

Structural Multi-Equation Macroeconomic Models: Identification-Robust Estimation and Fit*

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Abstract

Weak identification is likely to be prevalent in multi-equation macroeconomic models such as in dynamic stochastic general equilibrium setups. Identification difficulties cause the breakdown of standard asymptotic procedures, making inference unreliable. While econometric techniques catering to weak-instrument problems initially gained relevance in macroeconomics due to their application to single equations such as the New Keynesian Phillips Curve, more work is currently emerging on multi-equation set-ups.

In this paper, we develop a set of identification-robust econometric tools that, regardless of the model's identification status, are useful for estimating and assessing the fit of a system of structural equations. We consider two approaches, one based on the econometric specification defined by the closed form rational-expectations-consistent solution of the DSGE model (MC-FI), and the other based on the econometric model defined by the orthogonality restrictions associated with the Euler Errors of the considered model (EE-GMM). Both methods rely on inverting identification-robust tests.

We apply our methodologies, using U.S. data, to the standard New Keynesian model such as the one studied in Clarida, Gali, and Gertler (1999). We find that, despite the presence of identification difficulties, our proposed methods are able to shed some light on the fit of the considered model and, particularly, on the nature of the NKPC. Notably our results show that (i) confidence intervals obtained using our system-based approach are generally tighter, and thus more informative, than their univariate-based counterparts, (ii) model coefficients except some related to the output gap are significant at conventional levels, and (iii) when we use the EE-GMM approach and a real-time output gap measure, the NKPC is preponderantly forward-looking.

JEL classification: C52, C53, E37

1. Introduction

Optimization-based macroeconomic models, and, in particular, dynamic stochastic general equilibrium (DSGE) setups, are popular nowadays for analyzing a multitude of macroeconomic questions such as the effects of monetary policy. But as models of this sort become increasingly complex, featuring many types of markets, various rigidities, and different nonlinearities, the decision of whether to use a *limited* or *full* information (*LI* or *FI*) approach for estimation becomes a central question for model developers. Indeed, there appears to be a conflict in the conclusions of available published studies based on one or the other method. A striking example is the ongoing debate in the sizeable empirical literature with regard to the importance of the forward-looking component of the New Keynesian Phillips Curve (NKPC) equation. Recent contributions to this discussion include Galí, Gertler, and Lopez-Salido (2005) that uses LI methods, and Linde (2005) that uses FI methods, and which report opposite outcomes with respect to the forward-looking nature of the curve.

The LI/FI trade-off is an enduring econometric problem, often presented as one of weighing specification bias versus efficiency, but there are also other concerns. In particular, advances in econometrics regarding weak-instruments and weak-identification have revealed that the latter plague LI and FI methods equally, thus presenting a set of new challenges for applied researchers.

The macroeconomic literature acknowledges the LI/FI trade-off to some extent, often presenting it as one of deciding between Instrumental Variable (IV) or maximum likelihood estimation (MLE); see, for example, Jondeau and LeBihan (2008). Furthermore, published studies in the field are also familiar with the fact that weak instruments effects are critical to IV-based model performance. However, the implications of weak-identification on MLE seem to be less understood, and indeed often confused with issues related to very large estimated standard errors or poorly-approximated test cut-off points. While it may be argued that likelihood-ratio (LR) criteria have more attractive finite sample properties than, for example, IV-based Wald-type ones, and in particular, size correction techniques have a much better chance of success with LR statistics (see Dufour 1997), it should be emphasized that standard MLE and full-information maximum likelihood (FIML) inference are not immune to weak-identification problems.

The complications arise largely because nonlinearities can impose discontinuous param-

eter restrictions that cause the breakdown of standard asymptotic procedures. Given the connection between the parameters of the underlying theoretical model and those of the estimated econometric model¹, and given the identifying constraints imposed on the model, econometric versions of macroeconomic models are often highly nonlinear.² The more rich and complex the macroeconomic model, the more likely it is that standard regularity conditions will not fully hold. In this case, even when MLE is used for the estimation, resorting to usual t -type significance tests or Wald-type confidence intervals will lead to the same problems that plague GMM and linear or nonlinear IV;³ see the surveys of Stock, Wright, and Yogo (2002) and Dufour (2003). As may be checked from these studies, identification difficulties will not always lead to huge regular standard errors that would alert the researcher to the problem. Instead, spuriously tight confidence intervals could occur, often concentrated on wrong parameter values, thus leading to wrong inference.

Weak-instruments and weak-identification concerns have led to the development of so-called *identification-robust* procedures, *i.e.* procedures that achieve significance or confidence-level control (at least asymptotically) whether the statistical model is weakly or strongly identified, or whether instruments are weak or strong.⁴

In the analysis of DSGE-type models, problems such as errors-in-variables, underidentification, weak instruments, and specification concerns cannot be avoided. Furthermore, it is quite difficult to deal with these econometric problems simultaneously and convincingly using traditional inference methods.⁵ At the same time, econometric techniques catering

¹See Galí, Gertler, and Lopez-Salido (2005) and Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007) on the importance of maintaining these constraints.

²Even within the context of a single linear simultaneous equation, where identification is achieved through “exclusion” restrictions, the latter imply nonlinearity. This is easy to see when one derives the reduced-form or the structural likelihood function.

³We specifically mean Wald-type confidence intervals of the form [estimate \pm (asymptotic standard error) \times (asymptotic critical point)], intervals based on the *delta*-method, and even ones based on various bootstraps.

⁴See, for example, Dufour (1997), Dufour (2003), Staiger and Stock (1997), Wang and Zivot (1998), Zivot, Startz, and Nelson (1998), Dufour and Jasiak (2001), Kleibergen (2002), Kleibergen (2005), Stock, Wright, and Yogo (2002), Moreira (2003), Dufour and Taamouti (2005), Dufour and Taamouti (2007), and Andrews, Moreira, and Stock (2006).

⁵For comprehensive surveys on dealing with some of these issues in the presence of identification problems, see Stock, Wright, and Yogo (2002) and Dufour (2003). See also Dufour (1997), Staiger and Stock (1997), Wang and Zivot (1998), Zivot, Startz, and Nelson (1998), Dufour and Jasiak (2001), Kleibergen (2002), Kleibergen (2005), Dufour and Taamouti (2005), Dufour and Taamouti (2007), Andrews, Moreira, and Stock (2006), Hoogerheide, Kaashoek, and van Dijk (2007), Joseph and Kiviet (2005), Kiviet and Niemczyk (2007),

to weak-instrument problems have gained relevance in macroeconomics, initially due to their application to the NKPC and more recently to more general multi-equation set-ups.⁶

In this paper, we contribute to the literature concerned with identification in multi-equation systems and DSGE-type set-ups. First, we develop a set of identification-robust econometric tools that are useful for estimating and assessing the fit of a system of structural equations. We consider two approaches, one based on the econometric specification defined by the closed form rational-expectations-consistent solution of the DSGE model (MC-FI), and the other based on the econometric model defined by the orthogonality restrictions associated with the Euler Errors of the considered model (EE-GMM). Both methods rely on inverting identification-robust tests. While our EE-GMM method relates to the S-sets from Stock and Wright (2000), our MC-FI is, to the best of our knowledge, new to econometrics.

Second, we apply these tools, using U.S. data, to assess the fit of the standard New Keynesian model. This fundamental structure has been extensively studied in the literature (see, for example, Clarida, Gali, and Gertler 1999), and forms the building block of many other more complex models (see, for instance, Woodford 2003, Christiano, Eichenbaum, and Evans 2005, Del Negro, Schorfheide, Smets, and Wouters 2007, to mention a few.) To allow for comparisons between our newly-proposed system-based multivariate method and univariate ones, we also consider the univariate method applied by Dufour, Khalaf, and Kichian (2006), and the univariate linear IV method from Dufour and Taamouti (2005). Each method integrates and assesses, to a different degree, the model's structural restrictions.

The paper is organized as follows. In section 2, we introduce the model that we assess. Our methodology is discussed in section 3. Data and empirical results are presented in section 4. We conclude in section 5. A technical Appendix complements the methodology section.

Bolduc, Khalaf, and Moyneur (2008), and Chaudhuri and Zivot (2008).

⁶See, for example, Ma (2002), Mavroeidis (2004), Mavroeidis (2005), Mavroeidis (2006), Dufour, Khalaf, and Kichian (2006), Canova and Sala (2009), Nason and Smith (2008), Komunjer and Ng (2009a), Komunjer and Ng (2009b) Kleibergen and Mavroeidis (2009), Kleibergen and Mavroeidis (2010), Iskrev (2010), Guerron-Quintana, Inoue, and Kilian (2009), Moon and Schorfheide (2009), Granziera, Lee, Moon, and Schorfheide (2009), Schorfheide, Moon, Granziera, and Lee (2009), Mavroeidis (2009), and Magnusson and Mavroeidis (2010).

2. Framework

Though our method is applicable to more complex structures, we consider here a variant of the standard New Keynesian model. The latter, extensively studied by Clarida, Gali, and Gertler (1999), forms the building block of numerous recent fundamental models, and for our purposes is tractable enough to allow comparisons between our proposed multivariate approach and available univariate ones (see later sections). Specifically, we follow the setup in Linde (2005) that consists of a system of three equations: an NKPC equation, an aggregate demand equation and an interest rate rule:

$$\begin{aligned} \pi_t &= \omega_f E_t \pi_{t+1} + (1 - \omega_f) \pi_{t-1} + \gamma y_t + \varepsilon_{\pi,t} \\ y_t &= \beta_f E_t y_{t+1} + \sum_{i=1}^4 (1 - \beta_f) \beta_{y,i} y_{t-i} - \beta_r^{-1} (R_t - E_t \pi_{t+1}) + \varepsilon_{y,t} \\ R_t &= \gamma_\pi \left(1 - \sum_{i=1}^3 \rho_i\right) \pi_t + \gamma_y \left(1 - \sum_{i=1}^3 \rho_i\right) y_t + \sum_{i=1}^3 \rho_i R_{t-i} + \varepsilon_{R,t} \end{aligned} \quad (1)$$

where, for $t = 1, \dots, T$, π_t is aggregate inflation, y_t is the output gap, and R_t is the nominal interest rate, and $\varepsilon_t = (\varepsilon_{\pi,t}, \varepsilon_{y,t}, \varepsilon_{R,t})'$ is a zero-mean disturbance with variance/covariance matrix Ω ; we assume that Ω is invertible, and is not necessarily diagonal.⁷ Model (1) differs from Linde's specification since, as in e.g. Benati (2008), the coefficient on $(R_t - E_t \pi_{t+1})$ in the output equation corresponds to the inverse of the intertemporal elasticity of substitution. We estimate the intertemporal elasticity of substitution (denoted β_r), to reflect the recent DSGE literature. It is worth noting that Linde (2005) assumes a diagonal covariance matrix. For notational clarity, we will call the vectors

$$\theta = \left(\omega_f, \gamma, \beta_f, \beta_r, \gamma_\pi, \gamma_y, \rho_1, \rho_2, \rho_3 \right)' \quad (2)$$

$$\phi = \left(\theta', \beta_{y,1}, \beta_{y,2}, \beta_{y,3}, \beta_{y,4} \right)' \quad (3)$$

the model's “deep” parameters, and we let Θ and Φ denote the associated parameter spaces.

For estimation purposes, we consider, in turn, two empirical approaches based on: (i) the econometric specification defined by the closed form rational-expectations-consistent solution of (1), and (ii) the econometric model defined by the orthogonality restrictions associated with the three-dimensional Euler Errors from (1). We denote the former a Model-Consistent [MC] Full-Information [or MC-FI] approach, and the latter an Euler-Error [EE] based GMM [or EE-GMM] approach. Both methods treat the three equations as a system [that is we do

⁷It is worth noting that Linde (2005) assumes a diagonal covariance matrix.

not use a limited information equation by equation GMM or LIML method], and both exploit a systems-based metric that gauges statistical fit (discussed in the next section). While our EE-GMM method relates to the S-sets from Stock and Wright (2000), our MC-FI is to the best of our knowledge new to econometrics.

Assuming *i.i.d.* Gaussian errors and a closed specification, (5) [under certainty equivalence] can be written in the state-space form

$$\Gamma_{\mathbf{L}}(\phi) Y_{t-1} + \Gamma_{\mathbf{C}}(\theta) Y_t + \Gamma_{\mathbf{F}}(\theta) Y_{t+1} + C(\Omega) e_t = 0, \quad (4)$$

where $\Gamma_{\mathbf{L}}(\phi)$, $\Gamma_{\mathbf{C}}(\theta)$, $\Gamma_{\mathbf{F}}(\theta)$ and $C(\Omega)$ are (for presentation ease) defined in the Appendix and

$$Y_t = \left(\pi_t \quad y_t \quad y_{t-1} \quad y_{t-2} \quad y_{t-3} \quad R_t \quad R_{t-1} \quad R_{t-2} \right)'$$

With conventional restrictions on the model parameters, (4) can be solved into the VAR form

$$Y_t = B(\phi) Y_{t-1} + S(\phi, \Omega) e_t \quad (5)$$

where the error terms e_t are three-dimensional *i.i.d.* multivariate standard normal. We use Anderson and Moore type solvers.

Alternatively, a GMM-based estimation strategy may be pursued, that avoids solving the model. Specifically, model (1) implies that the vector $(\pi_t^*(\theta), y_t^*(\theta), R_t^*(\theta))'$ where

$$\begin{aligned} \pi_t^*(\theta) &= \pi_t - \omega_f \pi_{t+1} - (1 - \omega_f) \pi_{t-1} - \gamma y_t, \\ y_t^*(\theta) &= y_t - \beta_f y_{t+1} + \beta_r (R_t - \pi_{t+1}), \\ R_t^*(\theta) &= R_t - \left(1 - \sum_{i=1}^3 \rho_i \right) (\gamma_\pi \pi_t + \gamma_y y_t) - \sum_{i=1}^3 \rho_i R_{t-i}, \end{aligned}$$

interpreted as 3-dimensional Euler error, is uncorrelated with available instruments for the true value of $\theta \in \Theta$, leading to testable orthogonality conditions.

For further reference, we define

$$\begin{aligned} Z_{1t} &= (\pi_{t-1}, R_{t-1}, R_{t-2}, R_{t-3})', \\ Z_{2t} &= (y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4})'. \end{aligned}$$

The elements of Z_{1t} and Z_{2t} are the predetermined variables in the system, that is, correspond to the elements the elements of Y_{t-1} , to which we refer to as "intra-model" instruments.

Because closing the model is not necessary from a GMM perspective, additional instruments may also be introduced to correct for *e.g.* measurement errors or simply to expand the available information set. We refer to the vector of these "extra-model" instruments as \tilde{Z}_t .

For a critical discussion of the FI versus GMM methods, the reader may refer to Canova (2007) (Chapters 4-6). Both estimation strategies have advantages and disadvantages, none restricted to the model under consideration. However, both methods share the following difficulty. If the confidence intervals and hypothesis tests that result from both estimation strategies are, as is typically the case, validated through the use of standard asymptotic arguments, they can easily become unreliable when there are identification difficulties.

It is important to understand the fundamental reason behind such failures.⁸ Nonlinear constraints complicate statistical analysis in a non-trivial way because associated transformations may be discontinuous. That is, some or all of the parameters may become identifiable only on a subset of the parameter space. To reflect the resulting parameter uncertainty, estimation uncertainty cannot be controlled unless estimated confidence regions are allowed to be unbounded. Stated differently, one cannot keep error probability margins within any desired level using any method that leads to a confidence interval with bounded limits. Therefore, intervals of the form $\{\text{estimate} \pm (\text{standard error}) \times (\text{critical point})\}$, including the delta-method, cannot be size-corrected because they have bounded limits by construction. Furthermore, weak identification does not necessarily imply that estimated standard errors are very large; indeed, the opposite may occur, with tight confidence intervals concentrated on wrong parameter values.

3. Methodology

To relate our methodology to existing methods, we first present our EE-GMM type approach. For clarity of presentation our discussion is mostly descriptive but nonetheless formal; complete formulae and further references are relegated to the Appendix. Our presentation is organized as follows. We first explain, in sections 3.1 and 3.2, the motivation underlying each approach. Subsequently, sections 3.3 and 3.4 present the statistics we use in each case. Both approaches share the following fundamental principles.

⁸Regularity conditions do not hold or hold only weakly when there are identification difficulties. Please refer to the econometric literature cited in the introduction for further formal discussions.

In traditional estimation methodology, a point estimate is found first, and confidence intervals are then constructed. The approach we use proceeds in reverse: first we build a confidence region then we obtain a point estimate from the latter. The confidence region at a given level (say 95%) is obtained by "inverting" a test designed such that its significance level can be kept within a desired level (say 5%) whether identification is weak or strong. A test that satisfies this property and its associated confidence region are called "identification-robust".⁹

Inverting a test means assembling, analytically or numerically, the set of parameter values that are not rejected by this test. This requires solving a set of inequalities in the model parameters, defined through the test's rejection region.¹⁰ Assembling the values of the parameter that are not rejected by this statistic requires solving for the parameter values such that the corresponding t-statistic is below its critical point; it is easy to see that the above defined interval provides the solution to this inequality.

To see this, observe that the statistic we plan to invert is a function of the data as well as the parameter value under test. To facilitate our exposition, denote the function in question as $\mathcal{T}(\phi_0)$ where ϕ_0 is the candidate value of ϕ under test, and ϕ is the above defined [see (2)-(3)].¹¹ Furthermore, let \mathcal{T}_c refer to the test's 5% critical point, $p_{\mathcal{T}}(\phi_0)$, the p-value associated with $\mathcal{T}(\phi_0)$ and assume that the test is right-tailed. Inverting this test produces the set of ϕ_0 values that satisfy the following inequality:

$$\phi_0 : \mathcal{T}(\phi_0) < \mathcal{T}_c, \quad \text{or equivalently,} \quad \phi_0 : p_{\mathcal{T}}(\phi_0) > 5\%.$$

The solution to this inequality may or may not be a bounded set. As explained above, this ensures that the confidence regions so obtained will cover the unknown true values of ϕ . Relatively wide confidence sets reveal weak identification.

The least-rejected parameters or alternatively, the parameters associated with the largest

⁹A test whose significance level can be kept within a desired level whether identification is weak or strong or even when identification does not hold is also referred to, in the econometric literature, as a test that "does not require identification".

¹⁰For example, the usual confidence interval of the form [estimate \pm (standard error) \times (critical point)] results from inverting a t-statistic of the form: parameter estimate minus hypothesized parameter value divided by estimated standard error

¹¹For simplicity, we emphasize dependence on ϕ_0 whereas dependence on the data is implicit in our notation.

test p-values, that is

$$\hat{\phi} = \arg \max_{\phi_0} p_{\mathcal{T}}(\phi_0)$$

provide point estimates.¹² Test inversion may also generate an empty confidence region, if $\mathcal{T}(\phi_0) > \mathcal{T}_c$ for all relevant ϕ_0 . This can be interpreted as a significant J-type test, providing an overall assessment of the structural model restrictions.

While a large econometric literature has documented the superiority of such methods¹³, multi-equation models have not been directly addressed. In this paper, we propose two tests for inversion purposes that are adapted to models such as (1). In what follows, we first introduce the tests that are inverted. We next explain how the non-rejected parameter values are collected so confidence regions are formed.

3.1 Euler-Error GMM-based estimation: motivation

Our EE-GMM based method produces a confidence region with level $1 - \alpha$ for the deep parameter θ , that inverts an identification-robust test (presented below) associated with the null hypothesis

$$H_{01} : \theta = \theta_0 \tag{6}$$

where θ_0 is given by

$$\theta_0 = \left(\omega_f^0, \gamma^0, \beta_f^0, \beta_r^0, \gamma_\pi^0, \gamma_y^0, \rho_1^0, \rho_2^0, \rho_3^0 \right)',$$

and where the parameter values with the zero superscript are known values. We introduce a 3-equation system of artificial regressions

$$\begin{aligned} \pi_t^*(\theta_0) &= Z'_{1t} \vartheta_{1,\pi} + Z'_{2t} \vartheta_{2,\pi} + \tilde{Z}'_t \eta_\pi + u_{\pi,t}, \\ y_t^*(\theta_0) &= Z'_{1t} \vartheta_{1,y} + Z'_{2t} \vartheta_{2,y} + \tilde{Z}'_t \eta_y + u_{y,t}, \\ R_t^*(\theta_0) &= Z'_{1t} \vartheta_{1,R} + Z'_{2t} \vartheta_{2,R} + \tilde{Z}'_t \eta_R + u_{R,t}, \end{aligned} \tag{7}$$

where $u_t = (u_{\pi,t}, u_{y,t}, u_{R,t})'$ is a zero-mean disturbance with variance/covariance matrix Σ_u . Observe that Σ_u embeds the cross-equation restrictions from (1).

¹²These are the so-called Hodges-Lehmann point estimates (see Hodges and Lehmann 1963, 1983, and Dufour, Khalaf, and Kichian 2006).

¹³See, for example, Dufour (1997), Dufour (2003), Staiger and Stock (1997), Wang and Zivot (1998), Zivot, Startz, and Nelson (1998), Dufour and Jasiak (2001), Dufour and Taamouti (2005), and Dufour and Taamouti (2007).

In this context, (6) implies that the following restrictions hold

$$H_{01}^* : \vartheta_{1,\pi} = \vartheta_{1,y} = \vartheta_{1,R} = 0, \quad \vartheta_{2,\pi} = \vartheta_{2,R} = 0, \quad \eta_\pi = \eta_y = \eta_R = 0. \quad (8)$$

Hence, testing for H_{01}^* within (7) provides a test of (6). H_{01}^* consists of exclusion restrictions of the Seemingly Unrelated Regression [SURE] form, that is, the restrictions differ with the different equations in (7). In particular, the structure does not restrict the lags of output out from the second equation (the output equation) of (7), so H_{01}^* involves 20 exclusion restrictions when no extra-model instruments are used.

Mapping H_{01} into H_{01}^* is convenient because (7) does not require statistical identification [the right-hand side regressors are not “endogenous”] so usual statistics for testing the exclusion of regressors can be applied in a straightforward manner. Furthermore, since the left-hand-side of (7) is the stacked 3-dimensional Euler error conforming with (1), the exclusion restrictions in H_{01}^* embody the usual GMM orthogonality restrictions. In addition, the contemporaneously correlated disturbances within (7) embeds the correlation structure of the considered Euler errors.

To assess H_{01}^* , traditional criteria may be used. Indeed, this is the intuition exploited by Stock and Wright (2000) leading to inverting the GMM objective function; related arguments also underlie the procedures analyzed in Kleibergen and Mavroeidis (2009), in the context of a *single-equation* New Keynesian Phillips Curve. A weighting matrix treated as a function of θ matters importantly in this context. An optimal weighting matrix and continuous updating is required for some of the efficient methods proposed by Kleibergen and Mavroeidis (2009).

Our test criterion (defined below) for assessing the exclusion restrictions in H_{01}^* differs from traditional GMM practices in the following ways. (1) We use a weighting function that depends explicitly on the tested θ_0 value of the parameter for efficiency and identification robustness purposes, yet we avoid the iterative continuous updating optimal GMM.¹⁴ (2) We embed the cross-equation restrictions from (1) into the test, via analytically tractable estimates of $\Sigma_u(\theta_0, \Omega)$. Since the Euler-Errors bypass the rational-expectation (RE) solution of (1), the fact remains that all cross-equation restrictions from (1) cannot be accounted for

¹⁴Our method does not require iterating the GMM objective function, because we rely on the SURE artificial regression framework. In this case it is well known [see Dufour and Khalaf (2002a-b, 2003) and the references therein] that iterative GLS does not pay off (that is, does not improve efficiency) in practice. Moving from the GMM to the SURE-GLS context via an artificial regression that captures the same orthogonality restrictions decreases numerical burdens immensely without efficiency costs.

via any GMM-type method. The FI statistic we introduce next achieves this objective.

3.2 Model-consistent Full-Information estimation: motivation

Our MC-FI method produces a confidence region with level $1 - \alpha$ for the deep parameter ϕ , that inverts an identification-robust test (presented below) associated with the null hypothesis

$$H_{02} : \phi = \phi_0 \quad (9)$$

where $\phi_0 = \left(\theta'_0, \beta_{y,1}^0, \beta_{y,2}^0, \beta_{y,3}^0, \beta_{y,4}^0 \right)'$, θ_0 is given by

$$\theta_0 = \left(\omega_f^0, \gamma^0, \beta_f^0, \beta_r^0, \gamma_\pi^0, \gamma_y^0, \rho_1^0, \rho_2^0, \rho_3^0 \right)'$$

and the parameter values with the zero superscript are known values that, in contrast with our EE-GMM type test, are restricted so that an associated rational expectation [RE solution exists.

Following our EE-GMM rationale, we define the artificial regression denoted

$$\mathbf{Y}_t^*(\phi_0) = \Pi' Y_{t-1} + U_t \quad (10)$$

where

$$\mathbf{Y}_t^*(\phi) = \left(\pi_t^*(\theta) \quad y_t^*(\theta) - \sum_{i=1}^4 (1 - \beta_f) \beta_{y,i} y_{t-i} \quad R_t^*(\theta) \right)', \quad (11)$$

and U_t is a three-dimensional zero-mean disturbance with variance/covariance matrix Σ_U . Here again, Observe that Σ_U embeds the cross-equation restrictions from (1).

In this context, (9) implies that the following restrictions hold

$$H_{02}^* : \Pi = 0. \quad (12)$$

Indeed, the left-hand-side of (10) coincides with the components of the vector

$$\mathbf{Y}_t(\phi_0) = (\Gamma_{\mathbf{L}}(\phi_0) Y_{t-1} + \Gamma_{\mathbf{C}}(\theta_0) Y_t + \Gamma_{\mathbf{F}}(\theta_0) Y_{t+1})$$

that do not correspond to identities in (4)-(5) which under the null reduces to

$$Y_t - B(\phi_0) Y_{t-1} = S(\phi_0, \Omega) e_t \quad (13)$$

Hence, testing for H_{02}^* within (10) provides a test of (9) so long as the RE solution exists. One may thus focus on testing, within the subspace restricted by the existence of a model-consistent solution, the 24 exclusion restrictions in H_{02}^* .

3.3 An Identification-Robust GMM-type Test Criterion

In the context of (7), the test criterion [denoted $\mathcal{W}(\theta_0)$] that we use is one of the most popular F-type Wald statistics in SURE analysis, and is given in equation (17) of the Appendix. we use an F approximate distribution [with degrees-of-freedom that depend on the sample size and on the number of predetermined variables and instruments used in the test] to obtain associated p -values.¹⁵ The approximate limiting null distribution for $\mathcal{W}(\theta_0)$ is $F(m, n(T-k))$ with

$$n = 3, \quad m = 2(8 + q) + (4 + q)$$

where n is the dimension of the Euler error, q as is the number external instruments in \tilde{Z}_t [if any are used and if not $q = 0$] used and $k = 8 + q$ is the number of regressors per equation in (7). This null distribution is thus standard and does not depend on unknown parameters even if the instruments used are weak. This result holds because (7) is a classical multivariate linear regression. $\mathcal{W}(\theta_0)$ applied to (7) differs from Stock and Wright's GMM-objective-function based statistic and the usual continuously updated GMM-objective-function based statistic mainly via its weighing matrix.¹⁶

Furthermore, the resulting cut-off points for the $\mathcal{W}(\theta_0)$ statistic is the same for any value θ_0 under test. So if we define

$$\overline{\mathcal{W}} = \min_{\theta_0} \mathcal{W}(\theta_0),$$

referring the latter to an $F(m, 3(T-k))$ cut-off point (say at level α) provide an identification robust J-test, since

$$\min_{\theta_0} \mathcal{W}(\theta_0) \geq F_\alpha(m, n(T-k)) \Leftrightarrow \mathcal{W}(\theta_0) \geq F_\alpha(m, n(T-k)), \quad \forall \theta_0 \quad (14)$$

where $F_\alpha(\cdot)$ denotes the α -level cut-off point under consideration. In other words, (14) implies that the $F(m, n(T-k))$ distribution provides valid and identification-robust conservative bounds on the null distributions of $\overline{\mathcal{W}}$.

The latter specification check can be carried out before the test inversion step to save computation time; if the outcome is not significant [*i.e.* if $\min_{\theta_0} \mathcal{W}(\theta_0) < F_\alpha(m, n(T-k))$],

¹⁵See (Theil, 1971, Chapter 6) and (Srivastava and Giles, 1987, Chapter 10) and the references therein. The statistic was analyzed by Dufour and Khalaf (2003). In a system with three equations, its F-based asymptotic approximation was shown to be relatively more stable (in terms of size control) than the χ^2 counterpart; the statistic also performed better than its likelihood ratio counterpart, the latter requiring bootstrapping to stabilize its size. In this paper, we rely on these result.

¹⁶Both statistics are asymptotically equivalent under the regularity conditions in Stock and Wright (2000).

then we can be sure that the associated confidence sets for θ will not be empty. Such specification tests can clearly be very useful tools for modelers, whether they are applied on their own or in conjunction with the test inversion problem. In view of the underlying nonlinearity, the latter minimizations must be performed numerically. We recommend a global optimization procedure such as Simulated Annealing because there is no reason to expect that $\mathcal{W}(\theta_0)$ is a smooth function of θ_0 .

We also show in the Appendix that missing instruments will not invalidate our fundamental results. In other words, if our test does not account for all explanatory variables that define the reduced form, the significance level of the test will not be affected. This also means that a full definition of the fundamental DGP is not required. This is formally shown in the Appendix where we also demonstrate robustness (in the sense of significance level control) to misspecification of the DGP underlying model (5). In particular, we show that the $\mathcal{W}(\theta_0)$ statistic is compatible with a general class of reduced forms. For instance, all we need is: (i) to assume that inflation, output, and the interest rate variable can jointly be explained, up to possibly contemporaneously-correlated disturbances, by their own lags (via some linear or nonlinear VAR form), (ii) a number of predetermined variables, which may or may not come from the theoretical model (intra-model or external instruments), and (iii) possibly a set of further exogenous or predetermined variables which were not included in the test [i.e. exogenous or predetermined variables that intervene in the fundamental data generating process yet were “missed” in the sense or “not considered” by the econometrician. Most importantly, our exposition in the Appendix implies that the latter missing instruments have no incidence on the test’s validity.

Since the model we consider does not imply cross-equation constraints, it is possible (and valid) to apply the univariate approach of Dufour, Khalaf, and Kichian (2006) on an equation-by-equation basis. Of course, the derived confidence sets across equations will not be simultaneous (global size control is not warranted). For the same reason, one may also relax all constraints and estimate each regression equation from the system (5) as a linear simultaneous equation, using the methodology from Dufour and Taamouti (2005). In this fashion, it is possible to analyze how results are affected as more restrictions are relaxed while still maintaining endogeneity and possible errors-in-variables.

3.4 An Identification-Robust Model-Consistent Full-Information Test Criterion

In the context of (10), the test criterion [denoted $\mathcal{L}(\phi_0)$] that we use is one of the most popular likelihood based statistics in multivariate regression analysis, and is given in equation (22) of the Appendix. we use an F approximate distribution [with degrees-of-freedom that depend on the dimension of the system (here $n = 3$), the sample size and on the number of predetermined variables used in the test] to obtain associated p -values.¹⁷ The approximate limiting null distribution for $\mathcal{L}(\phi_0)$ is $F(Kn, \mu\tau - 2\lambda)$ where $K = 8$ is the number of predetermined variables in the system and μ, τ and λ are given in (23)-(25) and depend only on n, T and K . This null distribution is again standard and does not depend on unknown parameters even if identification is weak. This result holds because (10) is a classical multivariate linear regression.

Here again, the resulting cut-off points for the $\mathcal{L}(\phi_0)$ statistic is the same for any value ϕ_0 under test. So if we define

$$\bar{\mathcal{L}} = \min_{\phi_0} \mathcal{L}(\phi_0),$$

referring the latter to a F cut-off point (say at level α) with degrees-of-freedom as (23)-(25), in provide an identification robust J-test, since

$$\min_{\phi_0} \mathcal{L}(\phi_0) \geq F_\alpha(Kn, \mu\tau - 2\lambda) \Leftrightarrow \mathcal{L}(\phi_0) \geq F_\alpha(Kn, \mu\tau - 2\lambda), \quad \forall \phi_0 \quad (15)$$

where $F_\alpha(\cdot)$ denotes the α -level cut-off point under consideration. Here again, this specification check can be carried out before the test inversion step to save computation time.

The above inference method is system-based and has a likelihood-based justification, yet is not strictly FIML.

3.5 Test Inversion Procedure

The test inversion procedure that we present in this section must also be conducted numerically. First, using a grid search over the economically-meaningful set of values for θ , in the

¹⁷The statistic which corresponds to a monotonic transformation of the the LR criterion was analyzed by Dufour and Khalaf (2002). In a system with three equations, its F-based asymptotic approximation was shown to be relatively more stable (in terms of size control) than its χ^2 counterpart, the latter requiring bootstrapping to stabilize its size. In this paper, we rely on these result.

case of $\mathcal{W}(\theta)$ and for ϕ in the case of $\mathcal{L}(\phi)$, we sweep, in turn, the choices for θ_0 and ϕ_0 . In the case of $\mathcal{L}(\phi)$, the choices for ϕ_0 are restricted to satisfy the RE solution exists. Formally, we accept ϕ_0 values for which the above defined $B(\phi_0)$ and $S(\phi_0, \Omega)$ exist. To do so, we check for the usual existence conditions, for every candidate ϕ_0 value, and to impose H_{02} , we replace Ω in the Anderson and Moore algorithm by the empirical variance/covariance matrix of the vector $Y_t^*(\phi_0)$, that is by $\hat{U}_0' \hat{U}_0 / T$.

For each choice considered, we compute the test relevant test statistic, $\mathcal{W}(\theta_0)$ or $\mathcal{L}(\phi_0)$ and its associated p -values using the above defined relevant F distribution. The parameter vectors for which the p -values are greater than the level α thus constitute a confidence set with level $1 - \alpha$.

Alternatively, it is possible to construct projection-based confidence sets. These can be obtained for any function $g(\theta)$ [or $h(\phi)$] by minimizing and maximizing (we use simulated annealing) the functions $g(\theta)$ over θ [or $h(\phi)$ over ϕ] such that $\mathcal{W}(\theta) < F_\alpha(m, n(T - k))$ [or $\mathcal{L}(\phi_0) < F_\alpha(Kn, \mu\tau - 2\lambda)$].

Components of θ are defined as a linear combination of θ , of the form $a'\theta$ where a is a conformable selection vector (consisting of zeros and ones).¹⁸ For example, $\omega_f = \begin{pmatrix} 1, & 0 & \dots & 0 \end{pmatrix} \theta$. So confidence intervals for all elements of θ are obtained by numerical maximization of the associated $a'\theta$ function over θ such that $\mathcal{W}(\theta) < F_\alpha(m, n(T - k))$. The same applies in the case of ϕ .

To find point estimates within our confidence region, we look for the values of θ_0 [or of ϕ_0] that lead to the largest p -value. These values are the most compatible with the data, or, alternatively, correspond to the “least rejected” model. We run the grid search check to verify the model rejections reported in the next section.

4. Empirical Results

We conduct our applications using U.S. data for the sample extending from 1962Q1 to 2005Q3. We use the GDP deflator for the price level, P_t , and the Fed Funds rate as the short-run interest rate. For the output gap, we consider two measures. The first is a real-time measure of the output gap, in the sense that the gap value at time t does not use information beyond that date. This ensures that the lags of the output gap are valid for use

¹⁸For a description of a similar procedure in a univariate setting, see e.g. Dufour and Jasiak (2001).

as instruments. Thus, as in Dufour, Khalaf, and Kichian (2006), we proceed iteratively: to obtain the value of the gap at time t , we detrend GDP with data ending in t . The sample is then extended by one observation and the trend is re-estimated. The latter is used to detrend GDP, and yields a value for the gap at time $t + 1$. This process is repeated until the end of the sample. A quadratic trend is used for this purpose. The second measure is the standard quadratically-detrended output gap as in Linde (2005), and which is included for comparison purposes. We then take the log of both these output gap series. We also use the marginal cost.

Our EE-GMM based estimations can be conducted using either intra-model instruments, or intra-model instruments supplemented with external ones. As external instruments, we consider lags 2 and 3 of both wage and commodity price inflation. Finally, as in Linde (2005), all our data is demeaned prior to estimation. All estimation with the standard quadratically-detrended output gap lead to empty confidence sets, leading to reject the model.

Recent studies on identification robust method have shown that test rejections may be driven by wrongly using instruments that are correlated with error terms; see Doko-Tchatoka and Dufour 2008. So spurious rejections would occur when, in the data, shocks are uncorrelated with time $t - \bar{\tau}$ information, yet time $t - (\bar{\tau} - 1)$ variables are used as instruments. The standard gap measure uses forward-looking data, so one cannot ensure that its lags are provably legitimate instruments.

Indeed, from Table 1, we can first see that the model is rejected with the standard output gap measure whether the EE-GMM or the MC-FI is applied. The model is not rejected with the other two measures, namely the real-time output gap and the marginal cost measure. Table 2 reports parameter estimates and associated projections for these two cases.

Notice that projections are generally wide around estimated parameter values, sometimes covering all of the considered parameter space. At the same time, we find, for some parameters, that projections are often tighter with the GMM-type approach, specially when extra-model instruments are used. In this respect, we can conclude that the NKPC is forward-looking based on the EE-GMM approach and the real-time output gap measure. The projections are (0.57-0.99) when only the intra-model instruments are used, and they are tighter, at (0.66, 0.85), when extra-model instruments are also used. On the other hand, when the marginal cost measure is used, the projections on ω_f are tighter when the MC-FI method is used, leading to the same conclusion that the NKPC is forward-looking.

Table 1. Systems Test Results

Equation	Test	GMM-type	GMM-type (intra-model)	FI-type
Instruments		Intra-model	Intra and Extra-model	Intra-model
Standard	J-type statistic	2.148		2.138
Output Gap	p-value	0.003		0.002
Real-time	J-type statistic	1.080	1.005	1.356
Output Gap	p-value	0.083	0.057	0.122
Marginal	J-type statistic	0.844	1.042	1.512
Cost	p-value	0.660	0.407	0.058

An important outcome is that links between the real and nominal sides of the economy are not significant. In particular, the parameter on the output gap/marginal cost measure in the NKPC equation is not significant.

Finally, comparisons with single-equation-based identification-robust methods, shown in Table 4, reveal that more information can be obtained on the NKPC using our system-based approaches. With the single-equation approaches, though the Hodges-Lehmann point estimates, using either the real-time output gap or marginal cost measures, stand at values of above 0.5, indicating a forward-looking curve, the projections around the estimates cannot rule out backward-looking behaviour. However, as we discuss above, with the system-based approaches, we can conclude that the NKPC is forward-looking.

Table 2. Parameter estimates and Projection-based confidence Regions				
Equation		GMM-type	GMM-type (intra-model)	FI-type
Instruments		Intra-model	Intra and Extra-model	Intra-model
Real-time output Gap				
NKPC	ω_f	0.821 [0.571, 0.990]	0.748 [0.657, 0.848]	0.787 [0.396, .990]
	γ	-0.016 [-0.005, -0.000]	-0.011 [-0.003, -0.000]	-0.013 [-0.007, 0.002]
Output	β_f	0.374 [0.016, 0.542]	0.471 [0.039, 0.556]	0.260 [0.000, 0.460]
	β_r	30.0 [13.858, 30.0]	30.0 [24.591, 30.0]	25.919 [10.448, 30.0]
Taylor Rule	γ_π	.960 [.914, 1.000]	.959 [.934, 0.983]	.961 [0.881, 1.031]
	γ_y		1.074 [1.049, 1.101]	1.064 [0.994, 1.154]
	ρ_1	1.067 [0.954, 1.179]	1.064 [1.002, 1.125]	1.142 [0.882, 1.269]
	ρ_2	-.417 [-0.573,-0.262]	-.424 [-0.511,-0.337]	-0.566 [-0.708,-0.144]
	ρ_3	.241 [-0.262, 0.344]	.248 [0.190, 0.305]	0.321 [0.056, 0.429]
Marginal Cost				
NKPC	ω_f	0.818 [0.279, 0.990]	0.818 [0.419, 0.990]	0.813 [0.606, .999]
	γ	0.006 [-0.010, 0.009]	0.006 [-0.009, 1.012]	0.000 [-0.004, 0.000]
Output	β_f	0.702 [0.000, 0.990]	0.702 [0.000, 0.990]	0.817 [0.000, 0.909]
	β_r	25.588 [4.343, 30.0]	25.588 [5.445, 30.0]	29.981 [22.664, 30.0]
Taylor Rule	γ_π	.912 [.742, 1.084]	.912 [.747, 1.057]	.921 [0.842, 0.984]
	γ_y	1.161 [1.007, 1.338]	1.161 [1.302, 1.350]	1.160 [1.088, 1.255]
	ρ_1	1.152 [0.861, 1.451]	1.152 [0.876, 1.419]	1.113 [1.044, 1.329]
	ρ_2	-.502 [-0.924,-0.076]	-.502 [-0.910,-0.127]	-0.387 [-0.753,-0.311]
	ρ_3	.273 [-0.009, 0.548]	.273 [-0.026, 0.550]	0.201 [0.173, 0.384]

Table 3. Estimates of output lags in the output equation

	FI-type	FI-type
	Real-time Gap	Marginal Coast
$\beta_{y,1}$	0.000 [0.000, 0.053]	0.004 [0.000, 0.043]
$\beta_{y,2}$	1.106 [0.880, 1.295]	1.141 [-0.282, 1.108]
$\beta_{y,3}$	0.006 [-0.279, 0.354]	0.854 [-0.019, 1.648]
$\beta_{y,4}$	-0.112 [-0.311,-0.073]	0.001 [-1.114,0.123]

Table 4: Single-Equation Methods; Inflation Equation

Coefficient	Intra-Model Instruments (RE case)					
	Linde-gap		RT-gap		MC	
	GAR	DT	GAR	DT	GAR	DT
ω_f	0.805 (0.275,0.995)	(0.892,1.379)	0.795 (0.045,0.995)	(0.866,1.440)	0.805 (0.275,0.995)	(0.720,1.994)
γ	0.015 (-0.095,0.135)	(-0.137,0.026)	-0.015 (-0.095,0.055)	(-0.095,0.082)	0.015 (-0.095,0.135)	(-0.499,0.180)
Coefficient	Intra- and Extra-Model Instruments (GMM-type case)					
	Linde-gap		RT-gap		MC	
	GAR	DT	GAR	DT	GAR	DT
ω_f	0.735 (0.315,0.995)	(0.891,1.230)	0.730 (0.310,0.995)	(0.865,1.191)	0.745 (0.345,0.995)	(0.776,1.294)
γ	-0.015 (-0.10,0.055)	(-0.115,0.026)	-0.010 (-0.09,0.060)	(-0.090,0.054)	0.015 (-0.095,0.125)	(-0.168,0.170)

MC is marginal cost. ω_f is the coefficient on π_{t+1} and γ is the coefficient on the gap or marginal cost term. DT results are for the no-constraints Dufour-Taamouti test case.

5. Conclusion

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Appendix

A.1 The Euler-Error GMM-type statistic

Consider the multivariate regression (7) rewritten in stacked form as

$$\epsilon(\theta_0) = (I_n \otimes X)b + u \quad (16)$$

where $n = 3$, $X = [X_1, \dots, X_T]'$ is the $T \times k$ matrix of instruments with t -th row equal to $X'_t = (Z'_t, \tilde{Z}'_t)$ so $k = 8 + q$,

$$\epsilon(\theta_0) = (\pi_1^*(\theta_0), \dots, \pi_T^*(\theta_0), y_1^*(\theta_0), \dots, y_T^*(\theta_0), R_1^*(\theta_0), \dots, R_T^*(\theta_0))'$$

is the nT -dimensional vector of Euler Errors evaluated at θ_0 , and

$$u = (u_{\pi,1}, \dots, u_{\pi,T}, u_{y,1}, \dots, u_{y,T}, u_{R,1}, \dots, u_{R,T})'$$

with variance/covariance matrix $(\Sigma_u \otimes I_T)$. In this context, the transformed null hypothesis (8) may be tested using the usual SURE-type F tests. We consider the statistic:

$$\mathcal{W}(\theta_0) = \left(\frac{n(T-k)}{m} \right) \frac{(A\hat{b})' \left[A \left(\hat{\Sigma}_u \otimes (X'X)^{-1} \right) A' \right]^{-1} (A\hat{b})}{(e(\theta_0) - (I_n \otimes X)\hat{b})' \left(\hat{\Sigma}_u^{-1} \otimes I_n \right) (e(\theta_0) - (I_n \otimes X)\hat{b})} \quad (17)$$

where \hat{b} and $\hat{\Sigma}_u$ denote the OLS coefficient and variance/covariance estimators from (7) and A is the $m \times 3k$ selection matrix with $m = 2k + 4 + q$

$$A = \begin{bmatrix} A_\pi \\ A_y \\ A_R \end{bmatrix}, \quad A_\pi = \begin{bmatrix} I_{(k)} & \text{zeros}(k, 2k) \\ \text{zeros}(4+q, k) & \bar{A} & \text{zeros}(4+q, k) \\ \text{zeros}(k, 2k) & I_{(k)} \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} I_{(2)} & 0 & 0 \\ 0 & 0 & I_{(q+3)} \end{bmatrix}.$$

Observe that \hat{b} and $\hat{\Sigma}_u$ depend on θ_0 . The statistic \mathcal{W} corresponds to the z statistic in equation (10.11) of (Srivastava and Giles, 1987, Chapter 10) and to equation (49) in (Dufour and Khalaf, 2003, equation (49)).

A.2 Invariance of the The Euler-Error GMM-type method to missing instruments

The econometric model underlying our GMM-type approach can be rewritten as

$$\begin{aligned}
\pi_t &= [\omega_f \pi_{t+1} + (1 - \omega_f) \pi_{t-1} + \gamma y_t] + \epsilon_{\pi,t} \\
y_t &= [\beta_f y_{t+1} + \sum_{i=1}^4 (1 - \beta_f) \beta_{y,i} y_{t-i} - \beta_r (R_t - \pi_{t+1})] + \epsilon_{y,t} \\
R_t &= [\gamma_\pi (1 - \sum_{i=1}^3 \rho_i) \pi_t + \gamma_y (1 - \sum_{i=1}^3 \rho_i) y_t + \sum_{i=1}^3 \rho_i R_{t-i}] + \epsilon_{R,t}
\end{aligned} \tag{18}$$

where $\epsilon_t = (\epsilon_{\pi,t}, \epsilon_{y,t}, \epsilon_{R,t})'$ is a zero-mean contemporaneously correlated disturbance vector that integrates rational expectation error. By conducting the test of (6) in the context of (18) as a test of (8) in the context of (7), as described, we obtain a p -value that it is robust to the specification of the fundamental DGPs under consideration, to measurement errors and excluded instruments. Indeed, the test conducted in this framework only requires that the (unrestricted) reduced form for the system is given, up to an error term, by some function of: (i) the predetermined variables in the system [here, the considered lags of π_t , y_t and R_t], (ii) any extra instruments \tilde{Z}_t used in the test, and (iii) possibly a set of further explanatory variables, denoted \tilde{Q}_t , which were not used in the test; these may include further lags of the endogenous variables, and/or further predetermined or exogenous variables that are omitted from the test, that is, are missing from the multivariate regression (7). To see this, suppose that the reduced form takes the unrestricted VAR specification

$$\begin{aligned}
\pi_t &= a_\pi \pi_{t-1} + \sum_{i=1}^3 b_{\pi,i} R_{t-i} + \sum_{i=1}^4 c_{\pi,i} y_{t-i} + \varpi'_\pi Q_t + \nu_{\pi,t}, \\
y_t &= a_y \pi_{t-1} + \sum_{i=1}^3 b_{y,i} R_{t-i} + \sum_{i=1}^4 c_{y,i} y_{t-i} + \varpi'_y Q_t + \nu_{y,t}, \\
R_t &= a_R \pi_{t-1} + \sum_{i=1}^3 b_{R,i} R_{t-i} + \sum_{i=1}^4 c_{R,i} y_{t-i} + \varpi'_R Q_t + \nu_{R,t},
\end{aligned} \tag{19}$$

where $Q_t = (\tilde{Z}'_t, \tilde{Q}'_t)'$. It is straightforward to check that substituting the right-hand of (19) into the right-hand-side of (7) still leads, under the null hypothesis (6), to

$$\pi_t^*(\theta_0) = \epsilon_{\pi,t}, \quad y_t^*(\theta_0) = \sum_{i=1}^4 \beta_{y,i} (1 - \beta_f) y_{t-i} + \epsilon_{y,t}, \quad R_t^*(\theta_0) = \epsilon_{R,t}, \tag{20}$$

which justifies the tests we apply.

A.3 The State-Space and solved model form

Setting $\phi_{y,i} = \beta_{y,i}(1 - \beta_f)$, we have

$$\begin{aligned}
 \Gamma_{\mathbf{L}}(\phi) &= \begin{bmatrix} (1 - \omega_f) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_{y,1}(1 - \beta_f) & \beta_{y,2}(1 - \beta_f) & \beta_{y,3}(1 - \beta_f) & \beta_{y,4}(1 - \beta_f) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_1 & \rho_2 & \rho_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\
 \Gamma_{\mathbf{C}}(\theta) &= \begin{bmatrix} -1 & \gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -\beta_r^{-1} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ (1 - \sum_{i=1}^3 \rho_i) \gamma_\pi & (1 - \sum_{i=1}^3 \rho_i) \gamma_y & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \\
 \Gamma_{\mathbf{F}}(\theta) &= \begin{bmatrix} \omega_f & 0 & 0 & \cdots & 0 \\ \beta_r^{-1} & \beta_f & 0 & \cdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \\ \vdots & & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

and

$$C(\Omega) = \begin{bmatrix} c_1 & 0 & 0 \\ c_2 & c_3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ c_4 & c_5 & c_6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where c_1, \dots, c_6 are the non-zero elements of the Cholesky factor of Ω . The diagonal case corresponds to $c_2 = c_4 = c_5 = 0$. From there on, conventional solution methods yield the solved model (5) where $B(\cdot)$ is 8×8 , $S(\cdot)$ is 8×3 and both are obtained numerically.

A.4 The Model-Consistent Full-Information statistic

Consider the multivariate regression (10) rewritten in matrix form as

$$\mathbf{Y}^*(\phi_0) = \mathbf{X}\Pi + U \quad (21)$$

where $\mathbf{Y}^*(\phi_0)$ is the $T \times n = 3$ matrix with row $\mathbf{Y}_t^*(\phi_0)$, \mathbf{X} is the $T \times K = 8$ matrix with row \mathbb{Y}'_t , and U is the $T \times n$ matrix with row U'_t . Let

$$\begin{aligned} \hat{U} &= \mathbf{Y}^*(\phi_0) - \mathbf{X}\hat{\Pi}, & \hat{\Pi} &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}^*(\phi_0), \\ \hat{U}_0 &= \mathbf{Y}^*(\phi_0), \end{aligned}$$

so $\hat{U}'_0\hat{U}_0$ and $\hat{U}'\hat{U}$ give the constrained [imposing $\Pi = 0$] and unconstrained sum of squared errors (SSE) matrices from (21). The statistic we use is

$$\mathcal{L}(\phi_0) = \left(\frac{\mu\tau - 2\lambda}{Kn} \right) \frac{1 - \left(|\hat{U}'\hat{U}| / |\hat{U}'_0\hat{U}_0| \right)^{1/\tau}}{\left(|\hat{U}'\hat{U}| / |\hat{U}'_0\hat{U}_0| \right)^{1/\tau}}. \quad (22)$$

This statistic has been shown to perform well in the multivariate regression literature, with the following approximate null distribution

$$\mathcal{L}(\phi_0) \sim F(Kn, \mu\tau - 2\lambda) \quad (23)$$

$$\mu = T - K - \frac{(n - K + 1)}{2}, \quad \lambda = \frac{nK - 2}{4} \quad (24)$$

$$\tau = \begin{cases} [(K^2n^2 - 4)/(K^2 + n^2 - 5)]^{1/2} & , \text{ if } K^2 + n^2 - 5 > 0 \\ 1 & , \text{ otherwise.} \end{cases} \quad (25)$$

$|\hat{U}'\hat{U}|/|\hat{U}'_0\hat{U}_0|$ is the well known Wilks statistic; see Dufour and Khalaf (2002, equation A.1). Observe that \hat{U} and \hat{U}_0 depend on ϕ_0 .