# Life Cycle Uncertainty and Portfolio Choice Puzzles

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**Abstract**

The standard portfolio-choice theory is hard to reconcile with following facts. (i) Despite a high rate of returns, the average household holds a low share of risky assets (equity premium puzzle). (ii) The share of risky assets is disproportionately larger for richer households. (iii) The share of risky assets increases with age. We show that a simple life-cycle model with Bayesian learning about earnings ability can account for all three facts. Young households whose incomes are skewed toward labor earnings face a large uncertainty about their ability in the market. To diversify highly risky human capital young workers—despite a longer investment horizon—-invest on safe financial assets in the early stage of life. As their abilities in the labor market are revealed over time, they can take on more risks in financial investment. On average, income of poor (as well as young) households are heavily skewed to labor earnings, making them to allocate savings towards safe financial assets to protect against labor market uncertainty.

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# 1 Introduction

While the portfolio choice across different types of households has received keen attention in the last decade, several puzzles remain unresolved. According to Haliassos and Michaelides (2002), there are three empirical facts that the standard model cannot reconcile: (i) Despite a high rate of returns, the average household holds a low share of risky assets (equity premium puzzle). (ii) The share of risky assets is disproportionately larger for richer households. (iii) The share of risky assets is increasing in age. The last fact is striking given that financial planners strongly advise taking more risks in the early stage of life.

We show that a simple life-cycle model with a gradual learning about ability can account for all three facts. Workers do not learn about their ability immediately upon entering the labor market. They learn about themselves gradually through the successive accumulation of labor market outcomes. Perceived risk of human capital is much higher for younger workers because they faces a larger uncertainty about the labor market outcomes. To diversify highly risky human capital young workers—despite a longer investment horizon—-invest on safe financial assets in the early stage of life. As their ability is revealed over time, they can take on more risks in financial investment. We show that if the learning about ability is sufficiently slow, the share of risky financial assets increases with age. On average, income of poor (as well as young) households are heavily skewed to labor earnings, enticing their investment towards safe assets to protect against risky human capital.

We quantitatively test these predictions by incorporating a Bayesian learning (about ability) into a standard life cycle model with portfolio choice. Agents face labor income risk but do not have access to a complete set of contingent claims. They have access to two savings instruments only: a risk-free bond and a stock that pays off a higher average return with uncertainty (equity premium). The key is that agents do not have perfect information about their wage profiles. Agents cannot distinguish true ability from noise and form beliefs (priors) about their ability. The beliefs are updated every period in a Bayesian fashion. As agents grow older, their (present value) of life-time labor income becomes less risky as they update their ability as well as the duration of working life becomes shorter.

The uncertainty in human capital is calibrated to reproduce the cross-sectional variance of log-earnings along the life cycle in the PSID. Agents work from the age of 20 to 65. They receive a social security benefit upon retiring. The returns to the two savings instruments, bonds and stocks, match the equity premium between the rate of return of S&P 500 and Treasury Bill. The model successfully reproduces 3 empirical facts documented from the Survey of Consumer Finance.

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To distinguish the role of imperfect learning about the ability (which is referred to as partial information model), we also simulate the model with perfect information about ability (full information model). The partial information model first, lowers the average risky share to 46% as opposed to 76% in the model with full information. Second, it produces a positive correlation between wealth and risky share. For example, the average risky share is 43.03% for people at the first quartile and 48.49% for people at the fourth quartile. At the full information model these numbers are 85.85% and 59.38% respectively, creating a negative relationship between assets and risky share. Lastly, it produces a weakly increasing relationship between the risky share and the age. In the full information model the profile is decreasing.

**Links to the Literature**

This is not the first one to recognize the effect of labor income risk for portfolio composition. Jagannathan and Kocherlakota (1997) find that a higher share of risky assets at young ages is optimal provided the labor income has very little volatility. Cocco, Gomes and Maenhout (2005) obtains a low average share of risky investment and its positive correlation with age based on a high risk aversion and disastrous labor income shocks. Ball (2008) studies the portfolio choice in the prsence of a social security system. Gomes and Michaelides (2005) reproduces a participation rates in the stock market in the data by introducing Epstein-Zin preferences. Wachter and Yogo (2010) show that nonhomothetic preferences can explain the above puzzles.

The second strand of literature focuses on the correlation between labor income risk and stock returns. Benzoni, Collin-Dufresne and Goldstein (2007) show how the labor income and stock market returns move together at a longer time horizon. In their model stocks are much riskier for young workers than for old workers. This generates a limited participation in the stock market and increasing age profile of risky share. Storesletten, Telmer and Yaron (2004) obtains a hump shaped investment profile by introducing a countercyclical idiosyncratic risk in labor income. Lynch and Tan (2010) show that the countercyclical variation in volatility of labor income growth plays an important role in portfolio choice. By calibrating the first two moments of labor income growth to match the countercyclical volatility and pro-cyclical mean found in U.S. data they can reduce the stock holdings by the low-wealth and young households.

The paper is organized as follows. Section 2 reports empirical regularities in portfolio choices across households (by age and wealth) based on the Survey of Consumer Finances. Section 3 provides illustrative examples to highlight the economic intuition behind our model. In Section 4 we discuss the Bayesian learning about worker's ability. Section 5 presents the quantitative analysis: 5.1 describes the calibration; 5.2 presents the optimal portfolio choice in the presence of imperfect information; 5.3 compares the simulation results from our model to the data. Section 6 concludes.

## 2 Data: Portfolio Choices across Households

Using the the data from the Survey of Consumer Finances (SCF) since 1998, we examine the portfolio choices across households by age and wealth. The SCF, conducted every three years, provides information on the structure of households' characteristics and investment decisions. While we provide the detailed information about definitions of assets and household characteristics below, the upshot of investment patterns from our analysis are:

- Nearly half of the population does not own risky assets in any form of savings account.
- The percentage of households holding a risky assets increases with age until retirement.
- For those investing in risky assets, the share of risky assets increases weakly over the life cycle.
- Wealthier people tend to allocate a larger fraction of their savings towards risky assets.

**Definitions** We first group assets by the degree of riskiness. To simplify the empirical analysis and link our findings to a portfolio choice model we allocate assets into two categories, namely "safe" and "risky" assets. Some assets can be easily allocated to one of these groups. For example, checking and savings accounts are clearly riskless investments while direct stock holdings clearly involve some risk taking. However, other accounts are invested in a bundle of risk-free and risky instruments. These involve mostly mutual funds and retirement accounts. Fortunately, the Survey of Consumer Finances provides information on how these accounts are invested. The respondents are asked not only how much money they have in an account but also about the way these money are invested. To this end we break these type of accounts into subgroups of ”safe" and “risky" type depending on the respond. If (s)he reports that most of the money are in bonds, money market accounts or other risk-free instruments we categorize the account as a risk-free. If the money are invested in some form of stocks we categorize the account as risky. Most of the accounts involve investments in both risk free and risky instruments in which case we assign half of the money into each category.

The assets considered as safe are checking accounts, savings accounts, money market accounts, certificates of deposit, cash value of life insurance, U.S. government or state bonds, mutual funds invested in tax-free bonds or government backed bonds, trusts and annuities invested in bonds, money market accounts, or life insurance and finally pension plans. On the other hand assets like stocks, brokerage accounts, mortgage-backed bonds, foreign and corporate bonds, mutual funds invested in stock funds, trusts and annuities invested in stocks or real estate and pension plans that are a thrift, profit sharing or stock purchase plan are considered risky. In Table 1 we report the average amount (in '98 dollars) held in specific accounts as well as the portion of the population who holds some positive amount of dollars in this account. We restrict the sample to those who have at least some positive amount of assets. One thing stands out: people show a strong preference towards holding safe assets. While 86.9% of people hold a checking account and 60.2% holds a savings account, only 20.6% own directly stocks. At the aggregate nearly everyone some form of safe asset while less than half of the population holds some form of a risky instrument.

Table 1: **Basic Accounts**

| Type of Account | Average($) | Participation |
|---|---|---|
| Checking account | 3,483$ | 86.9% |
| Savings account | 4,746$ | 60.2% |
| Savings bond (safe) | 5,916$ | 22.4% |
| Life insurance | 8,955$ | 31.7% |
| Retirement Accounts (safe) | 12,103$ | 38.1% |
| Total safe assets | 64,126$ | 99.8% |
| Stocks | 32,831$ | 20.6% |
| Trust (risky) | 6,121$ | 1.2% |
| Mutual fund (risky) | 12,574$ | 16.3% |
| Retirement Accounts (risky) | 26,376$ | 45.2% |
| Total risky assets | 89,336$ | 56.7% |
| Total financial assets | 153,462$ | 100% |

**Portfolio choice by age groups** In Table 2 we document three statistics for the portfolio choice by age groups. First, the participation decision- the fraction of investors who hold at least one risky asset. Second, the risky share of financial assets- the fraction of total assets allocated towards risky accounts. We document this statistic for the whole population (unconditional share) and for the set of investors who hold at least some positive amount of risky assets (conditional share). Naturally, the conditional share is at least as large as the unconditional share since it excludes from the sample investors with zero money in risky accounts. Table 2 reveals that risk taking behavior changes significantly with age. Younger people are more reluctant to hold risky assets than older people. Nearly 45% of individuals at their 20's hold a risky asset. Participation peaks between the age of 51-60, where nearly 69% of people hold a risky account, and decreases after the age of 60. The risky share of financial assets naturally follows this pattern. Individuals at their 20's invest nearly 18% of their total assets in risky accounts while individuals between the age of 51-60 invest nearly 34%. The evolution of the risky share for those who hold at least one risky account features the same pattern. The risky share is weakly increasing from nearly 40% at early stages of the life cycle to 50% closer to retirement.

Table 2: **Portfolio Choice by Age**

| Age groups | Participation | Risky Share Unconditional | Risky Share Conditional |
|---|---|---|---|
| 21-30 | 45.2% | 18.1% | 40.0% |
| 31-40 | 63.1% | 27.8% | 44.1% |
| 41-50 | 66.4% | 32.2% | 48.4% |
| 51-60 | 69.2% | 34.4% | 49.7% |
| 61-70 | 50.8% | 25.4% | 49.9% |
| Average | 56.7% | 26.5% | 46.7% |

**Econometric Analysis**

We estimate the relation between portfolio share, age and financial assets using a Tobit regression model.

$$risky\ share = \beta_0\, X + \sum_{j=1}^{4} \beta_j \;\; age_{20+10i-30+10i} + \beta_5 \log assets + e$$

The dependent variable is the risky share of financial assets. Regressor $X$ includes several variables like education, sex, marital status, number of children and total income. We also include dummy variables for each age group: 21-30 (omitted), 31-40, 41-50, 51-60 and 61-70. Lastly we include the log of total financial assets. The results from the regression are given in the Appendix. Controlling for financial assets, income and demographic characteristics ages after 30 invest a higher share of financial assets to risky accounts while ages after 60 almost the same. Specifically, ages 31-40 invest on average 9.0 percentage points more in risky accounts than the base group of 21-30. Ages 41-50 invest on average 6.2 percentage points more and ages 51-60 8.3 percentage points more. Lastly, investors between the age of 61-70 invest 0.5 percentage points less in risky accounts than the base group of 21-30. Our regression reveals a positive relation between financial assets and risky share. A 1% increase in financial assets increase the risky share by 0.07%. Bertaut and Starr-McCluer (2002) also find that risky shares are hump shaped in age after conditioning on log-assets. They also find a positive relationship between the risky share and the amount of financial assets hold by the investor.

# 3 Simple Portfolio Choice Theory

The purpose of this section is to provide a quick introduction on the basic mechanics of the portfolio choice. We build two and three period models where agents split their savings between bonds and stocks. At the same time agents face stochastic labor income. We use our findings to relate the risky share of financial assets to total assets, time horizon and labor income risk. We show that under certain conditions the model fails to reproduce the..

**Model** The investor lives for $T$ periods. Each period $t$ she receives income $y_t$. Income is an i.i.d random variable with probability function $f(y_t)$. Preferences are given by

$$U = E\sum_{t=1}^{T} \beta^{t-1} \frac{{c_t}^{1-\sigma}}{1-\sigma}$$

where $\sigma$ is the coefficient of risk aversion. There are two available savings instruments. A bond $b_t$ paying gross return $R$ after one period and a stock $s_t$ whose gross payoff is stochastic and equals $R_s = R + \mu + \eta$. $\mu$ is the risk premium which induces the agent to undertake the risky investment and $\eta$ is the innovation to excess return which is distributed as $N(0, \sigma_\eta^2)$. We denote the associated probability function as $\pi(\eta)$.

**Recursive language** The investor divides her current output between consumption $c$ and savings $b' + s'$. To make the problem easier we collapse total wealth into a single state variable $W = bR + sR_s$. We do not allow people to borrow. The problem can be written as follows.

$$V_j(W,y) = \max_{c,s',b'} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta \int_{\eta'} \int_{y'} V_{j+1}(W',y') df(y') d\pi(\eta') \right\}$$

$$\text{s.t.} \quad c + s' + b' = y + W$$
$$c \geq 0, \quad s' \geq 0, \quad b' \geq 0$$

**Zero future labor income** The first case we consider is that of zero labor income. We do not specify the length of the time horizon since results can be generalized into more than two periods. In this case the risky share given by Samuelson (1963) rule:

$$\frac{s'}{s' + b'} \approx \frac{1}{\sigma} \frac{\mu}{\sigma_\eta^2}$$

The risky share 1) increases in the risk premium 2) decreases in the risk aversion 3) decreases in the volatility of stock returns. The investor wants to divide her future total income in constant shares of risky and riskless asset. This is a basic property of CRRA preferences. Since total income consists only of financial assets the risky share is independent of cash in hand. In this case time horizon (or age) is also irrelevant. What matters is the length of time between rebalancing not the investment horizon itself (Jagannathan and Kocherlakota, 1997). Assets and time horizon can matter only implicitly through the presence of labor income.

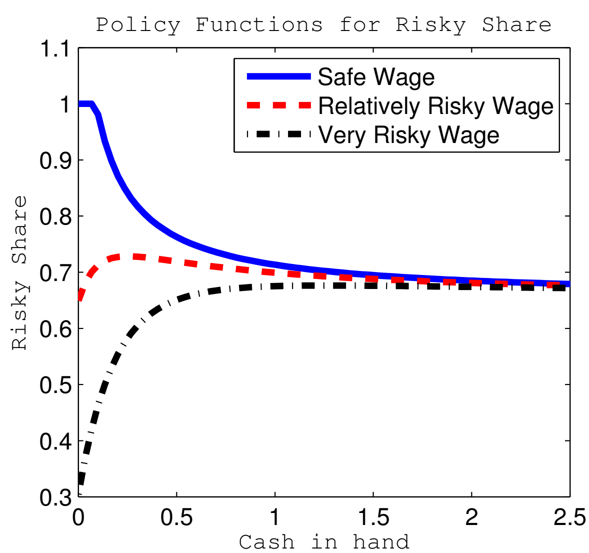


**Figure 1**: Two period example: risky share for different dispersions of labor income.

**A two-period model with positive future labor income** To understand portfolio choice in the presence of labor income, we should have in mind that the investor treats future labor income as an implicit asset. The first case we will consider is that of deterministic labor income. In this case the investor treats his future wage as a *riskless* asset. Less wealthy investors will save less so that both $s'$ and $b'$ decrease. However, since the agent is already endowed with a riskless asset (the wage), she can afford to decrease her stock holdings *less* than her bond holdings.[1] This property can be seen in Figure 1. We plot the risky share of assets $\frac{s'}{s'+b'}$ as a function of cash in hand $W+y$. Risky share decreases in cash in hand if tomorrow's wage is known with certainty. Note that the policy function hits the upper bound of one for very small values. This has to do with the no borrowing constraint. Optimally the agent would like to borrow on bonds and save on stocks but we do not allow this case. In addition, note that the policy function converges to a constant number. Intuitively, if the wage is only a small part of present discounted income, we go back to the case where the risky share is given by the Samuelson rule. Figure 1 also plots the risky share if the wage is relatively risky and very risky. In this cases we increased the dispersion of the probability distribution of future wage $f(y')$. Labor income risk has two effects on the policy function. First, at a given level of cash in hand, it decreases the risky share of assets. Intuitively, if the wage is risky the investor has one more source of income risk. To compensate for the extra risk she will allocate less fraction of her savings to stocks. Second, labor income risk alters the relation between cash in hand and risky share. In the case of risky labor income the investor has an implicit *risky* asset. Less wealthy investors will save less so that both $s'$ and $b'$ decrease. However, since the agent is already endowed with a risky asset, she minimizes her risk exposure by decreasing her stock holdings *more* than her bond holdings. As as result the risky share decreases if wealth decrease. This positive relation is more evident in the case of very risky wage. In the case of relatively risky wage we observe a positive relation between cash in hand and risky share only for small values of the former. This shows that the level of current wealth affects the perception of the investor about the riskiness of her wage. Lastly, note that all policy functions converge to the same value. Once again, if the wage is only a small part of present discounted income, the degree of riskiness becomes unimportant in determining the risky share.

**A three period model: the effect of time horizon** In this part we examine the same

[1] This result is linked to constant relative risk aversion. The investor wants to have constant shares of total riskless and risky assets. If wealth decreases the investor will decrease both bonds and stocks savings. However a part of the riskless assets (the wage) remains constant. To keep the *total* riskless share constant (including the wage) the investor will decrease the bond holdings more.

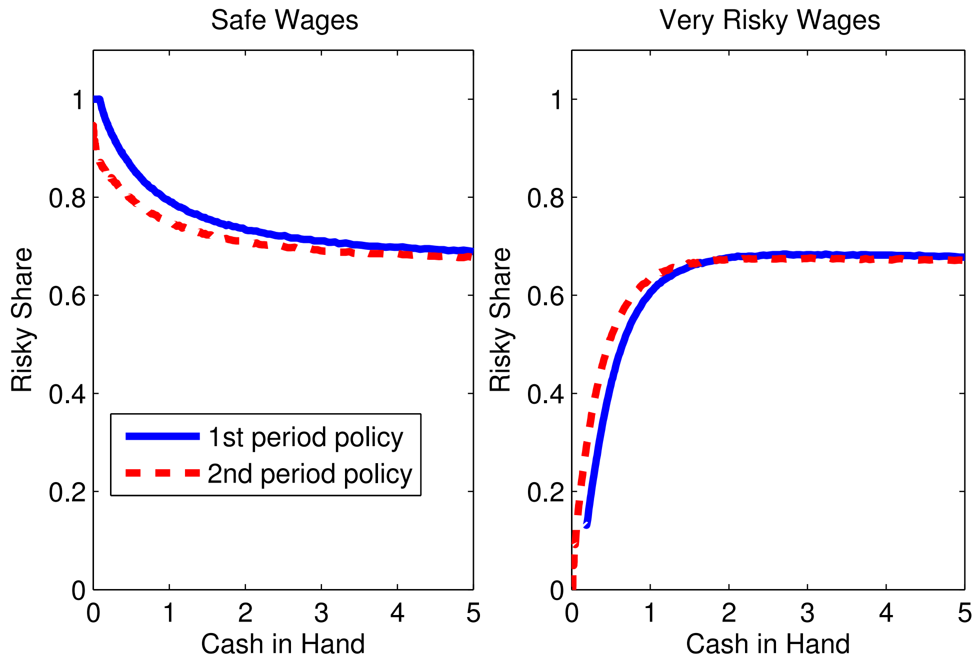


**Figure 2**: Three period example: risky share for different dispersions of labor income.

model for $T = 3$. This allows to analyze the savings decision both at the first period and at the second period. The second period of this model corresponds to the first period analyzed above. The mechanisms analyzed in the two period case also explain the relation between time horizon and risky share. The left panel of Figure 2 plots the risky share as a function of cash in hand at period 1 and 2 if labor income is deterministic. As expected the policy functions are decreasing in cash in hand since the investor treats her labor income a risk-free asset. More importantly the risky share is higher for younger cohorts than older cohorts for a given amount of cash in hand. To understand this property we should have in mind that wage is a riskless asset. In period 1 the investor is endowed with a larger stream of this riskless asset (in present discounted terms). Hence, she can afford to take extra risk by saving more in stocks. In period 2 her riskless endowment has shrunk. To compensate for this loss she switches her portfolio into riskless financial assets. To test this intuition we plot the same functions in case of larger labor income risk. As before, with high wage risk the risky share is increasing in cash in hand. In this case people save a higher fraction of their savings in stocks as they grow older. The reason is that second period's investors anticipate only on period risky wages ahead of them. Hence, their risk exposure has declined compared to the investors in the first period.

## 4 The Bayesian Learning About Ability

Labor economists have documented large variations in earnings both within and across age cohorts. Although people face a lot of heterogeneity in their earnings it is yet unclear what percent of future income is known. In this section we display the mechanics of an optimal learning model. The agent has some prior belief about the components of current wage and updates this belief after she observes her income realization. This framework has two useful properties. First, since the signals are noisy, the agent will never be absolute certain about her true type. Hence, from the workers' perspective labor income is much riskier than in a full information model. Second, people get a better sense of their true type as signals accumulate. As a result, younger cohorts face higher labor income variance than older cohorts.

The underlying assumption is that the agent of age $j$ observes her wage $y_j$, but cannot decompose it into its components. We assume that the log-wage is given by

$$y_j = a + x_j + \varepsilon_j \tag{1}$$

where $a$ is the fixed effect, $x_j$ is a transitory shock that follows an autoregressive process and $\varepsilon_j$ is a i.i.d. shock which adds noise to the system.[2] The AR(1) process is given by

$$x_j = \rho x_{j-1} + \nu_j, \qquad \text{with} \quad \nu_j \sim \text{ iid } N(0, \sigma_\nu^2) \tag{2}$$

Being able to tell whether the wage originates from the persistent shocks (fixed effect or autoregressive shock) or the i.i.d. shock is important because it affects the expectation about future wages. If the agent believes that high current wages are associated with a good fixed effect she will follow a completely different investment strategy than if she believed that high wages are caused by the i.i.d. component. Formally, the vector $\mathbf{S}_j = (a, x_j)$ is incompletely known through some noisy signals $y_j$ where

$$y_j = a + x_j + \varepsilon_j = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ x_j \end{bmatrix} = \mathbf{H}_j' \mathbf{S}_j + \varepsilon_j \tag{3}$$

and through some knowledge of the dynamic evolution of the system

[2] In the full model we also assume a deterministic life cycle trend of wages. We skip this now for convenience.

$$\mathbf{S}_{j+1} = \begin{bmatrix} a_{j+1} \\ x_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} a_j \\ x_j \end{bmatrix} + \begin{bmatrix} 0 \\ \nu_{j+1} \end{bmatrix} = \mathbf{F}\mathbf{S}_j + \mathbf{U}_{j+1} \tag{4}$$

The first equation links the noisy signals with the state variables while second set of equations are used to predict the next period state variables given today's state. The problem is to obtain the best possible estimate of next period's state variables given all available information. That is, given the signal $y_j$ as well as a belief about the states $\mathbf{S}_j = (a_j, x_j)$, what is the best possible estimate of $\mathbf{S}_{j+1} = (a_{j+1}, x_{j+1})$?

Belief is nothing more than a probability distribution. The agent has prior beliefs about the distribution of the fixed effect summarized by the first two moments $\{\mu_a, \sigma_a^2\}$. Similarly, the agent has prior beliefs about the distribution of the AR(1) shock summarized by $\{\mu_x, \sigma_x^2\}$. The agent has beliefs over the cross correlation as well $\{\sigma_{ax}\}$. In matrix form:

$$\mathbf{M}_{j|j-1} = \begin{bmatrix} \mu_a \\ \mu_x \end{bmatrix}_{j|j-1} \qquad \mathbf{V}_{j|j-1} = \begin{bmatrix} \sigma_a^2 & \sigma_{ax} \\ \sigma_{ax} & \sigma_x^2 \end{bmatrix}_{j|j-1} \tag{5}$$

Henceforth, the subscript $j|j-1$ will denote prior beliefs about a variable at age $j$ before signal $y_j$ is realized. Similarly, the subscript $j|j$ will denote posterior beliefs about the variable that use the signal $y_j$ as information. The updating occurs in a Bayesian fashion. In particular posterior means are given by

$$\mathbf{M}_{j|j} = \mathbf{M}_{j|j-1} + \mathbf{G}(y_j - \mathbf{H}_j'\mathbf{M}_{j|j-1}) \tag{6}$$

According to this formula the posterior belief is the prior belief plus a term that depends on the difference between the signal ($y_j$) and the agent's belief about her wage before the realization ($\mathbf{H}_j'\mathbf{M}_{j|j-1}$). If her current wage is different than what she expected she will update a lot her beliefs about the income components. If the wage is about what she expected then the update will be small. In both cases the update will be discounted by the term $\mathbf{G}$. This term depends on the prior variance of the distributions.[3]

If the agent is very uncertain about the distribution of a parameter she will place a smaller weight to the new information conveyed by the observed signal and vice versa. The posterior variance is given by

$$\mathbf{V}_{j|j} = \mathbf{V}_{j|j-1} - \mathbf{G}\mathbf{H}_j'\mathbf{V}_{j|j-1} \tag{7}$$

[3] In particular the term will be equal to $\mathbf{G} = \begin{bmatrix} \frac{\sigma_a^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \\ \frac{\sigma_x^2+\sigma_{ax}}{\sigma_a^2+2\sigma_{ax}+\sigma_x^2+\sigma_\varepsilon^2} \end{bmatrix}$.

Note that the variance decreases the more signals are revealed, hence the minus at the right hand side. Older cohorts face smaller uncertainty simply because they have more information about their income components. The evolution of the variance is deterministic as it does not depend on the current income $y_j$. This is a standard property of the optimal learning model.

The next step is to use the posterior distributions to forecast next period's income. The agent will enter period $j+1$ having as prior beliefs

$$\mathbf{M}_{j+1|j} = \begin{bmatrix} \mu_a \\ \mu_x \end{bmatrix}_{j+1|j} = \begin{bmatrix} \mu_a \\ \rho\mu_x \end{bmatrix}_{j|j} \tag{8}$$

$$\mathbf{V}_{j+1|j} = \begin{bmatrix} \sigma_a^2 & \sigma_{ax} \\ \sigma_{ax} & \rho^2\sigma_x^2 + \sigma_\nu^2 \end{bmatrix}_{j|j} \tag{9}$$

Using the above information next period's income will be distributed as

$$F(y_{j+1}|y_j) = N(\mathbf{H}'_{j+1}\mathbf{M}_{j+1|j}, \mathbf{H}'_{j+1}\mathbf{V}_{j+1|j}\mathbf{H}_{j+1} + \varepsilon_j) \tag{10}$$

**Learning Adjustment** The speed of learning is very important in our model. An increasing age profile of the risky share would be hard to be justified if investors found out their type quickly. Unfortunately, in the model uncertainty declines pretty fast. To slow things down we assume an exogenous adjustment parameter

$$\mathbf{A} = \begin{bmatrix} \lambda & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

that premultiplies the weight assigned to the innovations **G**. We rationalize this parameter on the basis of general and firm specific human capital. An agent entering the labor market is more likely to learn quickly about his ability to perform well within the firm she works. In contrast, her general abilities require a series of observations. Parameter $\lambda$ captures this fact. Mechanically the parameter controls the evolution of both the mean and the variance of the distribution of the fixed effect.

# 5 Quantitative Analysis

## 5.1 Model

This section describes the basic elements of the model. The question is whether we can match better the investment behavior over the life cycle by restricting the information set of the agents about their labor income. The model is solved in partial equilibrium and features endogenous portfolio choice, finite horizon and heterogeneous agents. We develop two versions of the same model. In the first the agent has full information about her labor income process (Full Information Model). In the second, she cannot decompose income innovations to its permanent and transitory components (Partial Information Model). The computation techniques to solve this model are described in the Appendix.

**Demographics** Every agent enters the labor market at age $j = 1$ and lives for J periods. There is no population growth in the model. At age $j_R$ all agents have to retire. Retirees receive a social security benefit $ss$ which is financed by proportional labor income taxes $\tau_{ss}$.

**Preferences** Agents derive utility from consumption. Leisure is not valued. Hence, agents devote their one unit of productive time to work. As in our simple example we assume CRRA preferences. Preferences are representable by a time separable utility function of the form :

$$U = E\sum_{t=1}^{T}\beta^{t-1}\frac{(c_t)^{1-\sigma}}{1-\sigma}$$

where $\beta$ is the discount factor and $\sigma$ captures both the intertemporal elasticity of substitution and the risk aversion. In the utility function we have included a consumption floor $\bar{c}$. The consumption floor makes low labor income shocks more costly. The literature has focused on alternative preferences to address the portfolio choice puzzles. For example, Gomes and Michaelides (2005) use Epstein-Zin preferences with heterogeneity in both risk aversion and intertemporal elasticity of substitution. Gomes and Michaelides (2005) use habit formation in their preferences. Wachter and Yogo (2008) use nonhomothetic preferences. Our approach is to use standard preferences with constant relative risk aversion. This way we can highlight the potential of labor income risk and learning to solve the portfolio choice puzzles.

**Wages** In the model investors face different wages along their lifetime. Many factors account for this heterogeneity. First, agents face permanent differences in productivity (fixed effects). Permanent ability is denoted $a$ and is distributed as $a \sim N(0, \sigma_a^2)$. Second, investors experience different productivity at different stages of their life cycle. The age productivity profile is denoted $\beta j$. This profile evolves deterministically along the life cycle and peaks around the age of 50. We assume that all agents face a common life cycle income profile.[4] Workers also experience idiosyncratic wage shocks. These follow an AR(1) process in logs:

$$x_j = \rho x_{j-1} + \nu_j, \quad \text{with} \quad \nu_j \sim \text{ iid } N(0, \sigma_\nu^2) \tag{12}$$

represented by as a finite state Markov chain $\Gamma(x_j | x_{j-1})$. Lastly, workers face independently distributed idiosyncratic shocks denotes as $\varepsilon_j$. The probability distribution over these shocks is denoted $f(\varepsilon)$. The natural logarithm of wages for agent $i$ of age $j$ is given by

$$y_{ij} = a_i + \beta j + x_j + \varepsilon_j \tag{13}$$

**Asset Markets** We assume an incomplete markets model. Agents cannot fully insure against labor income shocks but can partially insure using tow savings instruments. A risk free bond $b$ paying gross return $R$ in terms of consumption units after one period. And a stock $s$ which has payoff $R_s = R + \mu + \eta$. $\mu$ is the equity premium while $\eta$ is the innovation to excess return and is distributed as $N(0, \sigma_\eta^2)$. That is on average the stock pays more to convince investors to bear the extra risk.[5] We assume that investors can costlessly rebalance their portfolio each period. In the model savings takes place for two main reasons. First, investors save to prepare for retirement (life cycle savings). Second they save to insure against negative labor income shocks (precautionary savings). At the same time we do not allow investors to borrow. This constraint can be rationalized on the basis of standard moral hazard and adverse selection arguments.

**Participation cost** To participate in the stock market investors have to pay a fixed monetary cost. A common explanation for these costs is information costs. Stock market participation often requires a significant amount of time to monitor firms' performance, analyze stock market trends and acquire information about growth prospects. At the same time, most portfolios are handled by intermediaries who demand monetary payments.

[4] We find that incorporating heterogeneity in income profiles has little difference in the results. At the same time, the computational cost of an extra state variable is large.

[5] We do not allow innovations to be correlated with the component of permanent labor income.

The fixed cost is denoted $FC$.

**Government** The government engages only in one activity: it runs a balanced social security system. The tax rate $\tau_{ss}$ is set to assure the system's budget balance. In the presence of the social security system investors expect a stream of certain labor income, that is, independent of their past labor shocks, once they reach retirement. This allows them to take more risk throughout their life cycle especially as they approach retirement.

**Information Structure** We will analyze two models. The first is a full information model. In this case agents can observe both the wage $y$ and its separate components $a, \beta, x, \varepsilon$. Labor income is risky because of the innovations to the transitory shock $\nu$ and the i.i.d. component $\varepsilon$. The second model is a partial information model. In this case agents can only observe the wage $y$ and the life cycle component $\beta$ but not the other income components separately. Every period they make predictions based on their current beliefs and the current signal. Income uncertainty is much higher in this model since the investor does not know her fixed effect $a$ or the accumulated innovations up to her age as summarized by component $x$.

**Investors' problem - Full Information Model** We describe the problem recursively. As stated before, in this case the investor observes each component separately. The state variables are total wealth $W = bR + sR_s$, the fixed effect $a$, the transitory component $x$ and the i.i.d component $\varepsilon$. The $j$ period value function is denoted as $V_j(W, a, x, \varepsilon)$. The continuation value $V_{j+1}$ is stochastic for two reasons. First, the income is stochastic both because of the innovations $\nu$ and $\varepsilon$. Second, next period's wealth $W'$ is stochastic because the stock return $R_s$ is random. This interaction between stochastic labor income and stochastic wealth is the centerpiece of our analysis. Every period investors divide their total income $(y+W)$ to consumption $(c)$ bond savings $(b')$ and stocks savings $(s')$. After the retirement age they don't have any labor income but they do receive a social security benefit $ss$.

$$V_j(W,a,x,\varepsilon) = \max_{c,s',b'} \left\{ u(c) + \beta \int_{\eta'} \int_{x'} \int_{\varepsilon'} V_{j+1}(W',a',x',\varepsilon') df(\varepsilon') d\Gamma(x'|x) d\pi(\eta') \right\} \tag{14}$$

$$\text{s.t.} \quad c + s' + b' = (1-\tau_{ss})e^{y_j} + ss\{j \geq j_R\} - FC\{s' > 0\} + W \tag{15}$$

Equation (15) is the worker's budget constraint if she participates in the stock market. Labor income net of social security contributions and current wealth is divided among consumption and stock savings and bond savings. If stock savings are positive the investor has to pay the fixed cost $FC$. Retired investors receive only the social security benefit.

**Investors' problem - Partial Information Model** In the partial information case the income components cannot be separately observed. The state variables are current income $y_j$, and the probability distributions over the vector $\mathbf{S}_j = (a, x_j)$. These distributions are described by the first two moments $\mathbf{M}_{j|j-1}$ and $\mathbf{V}_{j|j-1}$. However, we do not have to explicitly include the second moment in the value function since age is a sufficient statistic to describe its evolution.[6] The $j$ period value function is denoted as $V_j(W, y, \mathbf{M}_{j|j-1})$.

$$V_j(W, y, \mathbf{M}_{j|j-1}) = \max_{c, s', b'} \left\{ u(c) + \beta \int_{\eta'} \int_{y'} V_{j+1}(W', y', \mathbf{M}_{j+1|j}) dF_j(y'|y) d\pi(\eta') \right\}$$

$$\text{s.t.} \quad c + s' + b' = (1 - \tau_{ss}) e^{y_j} + ss\{j \geq j_R\} - FC\{s' > 0\} + W \tag{16}$$

The probability distribution $F_j(y'|y)$ is given by (10). Both the current signal $y_j$ and the current belief $\mathbf{M}_{j|j-1}$ form an expectation about next period's income $\mathbf{H}'_{j+1}\mathbf{M}_{j+1|j}$. The dispersion of values along that mean depends on the variance $\mathbf{V}_{j|j-1}$ i.e. on the number of signals accumulated up to age $j$. Note that beliefs affect current decisions only through changing expectations about tomorrow's income.

This problem is solved as follows

- Using her signal and her beliefs the agent updates the priors based on (6) and (7).

- The posterior distributions describe next period's states based on (8) and (9)

- The expectation about next period's income is given by (10)

[6] This will be true if agents begin the life cycle as having the same beliefs. We assume this in the simulation.

## 5.2 Calibration

This section discusses the parametrization of the model. There are three set of parameters in our model.

- Preference parameters $\{\sigma, \beta\}$.

- Parameters relating to the asset market structure $\{\mu, \sigma_\eta^2, r^b, \tau_{ss}\}$

- Parameters that govern the labor income process $\{\rho, \sigma_a^2, \sigma_\varepsilon^2, \sigma_\nu^2, \lambda\}$ .

The coefficient of risk aversion is an important determinant of the risky share. Higher values of risk aversion deter investors from saving into risky assets. Mehra and Prescott (1985) show that one needs an extremely high value of risk aversion in order to rationalize the high equity premium observed in the data. Cocco, Gomes and Maenhout (2005) use a value of 10. We use a value of $\sigma = 5$. The average risky share also depends on the discount factor $\beta$. In Figure 2 we showed how the risky share depends on the amount of assets being hold by the investor. Hence, matching the average asset holdings is crucial for making good predictions about the risky share. We endogenously calibrate the discount factor using the capital output ratio equal to 3. Labor income risk is different between the FIM and PIM. Hence, we recalibrate the discount factor in each case. The FIM model produces a value $\beta = 1.05$. The PIM leads to a lower value of 0.97. This is because the latter model generates more precautionary savings. The consumption floor is set at the value of the lowest productivity shock. Following CGM (2005) we set the equity premium equal to $\mu = 4\%$. This moderate value is common in the literature and represents the equity received by the investors net of taxes. The risk free rate is set to $R = 1.02$ based on the average real rate of US 3-month treasury bills in the post war period. The standard deviation of the innovations to the risky asset is set at 0.18 based on Gomes amd Michaelides (2005). These values combined produce a risky share of about 24% for investors whose assets are so large so that labor income becomes insignificant. This value is calculated based on the the Samuelson rule. Finally the social security tax is set at $\tau_{ss} = 10\%$.

The last set of parameters involves the labor income process as well as the prior beliefs of the investor. Our numbers are based on Storesletten, Telmer and Yaron (2004) who use the variance of log-earnings along the life cycle to match the dispersion in the fixed effect, the dispersion in the transitory shock, the persistence of the AR(1) process and the

| Parameter | Notation | Value |
|---|---|---|
| Risk Aversion | $\sigma$ | 5 |
| Discount Factor: FIM (PIM) | $\beta$ | 1.05 (0.97) |
| Risk free rate | $R$ | 1.02 |
| Equity Risk Premium | $\mu$ | 0.04 |
| Stock Return Volatility | $\sigma_\eta$ | 0.18 |
| Transition probability for stock | $\pi(\eta)$ | 0.5 |
| Social Security Tax | $\tau_{ss}$ | 0.10 |
| Variance of fixed effect | $\sigma_a^2$ | 0.2173 |
| Variance of transitory component | $\sigma_v^2$ | 0.035 |
| Persistence parameter | $\rho$ | 0.985 |
| Variance of iid component | $\sigma_\varepsilon^2$ | 0.063 |
| Learning Parameter | $\lambda$ | 0.5 |

distribution of the iid component. Specifically, the variance of the fixed effect is set at $\sigma_a^2 = 0.21$ to match the variance of log earnings at the initial stages of the working life cycle. The variance of the transitory component is set at $\sigma_v^2 = 0.035$ to match the variance of log earnings at the later stages of the working life cycle while the persistence parameter $\rho$ is set to 0.985 to match the linear transition between these two stages. For the variance of the i.i.d component we use the value of $\sigma_\varepsilon^2 = 0.06$. The value of $\lambda$ is set at 0.5. It is less clear how to determine the amount of prior beliefs of the investor. The empirical distribution of the fixed effect provides an upper bound for the amount of uncertainty faced by the individual upon entering the labor market. At the same time it would be reasonable to assume that although uncertain the investor still knows more about her type than the econometrician. However, in order to understand the relative strength of the learning mechanism we choose the priors based on the empirical distributions.

$$\mathbf{V}_{j+1|j} = \begin{bmatrix} 0.21 & 0 \\ 0 & 0.035 \end{bmatrix}_{j|j} \tag{17}$$

## 5.3 Policy Functions

Before simulating the model it is instructive to look at the policy functions, in particular the risky share of financial assets. We will compare the risky share rules for both the FIM and the PIM. This will provide intuition about the differences in the results between the two models.

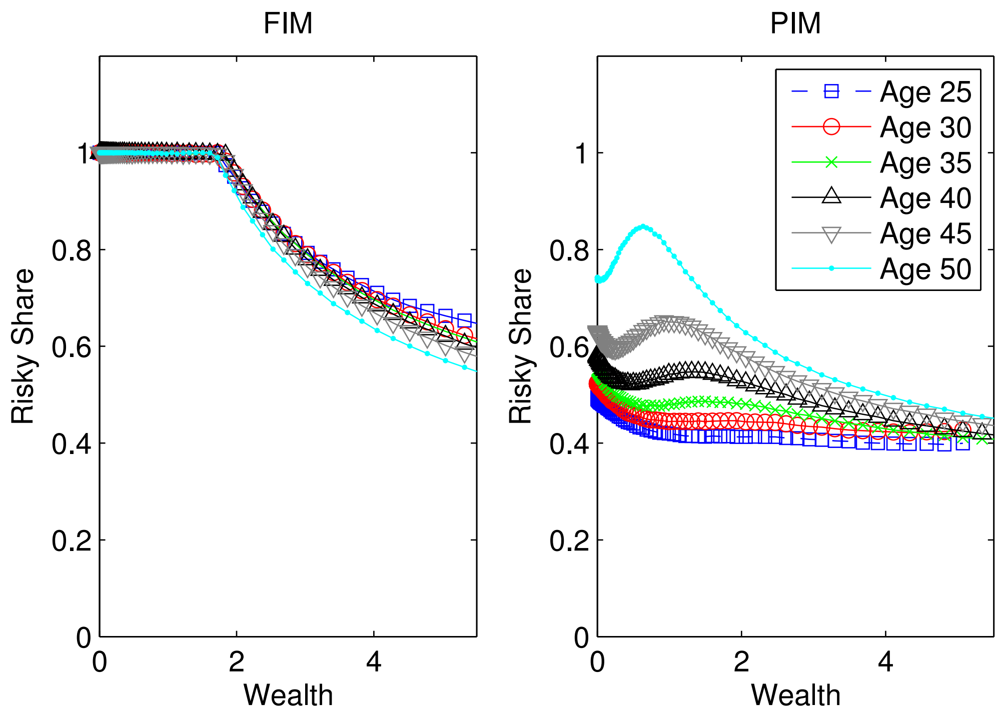


**Figure 3**: Risky Share and Wealth: *Left Panel:* Full Information model. *Left Panel:* Partial Information model.

In Figure 3 we plot the policy functions for several age groups for both the FIM (left panel) and the PIM (right panel). In the left panel we observe that 1) the policy functions are decreasing in wealth and 2) the policy functions are decreasing in age. As shown in Section 2 both properties are related to labor income risk. In the FIM agents are aware of their type from the beginning of the life cycle. The only uncertainty they face comes from the stochastic innovations of the transitory shock and the i.i.d. shock. With small uncertainty labor income is an implicit safe asset. Wealthier people will decrease their risky share to avoid extreme risk exposure while older people will decrease their risky share since they expect a smaller stream of safe labor income. At the right panel we plot policy functions for the same ages and productivity when beliefs are approximately correct. Both properties change in this case. First, especially for young people the policy functions are flat or weakly decreasing. At the same time older people have higher risky share since for those ages much of the uncertainty has been resolved.

### 5.4 Simulation Results

In this part we examine the implications of the two models for the portfolio choice. To this end we simulate a panel of 10,000 individuals and track them over time. Once again we make comparisons between the two models.

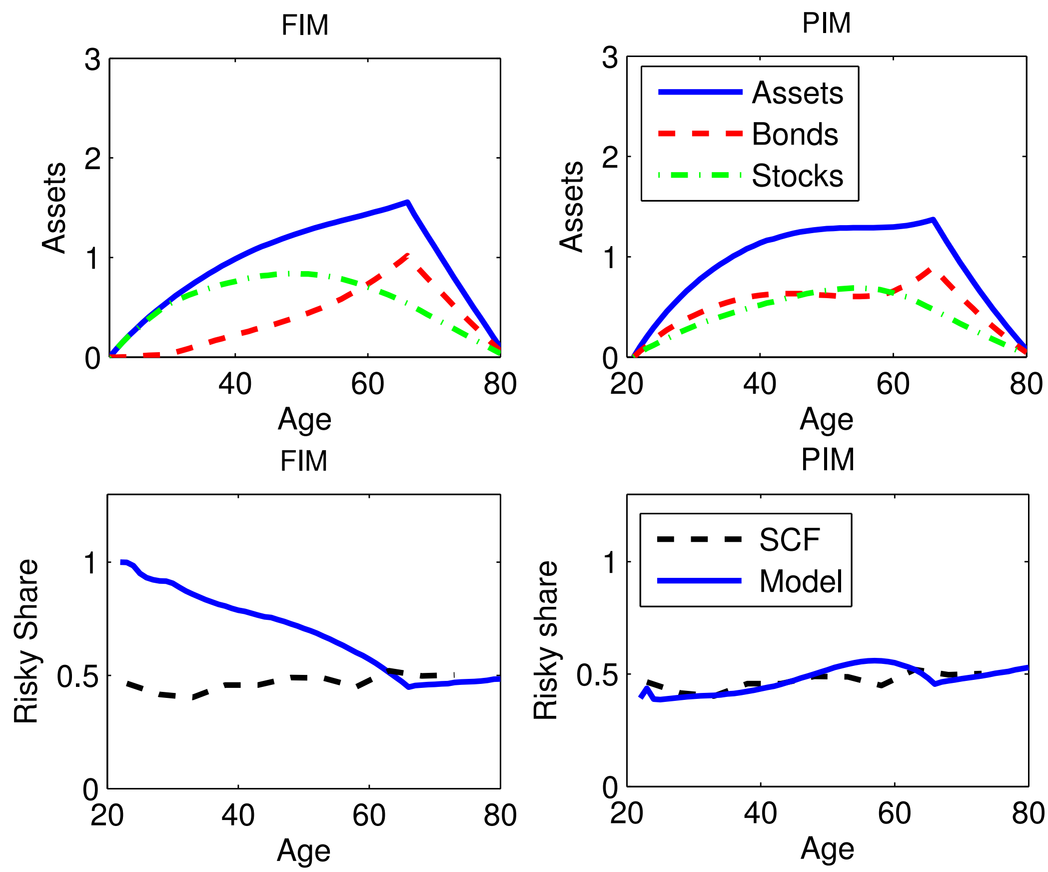


**Figure 4**: Life Cycle Profiles: *Left Panel:* Full Information model. *Left Panel:* Partial Information model.

**Life Cycle Profiles** The upper panel of Figure 4 plots the life cycle profiles for bond holdings, stock holdings and total assets for both the FIM and the PIM. The lower panels plot the risky share of total financial assets for the two models as well as the risky share as calculated in the SCF. Looking at the upper panels we observe a different investment strategy across the models. In the left upper panel, which represents the FIM, investors accumulate too much of stocks and too little of bond holdings early in life. After the age of 45 they rebalance their portfolio towards bond holdings and decrease their stocks. This leads to a decreasing risky share of assets along the life cycle (lower left panel). So why do investors decrease on average their risky share in this model? For two reasons. The first is that wealthier people tend to decrease their risky share (Figure 1). As people approach retirement their average savings increase. Hence this is movement along a decreasing policy function. The second reason has to do with the age effects on the risky share. In the FIM people are aware of their type. Younger people expect a longer stream of certain (high or low) income than people closer to retirement. As a result, they invest relatively more in stocks. This is a movement between policy functions. The PIM predicts a different investment behavior. People start accumulating bonds even at early stages of the life cycle. At the same time stock holdings continue to increase up to the age of 60. This results in a weakly increasing risky share along the life cycle. To understand this behavior we should take another look at the policy functions in Section 7. In the PIM wealthier people increase or decrease less their risky share. This reflects the larger uncertainty about their labor income. As a result the movement along the policy function does not generate large decrease in the risky share. In addition, at the PIM older people face higher labor income uncertainty compared to younger because of the information update. This leads to an upward shift in policy functions as investors approach retirement. Comparing the risky shares we can see that the PIM does a much better job matching the empirical risky share.

Table 3: The Risky Share of Financial Assets: FIM

| **Risky share** | **Q1** | **Q2** | **Q3** | **Q4** | **Average** |
|---|---|---|---|---|---|
| **Age 21-30** | 98.21% | 89.76% | 92.91% | 99.17% | 94.81% |
| **Age 31-40** | 79.71% | 85.44% | 85.82 % | 79.06% | 82.50% |
| **Age 41-50** | 78.17% | 82.61% | 68.83% | 66.19% | 74.00% |
| **Age 51-60** | 76.82% | 69.48% | 55.80% | 50.92% | 62.93% |
| **Age 61-65** | 68.93% | 56.60% | 40.58% | 36.82% | 50.31% |
| **Average** | 85.85% | 80.43% | 79.50% | 56.38% | 75.48% |

**Risky Share Decomposition** To compare further the ability of the models to match the portfolio behavior we report the share of financial assets held in stocks *within* age and wealth groups. Table 4 performs such a decomposition. The FIM predicts a decreasing risky share both across age and across wealth groups. The average risky share for people at the second quartile is 80.43% in the FIM but 44.01% in the SCF (Table 2). The average risky share at the fourth quartile is 56.38% in the FIM while 63.3% in the SCF. This reflects the shape of the policy functions for the full information case. Wealth-poor people allocate

Table 4: The Risky Share of Financial Assets: PIM

| Risky Share | Q1 | Q2 | Q3 | Q4 | Average |
|---|---|---|---|---|---|
| **Age 21-30** | 39.07% | 37.34% | 39.06% | 43.65% | 39.88% |
| **Age 31-40** | 35.60% | 40.71% | 44.93 % | 46.40% | 42.06% |
| **Age 41-50** | 45.41% | 48.91% | 47.83% | 49.74% | 48.06% |
| **Age 51-60** | 60.31% | 56.41% | 48.70% | 55.23% | 55.07% |
| **Age 61-65** | 67.55% | 52.74% | 42.04% | 42.46% | 50.96% |
| **Average** | 43.03% | 48.74% | 46.31% | 48.99% | 46.83% |

almost 100% of their savings to stocks. This happens because labor income risk is low and savings are only a very small part of future income. Hence, these groups can afford to take extra risk. Wealth rich people allocate a much smaller part in stocks. For these groups assets are a significant part of total future income, so they are more cautious. In contrast, the PIM predicts a positive correlation between wealth and risky share. The average risky share for people at the second quartile is 48.74% in the PIM but 44.01% in the SCF (Table 2). The average risky share at the fourth quartile is 48.49% in the PIM while 63.3% in the SCF. The model succeeds in capturing the relation between wealth and risky share for reasons already explained. If labor risk is high then wealthier people can afford to save more in stocks than poorer agents if they expect the same amount of wages. If they didn't they would have taken too less risk. We next turn our attention to the relation between the risky share and time horizon, within wealth groups. Within the wealth groups the risky share decreases along the life cycle for the FIM but increases for the PIM. This weakly increasing pattern is also seen in the SCF (Table 3). Once again, this results from younger people facing higher uncertainty than older people in the PIM.

# 6 Conclusion

We develop a life-cycle model with Bayesian learning about labor-market ability to account for three portfolio choice puzzles in the literature: (i) investors hold a substantially low share of risky assets; (ii) the share of risky assets is disproportionately larger for richer households, (iii) the risky-asset share increases with age. Our model can explain all three puzzles. A risky nature of human capital (uncertainty about labor market outcomes) moves households towards a less risky financial portfolio. The perceived life-time labor-market uncertainty is substantially larger for young households as they have not learned about themselves. Over time, the uncertainty in the labor market is gradually resolved and households can take on more risk in investment on financial market. On average, poor (as well as young) households rely heavily on labor earnings, making them to hold safe assets. By comparing our benchmark to the model with full information (perfect information about the labor market ability), we find that learning is essential in generating these results. Recognizing the labor-market risk at different stages of life helps us to understand the household behavior in the financial market. Our results strongly suggest an important interaction in risks between financial and labor markets.

# References

**Alan, Sue (2006)**. “Entry costs and stock market participation over the life cycle." *Review of Economic Dynamics*, 9(4): 588-611.

**Bertaut and Starr-McCluer (2002)**. “Household Portfolios in the US," in *Household Portfolios*, Cambridge,MA MIT Press.

**Benzoni, Luca, Pierre Collin-Dufresne and Robert S. Goldstein (2007)**. “Portfolio choice over the life cycle when the stocks and labor markets are cointegrated." *Journal of Finance*, 62(5): 2123-2167.

**Cocco, Joao F., Francisco J.Gomes, and Pascal J. Maenhout (2005)**. “Consumption and portfolio choice over the life cycle." *Review of Financial Studies*, 18(2): 491-533.

**Gomes Francisco J., and Alexander Michealides (2005)**. “Optimal life cycle asset allocation: Understanding the empirical evidence." *Journal of Finance*, 60(2): 864-904.

**Guiso Luigi, Michael Haliassos, and Tullio Jappelli (2002)**. “Household Portfolios". Cambridge, MA: MIT Press

**Guvenen Fatih (2007)**. “Learning you Earning: Are Labor Income Shocks Really Very Persistent?" *American Economic Review*, 97 (June): 687-712.

**Huggett Mark, Gustavo Ventura and Amir Yaron (2011)**. “Sources of Lifetime Inequality" *American Economic Review*, 101 (December): 2,923-54.

**Jagannathan, Ravi and Narayana R.Kochrlakota (1996)**. “Why should older people invest less ins stocks than younger people?" *Federal Reserve Bank of Minneapolis Quarterly review*, 20(3): 11-23.

**Lynch, Anthony W. and Sinan Tan (2008)**. “Labor income dynamics at business cycle frequencies: Implications for portfolio choice."*Journal of Financial Economics*, forthcoming.

**Mehra Rajnish and Edward C. Prescott (1985)**. “ The equity premium: A puzzle." *Journal of Monetary Economics*, 15(2), 145-161.

**Storesletten, Kjetil, Chris Telmer and Amir Yaron (2004)**. "Consumption and risk sharing over the life cycle." *Journal of Monetary Economics*, vol. 51, pp. 609-633, 144: 2277-2292.

**Storesletten, Kjetil, Chris Telmer and Amir Yaron (2004)**. "Asset pricing with idiosyncratic risk and overlapping generations." *Review of Economic Dynamics*, 10(4): 519-548.

**Storesletten, Kjetil, Chris Telmer and Amir Yaron (2004)**. "Cyclical dynamics in idiosyncratic labor market risk." *Journal of Political Economics*, 112(3): 695-717.

**Wachter, Jessica A. and Motohiro Yogo (2010)**. "Why Do Household Portfolio Shares Rise in Wealth?." *Review of Financial Studies*, 23(11): 3929-3965.

# Appendix

Table 5: Econometric Analysis

| Explanatory Variable | Coefficient |
|---|---|
| Education | -.0287 |
| | (.0068) |
| Male | .0233 |
| | (.0068) |
| Log Income | .0623 |
| | (.0041) |
| Married | -.0166 |
| | ( .0073 ) |
| Children | -.0157 |
| | ( .0017 ) |
| Age 31-40 | .1260 |
| | ( .0083 ) |
| Age 41-50 | .0877 |
| | ( .0080 ) |
| Age 51-60 | .1169 |
| | ( .0088 ) |
| Age 61-70 | -.0079 |
| | ( .0103 ) |
| Log Assets | .1068 |
| | ( .0016 ) |
| Observations | 21,525 |