate productive values of GW. In fact, with this kind of limitation, GW should not too negatively affect crop production to remain attractive but should not have too good characteristics to leave RW sufficiently attractive, at least for a group of farmers. More precisely, it is obvious that as long as the productive ratio is higher than the long term cost ratio, we may observe that the pure GW equilibrium and the asymmetric equilibrium coexists. Furthermore, if this productive ratio is smaller than the short term cost ratio, then these two stationary Nash equilibria may occur simultaneously. Finally, since the conjunctive use equilibrium occurs when these two ratios (in terms of production and cost) are equal (cf. equation (20)), we can therefore observe these three stationary Nash equilibria simultaneously.

The following picture summarizes all the above discussion.

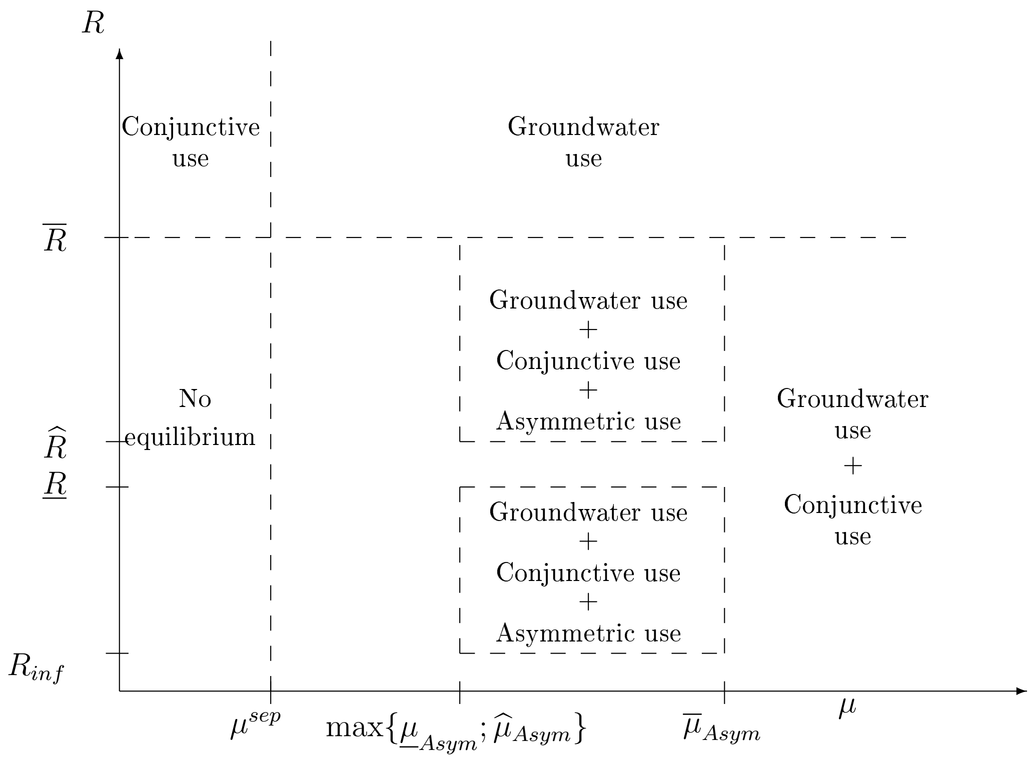

Figure 1:

5 The Feedback Nash Equilibria

We suggest now the alternative modeling choice by analyzing the feedback equilibrium. In other terms, we claim that water users can adjust their behavior (extraction rate and RW storage) at each moment to the level of the GW stock. For this kind of equilibria, remark the Hamiltonian is written as :

$$\begin{aligned} H_i &= F\left(W(t)\right) - C(h(t), w_g^i(t)) - Kw_r^i(t) \\ &+ p^i \left[\rho \left(R - w_r^i(t) - \sum_{j\neq i}^{N} w_r^j(h(t))\right) - w_g^i(t) - \sum_{j\neq i}^{N} w_g^j(h(t))\right] \end{aligned} \tag{40}$$

and the necessary conditions

$$\frac{\partial L_i}{\partial w_g^i} = \mu F'\left(W(t)\right) - \frac{\partial C(h(t), w_g^i(t))}{\partial w_g^i} - p^i(t) + \lambda_g^i(t) = 0 \tag{41}$$

$$\frac{\partial L_i}{\partial w_r^i} = \Theta F'\left(W(t)\right) - K - \rho p^i(t) + \lambda_r^i(t) = 0 \tag{42}$$

$$\dot{p^i} = p^i \left(\delta + \frac{\partial w_g^j}{\partial h} + \rho \frac{\partial w_r^j}{\partial h}\right) + \frac{\partial C(h(t), w_g^i(t))}{\partial h} \tag{43}$$

$$\dot{h} = \rho R - \rho w_r^i(t) - \rho \sum_{j\neq i}^{N} w_r^j(h(t)) - w_g^i(t) - \sum_{j\neq i}^{N} w_g^j(h(t)) \tag{44}$$

Except equation (43), the conditions are similar to those obtained in the open-loop equilibrium. The new term $p^i\left(\frac{\partial w_g^j}{\partial h} + \rho\frac{\partial w_r^j}{\partial h}\right)$ appears because water users employ feedback strategy and therefore adjust there water use to the level of water table. Within this context, each agents takes into account the optimal reaction of the others by deducing it from the evolution of the water table, which is observable.

We can therefore easily rewrite propositions 1-3 given the stationary equilibrium of the in-situ scarcity rent p^i captures the optimal reaction of others water users. This modifies the value of the long-term full pumping marginal cost : $\frac{\partial C(h^*, w_g^{i*})}{\partial w_g^{i*}} - \frac{1}{\left(\delta + \frac{\partial w_g^{j*}}{\partial h} + \rho\frac{\partial w_r^{j*}}{\partial h}\right)} \cdot \frac{\partial C(h^*, w_g^{i*})}{\partial h}$.

5.1 Some comparisons between open loop and Nash equilibrium at the steady state

Remark that when $N = 2$, the asymmetric equilibrium in feedback coincides with the asymmetric equilibrium in open loop.

For the pure GW equilibrium, Negri [12] proved that the water table steady state in the open loop case is greater than the steady state equilibrium in the feedback case. However, it is not the case for the conjunctive use equilibrium as we show in the following example.

We choose the following hypothetical parameter values : $c = 50$, $\delta = 0.02$, $K = 2.7$, $M = 1$, $\mu = 11.65$, $N = 2$, $R = 0.135$, $\rho = 0.2$, $\theta = 2.7$.

Table 2: Comparison between the open loop and feedback steady state

	The pure GW equilibrium	The conjunctive use equilibrium
Open loop	$h = 40.8572$ $w_g = 0.0135$	$h = 40.5182$ $w_g = 0.0015$ $w_r = 0.0595$
Feedback	$h = 40.6977$ $w_g = 0.0135$	$h = 40.5291$ $w_g = 0.0046$ $w_r = 0.0437$

In conclusion, we observe $h^{GWOL} > h^{GWF} > h^{cF} > h^{cOL}$.

6 Conclusion and Further Discussion

This paper extends the literature on groundwater management in the presence of strategic behavior, namely the part of the literature focusing on conjunctive use between two water sources, by providing evidence that there may exist a multiplicity of equilibria. This multiplicity occurs because resource users can choose to use either only one of the two water sources or the two sources simultaneously. Depending on difference between the productive ratio and the cost ratio, they will opt for one of these three irrigation strategies : (i) the pure groundwater strategy, (ii) the conjunctive use strategy and (iii) the asymmetry strategy (a group of resource users opts for rainwater storage while the other group opts for groundwater extraction). However, this comparison can leave the selection of an equilibrium undetermined, meaning that several equilibria can occur simultaneously. The comparison of the gains provided by each of these equilibria allows us to suggest a ranking of irrigation strategy.

Moreover, the second cornerstone of this study is the possibility to have an asymmetric equilibrium whereas all economic agents are symmetric in terms of their productive activity. This framework is similar to that of anti-coordination games. Some of the resource users may want to avoid the usual cost pumping externality, i.e. avoid to be subjected to an increase in pumping cost because of withdrawals by all other users. To this end, they directly harvest some part of rainfalls for irrigation instead of extracting groundwater beneath to their land. Others continue to exclusively pump water from the ground. We can find in numerical simulations that sometimes the first group gets a higher payoff than the group of GW users.

We want to point out that in this paper we consider a simplified groundwater dynamics. We namely assume that there is no irrigation return flow to the aquifer. It is usual to consider that some proportion of the water that is not consumed by crops percolates to the aquifer. In that case, we could generate an intermediate water productivity resulting from the irrigation percolation of potentially clean irrigation water to potentially brackish GW. The productive ratio may be therefore modified leading to extend the interval of the occurrence of one of the equilibria.

Moreover, we want to complete, like in the open loop case, the feedback analysis and to provide a comparison between the various equilibria with more relevant economic intuitions and to present the stability results for both cases.

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Appendix

Proof of Proposition 1

It is straightforward by combining equation (15) and equation (17).

Proof of Proposition 2

We have to check the necessary conditions on μ to obtain $w_r^c > 0$.

1. If $\{A < 0;\ B < 0\}$,

$$\begin{aligned}
&w_r^{c*} > 0 \Rightarrow \mu < \frac{N}{\rho R}\left(\frac{\theta - K}{\theta}\right) - \frac{\rho}{\delta\theta} \equiv \mu^{sep}\\
&\text{With equation 46, we know that : } \mu < \mu^g\\
&\text{Since } \ B < 0, \text{here, we observe } \ \mu^{sep} - \mu^g < 0\\
&\text{Therefore } \ \mu < \mu^{sep}\\
&\text{With equation 46, we know that } \ B > 0 \Leftrightarrow R > \overline{R}\\
&\text{But } \ \mu < \mu^{sep} \ \text{ requires } \ \mu^{sep} > 0 \Leftrightarrow \breve{R} \equiv \frac{\delta N(\theta - K)}{\rho^2} > R\\
&\text{Compute } \ \breve{R} - \overline{R} = \frac{N(\theta - K)(\rho^2 - \delta\theta^2)}{\rho^2\delta\theta^2}\\
&\text{We need } \ \rho^2 - \delta\theta^2 > 0 \ \text{ to ensure a solution}
\end{aligned}$$

2. If $\{A > 0;\ B > 0\}$,

$$\begin{aligned}
&w_r^{c*} > 0 \Rightarrow \mu > \frac{N}{\rho R}\left(\frac{\theta - K}{\theta}\right) - \frac{\rho}{\delta\theta} \equiv \mu^{sep}\\
&\text{With equation 48, we know that } \ \mu > \mu^g\\
&\text{Compute : } \mu^{sep} - \mu^g = \frac{N}{\rho\theta R}B > 0\\
&\text{Therefore } \ \mu > \mu^{sep}
\end{aligned}$$

Proof of Proposition 3

It is straightforward by combining equation (31) and equation (32).

Details of Remark 3

Consider two situations:

$$\begin{aligned}
w_g^c > 0 \quad &\Leftrightarrow \quad \{A < 0; B < 0\} &(45)\\
&\Leftrightarrow \quad \left\{\mu < \frac{\theta}{\rho} - \frac{\rho}{\theta\delta} \equiv \mu^g; \quad R > \frac{N(\theta - K)}{\theta^2} \equiv \overline{R}\right\} &(46)
\end{aligned}$$

and

$$w_g^c > 0 \quad\Leftrightarrow\quad \{A > 0; B > 0\} \tag{47}$$

$$\Leftrightarrow\quad \left\{\mu > \frac{\theta}{\rho} - \frac{\rho}{\theta\delta} \equiv \mu^g; \quad R < \frac{N(\theta - K)}{\theta^2} \equiv \overline{R}\right\} \tag{48}$$

Proof of Remark 4

(i) Remember that

$$\lambda_r^{a*} = \left(\frac{\rho}{\delta} + \mu\theta\right)\frac{\rho}{M}\left[R - (N-M)\frac{\theta - K}{\theta^2}\right] - (\theta - K)$$

$$\lambda_g^{a*} = \mu\frac{\theta - K}{\theta} - \left(\frac{1+\mu^2\delta}{\delta}\right)\frac{\rho}{M}\left[R - (N-M)\frac{\theta - K}{\theta^2}\right]$$

$\lambda_r^{a*} > 0 \Rightarrow -\frac{\rho}{M}\left[R - (N-M)\frac{\theta-K}{\theta^2}\right] < -\frac{\theta-K}{\frac{\rho}{\delta}+\mu\theta}$, then $\lambda_g^{a*} < \left[\frac{\mu}{\theta} - \frac{1+\mu^2\delta}{\rho+\delta\mu\theta}\right](\theta - K)$. This last quantity must be positive (otherwise $\lambda_g^{a*} < 0$) iff $\mu\rho - \theta > 0$.

(ii) First, one can easily check that $\lambda_r^a > 0 \Leftrightarrow \mu > \frac{1}{\theta}\left[\frac{M(\theta-K))}{\rho\left[R-(N-M)\frac{\theta-K}{\theta^2}\right]} - \frac{\rho}{\delta}\right] \equiv \underline{\mu}_a$. Second, notice that λ_g^a is a polynomial equation of second degree that can be rewritten as $\lambda_g^{a*} = (\widehat{\mu}_a - \mu)(\mu - \overline{\mu}_a)$ with

$$\begin{aligned}
\widehat{\mu}_a &= \left(\frac{\theta - K}{\theta} - \sqrt{\Delta}\right)\frac{M}{2\rho\left[R - (N-M)\left(\frac{\theta-K}{\theta^2}\right)\right]} > 0,\\
\overline{\mu}_a &= \left(\frac{\theta - K}{\theta} + \sqrt{\Delta}\right)\frac{M}{2\rho\left[R - (N-M)\left(\frac{\theta-K}{\theta^2}\right)\right]} > 0,\\
\text{where } \Delta &= \left(\frac{\theta - K}{\theta}\right)^2 - \frac{4\rho^2}{\delta M^2}\left[R - (N-M)\left(\frac{\theta - K}{\theta^2}\right)\right]^2.
\end{aligned}$$

A necessary condition such that the two critical values to exist is $\Delta > 0 \Leftrightarrow R^{sup} \equiv \left(\frac{\theta-K}{\theta^2}\right)\left[N - M\left(\frac{2\rho-\theta\sqrt{\delta}}{2\rho}\right)\right] > R$. We conclude that $\lambda_g^{a*} > 0 \Leftrightarrow \{R^{sup} > R$ and $\mu \in]\widehat{\mu}_a, \overline{\mu}_a[\}$.

Now, let us show a necessary condition for the interval $\left]\underline{\mu}_a, \overline{\mu}_a\right[$to be non empty. We have:

$$\begin{aligned}
&\overline{\mu}_a - \underline{\mu}_a > 0\\
\Leftrightarrow\quad &\sqrt{\Delta} > \frac{\theta - K}{\theta} - \frac{2\rho^2}{\delta\theta M}\left[R - (N-M)\left(\frac{\theta - K}{\theta^2}\right)\right] > 0\\
\Leftrightarrow\quad &\left\{\begin{array}{l} \frac{2\rho^2}{\delta M}\left[R - (N-M)\left(\frac{\theta-K}{\theta^2}\right)\right] > \theta - K \text{ and}\\ \Delta > \left(\frac{\theta-K}{\theta} - \frac{2\rho^2}{\delta\theta M}\left[R - (N-M)\left(\frac{\theta-K}{\theta^2}\right)\right]\right)^2 \end{array}\right.\\
\Leftrightarrow\quad &\left\{ \widehat{R} \equiv \frac{(\theta-K)}{\theta^2}\left[N - M\left(1 - \frac{\delta\theta^2}{2\rho^2}\right)\right] < R < \frac{(\theta-K)}{\theta^2}\left(N - M\frac{\rho^2}{\rho^2+\delta\theta^2}\right) \equiv \underline{R}\right.
\end{aligned}$$

Moreover, we have different bounds for the level of the recharge. First, we therefore have to check if the upper-bound R^{sup} is higher than $\underline{R}$ and then we have to check if this last bound is higher than the lower bound $\widehat{R}$.

$$\begin{aligned} R^{sup} - \underline{R} &= \frac{M\theta\sqrt{\delta}\left(\rho - \theta\sqrt{\delta}\right)}{2\rho^2} > 0 \\ \underline{R} - \widehat{R} &= M\frac{\delta\theta^2\left(\rho^2 - \delta\theta^2\right)}{2\rho^2\left(\rho^2 + \delta\theta^2\right)} > 0 \end{aligned}$$

And we obviously have $\widehat{R} - R_{inf} > 0$ under remark 1, i.e. $w_g^{a*} > 0$.

Proof of Proposition 4

It is clear from propositions 1, 2 and 3.

Proof of Proposition 5

From Proposition 1 and 2, it is straightforward that the pure GW equilibrium and the conjunctive equilibrium cannot coexist.

Proposition 3 requires that $\mu \in \left]\max\left\{\underline{\mu}_a; \widehat{\mu}_a\right\}; \overline{\mu}_a\right[$ if an asymmetric equilibrium exists. Let us check:

$$\begin{aligned} \underline{\mu}_a - \mu^{sep} &= \frac{1}{\theta}\left[\frac{M\left(\theta - K\right))}{\rho\left[R - \left(N - M\right)\frac{\theta - K}{\theta^2}\right]} - \frac{\rho}{\delta}\right] - \left(\frac{N}{\rho R}\left(\frac{\theta - K}{\theta}\right) - \frac{\rho}{\delta\theta}\right) \\ &= \left(\frac{\theta - K}{\rho\theta}\right)\left(\frac{M}{R - \left(N - M\right)\frac{\theta - K}{\theta^2}} - \frac{N}{R}\right) \\ &= \left(\frac{\theta - K}{\rho\theta}\right)\frac{\left(N - M\right)\left(N(\theta - K) - R\theta^2\right)}{R\left[R\theta^2 - \left(N - M\right)\left(\theta - K\right)\right]} \end{aligned}$$

Remember that $R < \overline{R}$, therefore $N(\theta - K) - R\theta^2 > 0$ and $R\theta^2 - \left(N - M\right)\left(\theta - K\right) > 0$ because $w_g^a > 0$. We conclude that $\underline{\mu}_a^{sep} > \mu^{sep}$ which entails $\max\left\{\underline{\mu}_a; \widehat{\mu}_a\right\} > \mu^{sep}$.

Proof of Proposition 6

By observing remark 4, it is straightforward that the asymmetric equilibrium cannot exist simultaneously within the interval $\mu \in \left]\mu^{sep}; \max\left\{\underline{\mu}_a; \widehat{\mu}_a\right\}\right[\bigcup\left]\overline{\mu}_a; +\infty\right[$ when $R < \underline{R}$.