Optimal Fiscal Policy in the Neoclassical Growth Model Revisited*

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Abstract

This paper studies optimal Ramsey taxation in a version of the neoclassical growth model in which investment becomes productive within the period, thereby making the supply of capital elastic in the short run. Because taxing capital is distortionary in the short run, the government's ability/desire to raise revenues through capital income taxation in the initial period or when the economy is hit with a bad shock is greatly curtailed. Our timing assumption also leads to a tractable Ramsey problem without state-contingent debt, which gives rise to debt-financed budget deficits during recessions.

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1 Introduction

This paper studies optimal fiscal policy in a version of the neoclassical growth model in which capital is elastically supplied even in the short run. This is accomplished by letting investment in capital become productive within the period.

It is well understood that the conventional timing in the neoclassical growth model, in which the size of the capital stock today is the result of past investment decisions, implies that capital is inelastically supplied in the short run. It should be equally clear that a by-product of this conventional timing assumption—that capital is inelastically supplied in the short run—is at the heart of many well-established results within the optimal taxation literature. A prominent example is the well-known prescription to tax initial asset holdings at confiscatory rates, a result that Chamley (1986) and much of the subsequent literature tries to circumvent by imposing bounds on tax rates: without these exogenous bounds, a first-best allocation obtains, an obviously uninteresting problem. Tax rates over the business cycle are similarly dictated by the conventional timing of the neoclassical growth model. Every period, the government promises not to distort the return to investment while at the same time announcing that recessions will be financed with unusually high taxes on capital income, and *vice versa* during booms. This strategy is clearly optimal as the government can avoid distorting investment decisions *ex ante* while at the same time exploiting its ability to absorb shocks in a non-distortionary way by taxing/subsidizing the return to capital *ex post*.

This paper shows that changing the timing of events in the neoclassical growth model in such a way as to make the supply of capital elastic in the short run drastically alters the prescriptions that emanate from standard Ramsey problems. Our assumption that investment in capital becomes productive within the period gives individuals an alternative to supplying capital, namely consuming, which is not present under the conventional timing. Knowing that this alternative exists limits the ability and desire of the government to use capital income taxes to finance government expenditures, either in the initial period or over the business cycle.

One of our main results, already alluded to above, is that the solution to the Ramsey problem generally features a unique non-trivial level of distortions. While the level of distortions depends on individuals' initial asset holdings, it does not rely on the presence of bounds exogenously imposed on the Ramsey problem. As such, the trivial result that the solution to the Ramsey problem without imposing exogenous bounds is time-consistent does not hold in our environment.[1]

Next we offer a complete characterization of the behavior of tax rates in a stochastic environment in which the government has access to state-contingent debt. Under a class of utility functions in which consumption and leisure are separable, we show that neither the labor nor the capital income tax varies over time, and that the tax on capital is zero in all but the initial period. Under Cobb-Douglas utility, both tax rates become pro-cyclical, that is, they are low during recessions. In either case, the government uses state-contingent debt as a shock absorber, much like the *ex post* capital income tax is used for that purpose in Chari et al. (1994). As a result, debt and the primary deficit move in opposite directions, a counterfactual result which Marcet and Scott (2009) showed to be pervasive in models in which the government has access to state-contingent debt. This leads us to study a Ramsey problem under incomplete markets.

The Ramsey problem without state-contingent debt is a notoriously difficult problem to study (see Chari and Kehoe (1999)). However, this problem is quite tractable in our framework. Technically, this tractability emanates from the fact that our first order conditions can be expressed in terms of prices as functions of quantities. This allows us to write down a version of the Ramsey problem, known as the primal, in which the government chooses quantities subject to a sequence of *implementability* constraints which can be studied using techniques developed in Marcet and Marimon (1995). The upshot of this problem is that in this environment the government runs debt-financed primary deficits during recessions.

Our work complements that of Farhi (2010), who uses the conventional timing but imposes that the government set capital income tax rates one period ahead in order to mitigate the free lunch associated with volatile *ex post* capital income tax rates. While Farhi (2010)'s timing of tax announcement does reduce the size of *ex*

[1] The conventional solution entails taxing the initial return on capital at confiscatory rates, and to finance all future government expenditures through the return on that capital. This solution turns out to be highly distortionary in our environment. The contrast in results across the two environments is reminiscent of the Lucas (1980) vs Svensson (1985) timing issue in cash-in-advance models, as shown in Nicolini (1998).

post tax rates on capital income, these rates remain unrealistically large (in the order of $\pm$ 100% under his parameterization), presumably because most of the capital stock tomorrow is still given by history.[2] As a result, the government runs a primary surplus and the value of outstanding debt decreases during recessions. In our environment, the fact that individuals have the ability to build and use capital within the period makes capital comparatively more elastic, leading to capital income tax rates at least an order of magnitude smaller than those found in Farhi (2010). The upshot is that our environment gives rise to debt-financed primary deficits during recessions, in line with the empirical findings of Marcet and Scott (2009). In the latter paper, as well as in Scott (2007), capital income taxes are rules out altogether in order to focus on the implications of their model with and without state-contingent debt. They argue that ruling out contingent debt is key to bring the model's prescription closer to the data. In addition, Scott (2007) shows that under incomplete markets, government debt and the labor tax rate inherit a unit root component which, as emphasized by Aiyagari et al. (2002) in a model without capital, lends some support to Barro (1990)'s conjecture. Qualitatively, our simulations confirm that these results hold even when the government sets capital tax rates optimally.

Before moving to the description of our economic environment, our central assumption that investment becomes productive within the period deserves some comments.[3] First, we show in the appendix that this assumption can be viewed as the opposite from the equally extreme conventional assumption that today's investment only becomes productive in the next period. Second, we view this assumption more as a way to introduce some elasticity to the supply of capital rather than a way of improving the realism of the neoclassical growth model. There are countless issues for which the conventional timing assumption is either desirable or, at least, innocuous.[4] Optimal taxation is just not one of them.

The rest of the paper is organized as follows. The next section presents our

[2]This is reminiscent of the result that the exogenous upper bound imposed on capital income tax rates initially binds for several periods before tax rates settle around zero in the deterministic neoclassical growth model.

[3]Interestingly, a similar timing is commonly used in the housing literature, in which individuals move into their house in the same period in which the house is built: e.g. see Kiyotaki et al. (2007) or Fisher and Gervais (2010).

[4]In fact, the first-best allocations under both timing assumptions are essentially indistinguishable.

general economic environment, which consists of the neoclassical growth model with an alternative timing assumption. In Sections 3 and 4 we set up and analyze a deterministic and a stochastic Ramsey problem, respectively. Section 5 is devoted to the analysis of a Ramsey problem without state-contingent debt. A brief conclusion is offered in Section 6.

2 General Economic Environment

The economic environment we consider is similar to that of Chari et al. (1994): a stochastic version of the one-sector neoclassical growth model. As emphasized in the introduction, the main distinguishing feature of our environment is that current investment in capital becomes productive immediately. In this section, we introduce the general economic environment. We later study special cases of this environment, starting with a deterministic version, followed by stochastic versions with and without state-contingent government debt.

Time is discrete and lasts forever. Each period the economy experiences one of finitely many events $s_t \in S$. We denote histories of events by $s^t = (s_0, s_1, \ldots, s_t)$. As of date 0, the probability that a particular history s^t will be realized is denoted $\pi(s^t)$.

Production The production technology is represented by a neoclassical production function with constant returns to scale in capital (k) and labor (l):

$$y(s^t) = f\big(k(s^t), l(s^t), s_t\big) = A(s^t)k(s^t)^{\alpha}l(s^t)^{1-\alpha}, \tag{1}$$

where $A(s_t)$ represents the state of technology in period t, $y(s^t)$ denotes the aggregate (or per capita) level of output, and $k(s^t)$ and $l(s^t)$ denote capital and labor used in production. Output can be used either for private consumption ($c(s^t)$), public consumption ($g(s^t)$), or as investment ($i(s^t)$). Feasibility thus requires that

$$c(s^t) + g(s^t) + i(s^t) = f\big(k(s^t), l(s^t), s_t\big). \tag{2}$$

What distinguishes this paper from others in the literature is our law of motion for capital, defined via

$$i(s^t) = k(s^t) + \delta k(s^t) - k(s^{t-1}). \tag{3}$$

The important feature of this law of motion is that investment in capital becomes productive immediately, i.e. it is used in production and depreciates within the period.[5] In this way, the supply of capital is elastic even in the short run. Replacing the law of motion (3) into (2) results in the following feasibility constraint:

$$c(s^t) + g(s^t) + k(s^t) = f\big(k(s^t), l(s^t), s_t\big) - \delta k(s^t) + k(s^{t-1}). \tag{4}$$

The usual properties of the neoclassical growth model hold in our environemnt: the capital to labor ratio is independent of scale, firms make zero profits in equilibrium, and factors are paid their marginal products:

$$\hat{w}(s^t) = f_l\big(k(s^t), l(s^t), s_t\big) \qquad = f_l(s^t); \tag{5}$$

$$\hat{r}(s^t) = f_k\big(k(s^t), l(s^t), s_t\big) - \delta = f_k(s^t) - \delta, \tag{6}$$

where $\hat{w}(s^t)$ and $\hat{r}(s^t)$ denote before-tax wage and interest rates, respectively.

Households The economy is populated by a large number of identical individuals who live for an infinite number of periods and are endowed with one unit of time every period. Individuals' preferences are ordered according to the following utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U\big(c(s^t), l(s^t)\big), \tag{7}$$

where $c(s^t)$ and $l(s^t)$ represent consumption and hours worked at history s^t. We assume that the felicity function is increasing in consumption and leisure $(1 - l^{s^t})$, strictly concave, twice continuously differentiable, and satisfies the Inada conditions for both consumption and leisure.

Each period individuals face the budget constraint

$$c(s^t)+k(s^t)+\sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) = w(s^t)l(s^t)+r(s^t)k(s^t)+k(s^{t-1})+b(s_t|s^{t-1}) \tag{8}$$

where $w(s^t) = [1 - \tau^w(s^t)]\hat{w}(s^t)$ and $r(s^t) = [1 - \tau^k(s^t)]\hat{r}(s^t)$ denote after-tax wage and interest rates, respectively. The fiscal policy instruments τ^w and τ^k, as well as government debt $b(s_{t+1}|s^t)$, will be discussed in detail below. Notice that capital and

[5] Appendix A offers a micro-foundation of this timing assumption.

government debt are treated rather symmetrically in budget constraint (8), except of course for the fact that the size of the capital stock and its return cannot depend on tomorrow's state of the economy. In other words, today's price of one unit of capital tomorrow is $1 - r(s^t)$, much like today's price of a bond which pays one unit of consumption good tomorrow in state s_{t+1} is $q(s_{t+1}|s^t)$. As we will see later, the symmetry is even clearer without uncertainty or in the absence of state-contingent government debt.[6]

Letting $p(s^t)$ denote the Lagrange multiplier on the budget constraint at history s^t, the first order necessary (and sufficient) conditions for a solution to the consumer's problem are given by (8) and

$$\beta^t \pi(s^t) U_c(s^t) = p(s^t), \tag{9}$$

$$\beta^t \pi(s^t) U_l(s^t) = -w(s^t) p(s^t), \tag{10}$$

at all dates t and histories s^t for consumption and labor,

$$-p(s^t)\left(1 - r(s^t)\right) + \sum_{s_{t+1}} p(s^{t+1}) = 0, \tag{11}$$

at all dates t and histories s^t for capital,

$$-p(s^t) q(s_{t+1}|s^t) + p(s^{t+1}) = 0, \tag{12}$$

at all dates t, histories s^t, and all states s_{t+1} tomorrow for bond holdings, as well as the transversality conditions

$$\lim_{t\to\infty} p(s^t) k(s^t) = 0, \tag{13}$$

$$\lim_{t\to\infty} \sum_{s_{t+1}} p(s^{t+1}) b(s_{t+1}|s^t) = 0. \tag{14}$$

Under complete markets, it is well knows that these first order conditions and the budget constraint can be conveniently combined into a single present value constraint, as stated next:

Proposition 1 *Under complete markets, an allocation solves the consumer's problem if and only if it satisfies equations* (8)–(14)*, or, equivalently, if and only if it satisfies*

[6] In the appendix we show that if a period is composed of many sub-periods, then this budget constraint is one way to resolve the time-aggregation problem.

the implementability constraint[7]

$$\sum_{t,s^t} \beta^t \pi(s^t) \left[U_c(s^t)c(s^t) + U_l(s^t)l(s^t)\right] = A_0, \tag{15}$$

where $A_0 = U_c(s_0)[k_{-1} + b_{-1}]$*, and* k_{-1} *and* b_{-1} *are initial amounts of capital and government debt held by individuals.*

Proof. The proof is standard. [See for example Chari et al. (1994).] ■

The Government The government's role in this economy is to finance an exogenous stream of government expenditures, $g(s^t)$. The fiscal policy instruments available to the government consist of a proportional labor income tax $\tau^w(s^t)$; a proportional capital income tax $\tau^k(s^t)$; and issuance of new government debt $b(s_{t+1}|s^t)$. At date t, the government's budget constraint is as follows:

$$g(s^t) + b(s_t|s^{t-1}) = \sum_{s_{t+1}} q(s_{t+1}|s^t)b(s_{t+1}|s^t) + \tau^w(s^t)\hat{w}(s^t)l(s^t) + \tau^k(s^t)\hat{r}(s^t)k(s^t). \tag{16}$$

The government thus has to finance government expenditures as well as debt issued yesterday that promised to pay in the event that s_t would occur today. In addition to taxing capital and labor income, the government can raise revenues by issuing new (state-contingent) debt.

3 Deterministic Ramsey Problem

Before analyzing the general stochastic model introduced in the previous section, it will prove instructive to study a deterministic version of the model first. The intuition from this simpler model will in some sense carry over to the more complicated stochastic environment.

Accordingly, we set up a standard Ramsey problem for a deterministic version of the model. As is well known, there is an equivalence between choosing fiscal policy instruments directly and choosing allocations among an appropriately restricted

[7] To obtain the implementability constraint, multiply the budget constraint (8) by $p(s^t)$, add them up, and use the first order conditions (9)–(12) to replace prices.

set of allocations.[8] The government's problem consists of maximizing the utility of the representative individual (7) subject to the implementability constraint (15) and feasibility (4).[9] If we let λ denote the Lagrange multiplier associated with the implementability constraint, we can define the pseudo-welfare function W by

$$W\big(c_t, l_t, \lambda\big) = U\big(c_t, l_t\big) + \lambda \left(U_{c_t} c_t + U_{l_t} l_t\right).$$

The Lagrangian associated with the Ramsey problem, given k_{-1} and b_{-1}, is then given by:

$$L(k_{-1}, b_{-1}) = \min_{\lambda} \max_{\{c_t, l_t, k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t W\big(c_t, l_t, \lambda\big) - \lambda U_{c_0}(k_{-1} + b_{-1})$$

subject to the feasibility constraint

$$c_t + g_t + k_t = f(k_t, l_t) - \delta k_t + k_{t-1}.$$

It should be clear that one can replace the feasibility constraint into the objective function, and that the labor supply can be assumed to satisfy an optimality condition. Accordingly, slightly abusing notation, the Ramsey problem can be rewritten as

$$L(k_{-1}, b_{-1}) = \min_{\lambda} \max_{\{k_t\}_{t=0}^{\infty}} \left\{ W\big(k_{-1}, k_0, \lambda\big) - \lambda U_{c_0}(k_{-1}, b_{-1}) + \sum_{t=1}^{\infty} \beta^t W\big(k_{t-1}, k_t, \lambda\big) \right\}$$

Notice that the last term inside the maximand is a standard recursive problem: if we define $V(k, \lambda)$ via

$$V\big(k, \lambda\big) = \max_{k'} \Big\{ W\big(k, k', \lambda\big) + \beta V(k', \lambda) \Big\},$$

then the problem becomes

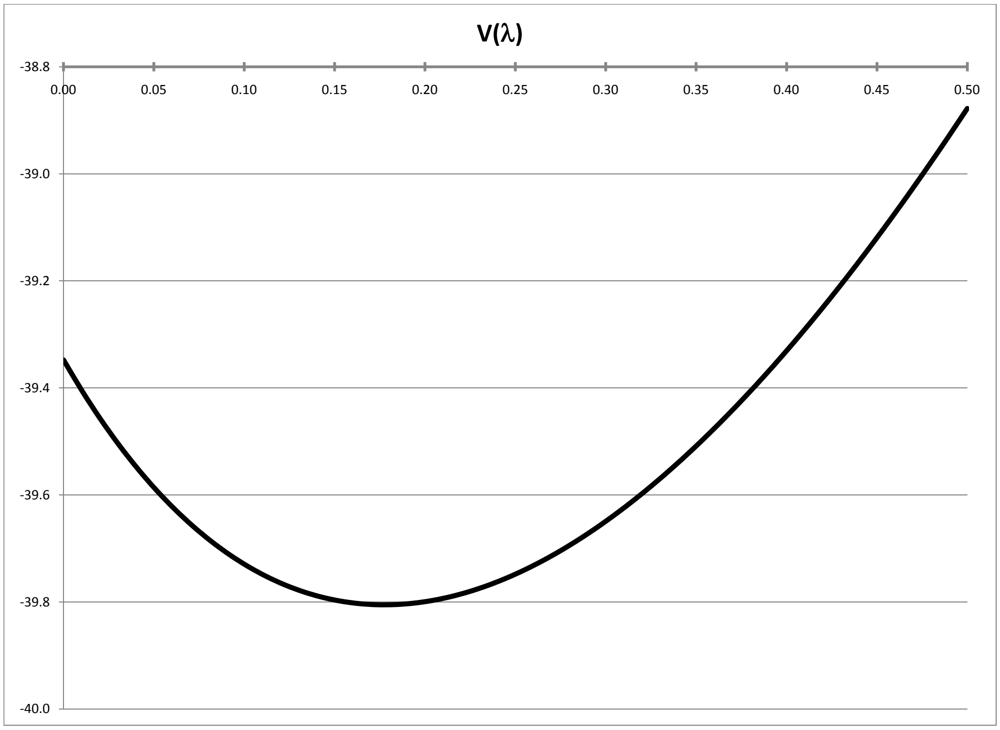
$$\begin{aligned} L(k_{-1}, b_{-1}) &= \min_{\lambda} \max_{k_0} \left\{ W\big(k_{-1}, k_0, \lambda\big) - \lambda U_{c_0} a_{-1} + \beta V\big(k_0, \lambda\big) \right\} \\ &= \min_{\lambda} \widehat{V}(k_{-1}, b_{-1}, \lambda), \end{aligned}$$

where $\widehat{V}$ is the value of the maximand evaluated at the optimum for any given value of λ.

[8] See Chari and Kehoe (1999) or Erosa and Gervais (2001).

[9] It is well known that if an allocation satisfies the implementability constraint and the feasibility constraint, it must also satisfy the government budget constraint (16)—e.g. see Chari and Kehoe (1999) or Erosa and Gervais (2001). Accordingly, we omit the proof.

Figure 1: Value function $\widehat{V}(\lambda)$



Note: The parameterization underlying this figure is as follows: Cobb-Douglas production function with a capital share of 1/3; capital depreciates at a rate of 7% per period; utility function additively separable and logarithmic in consumption and leisure; discount factor equal to 0.958; government spending such that it represents 17.5% of steady state GDP; initial capital is set to 1.5 (below its steady state value of 1.8) and initial debt is set to 0.

Figure 1 shows the shape of the value function $\widehat{V}$ as a function of λ, for a given value of initial assets. What this Figure shows is that without any restrictions on the fiscal policy instruments or otherwise, the optimal level of distortions, as represented by λ, is non-zero. Indeed, labor income is taxed at a rate of 19% in the long run. Capital income is not taxed in the long run: this can be shown formally as we will see in the next section.

The fact that it is optimal to distort this economy is in sharp contrast to results obtained under the more conventional timing whereby investment made during the period only becomes productive the next period. The reason is well known: under conventional timing, taxing initial assets represents a lump-sum way to raise revenues for the government, as these assets were accumulated in the past. Accordingly, the optimal fiscal policy entails taxing these initial assets at 'confiscatory' rates, or just enough for the government to finance the present value of its spending. In terms of Figure 1, the value function $\widehat{V}$ would be a strictly increasing function, with its minimum at exactly zero, meaning that a first-best outcome would be attained.

The intuition for our result comes directly from our timing assumption. Since investment becomes productive immediately, and its return is realize during the period, taxing capital at date zero becomes distortionary: individuals do not have to supply capital accumulated from the past. They can, and will, consume large amounts should the government tax their capital away. Realizing that fact, the government does not confiscate initial assets. Nevertheless, in the numerical example underlying Figure 1, the tax rate on capital is very high, close to 700%.[10] As a result, consumption at date 0 is around 50% higher than in period 1, which is itself slightly below its steady state level. The tax rate on labor at date 0, however, is negative 20%: this makes leisure relatively expensive in that period, thereby increasing the labor supply.[11]

The general message of this analysis is that the government's ability to use capital income taxes in a lump-sum fashion disappears once the supply of capital is elastic. This simple yet powerful message will also be at the heart of our findings in a stochastic economy, to which we now turn our attention.

4 Stochastic Ramsey Problem

To study optimal policy in this environment, we proceed as in the previous section and set up a standard Ramsey problem. With λ still denoting the Lagrange multiplier on the implementability constraint, the pseudo-welfare function W now reads

$$W\big(c(s^t), l(s^t), \lambda\big) = U\big(c(s^t), 1 - l(s^t)\big) + \lambda\left[U_c(s^t)c(s^t) + U_l(s^t)l(s^t)\right]. \qquad (17)$$

[10] While the capital income tax is very high in the initial period, it is far from being sufficiently high to eliminate all future distortions, as discussed above.

[11] We suspect, and will verify in a subsequent version of this paper, that the fiscal policy at date 0 is quite sensitive to the value of the intertemporal elasticity of substitution, which is unity in this example.

The Ramsey problem is thus as follows:

$$L(k_{-1}, b_{-1}) = \min_{\lambda} \max_{\{c(s^t), l(s^t), k(s^t)\}_{t,s^t}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) W\big(c(s^t), l(s^t), \lambda\big) \\ - \lambda U_c(s_0)[k_{-1} + b_{-1}]$$

subject to the feasibility constraint (4), keeping in mind that the government budget constraint must holds and so does not constrain the solution to this problem.

The government typically has more instruments than it needs in the stochastic neoclassical growth model, in the sense that many tax codes can decentralize any given allocation (e.g. see Zhu (1992) or Chari et al. (1994)). Such is not the case in our environment: our tax code is unique, in the sense that any given allocation can only be decentralized by a single tax system. Technically, this comes from the fact that the tax rate on capital income is pined down by the marginal product of capital (6) as well as the optimality conditions (9) and (11): for any implementable allocation, there exists a single value of the capital tax which makes these equations hold. Intuitively, the indeterminacy under conventional timing comes from the fact that an allocation can, for example, be implemented with a tax rate on capital income that varies with the state tomorrow and risk-free debt, or with a flat tax on capital income tomorrow and state-contingent debt. Here, the capital income tax applies to the return to investment made during the period, so it is uniquely determined even with state-contingent debt. It follows that ruling out state-contingent debt is not innocuous in our environment, as will be clear in the next section.

The optimality conditions for this Ramsey problem are quite simple, and can be analyzed analytically. Let $\beta^t \phi(s^t)$ represent the Lagrange multiplier on the feasibility constraint (4) at history s^t. The first order conditions with respect to consumption, labor, and capital are, respectively,

$$\pi(s^t) W_c(s^t) = \phi(s^t), \tag{18}$$

$$\pi(s^t) W_l(s^t) = -f_l(s^t)\phi(s^t), \tag{19}$$

$$\phi(s^t)\big[1 - (f_k(s^t) - \delta)\big] = \beta \sum_{s_{t+1}} \phi(s^{t+1}), \tag{20}$$

where W_c and W_l represent the derivative of the pseudo-welfare function W (17) with respect to consumption and the labor supply, respectively.

4.1 Optimal Fiscal Policy

The rest of this section is devoted to characterizing optimal fiscal policy. Our characterization, which requires making assumptions about the form of the utility function, involves in turn the labor income tax and the capital income tax. An important note concerning state-contingent debt concludes the section.

Our first two Propositions show that while the labor income tax does not depend on the state of the economy if the per-period utility function is separable between consumption and *labor* and both part exhibit constant elasticity of substitution (CES), it becomes pro-cyclical when individual care about leisure, even if the utility function is CES in *leisure*.

Proposition 2 *Assume that the felicity function is separable, $U(c, l) = u(c) + v(l)$, with $u(c)$ and $v(l)$ both exhibiting constant elasticity of substitution. Then the tax rate on labor income is invariant to the productivity shock.*

Proof. Combining the first order conditions with respect to consumption (18) and labor (19) from the Ramsey problem and using (5), we get

$$-\frac{W_l(s^t)}{W_c(s^t)} = \hat{w}(s^t). \tag{21}$$

The derivatives W_c and W_l are given by

$$\begin{aligned} W_c(s^t) &= (1+\lambda)U_c(s^t) + \lambda U_c(s^t)H_c(s^t), \\ W_l(s^t) &= (1+\lambda)U_l(s^t) + \lambda U_l(s^t)H_l(s^t), \end{aligned}$$

where

$$\begin{aligned} H_c(s^t) &= \frac{U_{c,c}(s^t)c(s^t) + U_{c,l}(s^t)l(s^t)}{U_c(s^t)}, \\ H_l(s^t) &= \frac{U_{l,c}(s^t)c(s^t) + U_{l,l}(s^t)l(s^t)}{U_l(s^t)}. \end{aligned}$$

Now pick two histories as of date t, s^t and $\tilde{s}^t$. From (21), it must be that

$$\frac{W_l(s^t)}{W_c(s^t)\hat{w}(s^t)} = \frac{W_l(\tilde{s}^t)}{W_c(\tilde{s}^t)\hat{w}(\tilde{s}^t)},$$

or, equivalently,

$$\frac{\left[1+\lambda+\lambda H_l(s^t)\right]U_l(s^t)}{\left[1+\lambda+\lambda H_c(s^t)\right]U_c(s^t)\hat{w}(s^t)} = \frac{\left[1+\lambda+\lambda H_l(\tilde{s}^t)\right]U_l(\tilde{s}^t)}{\left[1+\lambda+\lambda H_c(\tilde{s}^t)\right]U_c(\tilde{s}^t)\hat{w}(\tilde{s}^t)}.$$

Since the felicity function is separable, the functions H_c and H_l become

$$\begin{aligned} H_c(s^t) &= \frac{U_{c,c}(s^t)c(s^t)}{U_c(s^t)}, \\ H_l(s^t) &= \frac{U_{l,l}(s^t)l(s^t)}{U_l(s^t)}. \end{aligned}$$

And since the sub-utilities for consumption and labor are both from the constant elasticity of substitution class of utility functions, both $H_c(s^t)$ and $H_l(s^t)$ are constant. Accordingly, the last expression reduces to

$$\frac{U_l(s^t)U_c(\tilde{s}^t)}{U_c(s^t)U_l(\tilde{s}^t)} = \frac{\hat{w}(s^t)}{\hat{w}(\tilde{s}^t)}.$$

But the first order conditions for consumption and labor from the household's problem (equations (9) and (10)) at histories s^t and $\tilde{s}^t$ imply

$$\frac{U_l(s^t)U_c(\tilde{s}^t)}{U_c(s^t)U_l(\tilde{s}^t)} = \frac{w(s^t)}{w(\tilde{s}^t)} = \frac{(1-\tau^w(s^t))\hat{w}(s^t)}{(1-\tau^w(\tilde{s}^t))\hat{w}(\tilde{s}^t)}.$$

For the last two equations to hold it must be the case that $\tau^w(s^t) = \tau^w(\tilde{s}^t)$. ■

The intuition for this result is that because the elasticity of the labor supply does not vary with the shock, there is no reason for the government to tax labor at rates that vary with the shock.[12] Note that the previous result does not apply when individuals care about leisure, as opposed to disliking labor. The following proposition shows that indeed labor income taxes will in general not be constant when individuals care about leisure.

Proposition 3 *Assume that $\lambda > 0$ and that the felicity function is given by $U(c,l) = u(c)v(l)$, with $u(c) = (1-\sigma)^{-1}c^{1-\sigma}$ and $v(l) = (1-l)^{\nu(1-\sigma)} = (1-l)^{\eta}$, with $\sigma > 1$ and $\nu > 0$, and $\ln(c) + \eta\ln(1-l)$ for $\sigma = 1$. Assume that there exist two states s^t and $\tilde{s^t}$ such that $l(s^t) > l(\tilde{s}^t)$. Then $\tau^w(s^t) > \tau^w(\tilde{s}^t)$ if and only if*

$$\lambda < \frac{-1}{(1-\sigma)(1+\nu)}. \tag{22}$$

[12]Evidently, the same argument can be made using s^{t-1} and s^t as the two histories, which means that the tax rate on labor is not only state-independent, but also constant over time.

Proof. From equations (9)–(10) and (21), the tax rate on labor income can be expressed as

$$\tau^w(s^t) = \frac{\lambda\big(H_l(s^t) - H_c(s^t)\big)}{1 + \lambda + \lambda H_l(s^t)}. \tag{23}$$

Under the stated utility function, H_c and H_l are such that

$$H_l(s^t) - H_c(s^t) = \frac{-1}{1 - l(s^t)},$$
$$H_l(s^t) = -\sigma + \frac{1 - \eta l(s^t)}{1 - l(s^t)}.$$

Using these expression in equation (23) we have

$$\tau^w(s^t) = \frac{\lambda}{1 - \lambda(\sigma - 2) - l(s^t)\big(1 + \lambda(1 - \sigma)(1 + \nu)\big)}.$$

It follows that the tax rate is higher under state s^t than $\tilde{s}^t$ if the term multiplying labor in the denominator is positive, that is, if condition (22) holds. ■

Note that we need to assume that the economy is distorted ($\lambda > 0$), otherwise all taxes are zero. This Proposition establishes that whenever condition (22) is satisfied, if labor is pro-cyclical, so will the tax rate on labor income. Note that under logarithmic utility, i.e. when $\sigma = 1$, the condition is always satisfied. It becomes less likely to be satisfied as individuals become more risk averse, i.e. as σ increases. As such, this Proposition is useful to interpret the finding in Chari et al. (1994) that the correlation between the shock and labor taxes changes sign as they change the risk aversion parameter. Finally, note that what is key for the cyclicality of the labor tax, or lack thereof, is whether the utility function exhibits constant elasticity of substitution in labor or in leisure. When it is CES in leisure, the labor supply elasticity varies with the *level* of the labor supply, becoming more inelastic as the labor supply increases. This is in contrast to our previous proposition, where the labor supply elasticity was invariant to the level of the labor supply.

Our next results pertain to the tax on capital income. We first show that capital income should not be taxed if the utility function is separable and exhibits constant elasticity of substitution in consumption. We then argue that under non-separable preferences, the tax rate on interest income is likely to be pro-cyclical.

Proposition 4 *Assume that the felicity function is separable, $U(c,l) = u(c) + v(l)$, and that $u(c)$ exhibits constant elasticity of substitution. Then the capital income tax rate is zero at all dates and histories (other than the first period).*

Proof. Recall that the first order conditions (9) and (11) from the households' problem imply that

$$(1 - r(s^t)) = \sum_{s_{t+1}} \frac{\beta\pi(s^{t+1})U_c(s^{t+1})}{\pi(s^t)U_c(s^t)}. \tag{24}$$

Similarly, combining first order conditions (18) and (20) from the Ramsey problem we have

$$\big[1 - (f_k(s^t) - \delta)\big] = (1 - \hat{r}(s^t)) = \sum_{s_{t+1}} \frac{\beta\pi(s^{t+1})W_c(s^{t+1})}{\pi(s^t)W_c(s^t)}. \tag{25}$$

But under a separable utility function and constant elasticity of substitution in consumption,

$$W_c(s^t) = (1 + \lambda + \lambda H^c(s^t))U_c(s^t) = (1 + \lambda - \lambda\sigma)U_c(s^t),$$

where σ is the inverse of the intertemporal elasticity of substitution. Hence we can replace W_c with U_c in equation (25). But then the only way for both equation (24) and equation (25) to hold is if $\tau^k(s^t) = 0$. ■

This Proposition is in sharp contrast to the results in Chari et al. (1994), where the *ex post* tax rate on capital income is extremely volatile.[13] The intuition is that in their environment, the return on investment made today is taxed tomorrow. Since the investment decision has already been made when the tax authority sets the tax rate on capital income, this instrument is extremely useful to absorb shocks to the budget of the government. For example, if the economy experiences a bad shock today, then the government will tax capital income at a high rate to absorb the loss in revenue. The more persistent the shock is, the higher the tax rate. In fact, under standard parameter specifications, the increase in capital income taxes is so large that the government runs a primary surplus in the period of a negative shock, thereby absorbing the future path of low government revenues with very little change to the tax rate on labor income. Of course, the tax authority always promises individuals

[13] As pointed out at the beginning of this section, however, one should keep in mind that this statement implicitly picks one of many potential tax codes.

that *on average* capital income will not be taxed. This is what Chari et al. (1994) refer to as the *ex ante* tax rate on capital income, which, under the assumptions of our proposition 4, is zero.

In our environment, the return on capital is known at the time individuals make their investment decision, thereby eliminating the distinction between *ex ante* and *ex post* taxes on capital. In particular, the tax authority no longer has the ability to absorb shocks in a non-distortionary fashion through highly volatile capital income tax rates.

Under more general preferences, the tax rate on capital income will not be equal to zero in general. For instance, if $U(c,l) = u(c)v(l)$, with $u(c) = (1-\sigma)^{-1}c^{1-\sigma}$ and $v(l) = (1-l)^{\nu(1-\sigma)} = (1-l)^{\eta}$, with $\sigma > 1$ and $\nu > 0$, then capital income will tend to be subsidized in bad times and taxed in good times. To see this, note that the function $H_c(s^t)$ under this utility function is given by

$$H_c(s^t) = -\sigma - \eta \frac{l(s^t)}{1-l(s^t)},$$

which, since $\eta < 0$, is increasing in l. Now from equations (24) and (25), we have

$$\frac{1-r(s^t)}{1-\hat{r}(s^t)} = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\big(1+\lambda+\lambda H^c(s^t)\big)U_c(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)\big(1+\lambda+\lambda H^c(s^{t+1})\big)U_c(s^{t+1})}. \tag{26}$$

When this ratio is smaller than 1, capital income is subsidized, and capital income is taxed if the ratio is greater than 1. In particular, capital income is subsidized when $H_c(s^t)$ is relatively low, i.e. when the labor supply is relatively low. Much like the labor income tax, the capital income tax is thus likely to be pro-cyclical as long as labor is pro-cyclical.

The results of this section tell us that depending on the form of the utility function, labor and capital income taxes can either be acyclical or pro-cyclical. However, these results are silent as to the behavior of government debt over the business cycle, even if taxes are pro-cyclical. This is because with state-contingent government debt, it may be optimal for the government to commit to a policy that involves repaying a lower amount of debt during recessions—a partial default of debt in the words of Chari and Kehoe (1999). This can easily be established by deriving a present value budget constraint for the government. By substituting forward $b(s^{t+1}|s^t)$ into the government budget constraint (16), letting $ps(s^t) = \tau^w(s^t)\hat{w}(s^t)l(s^t) + \tau^k(s^t)\hat{r}(s^t)k(s^t) - g(s^t)$ denote the primary surplus, one obtains the following representation for debt:

$$b(s_t|s^{t-1}) = ps(s^t) + \sum_{\tau=t}^{\infty}\sum_{s_{\tau+1}} q(s^{\tau+1}|s^t)ps(s^{\tau+1}|s^t).$$

This equation states that a shock which reduces the present value of primary surpluses is associated with a low debt payment. In other words, the amount of debt that comes due following a shock that reduces the present value of primary surpluses must be lower than the amount of debt that comes due in the event of a shock that increases the present value of primary surpluses: state-contingent debt is used as a shock absorber. Whether this translates into an increase or a decrease in the value of debt outstanding is not clear (see equation (16)): while the government faces a primary deficit in bad times, it also wakes up with fewer bonds to repay. However, numerical results suggest that the change in the primary deficit is small relative to the relative size of debt repayed in good vs. bad times. As a result, the government issues less debt in bad times than in good times.[14]

To conclude, our model implies that while the primary deficit can be countercyclical (i.e. tax revenues are low in bad times and high in good times), the presence of state-contingent government debt can make government debt pro-cyclical and thus negatively correlated with the primary deficit, a phenomenon which we typically do not observe (see Marcet and Scott (2009)). Accordingly, we now turn our attention to a situation in which the government only has access to risk-free debt.

5 Ruling out State-Contingent Debt

Ruling out state-contingent debt in the standard neoclassical growth model has proven difficult (e.g. see Chari and Kehoe (1999)). In our framework, however, this task is quite tractable. To see this, consider the consumer's budget constraint without state-contingent debt:

$$c(s^t) + k(s^t) + q(s^t)b(s^t) = w(s^t)l(s^t) + r(s^t)k(s^t) + k(s^{t-1}) + b(s^{t-1}). \tag{27}$$

[14]Similar results are discussed in Chari and Kehoe (1999) and Marcet and Scott (2009).

It should be clear that the first order conditions for consumption, labor, and capital, equations (9)–(11), remain valid under this budget constraint. These equations imply that

$$w(s^t) = -\frac{U_l(s^t)}{U_c(s^t)};$$
$$1 - r(s^t) = \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)},$$

which can be replaced in the budget constraint to obtain

$$c(s^t) + (k(s^t) + b(s^t))\beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} = -\frac{U_l(s^t)}{U_c(s^t)} l(s^t) + k(s^{t-1}) + b(s^{t-1}). \quad (28)$$

Of course, without state-contingent debt these budget constraints can no longer be expressed as a single present-value budget constraint. Ruling out state-contingent debt amounts to imposing a sequence of budget or implementability constraints of the form above. The difficulty in the neoclassical growth model under conventional timing is that the interest rate cannot merely be substituted out because it appears within an expectation sign in the Euler equation.

Given the form of the implementability constraint (28), we can use the methodology developed in Marcet and Marimon (1995) to obtain the following Ramsey problem in Lagrangian form:

$$\begin{aligned} L(k_{-1}, b_{-1}) = \min_{\{\lambda(s^t)\}_{t,s^t}} \max_{\{c(s^t), l(s^t), k(s^t), b(s^t)\}_{t,s^t}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \Big\{ U\big(c(s^t), l(s^t)\big) \\ + \lambda(s^t) \left(c(s^t) + \frac{U_l(s^t)}{U_c(s^t)} l(s^t) - k(s^{t-1}) - b(s^{t-1}) \right) U_c(s^t) \\ + \lambda(s^{t-1}) \big(k(s^{t-1}) + b(s^{t-1}) \big) U_c(s^t) \Big\} \quad (29) \end{aligned}$$

subject to feasibility (4) at all dates and histories, given k_{-1} and b_{-1}, with $\lambda_{-1} = 0$.

5.1 Analysis

We first establish that the evolution of the multiplier λ, which reflects the distortionary nature of taxation over time, contains a permanent component—a result first discussed in Aiyagari et al. (2002) in a model without capital, and more recently by Scott (2007) in a model with capital in which capital income taxation is ruled out. To establish this result, notice that the first-order condition for government debt states that

$$\sum_{s^{t+1}|s^t} \beta^{t+1}\pi(s^{t+1})\left(\lambda(s^t)U_c(s^{t+1}) - \lambda(s^{t+1})U_c(s^{t+1})\right) = 0. \tag{30}$$

Since $\lambda(s^t)$ is known at history s^t, it can be taken out of the expectation, establishing that

$$\lambda(s^t) = \frac{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)U_c(s^{t+1})\lambda(s^{t+1})}{\sum_{s_{t+1}} \pi(s^{t+1}|s^t)U_c(s^{t+1})}, \tag{31}$$

so that the multiplier λ follows a risk-adjusted Martingale. An interesting special case, which we study in more detail below, is one where the felicity function is quasi-linear, i.e. $U(c, l) = c + v(l)$. In this case, the marginal utility of consumption is constant at unity, and so the stochastic process for the multiplier λ becomes a non-negative martingale. Indeed, Farhi (2010) shows that if the government faces natural debt limits and the stochastic process governing the state s_t converges to a unique (non-degenerate) stationary distribution, then λ_t converges to zero, which implies that the Ramsey allocation converges to a first-best allocation (i.e. all taxes are zero in the long run). This result holds in our economy as well.

In general not much can be said analytically about the behavior of optimal taxes in this environment. In particular, nothing can be said about the labor income tax, at least as far as we can tell. For the capital income tax, we establish one special case in which it is always zero. If we let $\beta^t\pi(s^t)\phi(s^t)$ be the multiplier on the feasibility constraint at history s^t, the first order condition with respect to capital reads

$$\sum_{s^{t+1}|s^t} \beta^{t+1}\pi(s^{t+1})\left(\lambda(s^{t+1}) - \lambda(s^t)\right)U_c(s^{t+1}) \\ + \beta^t\pi(s^t)\phi(s^t)\Big(1 - \big(f_k(s^t) - \delta\big)\Big) - \sum_{s^{t+1}|s^t} \beta^{t+1}\pi(s^{t+1})\phi(s^{t+1}) = 0,$$

which, given (30), implies that

$$1 - \big(f_k(s^t) - \delta\big) = 1 - \hat{r}(s^t) = \frac{\sum_{s^{t+1}|s^t} \beta\pi(s^{t+1})\phi(s^{t+1})}{\pi(s^t)\phi(s^t)}. \tag{32}$$

As usual, recalling equation (24)—which holds here as well—capital income should not be taxed ($\hat{r}(s^t) = r(s^t)$) if the shadow value of resources is equal to marginal utility of consumption at all dates and states, i.e. if $\phi(s^t) = U_c(s^t)$.[15] This will in general not be the case, even under a per-period utility function separable between consumption and leisure. In this case, the value of the multiplier ϕ, from the first order condition for consumption, is given by

$$\phi(s^t) = U_c(s^t)\left[1 + \lambda(s^t)\left(\frac{U_{cc}(s^t)c(s^t)}{U_c(s^t)} + 1\right)\right.$$
$$\left. - \big(\lambda(s^t) - \lambda(s^{t-1})\big)\frac{U_{cc}(s^t)}{U_c(s^t)}\Big(k(s^{t-1}) + b(s^{t-1})\Big)\right]. \quad (33)$$

Clearly, the term inside the square brackets will not be equal to one in general. There is, however, one special case under which we can establish that capital income should not be taxed, as we state in the following proposition.

Proposition 5 *If the per-period utility function is quasi-linear in consumption, i.e.* $U(c,l) = c + v(l)$, *then the tax rate on capital income is zero.*

Proof. First note that under this utility function, because the marginal utility of consumption is fixed at unity, (24) implies that $1 - r(s^t) = \beta$. From (33), the value of the multiplier on the feasibility constraint is given by $\phi(s^t) = 1 + \lambda(s^t)$. Furthermore, (31) implies that $\lambda(s^t) = \sum_{s_{t+1}} \pi(s^{t+1}|s^t)\lambda(s^{t+1})$. Using these facts in equation (32) imply that $1 - \hat{r} = \beta$. ■

5.2 Numerical Example

To gain more insight into the kind of prescription that emanate from the model without state-contingent debt, we resort to numerical results. While the model is conceptually tractable, solving it numerically is nontheless quite challenging for several reasons. First is the presence of 4 state variables (the exogenous shock, capital, debt, and the multiplier). Second, the solution, and therefore the appropriate state space, depends on initial conditions for debt and capital. Finally, and perhaps more importantly, the random walk component for the evolution of the multiplier λ discussed above makes the problem unstable unless one starts from very good initial guesses.

[15] Note that this is merely a sufficient condition, so there can be cases in which this condition does not hold yet the tax rate on capital income is nevertheless equal to zero. Indeed, this is the case in Proposition 5 below.

We thus proceed using homotopy from a deterministic model without capital under quasi-linear utility, successively adding shocks, curvature to the utility function, capital, and partial depreciation. We solve the model by iterating on policy functions approximated by cubic splines on a grid composed of 15 points each for k, b and λ, and three points for the shock A (for a total of 10,125 points on the state space). Although the grid may seem sparse, the solution is remarkably accurate even with fewer grid points, as we verified by comparing solutions for cases we can compute more precisely—such as the deterministic model.

We parameterize the model along the lines of Farhi (2010), who in turn follows Chari et al. (1994). A period is taken to represent a year. The discount factor β is set to 0.958. The utility function is given by $u(c, l) = \log(c) + \nu \log(1 - l)$, with $\nu = 1.5$. The production function is Cobb-Douglas with capital share α set to 0.33. Capital depreciates at a rate of 7 percent per period. Government spending g is equal to 0.1067, which implies an average spending to output ratio in the range of 16.7%.[16] Productivity is modeled as a three state first order Markov chain which approximates an AR 1 process with persistence equal to 0.5 and standard deviation of the innovations equal to 0.014.[17] We interpret the low state as a recession, the middle state as normal times, and the high state as a boom.

Perhaps the two most interesting aspects that simulations can clarify are the responses of fiscal policy instruments (tax rates and debt) to shocks, and the long run properties of the economy in general and of the multiplier λ in particular. While we know that λ converges almost surely to zero under a quasi-linear utility function, no such result can be established more generally, as one might expect given the numerical results from Aiyagari et al. (2002).

To study the response of fiscal policy instruments to shocks, we first let the economy 'converge to a steady state' under normal times. The economy then experiences a one-standard deviation shock for 2 periods, after which the economy return to normal

[16]This ratio depends on the level of distortions, which in turn depends on initial conditions.

[17]The persistence and variance of the shock matter for the results we present below. In particular, it is important to note that Farhi (2010) uses a process which is much more persistent and much more volatile than ours. The idea behind our choice of persistence was to have recessions that last 2 years on average. A future version of the paper will compare results under the alternative shock process.

times. The response of the economy to a negative shock is displayed in Figures 2.[18] Before describing that figure, it is important to note that these responses depend on the state of the economy at the time of the shock, i.e. the size of the capital stock, debt, and the multiplier λ. The position of these variables following a long sequence of neutral shocks depends on the initial conditions for debt and capital. Figure 2 was generated with $b_{-1} = 0$ and $k_{-1} = 1.5$, the same as in Figure 1. We discuss the impact of changing these initial conditions later in this section.

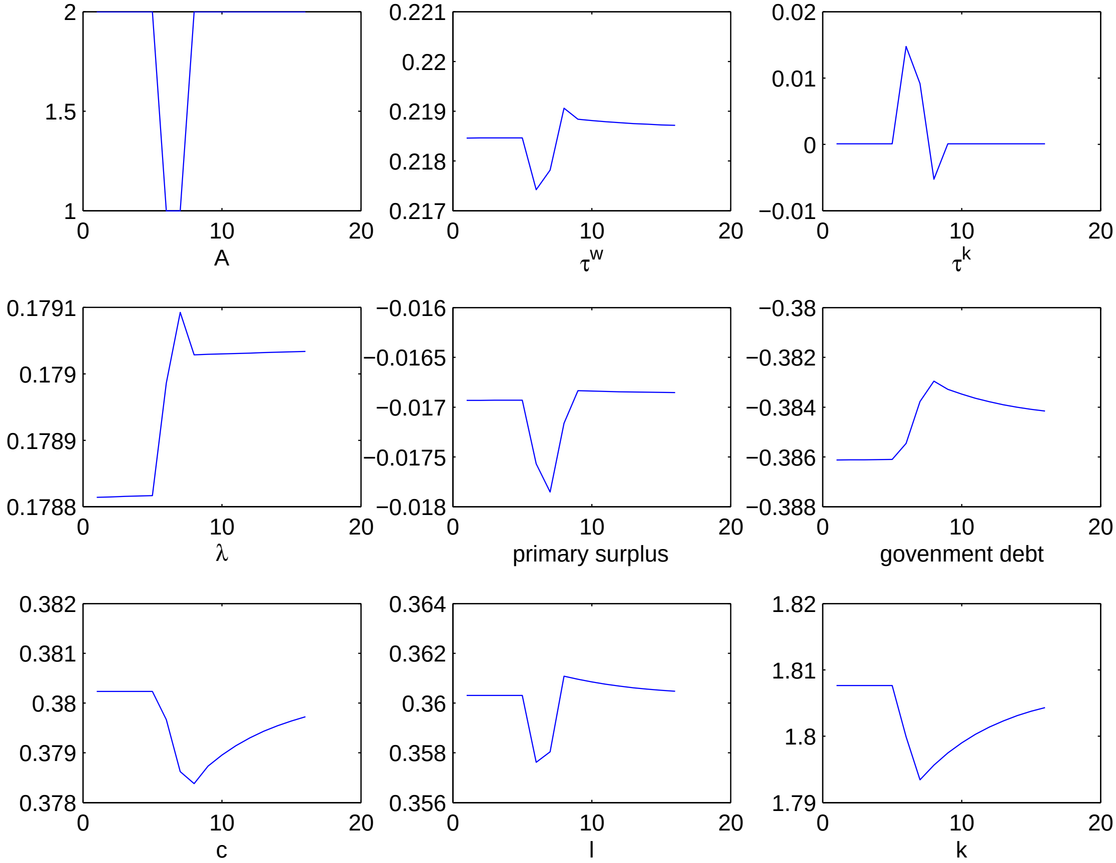
Figure 2 shows that while labor income tax decreases in recessions, the capital income tax increases. However, the magnitude of the increase in the capital income tax is minuscule relative to what Chari et al. (1994) report under the standard timing assumption, or relative to Farhi (2010) under a pre-announced capital income tax policy.[19] The reason of course is that the supply of capital in the short run is much more elastic under our timing assumption than it is in these other studies. And because of this small change in the capital income tax rate, the primary deficit of the government increases in recession. The rise in the deficit is financed by increasing the value of government debt. Furthermore, the tax on labor income increases and remains high once the economy is out of the recession, reflecting the fact that more debt needs to be financed. Similarly, the value of the multimplier λ—a measure of the extent of distortions in the economy—also remains high well after the recession is over.

To illustrate that the shape of these responses depends on the state of the economy at the outset of a recession, Figure 3 displays how the economy behaves in recessions when government debt is positive in the long run. This is generated by assuming a high level of government debt in the initial period, along with a relatively small stock of capital. After a long sequence of neutral shocks, this economy is much more distorted than the one we discussed above. This is reflected by a higher value of λ, a higher tax rate on labor, a primary deficit as opposed to a surplus, as well as lower consumption, labor, and capital. The behavior of this economy during a recession is similar to that shown in Figure 2 along most dimensions, but not for government debt. First, the increase in government debt is milder on impact, because the increase in the

[18] Since the response to a positive shock is essentially the mirror image, we omitted that figure.

[19] The fact that the labor tax is very stable is common, both in complete (Chari et al. (1994)) and incomplete markets settings (Farhi (2010)).

Figure 2: Response to a 2-Period Negative Shock

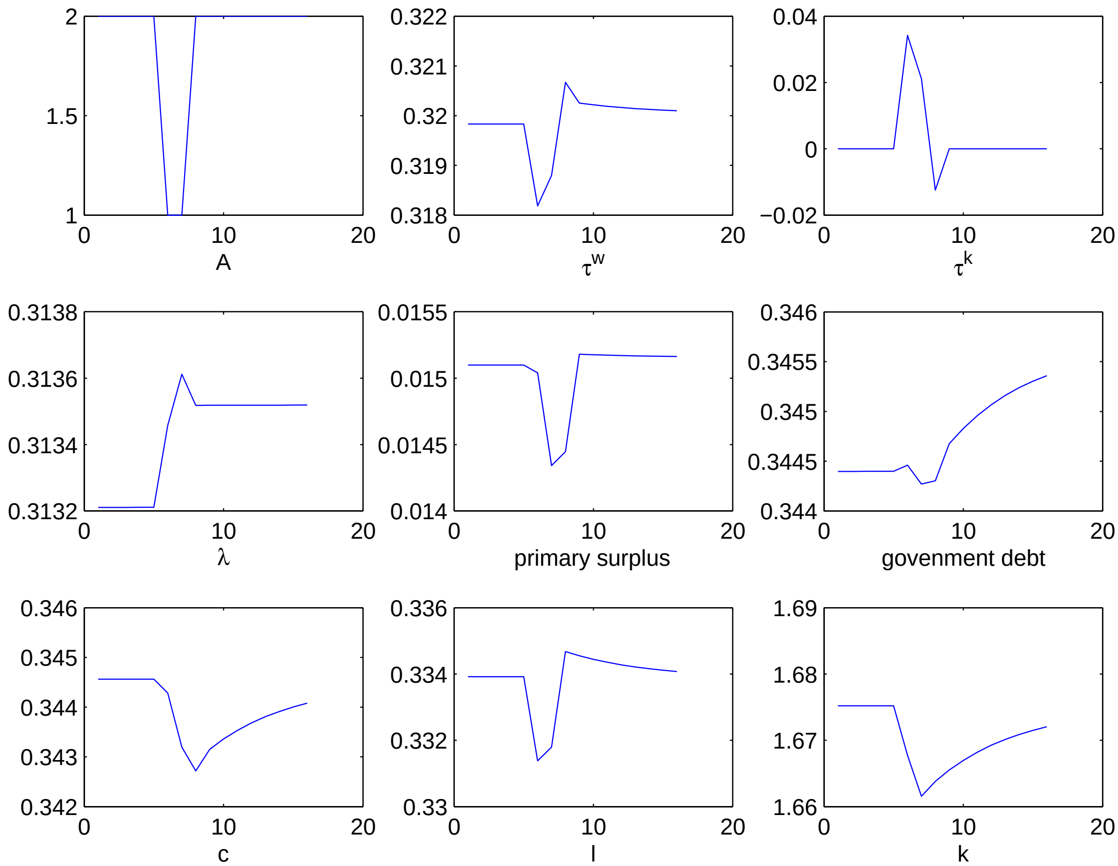


Note: The parameterization underlying this figure is discussed in the text. Initial capital is set to 1.5 and initial debt is set to 0. $A \in \{1, 2, 3\}$ refers to the shock; *primary surplus* means government spending minus tax revenues; *government debt* refer to the value of debt issued; other variables should be self-explanatory.

tax rate on capital income is more pronounced. Furthermore, the value of government debt actually falls for a couple periods before increasing again. Intuitively, while the value of debt issued in the period of the shock ($q(s^t)b(s^t)$) increases, part of the increase is due to an increase in the price of bonds in that period. As a result, the amount of debt that comes due the next period is relatively low, thereby offsetting the low primary surplus in that period. This effect disappears as the prices of bonds goes back down in normal times, i.e. as the interest rate goes back up. In the context of Figure 2, where government debt is negative, this price effect actually acts in the opposite direction, thereby amplifying the increase in the value of government debt.

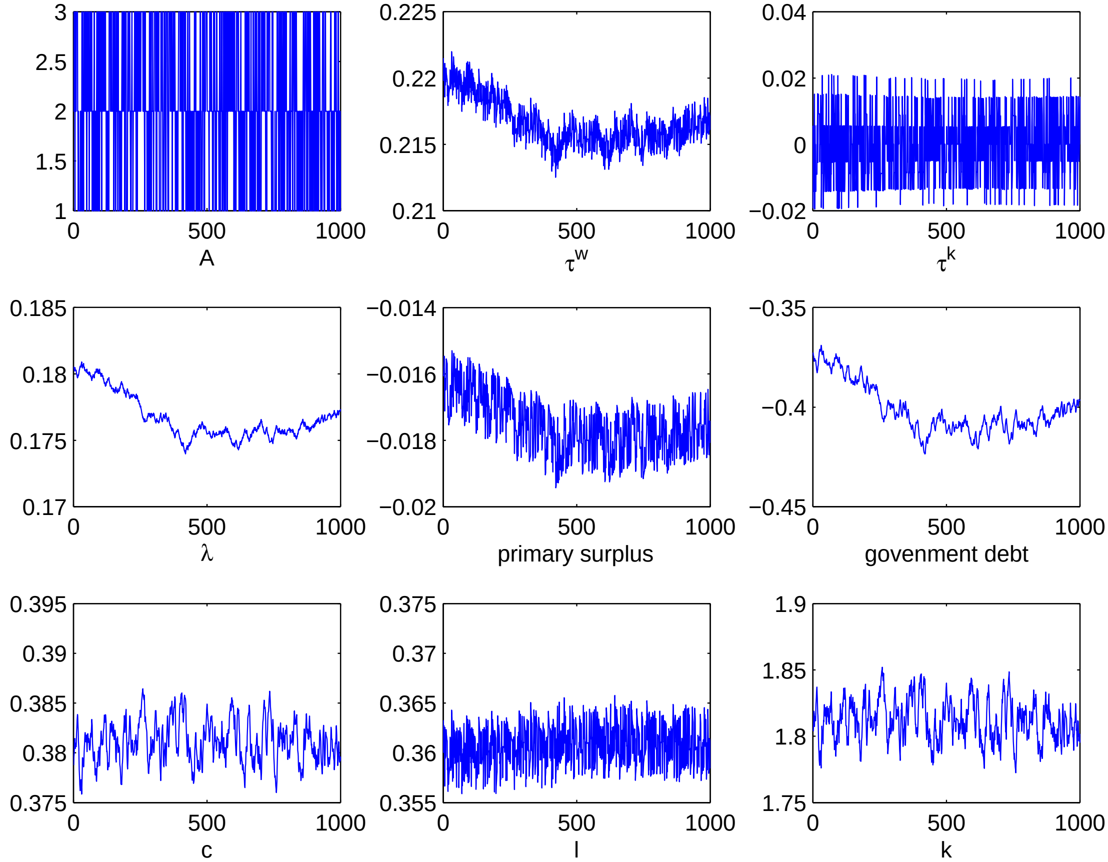
Moving to the behavior of the economy in the long run, Figure 4 plots the usual variables for the last 1000 periods of a 6000 period simulation. The first thing to notice is that the economy is still distorted ($\lambda > 0$) after 6000 periods. A second interesting aspect of this simulation concerns the relative persistence of variables over time. While all variables are persistent, the multiplier λ is noticeably more persistent than any other variable. The fact that distortions are persistent is reflected in the high persistence of debt, and, to a lesser extent, the tax on labor income. Finally, we note that the primary deficit is much less persistent than government debt, an empirical fact discussed at length in Marcet and Scott (2009).

Figure 3: Response to a 2-Period Negative Shock with Positive Debt



Note: The parameterization underlying this figure is discussed in the text. Initial capital is set to 1.0 and initial debt is set to 1.3. $A \in \{1, 2, 3\}$ refers to the shock; *primary surplus* means government spending minus tax revenues; *government debt* refer to the value of debt issued; other variables should be self-explanatory.

Figure 4: Long-Run Simulation



Note: The parameterization underlying this figure is discussed in the text. Initial capital is set to 1.5 and initial debt is set to 0. $A \in \{1, 2, 3\}$ refers to the shock; *primary surplus* means government spending minus tax revenues; government debt refer to the value of debt issued; other variables should be self-explanatory. The figure plots the last 1000 periods of a 6000 period simulation.

6 Conclusion

This paper studies optimal fiscal policy in a neoclassical growth model in which investment becomes productive within the period. We argue that in the context of Ramsey problems, this alternative timing is a useful assumption to avoid a perfectly inelastic supply of capital in the short run, which is at the heart of many results in the optimal taxation literature.

Our first result is that with an elastic supply of capital it is no longer optimal to confiscate initial asset holdings: the solution to the Ramsey problem features a unique non-trivial level of distortions without imposing exogenous bounds on tax instruments. A related result is that capital income taxes are no longer used as a shock absorber. However, state-contingent debt can be used for that purpose, leading to counterfactual movements between government debt and the primary deficit. This leads us to study a Ramsey problem without state-contingent debt, a typically hard problem which is considerably more tractable under our alternative timing assumption. The upshot of this problem is that the government runs debt-financed primary deficits during recessions.

A Timing Assumption

Imagine that any period t is divided into n sub-periods. During the first sub-period, the budget constraint is given by

$$c(s^t,1) + k(s^t,1) + \sum_{s_{t+1}} q(s^t,s_{t+1})b(s^t,s_{t+1})$$
$$= w(s^t,1)l(s^t,1) + \big(1 + r(s^t,1)\big)k(s^{t-1}) + b(s^t),$$

where $c(s^t,1)$ denotes consumption during the first sub-period, and similarly for other variables. Note that bonds are treated in an identical fashion as in the main text, that is, they are one period instruments. For sub-periods $i = 2,\ldots,n$, the budget constraint is then given by

$$c(s^t,i) + k(s^t,i) = w(s^t,i)l(s^t,i) + \big(1 + r(s^t,i)\big)k(s^t,i-1).$$

If we sum the sub-period budget constraints, we have

$$\sum_{i=1}^{n} c(s^t,i) + k(s^t,n) + \sum_{s_{t+1}} q(s^t,s_{t+1})b(s^t,s_{t+1})$$
$$= \sum_{i=1}^{n} \Big[w(s^t,i)l(s^t,i) + r(s^t,i)k(s^t,i-1)\Big] + k(s^{t-1}) + b(s^t).$$

This means that the conventional timing assumption boils down to assuming that

$$\sum_{i=1}^{n} \Big[(r(s^t,i))k(s^t,i-1)\Big] = r(s^t)k(s^{t-1}).$$

Accordingly, our timing corresponds to the opposite extreme assumption that

$$\sum_{i=1}^{n} \Big[(r(s^t,i))k(s^t,i-1)\Big] = r(s^t)k(s^t).$$

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