An Equilibrium Asset Pricing Model with Labor Market Search

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Abstract

Frictions in the labor market are important for understanding the equity premium in the financial market. We embed the Diamond-Mortensen-Pissarides search framework into a dynamic stochastic general equilibrium model with recursive preferences. The model produces realistic equity premium and stock market volatility, as well as low interest rate and interest rate volatility. The equity premium is also countercyclical, and forecastable with labor market tightness, a pattern we confirm in the data. Intriguingly, three key ingredients in the model (small profits, large job flows, and matching frictions) combine to give rise endogenously to rare economic disasters à la Rietz (1988) and Barro (2006).

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1 Introduction

Modern asset pricing research has been successful in specifying preferences and cash flow dynamics to explain the equity premium, its volatility, and its cyclical dynamics in endowment economies. Accounting for the equity premium in production economies with endogenous cash flows has proven more difficult (e.g., Rouwenhorst (1995); Jermann (1998)). However, prior studies treat consumption mostly as dividends. In the data labor income represents about two thirds, while dividends comprise only a small fraction of aggregate disposable income. As such, an equilibrium macroeconomic model of asset prices should take the labor market seriously.

We study aggregate asset prices by embedding search frictions in the labor market (e.g., Diamond (1982); Mortensen (1982); Pissarides (1985, 2000)) into a dynamic stochastic general equilibrium economy with recursive preferences. A representative household pools incomes from its employed and unemployed workers, and decides on optimal consumption and asset allocation. The unemployed workers search for job vacancies posted by a representative firm. The labor market is represented as a matching function that takes vacancies and unemployed workers as inputs to produce the number of new hires (filled vacancies). The rate at which a vacancy is filled decreases with the congestion in the labor market or labor market tightness, which is defined as the ratio of the number of vacancies over the number of unemployed workers. Deviating from Walrasian equilibrium, matching frictions create rents to be divided between the firm and the employed workers through the wage rate, which is in turn determined by the outcome of a generalized Nash bargaining process.

We report two major results. First, our economy provides a coherent account of aggregate asset prices. Quantitatively, the economy reproduces an equity premium of 5.70% and an average stock market volatility of 10.83% per annum. Both moments are adjusted for financial leverage, and are close to the moments in the data, 5.07% and 12.94%, respectively. The equity premium is also countercyclical in the model. The vacancy-unemployment ratio forecasts stock market excess returns with a significantly negative slope, a pattern we confirm in the data. In the model, the average interest rate is 2.90%, which is somewhat high relative to 0.59% in the data. The interest rate volatility is 1.34%, which is close to 1.87% in the data. Finally, the model is also broadly consistent with business cycle moments for aggregate quantities as well as labor market variables.

Second, our economy reproduces endogenously rare but deep disasters per Rietz (1988) and Barro (2006). In the model's simulated stationary distribution, the unemployment rate is positively skewed with a long right tail. The mean unemployment rate is 8.51%, and its skewness is 7.83. The 2.5 percentile is 5.87%, which is not far from the median of 7.30%, but the 97.5 percentile is far away, 19.25%. As such, output and consumption are both negatively skewed with a long left tail, giving rise endogenously to rare disasters. Applying Barro and Ursúa's (2008) peak-to-trough measurement on the simulated data, we find that the consumption and GDP disasters in the model have the same average magnitude, about 20%, as in the data. The consumption disaster probability is 3.08% in the model, which is close to 3.63% in the data. The GDP disaster probability is 4.66%, which is somewhat high relative to 3.69% in the data. However, both disaster probabilities in the data are within one cross-simulation standard deviation from the disaster probabilities in the model.

We show via comparative statics that three key ingredients of the model (small profits, large job flows, and matching frictions), when combined, are capable of producing disasters and a realistic equity premium. First, we adopt a relatively high value of unemployment activities, implying small profits (output minus wages). Also, a high value of unemployment makes wages relatively inelastic to labor productivity, giving rise to operating leverage. In recessions, output falls, but wages do not fall as much, causing profits to drop disproportionately more than output. As such, by dampening the procyclical covariation of wages, the small profits magnify the procyclical covariation (risk) of dividends, causing the equity premium to rise. Finally, the impact of the inelastic wages is stronger in worse economic conditions, when the profits are even smaller (because of lower productivity). This time-varying operating leverage amplifies the risk and risk premium, making the equity premium and the stock market volatility countercyclical.

Second, job flows are large in the model. The labor market is characterized by large job flows in and out of employment. In particular, whereas the rate of capital depreciation is around 1% per month (e.g., Cooper and Haltiwanger (2006)), the worker separation rate is 5% in the data (e.g., Davis, Faberman, Haltiwanger, and Rucker (2010)). As such, contrary to swings in investment that have little impact on the disproportionately large capital stock, cyclical variations in job flows cause large fluctuations in aggregate employment. Because capital (not investment per se) enters the production function, volatile but small investment flows have little impact on the output volatility. In contrast, the large job flows out of employment put a tremendous strain on the labor market to put unemployed workers back to work. Any frictions that disrupt this process in the labor market have a major impact on the macroeconomy. Consequently, economies with labor market frictions can be substantially riskier than baseline production economies without labor market frictions.

Third, matching frictions induce downward rigidity in the marginal costs of hiring. During the matching process, if one side of the labor market becomes more abundant than the other side, it will be increasingly difficult for the abundant side to meet and trade with the other side (which becomes relatively scarce). In particular, expansions are periods in which many vacancies compete for a small pool of unemployed workers. The entry of an additional vacancy can cause a pronounced drop in the probability of a given vacancy being filled. This effect raises the marginal costs of hiring, slowing down job creation flows and making expansions more gradual.

Conversely, recessions are periods in which many unemployed workers compete for a small pool of vacancies. Filling a vacancy with an unemployed worker occurs quickly, and the marginal costs of hiring are lower. However, the congestion in the labor market affects unemployed workers, rather than vacancies in recessions. The entry of an additional vacancy has little impact on the probability of any given vacancy being filled. As such, although the marginal costs of hiring can rise rapidly in expansions, the marginal costs decline only slowly in recessions. This downward rigidity is further reinforced by fixed matching costs per Mortensen and Nagypál (2007) and Pissarides (2009). By putting a constant component into the marginal costs of hiring, the fixed costs restrict the marginal costs from declining fast in recessions, further hampering job creation flows.

To see how the three key ingredients combine to endogenize disasters, consider a large negative shock hitting the economy. The profits, which are small to begin with, become even smaller as a result of lower labor productivity. Also, wages are inelastic, staying at a relatively high level, reducing the small profits still further. To make a bad situation worse, the marginal costs of hiring run into downward rigidity, an inherent attribute of the matching process, which is further buttressed by fixed matching costs. As the marginal costs of hiring fail to decline to counteract the impact of shrinking profits, the incentives of hiring are suppressed, stifling job creation flows. All the while, jobs continue to be destroyed at a high rate of 5% per month. Consequently, aggregate employment falls off a cliff, giving rise endogenously to economic disasters.

Our work provides two new insights to the macro finance literature. First, labor market frictions are important, if not essential, for equilibrium asset prices. In baseline production economies, often with capital as the only productive input, the amount of endogenous risk is too small, giving rise to a negligible and time-invariant equity premium.[1] We show that labor market frictions seem to

[1]Rouwenhorst (1995) shows that the standard real business cycle model fails to explain the equity premium because of consumption smoothing. With internal habit preferences, Jermann (1998) and Boldrin, Christiano,

overcome many of these difficulties in production economies. Second, labor market frictions are important for *endogenizing* rare disasters in production economies.[2] To our knowledge, most, if not all, studies in the existing disasters risk literature specify disasters exogenously either on aggregate total factor productivity or on both aggregate productivity and capital stock. However, while there exists some evidence on consumption and GDP disasters, direct evidence on productivity disasters seems scarce. In our model, productivity follows a standard autoregressive process with homoscedastic shocks. As such, our endogenous disaster mechanism helps reconcile the existing exogenous disaster models with the lack of direct evidence on productivity disasters.

Our work also adds to the labor search literature.[3] Methodologically, we solve the search model using a globally nonlinear projection algorithm with parameterized expectations à la Christiano and Fisher (2000). New to the search literature, this solution algorithm allows us to characterize disaster dynamics in the search model. In contrast, disaster dynamics have been missed so far in the search literature, in which models have traditionally been solved with localized linearization methods.

Our work is related to Danthine and Donaldson (2002), who show that the priority status of wages magnifies the risk of dividends to produce a high equity premium. Uhlig (2007) shows that wage rigidity helps explain the Sharpe ratio and the interest rate volatility in an external habit model.[4] Instead of specifying wages exogenously, we differ by using the standard search framework with period-by-period Nash bargaining to delink wages from marginal product of labor endogenously. We also go further in characterizing the cyclical dynamics of the equity premium, the stock market volatility, and dividends, as well as endogenous disaster dynamics. Bazdresch, Belo, and Lin (2009) show that labor adjustment costs help explain the cross section of expected stock returns. We differ by examining aggregate asset prices in an equilibrium search economy.

and Fisher (2001) use capital adjustment costs and cross-sector immobility, respectively, to restrict consumption smoothing to explain the equity premium. However, both models struggle with excessively high interest rate volatilities. Lettau and Uhlig (2000) show that external habit causes consumption to be excessively smooth relative to that in the data. Using recursive preferences to curb interest rate volatility, Tallarini (2000) and Kaltenbrunner and Lochstoer (2010) show that baseline production economies with and without capital adjustment costs still fail to match the equity premium and the stock market volatility (see also Campanale, Castro, and Clementi (2010)).

[2]Rietz (1988) argues that rare disasters explain the equity premium puzzle. Barro (2006), Barro and Ursúa (2008), and Barro and Jin (2011) examine long-term international data that include many disasters (see also Reinhart and Rogoff (2009)). Gourio (2010) embeds disasters exogenously into a production economy to examine the equity premium.

[3]Merz (1995) and Andolfatto (1996) embed search frictions into the real business cycle framework. Shimer (2005) argues that the unemployment volatility in the search model is too low relative to that in the data. Hagedorn and Manovskii (2008) use small profits, and Mortensen and Nagypál (2007) and Pissarides (2009) use fixed matching costs to address the Shimer puzzle. We instead study equilibrium asset prices.

[4]In addition, Gourio (2007) shows that operating leverage derived from labor contracting helps explain the cross-section of expected stock returns. Favilukis and Lin (2012) quantify the role of infrequent wage renegotiations in an asset pricing model with long run productivity risk and labor adjustment costs.

The rest is organized as follows. Section 2 constructs the model. Section 3 describes the calibration. Sections 4 and 5 present quantitative results on asset prices and disasters, respectively. Section 6 concludes. All the proofs, computational details, and supplementary results are in the appendices.

2 The Model

We embed the standard Diamond-Mortensen-Pissarides (DMP) search model into a dynamic stochastic general equilibrium economy with recursive preferences.

2.1 Search and Matching

The model is populated by a representative household and a representative firm that uses labor as the single productive input. Following Merz (1995), we use the representative family construct, which implies perfect consumption insurance. The household has a continuum (of mass one) of members who are, at any point in time, either employed or unemployed. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the members together before choosing per capita consumption and asset holdings.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers, U_t. Vacancies are filled via a constant returns to scale matching function, $G(U_t, V_t)$, specified as:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^\iota + V_t^\iota)^{1/\iota}}, \tag{1}$$

in which $\iota > 0$ is a constant parameter. This matching function, originated from Den Haan, Ramey, and Watson (2000), has the desirable property that matching probabilities fall between zero and one.

In particular, define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate), $f(\theta_t)$, is:

$$f(\theta_t) \equiv \frac{G(U_t, V_t)}{U_t} = \frac{1}{\left(1 + \theta_t^{-\iota}\right)^{1/\iota}}, \tag{2}$$

and the probability for a vacancy to be filled per unit of time (the vacancy filling rate), $q(\theta_t)$, is:

$$q(\theta_t) \equiv \frac{G(U_t, V_t)}{V_t} = \frac{1}{(1 + \theta_t^\iota)^{1/\iota}}. \tag{3}$$

It follows that $f(\theta_t) = \theta_t q(\theta_t)$ and $q'(\theta_t) < 0$, meaning that an increase in the scarcity of unemployed workers relative to vacancies makes it harder to fill a vacancy. As such, θ_t is labor market tightness from the firm's perspective, and $1/q(\theta_t)$ is the average duration of vacancies.

The representative firm incurs costs in posting vacancies. Following Mortensen and Nagypál (2007) and Pissarides (2009), we assume that the unit costs per vacancy, denoted κ_t, contain two components, the proportional costs, κ_0, and the fixed costs, κ_1. Formally,

$$\kappa_t \equiv \kappa_0 + \kappa_1 q(\theta_t), \tag{4}$$

in which $\kappa_0, \kappa_1 > 0$. The proportional costs are standard in the search literature. The fixed costs aim to capture matching costs, such as training, interviewing, negotiation, and administrative setup costs of adding a worker to the payroll, costs paid after a hired worker arrives but before wage bargaining takes place. The marginal costs of hiring arising from the proportional costs, $\kappa_0/q(\theta_t)$, increase with the mean duration of vacancies, $1/q(\theta_t)$. In contrast, the marginal costs from the fixed costs are "fixed" at κ_1 (independent of the duration of vacancies). The total marginal costs of hiring are given by $\kappa_0/q(\theta_t)+\kappa_1$. In expansions, the labor market is tighter for the firm (θ_t is higher), meaning that the vacancy filling rate, $q(\theta_t)$, is lower. As such, the marginal costs of hiring are procyclical.

Once matched, jobs are destroyed at a constant rate of s per period. Employment, N_t, evolves as:

$$N_{t+1} = (1-s)N_t + q(\theta_t)V_t, \tag{5}$$

in which $q(\theta_t)V_t$ is the number of new hires. The size of the population is normalized to be unity, $U_t = 1 - N_t$. As such, N_t and U_t are also the rates of employment and unemployment, respectively.

2.2 The Representative Firm

The firm takes aggregate labor productivity, X_t, as given. The law of motion of $x_t \equiv \log(X_t)$ is:

$$x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}, \tag{6}$$

in which $\rho \in (0,1)$ is the persistence, $\sigma > 0$ is the conditional volatility, and ϵ_{t+1} is an independently and identically distributed (i.i.d.) standard normal shock. The firm uses labor to produce output, Y_t, with a constant returns to scale production technology,

$$Y_t = X_t N_t. \tag{7}$$

To keep the model parsimonious to zero in on the impact of labor market frictions on asset prices, we abstract from physical capital in the production function. The absence of capital is unlikely to be important for our quantitative results. As noted, small (albeit volatile) investment flows have little impact on fluctuations in aggregate capital, which is largely fixed at business cycle frequencies. As a testimony to the quasi-fixity of capital, one has to assume an excessively large volatility for exogenous productivity shocks to match the output growth volatility in a baseline production economy, (e.g., Kaltenbrunner and Lochstoer (2010)). As such, the majority of cyclical variations in aggregate output is driven by movements in aggregate employment (e.g., Cogley and Nason (1995)). In addition, hiring decisions are mostly driven by movements in the marginal product of labor. Because capital, not investment, enters the marginal product of labor, volatile but small investment flows have little impact on the marginal product of labor (and hiring decisions). As such, it is not surprising that many important labor search studies abstract from capital (e.g., Shimer (2005); Mortensen and Nagypál (2007); Pissarides (2009)).

The dividends to the firm's shareholders are given by:

$$D_t = X_t N_t - W_t N_t - \kappa_t V_t, \tag{8}$$

in which W_t is the wage rate (to be determined later in Section 2.4). Let $M_{t+\triangle t}$ be the representative household's stochastic discount factor from period t to $t+\triangle t$. Taking the matching probability, $q(\theta_t)$, and the wage rate, W_t, as given, the firm posts an optimal number of job vacancies to maximize the cum-dividend market value of equity, denoted S_t:

$$S_t \equiv \max_{\{V_{t+\triangle t}, N_{t+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t \left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t} \left[X_{t+\triangle t} N_{t+\triangle t} - W_{t+\triangle t} N_{t+\triangle t} - \kappa_{t+\triangle t} V_{t+\triangle t} \right] \right], \tag{9}$$

subject to the employment accumulation equation (5) and a nonnegativity constraint on vacancies:

$$V_t \geq 0. \tag{10}$$

Because $q(\theta_t) > 0$, this constraint is equivalent to $q(\theta_t) V_t \geq 0$. As such, the only source of job destruction in the model is the exogenous separation of employed workers from the firm.[5]

[5] The nonnegativity constraint on vacancies has been ignored so far in the labor search literature, in which models are traditionally solved via linearization methods. Using a globally nonlinear projection algorithm, we find that the nonnegativity constraint is occasionally binding in the simulations from the search model, especially when profits are small. Because a negative vacancy does not make economic sense, we feel compelled to impose the nonnegativity constraint to solve the model accurately, albeit with higher computational costs. However, the constraint is not a central ingredient of the model. In simulations based on our benchmark calibration, the constraint only binds for 0.013% of the time, which is extremely rare. In addition, the zero-vacancy observations are more the effect than the cause. Small profits and large job flows are the causes. We have simulated the models with large profits, and separately, with small job flows. The constraint never binds in these economies. As such, the constraint per se is not the driving force of our results. Furthermore, relaxing this constraint in the form of endogenous job destruction is likely to strengthen, rather than weaken our results. Endogenous job destruction should rise during recessions,

Let λ_t denote the multiplier on the nonnegativity constraint $q(\theta_t)V_t \geq 0$. From the first-order conditions with respect to V_t and N_{t+1}, we obtain the intertemporal job creation condition:

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s) \left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda_{t+1} \right] \right] \right]. \tag{11}$$

Intuitively, the marginal costs of hiring at time t (with the nonnegativity constraint accounted for) equal the marginal value of employment to the firm, which in turn equals the marginal benefit of hiring at period $t+1$, discounted to t with the stochastic discount factor, M_{t+1}. The marginal benefit at $t+1$ includes the marginal product of labor, X_{t+1}, net of the wage rate, W_{t+1}, plus the marginal value of employment, which equals the marginal costs of hiring at $t+1$, net of separation. Finally, the optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t) V_t = 0. \tag{12}$$

Recalling S_t is the cum-dividend equity value, we define the stock return as $R_{t+1} \equiv S_{t+1}/(S_t - D_t)$. The constant returns to scale assumption implies that (see Appendix A for derivations):

$$R_{t+1} = \frac{X_{t+1} - W_{t+1} + (1-s)\left[\kappa_0/q(\theta_{t+1}) + \kappa_1 - \lambda_{t+1}\right]}{\kappa_0/q(\theta_t) + \kappa_1 - \lambda_t}. \tag{13}$$

Intuitively, the stock return is the quantitative tradeoff between the marginal benefit of hiring that accrues in period $t+1$ and the marginal costs of hiring incurred over period t, as in Cochrane (1991).

2.3 The Representative Household

The household maximizes utility, denoted J_t, over consumption using recursive preferences (e.g., Kreps and Porteus (1978); Epstein and Zin (1989)) by trading risky shares issued by the representative firm and a risk-free bond. Let C_t denote consumption. The recursive utility function is given by:

$$J_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left(E_t \left[J_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \tag{14}$$

in which β is time discount factor, ψ is the elasticity of intertemporal substitution, and γ is relative risk aversion. This utility function separates ψ from γ, allowing the model to produce a high equity premium and a low interest rate volatility simultaneously.

amplifying the disaster dynamics. Finally, even if we allow endogenous job destruction, the resulting economy is likely to be very similar to our benchmark economy. The level of labor productivity at which the representative firm would start to fire workers is lower than the level at which the firm does not post vacancies. As such, the frequency of endogenous job destruction should be even lower than 0.013%.

The household's first-order condition gives rise to the fundamental equation of asset pricing:

$$1 = E_t[M_{t+1}R_{t+1}]. \tag{15}$$

In particular, the stochastic discount factor, M_{t+1}, is given by:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{E_t[J_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \tag{16}$$

Finally, the risk-free rate is given by $R^f_{t+1} = 1/E_t[M_{t+1}]$.

2.4 Wage Determination

The wage rate is determined endogenously by applying the sharing rule per the outcome of a generalized Nash bargaining process between the employed workers and the firm. Let $\eta \in (0,1)$ be the workers' relative bargaining weight and b the workers' value of unemployment activities. The equilibrium wage rate is given by (see Appendix B for derivations):

$$W_t = \eta\left(X_t + \kappa_t\theta_t\right) + (1-\eta)b. \tag{17}$$

The wage rate is increasing in labor productivity, X_t, and in the total vacancy costs per unemployed worker, $\kappa_t\theta_t = \kappa_t V_t/U_t$. Intuitively, the more productive the workers are, and the more costly for the firm to fill a vacancy, the higher the wage rate is for employed workers. Also, the value of unemployment activities, b, and the workers' bargaining weight, η, affect the wage elasticity to labor productivity. The lower η is, and the higher b is, the more the wage rate is tied with the constant value of unemployment activities, inducing a lower wage elasticity to productivity.

2.5 Competitive Equilibrium

In equilibrium, the financial markets clear. The risk-free asset is in zero net supply, and the household holds all the shares of the representative firm. As such, the equilibrium return on wealth equals the stock return, and the household's financial wealth equals the cum-dividend equity value of the firm. The goods market clearing condition is then given by:

$$C_t + \kappa_t V_t = X_t N_t. \tag{18}$$

The competitive equilibrium in the search economy consists of vacancy posting, $V_t^\star \geq 0$; multiplier, $\lambda_t^\star \geq 0$; consumption, $C_t^\star$; and indirect utility, $J_t^\star$; such that (i) $V_t^\star$ and $\lambda_t^\star$ satisfy the

Table 1 : Parameter Values in the Benchmark Monthly Calibration

Notation	Parameter	Value
β	Time discount factor	$0.99^{1/3}$
γ	Relative risk aversion	10
ψ	The elasticity of intertemporal substitution	1.5
ρ	Aggregate productivity persistence	0.983
σ	Conditional volatility of productivity shocks	0.0077
η	Workers' bargaining weight	0.052
b	The value of unemployment activities	0.85
s	Job separation rate	0.05
ι	Elasticity of the matching function	1.25
κ_0	The proportional costs of vacancy posting	0.6
κ_1	The fixed costs of vacancy posting	0.4

intertemporal job creation condition (11) and the Kuhn-Tucker conditions (12), while taking the stochastic discount factor in equation (16) and the wage equation (17) as given; (ii) $C_t^\star$ and $J_t^\star$ satisfy the intertemporal consumption-portfolio choice condition (15), in which the stock return is given by equation (13); and (iii) the goods market clears as in equation (18).

3 Calibration and Computation

We calibrate the model in Section 3.1 and discuss our global solution algorithm in Section 3.2.

3.1 Calibration

We calibrate the model in monthly frequency. Table 1 lists the parameter values in the benchmark calibration. For the five preference and technology parameters, our general calibration strategy is to use values that are (largely) standard in the literature. In particular, following Bansal and Yaron (2004), we set the risk aversion, γ, to be 10, and the elasticity of intertemporal substitution, ψ, to be 1.5. Following Gertler and Trigari (2009), we set the time discount factor, β, to be $0.99^{1/3}$, the persistence of the (log) aggregate productivity, ρ, to be $0.95^{1/3}$, and the conditional volatility of the aggregate productivity, σ, to be 0.0077. In particular, the σ value is chosen so that the volatilities of consumption growth and output growth in the model are largely in line with those in the data.

For the labor market parameters, our general calibration strategy is to use existing evidence and quantitative studies (as much as we can) to restrict their values. For the parameters whose values are important in driving our quantitative results, we conduct extensive comparative statics to evaluate their impact and to understand the underlying mechanism. It is worthwhile pointing out that our calibration strategy differs from the standard practice in the search literature that relies only on steady state relations. In our highly nonlinear model, steady state restrictions hold very poorly in the model's simulations. This nonlinearity means that matching a given moment precisely in simulations is virtually impossible. As such, we exercise care in reporting a wide range of simulated moments to compare with moments in the data.

Our calibration of the workers' bargaining weight, η, and the value of unemployment activities, b, is in the same spirit as in Hagedorn and Manovskii (2008). Hagedorn and Manovskii calibrate η to be 0.052 to match the wage elasticity to labor productivity, which is estimated to be 0.49 in their sample. We set η to be the same value, which in turn implies a wage elasticity of 0.58 in the model. This value of η is somewhat conservative in the sense that we could have used a lower value to generate a lower wage elasticity than that in the benchmark calibration.

The calibration of b is more controversial in the labor search literature. Shimer (2005) pins down $b = 0.4$ by assuming that the only benefit for an unemployed worker is government unemployment insurance. However, Mulligan (2012) estimates that the ratio of the average monthly overall safety net benefit over the median monthly earnings of heads and spouses can be as high as 0.70.[6] Hagedorn and Manovskii (2008) also argue that in a perfectly competitive labor market, b should equal the value of employment. The value of unemployment activities measures not only unemployment insurance, but also the total value of home production, self-employment, disutility of work, and leisure. In the model, the average marginal product of labor is unity, to which b should be close. We set b to be 0.85, which is the same value in Rudanko (2011). Although relatively high, this value of b is not as extreme as 0.955 in Hagedorn and Manovskii.[7]

More generally, we view the high-b calibration only as a parsimonious modeling device to obtain

[6] Mulligan (2012, p. 29) reports the median monthly earnings of heads and spouses to be \$3,148, payroll taxes \$482, and the overall net monthly safety net benefit \$1,560 on average during fiscal year 2007 (and is \$300 per month greater in 2009 and 2010). As such, the (replacement) ratio can be calculated as $(1,560+300)/(3,148-482) = 0.70$.

[7] We have implicitly assumed that the value of unemployment activities, b, does not enter the resource constraint in equation (18). The part of b that is due to government unemployment benefits can be taken out of the resource constraint by assuming that the government finances the unemployment benefits via taxing the representative household. The part of b that is derived from, for example, home production does not enter the resource constraint because the output from home production is not marketable. One might value a home-cooked meal because it is prepared by one's spouse. However, such a home-made meal has a very low market value because other people would rather go to a real restaurant. By the same taken, home production might not enter the stochastic discount factor. Whether one had a nice dinner at home Sunday night or whether one shot under par on the golf course over the weekend should not affect how one prices stocks coming to work Monday morning.

small profits and inelastic wages, which are important for realistic labor market volatilities. We have nothing new to say about labor market volatilities. Rather, our key insight is that conditional on realistic labor market dynamics, a search model also has important implications for asset prices and disasters. The parsimony with the high-b calibration is valuable, both conceptually as a first stab in embedding the DMP structure into an equilibrium asset pricing framework, and pragmatically as a first step in solving the resulting model nonlinearly (see Section 3.2 for our algorithm). Other specifications with small profits and inelastic wages are likely to have similar implications. However, to what extent this statement is true, quantitatively, is left for future research.

We set the job separation rate, s, to 5%. This value, which is also used in Andolfatto (1996), is estimated in Davis, Faberman, Haltiwanger, and Rucker (2010, Table 5.4), and is within the range of estimates from Davis, Faberman, and Haltiwanger (2006). This estimate is higher than 3.7% from the publicly available Job Openings and Labor Turnover Survey (JOLTS). As pointed out by Davis, Faberman, Haltiwanger, and Rucker, the JOLTS sample overweights relatively stable establishments with low rates of hires and separations and underweights establishments with rapid growth or contraction (see also Hall (2010)). For the elasticity parameter in the matching function, ι, we set it to be 1.25, which is close to the value in Den Haan, Ramey, and Watson (2000). We also report comparative statics by varying its value to 0.9.

To pin down the two parameters in the vacancy costs, κ_0 and κ_1, we first experiment so that the unit costs of vacancy posting are on average around 0.8 in the model's simulations. This level of the average unit costs is necessary for the model to reproduce a realistic unemployment rate. The average unemployment rate for the United States over the 1920–2009 period is about 7%. However, flows in and out of nonparticipation in the labor force, as well as discouraged workers not accounted for in the pool of individuals seeking employment, suggest that the unemployment rate should be somewhat higher. In the simulations with the benchmark calibration, the mean unemployment rate is 8.51% (and the median is 7.3%). The evidence on the relative weights of the proportional costs and the fixed costs out of the total unit costs of vacancy seems scarce. To pin down κ_0 and κ_1 separately, we set the weight of the fixed costs to be 25%, meaning $\kappa_0 = 0.6$ and $\kappa_1 = 0.4$. We also report comparative statics in which the weight of the fixed costs is zero, meaning the unit costs of vacancy are constant, around 0.8.

Is the magnitude of the vacancy (hiring) costs in the model empirically plausible? The model implies that the marginal costs of vacancy posting in terms of labor productivity (output per worker) equal 0.815, which is the average of $\kappa_0 + \kappa_1 q(\theta_t)$ in simulations (the average labor productivity is unity). The marginal costs of hiring are on average 1.59, which is the average of $\kappa_0/q(\theta_t) + \kappa_1$. Merz and Yashiv (2007) estimate the marginal costs of hiring to be 1.48 times the average output per worker with a standard error of 0.57. As such, 1.59 is within the plausible range of empirical estimates. For the total costs of vacancy, $\kappa_t V_t$, the average in the model's simulations is about 0.73% of annual wages. This magnitude does not appear implausibly large. In particular, the estimated labor adjustment costs in Bloom (2009) imply limited hiring and firing costs of "about 1.8% of annual wages" and high fixed costs of "around 2.1% of annual revenue (p. 663)."

3.2 Computation

Although analytically transparent, solving the model numerically is quite challenging. First, the search economy is not Pareto optimal because the competitive equilibrium does not correspond to the social planner's solution. Intuitively, the firm in the decentralized economy does not take into account the congestion effect of posting a new vacancy on the labor market when maximizing the equity value, whereas the social planner does when maximizing social welfare. As such, we must solve for the competitive equilibrium from the optimality conditions directly. Unlike value function iterations, algorithms that approximate the solution to optimality conditions do not have convenient convergence properties. Second, because of the occasionally binding constraint on vacancy, standard perturbation methods cannot be used. As such, we solve for the competitive equilibrium using a globally nonlinear projection algorithm, while applying the Christiano and Fisher (2000) idea of parameterized expectations to handle the vacancy constraint.

Third, because of the model's nonlinearity and our focus on nonlinearity-sensitive asset pricing and disaster moments, we must solve the model on a large, fine grid to ensure accuracy. We must also apply homotopy to visit the parameter space in which the model exhibits strong nonlinearity. Because many economically interesting parameterizations imply strong nonlinearity, we can only update the parameter values very slowly to ensure the convergence of the projection algorithm.

The state space of the model consists of employment and productivity, (N_t, x_t). The goal is to solve for the optimal vacancy function: $V_t^\star = V(N_t, x_t)$, the multiplier function: $\lambda_t^\star = \lambda(N_t, x_t)$, and an indirect utility function: $J_t^\star = J(N_t, x_t)$ from two functional equations:

$$
\begin{aligned}
J(N_t, x_t) &= \left[(1-\beta)C(N_t, x_t)^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J(N_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} & (19)\\
\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda(N_t, x_t) &= E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right]\right]\right]. & (20)
\end{aligned}
$$

$V(N_t, x_t)$ and $\lambda(N_t, x_t)$ must also satisfy the Kuhn-Tucker conditions (12).

The standard projection method would approximate $V(N_t, x_t)$ and $\lambda(N_t, x_t)$ to solve equations (19) and (20), while obeying the Kuhn-Tucker conditions. With the occasionally binding constraint, the vacancy and multiplier functions are not smooth, making the standard projection method tricky and cumbersome to apply. As such, we adapt the Christiano and Fisher (2000) parameterized expectations method by approximating the right-hand side of equation (20):

$$
\mathcal{E}_t \equiv \mathcal{E}(N_t, x_t) = E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left[\frac{\kappa_0}{q(\theta_{t+1})} + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right]\right]\right]. \quad (21)
$$

We then exploit a convenient mapping from the conditional expectation function to policy and multiplier functions, thereby eliminating the need to parameterize the multiplier function separately. Specifically, after obtaining the parameterized $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) = \kappa_0/\left(\mathcal{E}_t - \kappa_1\right)$. If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ defined in equation (3), and $V_t = \theta_t(1-N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$. Appendix C contains additional details of our computational methods.

4 Quantitative Results: Asset Prices

We present basic business cycle and asset pricing moments in Section 4.1. In Section 4.2, we examine time-varying risk premiums by using labor market tightness to forecast stock market excess returns. We study the model's implications for long run risks and volatility risks in Section 4.3 as well as procyclical dividends in Section 4.4. Finally, Section 4.5 reports several comparative statics.

4.1 Basic Business Cycle and Financial Moments

Panel A of Table 2 reports the standard deviation and autocorrelations of log consumption growth and log output growth, as well as unconditional financial moments in the data. Consumption is annual real personal consumption expenditures, and output is annual real gross domestic product from 1929 to 2010 from the National Income and Product Accounts (NIPA) at Bureau of Economic Analysis. The annual consumption growth in the data has a volatility of 3.04%, and a first-order autocorrelation of 0.38. The autocorrelation drops to 0.08 at the two-year horizon, and turns negative, −0.21, at the three-year horizon. The annual output growth has a volatility of 4.93% and a high first-order autocorrelation of 0.54. The autocorrelation drops to 0.18 at the two-year horizon, and turns negative afterward: −0.18 at the three-year horizon and −0.23 at the five-year horizon.

We obtain monthly series of the value-weighted market returns including all NYSE, Amex, and Nasdaq stocks, one-month Treasury bill rates, and inflation rates (the rates of change in Consumer Price Index) from Center for Research in Security Prices (CRSP). The sample is from January 1926 to December 2010 (1,020 months). The mean of real interest rates (one-month Treasury bill rates minus inflation rates) is 0.59% per annum, and the annualized volatility is 1.87%.

The equity premium (the average of the value-weighted market returns in excess of one-month Treasury bill rates) in the 1926–2010 sample is 7.45% per annum. Because we do not model financial leverage, we adjust the equity premium in the data for leverage before matching with the equity premium from the model. Frank and Goyal (2008) report that the aggregate market leverage ratio of U.S. corporations is stable around 0.32. As such, we calculate the leverage-adjusted equity premium as $(1-0.32)\times 7.45\% = 5.07\%$ per annum. The annualized volatility of the market returns in excess of inflation rates is 18.95%. Adjusting for leverage (taking the leverage-weighted average of real market returns and real interest rates) yields an annualized volatility of 12.94%.

Panel B of Table 2 reports the model moments. To reach the model's stationary distribution, we always start at the initial condition of zero for log productivity and 0.90 for employment, and simulate the economy for 6,000 months. From the stationary distribution, we repeatedly simulate 1,000 artificial samples, each with 1,020 months. On each sample, we calculate the annualized monthly averages of the equity premium and the real interest rate, as well as the annualized monthly volatilities of the market returns and the real interest rate. We also time-aggregate the first 984 monthly observations of consumption and output into 82 annual observations. (We add up 12 monthly observations within a given year, and treat the sum as the year's annual observation.) For each data moment, we report the average as well as the 5 and 95 percentiles across the 1,000 simulations. The p-values are the frequencies with which a given model moment is larger than its data counterpart.

The model predicts a consumption growth volatility of 3.63% per annum, which is somewhat higher than 3.04% in the data. This data moment lies within the 90% confidence interval of the

Table 2 : Basic Business Cycle and Financial Moments

In Panel A, consumption is annual real personal consumption expenditures (series PCECCA), and output is annual real gross domestic product (series GDPCA) from 1929 to 2010 (82 annual observations) from NIPA (Table 1.1.6) at Bureau of Economic Analysis. σ^C is the volatility of log consumption growth, and σ^Y is the volatility of log output growth. Both volatilities are in percent. $\rho^C(\tau)$ and $\rho^Y(\tau)$, for $\tau = 1, 2, 3$, and 5, are the τ-th order autocorrelations of log consumption growth and log output growth, respectively. We obtain monthly series from January 1926 to December 2010 (1,020 monthly observations) for the value-weighted market index returns including dividends, one-month Treasury bill rates, and the rates of change in Consumer Price Index (inflation rates) from CRSP. $E[R - R^f]$ is the average (in annualized percent) of the value-weighted market returns in excess of the one-month Treasury bill rates, adjusted for the long-term market leverage rate of 0.32 reported by Frank and Goyal (2008). (The leverage-adjusted average $E[R - R^f]$ is the unadjusted average times 0.68.) $E[R^f]$ and σ^{R^f} are the mean and volatility, both of which are in annualized percent, of real interest rates, defined as the one-month Treasury bill rates in excess of the inflation rates. σ^R is the volatility (in annualized percent) of the leverage-weighted average of the value-weighted market returns in excess of the inflation rates and the real interest rates. In Panel B, we simulate 1,000 artificial samples, each of which has 1,020 monthly observations, from the model in Section 2. In each artificial sample, we calculate the mean market excess return, $E[R - R^f]$, the volatility of the market return, σ^R, as well as the mean, $E[R^f]$, and volatility, σ^{R^f}, of the real interest rate. All these moments are in annualized percent. We time-aggregate the first 984 monthly observations of consumption and output into 82 annual observations in each sample, and calculate the annual volatilities and autocorrelations of log consumption growth and log output growth. We report the mean and the 5 and 95 percentiles across the 1,000 simulations. The p-values are the percentages with which a given model moment is larger than its data moment.

	Panel A: Data	Panel B: Model			
		Mean	5%	95%	p-value
σ^C	3.04	3.63	1.92	8.12	0.46
$\rho^C(1)$	0.38	0.18	−0.04	0.46	0.10
$\rho^C(2)$	0.08	−0.14	−0.35	0.09	0.06
$\rho^C(3)$	−0.21	−0.13	−0.35	0.10	0.73
$\rho^C(5)$	0.06	−0.07	−0.28	0.14	0.14
σ^Y	4.93	4.13	2.47	8.39	0.17
$\rho^Y(1)$	0.54	0.19	−0.03	0.45	0.03
$\rho^Y(2)$	0.18	−0.13	−0.33	0.08	0.02
$\rho^Y(3)$	−0.18	−0.12	−0.33	0.09	0.68
$\rho^Y(5)$	−0.23	−0.08	−0.28	0.14	0.89
$E[R - R^f]$	5.07	5.70	4.87	6.52	0.89
$E[R^f]$	0.59	2.90	2.50	3.18	1.00
σ^R	12.94	10.83	9.94	11.82	0.00
σ^{R^f}	1.87	1.34	0.80	2.26	0.11

model's bootstrapped distribution with a bootstrapped p-value of 0.46. The model also implies a positive first-order autocorrelation of 0.18, but is lower than 0.38 in the data. At longer horizons, consumption growth in the model are all negatively autocorrelated. All the autocorrelations in the data are within 90% confidence interval of the model. The output growth volatility implied by the model is 4.13% per annum, which is somewhat lower than 4.93% in the data. Both the first- and the second-order autocorrelations in the data are outside the 90% confidence interval of the model. However, at longer horizons, the autocorrelations are negative in the model, consistent with the data.

The model is also broadly consistent with the business cycle moments of the labor market. The volatilities of unemployment and vacancies in the model are close to those in the data. However, the volatility of the vacancy-unemployment ratio in the model is somewhat lower than that in the data. Finally, the model also reproduces a Beveridge curve with a large negative correlation between unemployment and vacancies. See Appendix D for details.

The model seems to perform well in matching financial moments. The equity premium is 5.70% per annum, which is somewhat higher than the leverage-adjusted equity premium of 5.07% in the data. This data moment lies within the 90% confidence interval of the model's bootstrapped distribution. The volatility of the stock market return in the model is 10.83% per annum, which is close to the leverage-adjusted market volatility of 12.94% in the data. The volatility of the interest rate in the model is 1.34%, close to 1.87% in the data. However, the model implies an average interest rate of 2.90% per annum, which is somewhat high relative to 0.59% in the data. Overall, the model's fit of the financial moments, especially the stock market volatility, seems notable. As shown in Tallarini (2000) and Kaltenbrunner and Lochstoer (2010), baseline production economies with recursive preferences and capital adjustment costs struggle to reproduce a high stock market volatility.

4.2 Time-varying Risk Premiums

A large literature in finance shows that the equity premium is time-varying (countercyclical) in the data (e.g., Lettau and Ludvigson (2001)). In the labor market, vacancies are procyclical, and unemployment is countercyclical, meaning that the vacancy-unemployment ratio is strongly procyclical (e.g., Shimer (2005)). As such, the ratio should forecast stock market excess returns with a negative slope at business cycle frequencies. We document such predictability in the data.

We perform monthly long-horizon regressions of log excess returns on the CRSP value-weighted market returns, $\sum_{h=1}^{H} R_{t+3+h} - R^f_{t+3+h}$, in which $H = 1, 3, 6, 12, 24$, and 36 is the forecast horizon in months. When $H > 1$, we use overlapping monthly observations of H-period holding returns. We regress long-horizon returns on *two*-month lagged values of the vacancy-unemployment ratio. We impose the two-month lag to guard against look-ahead bias in predictive regressions.[8]

Panel A of Table 3 shows that the V/U ratio forecasts stock market excess returns at business cycle frequencies. At the one-month horizon, the slope is -1.43, which is more than 2.5 standard errors from zero. The standard error is adjusted for heteroscedasticity and autocorrelations of 12 lags per Newey and West (1987). The slopes are significant at the three-month and six-month horizons but turn insignificant afterward. The adjusted R^2s peak at 3.78% at the six-month horizon, and decline to 3.67% at the one-year horizon and further to 1.41% at the three-year horizon.

Panel B of Table 3 reports the model's quantitative fit for the predictive regressions. Consistent with the data, the model predicts that the V/U ratio forecasts market excess returns with a negative slope. At the one-month horizon, the predictive slope is -0.50 ($t = -2.06$). At the six-month horizon, the slope is -2.88 ($t = -2.29$). The slopes are smaller in magnitude than those in the data because the slopes are not adjusted for financial leverage. However, the model exaggerates the predictive power of the V/U ratio. Both the t-statistic of the slope and the adjusted R^2 peak at the six-month horizon but decline afterward in the data. In contrast, both statistics increase monotonically with the forecast horizon in the model, probably because it only has one shock.

Panel A of Figure 1 plots the cross-correlations and their two standard-error bounds between V_t/U_t and future market excess returns, $R_{t+H} - R^f_{t+H}$, for $H = 1, 2, \ldots, 36$ months in the data. No overlapping observations are used. The panel shows that the correlations are significantly negative for forecast horizons up to six months, consistent with the predictive regressions in Table 3. Panel B reports the cross-correlations and their two cross-simulation standard-deviation bounds from the model's bootstrapped distribution. Consistent with the data, the model predicts significantly negative cross-correlations between V_t/U_t and future stock market excess returns at short horizons. However, although the cross-correlations are insignificant at long horizons, the correlations decay

[8] We obtain seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the Bureau of Labor Statistics (BLS), and seasonally adjusted help wanted advertising index (the measure of vacancies) from the Conference Board. The sample is from January 1951 to June 2006. The Conference Board switched from help wanted advertising index to help wanted online index in June 2006. The two indexes are not directly comparable. As such, we follow the standard practice in the labor search literature in using the longer time series before the switch. The BLS takes less than one week to release monthly employment and unemployment data, and the Conference Board takes about one month to release monthly help wanted advertising index data. We verify this practice through a private correspondence with the Conference Board staff. Finally, to make the regression slopes comparable to those in the model, we scale up the V/U series in the data by a factor of 50 to make its average close to that in the model. The scaling is necessary because the vacancies and unemployment series in the data have different units.

Table 3 : Long-Horizon Regressions of Market Excess Returns on Labor Market Tightness

Panel A reports long-horizon regressions of log excess returns on the value-weighted market index from CRSP, $\sum_{h=1}^{H} R_{t+3+h} - R_{t+3+h}^{f}$, in which H is the forecast horizon in months. The regressors are two-month lagged values of the V/U ratio. We report the ordinary least squares estimate of the slopes (Slope), the Newey-West corrected t-statistics (t_{NW}), and the adjusted R^2s in percent. The seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics, and the seasonally adjusted help wanted advertising index (V) is from the Conference Board. The sample is from January 1951 to June 2006 (666 monthly observations). We multiply the V/U series by 50 so that its average is close to that in the model. In Panel B, we simulate 1,000 artificial samples, each of which has 666 monthly observations. On each artificial sample, we implement the exactly same empirical procedures as in Panel A, and report the cross-simulation averages and standard deviations (in parentheses) for all the model moments.

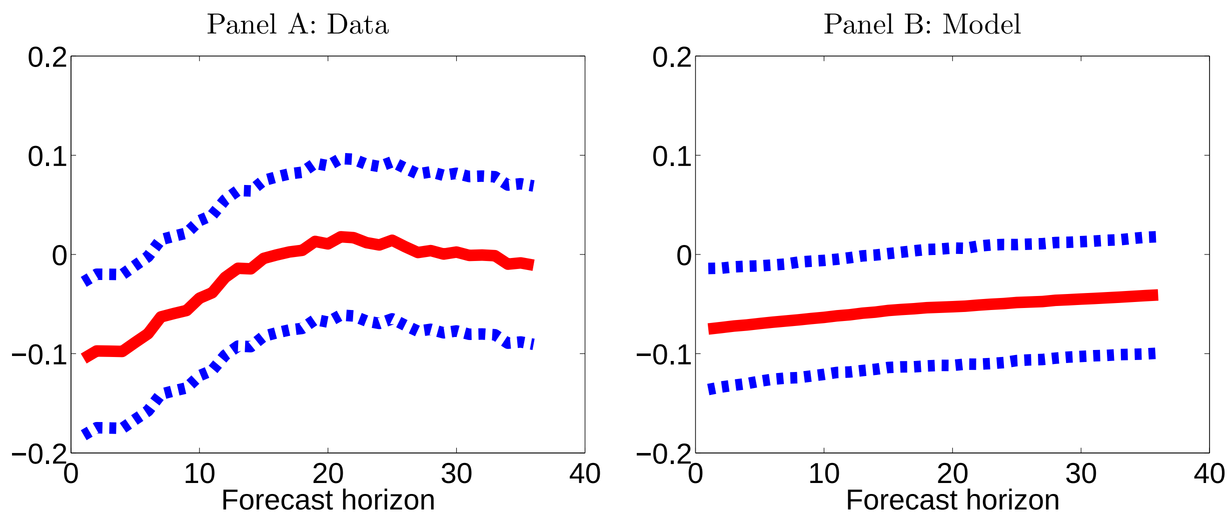
	Forecast horizon (H) in months					
	1	3	6	12	24	36
	Panel A: Data					
Slope	−1.43	−4.20	−7.30	−10.31	−9.02	−10.16
t_{NW}	−2.58	−2.55	−2.26	−1.70	−0.97	−0.86
Adjusted R^2	0.95	2.60	3.78	3.67	1.53	1.41
	Panel B: Model					
Slope	−0.50	−1.48	−2.88	−5.41	−9.62	−13.07
	(0.30)	(0.85)	(1.61)	(2.95)	(4.97)	(6.41)
t_{NW}	−2.06	−2.16	−2.29	−2.56	−3.22	−3.80
	(0.84)	(0.88)	(0.95)	(1.12)	(1.49)	(1.78)
Adjusted R^2	0.61	1.78	3.44	6.44	11.48	15.68
	(0.45)	(1.27)	(2.39)	(4.37)	(7.48)	(9.77)

more slowly than those in the data, consistent with Panel B of Table 3.

How does the model capture time-varying risk premiums? Panel A of Figure 2 plots the equity premium in annual percent in the state space. The equity premium is countercyclical in the model, low in expansions when employment and productivity are high, and high in recessions when employment and productivity are low. As noted, inelastic wages give rise to operating leverage, which amplifies risk and risk premium in recessions. In addition, the downward rigidity of the marginal costs of hiring, by suppressing the firm's incentives of hiring, further magnifies the risk dynamics. As a result, the stock market volatility is also countercyclical (Panel B). In contrast, V_t/U_t is procyclical. In expansions there are more vacancies and fewer unemployed workers, whereas in recessions there are fewer vacancies and more unemployed workers. The joint cyclicalities of the equity premium and V_t/U_t imply that the ratio should forecast market excess returns with a negative slope.

Figure 1 : Cross-Correlations between the V/U Ratio and Future Market Excess Returns

We report the cross-correlations (in red) between labor market tightness, V_t/U_t, and future market excess returns, $R_{t+H} - R^f_{t+H}$, in which $H = 1, 2, \ldots, 36$ is the forecast horizon in months, as well as their two standard-error bounds (in blue broken lines). In Panel A, V_t is the seasonally adjusted help wanted advertising index from the Conference Board, and U_t is the seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from the BLS. The sample is from January 1951 to June 2006. The market excess returns are the CRSP value-weighted market returns in excess of one-month Treasury bill rates. In Panel B, we simulate 1,000 artificial samples, each with 666 monthly observations. On each artificial sample, we calculate the cross-correlations between V_t/U_t and $R_{t+H} - R^f_{t+H}$, and plot the cross-simulation averaged correlations (in red) and their two cross-simulation standard-deviation bounds (in blue broken lines).



4.3 Endogenous Long Run Risks and Endogenous Time-varying Volatility

We also explore the model's implications for long run risks per Bansal and Yaron (2004) and uncertainty shocks per Bloom (2009). Bansal and Yaron propose long-run consumption risks to explain aggregate asset prices. Specifically, monthly consumption growth is assumed to follow:

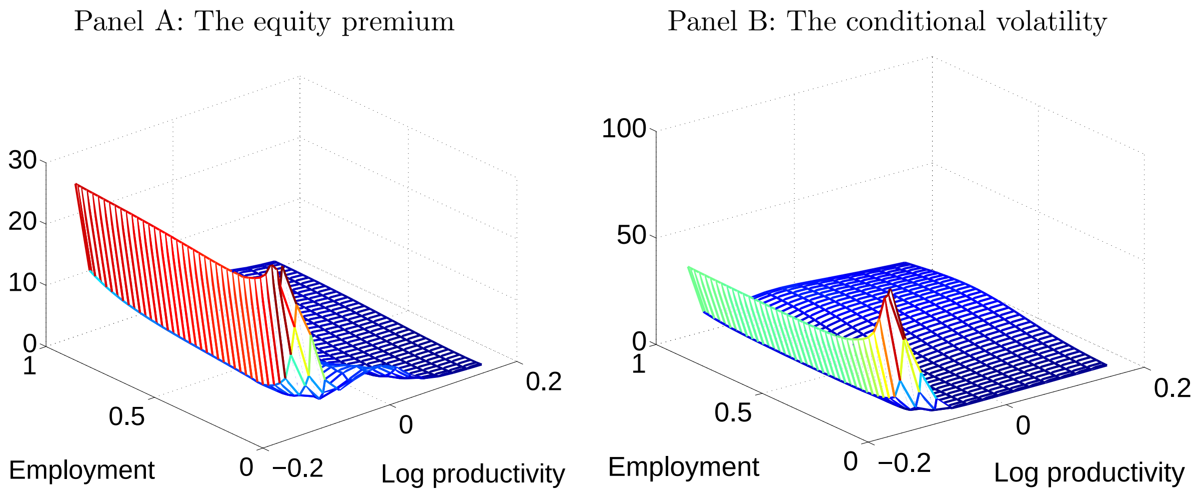
$$z_{t+1} = .979 z_t + .044 \sigma_t e_{t+1}, \tag{22}$$

$$g_{t+1} = .0015 + z_t + \sigma_t \eta_{t+1}, \tag{23}$$

$$\sigma^2_{t+1} = .0078^2 + .987(\sigma^2_t - .0078^2) + .23 \times 10^{-5} w_{t+1}, \tag{24}$$

in which g_{t+1} is the consumption growth, z_t is the expected consumption growth, σ_t is the conditional volatility of g_{t+1}, and $e_{t+1}, u_{t+1}, \eta_{t+1}$, and w_{t+1}, are i.i.d. standard normal shocks, which are mutually uncorrelated. Bansal and Yaron argue that the stochastic slow-moving component, z_t, of the consumption growth is crucial for explaining the level of the equity premium, and that

Figure 2 : The Equity Premium and the Conditional Stock Market Volatility, Both in Annual Percent



the mean-reverting stochastic volatility helps explain the time-variation in the risk premium.

Kaltenbrunner and Lochstoer (2010) argue that long-run risks can arise endogenously via consumption smoothing. Within a production economy with capital as the only productive input, Kaltenbrunner and Lochstoer (Table 6) show that the endogenous consumption growth follows:

$$z_{t+1} = .986 z_t + .093 \sigma e_{t+1}, \tag{25}$$

$$g_{t+1} = .0013 + z_t + \sigma \eta_{t+1}, \tag{26}$$

with transitory productivity shocks. With permanent productivity shocks, the z_t process follows:

$$z_{t+1} = .990 z_t + .247 \sigma e_{t+1}. \tag{27}$$

However, both versions of their model fail to reproduce time-varying volatilities.

We investigate how consumption dynamics in our search economy compare with those in the Kaltenbrunner and Lochstoer (2010) economy and with those calibrated in Bansal and Yaron (2004). This question is important because different parameterizations of the consumption process specified in equations (22)–(24) can be largely consistent with observable moments of consumption growth such as volatility and autocorrelations (see Table 2). Yet, different parameterizations can give rise to vastly different economic mechanisms for the stock market behavior.

From the model's stationary distribution, we simulate one million monthly periods. We calculate expected consumption growth and the conditional volatility of realized consumption growth in the state space, and use the solutions to simulate these moments. Fitting the consumption growth process specified by Bansal and Yaron (2004) on the simulated data, we obtain:

$$z_{t+1} = .697 z_t + .598 \sigma_t e_{t+1}, \tag{28}$$

$$g_{t+1} = z_t + \sigma_t \eta_{t+1}, \tag{29}$$

$$\sigma_{t+1}^2 = .0026^2 + .658(\sigma_t^2 - .0026^2) + 1.91 \times 10^{-5} w_{t+1}. \tag{30}$$

In addition, the unconditional correlation between e_{t+1} and η_{t+1} is 0.34, that between e_{t+1} and w_{t+1} is zero, and the correlation between η_{t+1} and w_{t+1} is 0.12.

Although consumption growth is not i.i.d. in our economy, the persistence in expected consumption growth is only 0.697, which is substantially lower than those in Bansal and Yaron (2004) and Kaltenbrunner and Lochstoer (2010). However, expected consumption growth is more volatile in our economy. The conditional volatility of expected consumption growth is about 60% of the conditional volatility of realized consumption growth. This percentage is higher than 9.3% and 24.7% in Kaltenbrunner and Lochstoer as well as 4.4% in Bansal and Yaron. For the stochastic variance, its persistence is 0.658 in our economy, which is lower than 0.987 in Bansal and Yaron. However, the volatility of our stochastic variance is more than eight times of theirs.

In all, the endogenous time-varying volatility is another dimension of the search economy that is distinctive from the baseline production economy in Kaltenbrunner and Lochstoer (2010).

4.4 Dividend Dynamics

Rouwenhorst (1995) shows that dividends are countercyclical in baseline production economies. Intuitively, dividends equal profits minus investment, and profits equal output minus wages. With a frictionless labor market, wages equal the marginal product of labor, meaning profits are proportional to, and as procyclical as output. Because investment is more procylical than output and profits due to consumption smoothing, dividends (profits minus investment) must be countercyclical.

This countercyclicality is counterfactual. Dividends in the production economies correspond to net payout (dividends plus stock repurchases minus new equity issues) in the data. Following Jermann and Quadrini (2010), we measure the net payout using aggregate data from the Flow of Funds Accounts of the Federal Reserve Board.[9] The sample is quarterly from the fourth quarter of 1951 to

[9] In particular, we calculate the net payout as net dividends of nonfarm, nonfinancial business (Table F.102, line 3) plus net dividends of farm business (Table F.7, line 24) minus net increase in corporate equities of nonfinancial business (Table F.101, line 35) minus proprietors' net investment of nonfinancial business (Table F.101, line 39).

the fourth quarter of 2010. From NIPA, we obtain quarterly real GDP and real consumption (Table 1.1.6) and quarterly implicit price deflator for GDP (Table 1.1.9) to deflate net payout. We use the Hodrick-Prescott (1997, HP) filter to detrend real net payout, real GDP, and real consumption as HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600.[10] We do not take logs because the net payout can be negative in the data. Consistent with Jermann and Quadrini, we find that the cyclical components of real net payout and real GDP have a positive correlation of 0.55. Also, real net payout and real consumption have a positive correlation of 0.53.

Our economy avoids the pitfall of countercyclical dividends in baseline production economies. Intuitively, wages in our economy are delinked from the marginal product of labor. Because of inelastic wages, profits are more procyclical than output. As noted, working as operating leverage, inelastic wages magnify the procyclicality of profits. This amplified procyclicality of profits is sufficient to overcome the procyclicality of total vacancy costs to turn dividends procyclical.

Quantitatively, the model largely replicates the dividend procyclicality. From the model's stationary distribution, we repeatedly simulate 1,000 artificial samples, each with 711 months (237 quarters). The sample size matches the quarterly series from the fourth quarter of 1951 to the fourth quarter of 2010 in Jermann and Quadrini (2010). On each sample, we time-aggregate monthly observations of dividends, output, and consumption into quarterly observations. After detrending the quarterly series as HP-filtered proportional deviations from the mean, we calculate the correlations between the cyclical components of dividends, output, and consumption. The correlation between dividends and output is 0.56, which is close to 0.55 in the data. The correlation between dividends and consumption is 0.66 with a cross-simulation standard deviation of 0.14. As such, the correlation of 0.53 in the data is within one standard deviation of the model's estimate.

We also compare the wage dynamics in the model to those in the data. Following Hagedorn and Manovskii (2008), we measure wages as labor share times labor productivity from BLS. The sample is quarterly from the first quarter of 1947 to the four quarter of 2010 (256 quarters). We take logs and HP-detrend the series with a smoothing parameter of 1,600. We find that the wage elasticity to labor productivity is 0.46, close to Hagedorn and Manovskii's estimate. To see the model's performance, we repeatedly simulate from its stationary distribution 1,000 artificial samples, each with 768 months (256 quarters). On each artificial sample, we take quarterly averages of

[10] Specifically, for any variable Z, the HP-filtered proportional deviations from the mean are calculated as $(Z-\bar{Z})/\bar{Z} - \mathrm{HP}[(Z-\bar{Z})/\bar{Z}]$, in which $\bar{Z}$ is the mean of Z, and $\mathrm{HP}[(Z-\bar{Z})/\bar{Z}]$ is the HP trend of $(Z-\bar{Z})/\bar{Z}$.

Table 4 : Comparative Statics

We report four experiments: (i) $b = .4$ is for the value of unemployment activities set to 0.4; (ii) $s = .035$ is for the job separation rate set to 0.035; (iii) $\kappa_t = .815$ is for the proportional costs of vacancy $\kappa_0 = .815$ and the fixed costs $\kappa_1 = 0$, in which .815 is the average κ_t in the benchmark calibration; and (iv) $\iota = .9$ is for the elasticity of the matching function set to .9. In each experiment, all the other parameters are identical to those in the benchmark calibration. See Table 2 for the description of Panel A. See the caption of Table 3 for the description of Panel B: (1) and (12) denote for forecast horizons of one and 12 months, respectively.

	Data	Benchmark	$b = .4$	$s = .035$	$\kappa_t = .815$	$\iota = .9$
	Panel A: Basic business cycle and financial moments					
σ^C	3.04	3.63	1.69	2.21	2.75	4.26
$\rho^C(1)$	0.38	0.18	0.13	0.15	0.15	0.22
$\rho^C(3)$	−0.21	−0.13	−0.10	−0.10	−0.11	−0.13
σ^Y	4.93	4.13	2.05	2.67	3.37	4.83
$\rho^Y(1)$	0.54	0.19	0.13	0.14	0.15	0.22
$\rho^Y(3)$	−0.18	−0.12	−0.10	−0.10	−0.11	−0.13
$E[R - R^f]$	5.07	5.70	0.12	0.00	2.34	6.07
$E[R^f]$	0.59	2.90	3.96	3.96	3.74	3.01
σ^R	12.94	10.83	3.87	11.47	12.29	10.79
σ^{R^f}	1.87	1.34	0.13	0.63	1.04	1.51
	Panel B: Forecasting market excess returns with the V/U ratio					
Slope(1)	−1.43	−0.50	−0.07	−0.19	−0.61	−0.62
Slope(12)	−10.31	−5.41	−0.79	−2.07	−6.63	−6.58
$t_{NW}(1)$	−2.58	−2.06	−0.67	−0.65	−1.71	−2.34
$t_{NW}(12)$	−1.70	−2.56	−0.87	−0.83	−2.13	−2.91

monthly wages and labor productivity to obtain quarterly series. Implementing the same empirical procedure used on the real data, we find that the wage elasticity to productivity is 0.58 in the model, which is somewhat high relative to 0.46 in the data.

4.5 Comparative Statics on Asset Prices

To shed further light on the intuition underlying the equity premium in the model, Table 4 conducts four comparative statics. (i) We change the value of unemployment activities, b, from 0.85 in the benchmark calibration to 0.4; (ii) we lower the job separation rate, s, from 0.05 to 0.035; (iii) we adjust the proportional costs of vacancy, κ_0, from 0.6 to 0.815, and simultaneously, adjust the fixed costs of vacancy, κ_1, from 0.4 to zero (0.815 is the average κ_t in the simulations from the benchmark economy); and (iv) we reduce the elasticity of the matching function, ι, from 1.25 to 0.9. In each experiment, all the other parameters remain the same as in the benchmark calibration.

In the first experiment, $b = 0.4$, which is the value in Shimer (2005). Because unemployment is less valuable to workers, the unemployment rate drops to 5%. A lower b also means that the wage rate is more sensitive to productivity shocks. The wage elasticity to productivity increases to 0.68 from 0.58 in the benchmark economy. As such, profits, vacancies, employment, and output are all less sensitive to shocks. The consumption growth volatility drops to 1.69% per annum and the output growth volatility to 2.05%. The equity premium drops to only 0.12% per annum, and the market volatility drops to 3.87%. The V/U ratio shows no predictive power for market excess returns.

As such, a high value of b is important for labor market volatilities as well as the equity premium. Intuitively, by dampening the procyclical covariation of wages with productivity, a high value of b magnifies the procyclical variation of profits and vacancies to increase the volatility of the V/U ratio (e.g., Hagedorn and Manovskii (2008)). This operating leverage mechanism also impacts asset prices as the high b amplifies the procyclical variation of dividends, raising the equity premium and the stock market volatility and making both financial moments countercyclical.

In the second experiment, we reduce the separation rate, s, from 5% to 3.5% per month. Because employment is destructed at a lower rate, the mean unemployment rate drops to 4.95% from 8.51% in the benchmark economy. In addition, the small job flows induce only small fluctuations in aggregate employment. As such, the consumption growth volatility drops to 2.21% per annum from 3.63% in the benchmark economy, and the output growth volatility drops to 2.67% from 4.13%. The equity premium is close to zero, and is largely time-invariant.

In the third experiment, we remove fixed matching costs, while maintaining the same level of average unit costs of vacancy in the benchmark economy. As noted, the fixed costs affect the economy by reinforcing the downward rigidity of the marginal costs of hiring. Removing the fixed costs weakens the downward rigidity, allowing the firm to create more jobs in recessions. Table 4 shows that the output growth volatility falls to 3.37% per annum, the consumption growth volatility to 2.75%, and the equity premium to 2.34%.[11]

As noted, we follow Den Haan, Ramey, and Watson (2000) to set the elasticity to 1.25 in the benchmark economy. In the final experiment, we quantify the impact of this parameter by varying it to 0.9. The last column of Table 4 shows that, sensibly, lowering the elasticity of the matching function strengthens the risk dynamics in the model. As the labor market becomes more frictional in

[11]There exists a tradeoff between the level and the cyclical dynamics of the unit costs of vacancy, κ_t, in terms of producing our quantitative results. In an earlier draft, we report that with the proportional unit costs, κ_0, raised to 0.975, all our quantitative results in the benchmark economy subsist, even without fixed matching costs.

matching vacancies with unemployed workers, the consumption growth volatility goes up to 4.26% per annum, and the output growth volatility to 4.83%. The equity premium increases to 6.07%.

5 Quantitative Results: Endogenous Disasters

The search economy gives rise endogenously to rare disasters à la Rietz (1988) and Barro (2006).

5.1 Disasters in the Benchmark Economy with Recursive Preferences

We simulate one million monthly periods from the model's stationary distribution. Figure 3 reports the empirical cumulative distribution functions for key quantities and asset pricing moments. From Panel A, unemployment is positively skewed with a long right tail. As the population moments, the mean unemployment rate is 8.51%, the median is 7.30%, and the skewness is 7.83. The 2.5 percentile of unemployment is close to the median, 5.87%, whereas the 97.5 percentile is far away, 19.25%. As a mirror image, the employment rate is negatively skewed with a long left tail. As a result, output, consumption, and dividends all show rare but deep disasters (Panels B, C, and D, respectively). With small probabilities, the model economy falls off a cliff.

The disasters in macroeconomic quantities reflect in asset prices as rare upward spikes in the equity premium, $E_t[R_{t+1} - R^f_{t+1}]$, and in the conditional stock market volatility, σ^R_t. From Panel E, the stationary distribution of the equity premium is positively skewed with a long right tail. The equity premium has a median of 6.16% per annum and the 2.5 and 97.5 percentiles of 2.34% and 10.04%, respectively. However, with small probabilities, the conditional equity premium can reach close to 25%. Panel F shows that the conditional volatility hovers around its median about 11% per annum. However, with small probabilities, the volatility can jump to more than 35%.

Do the economic disasters arising endogenously from the model resemble those in the data? Barro and Ursúa (2008) apply a peak-to-trough method on international data from 1870 to 2006 to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. For consumption disasters, Barro and Ursúa estimate the disaster probability to be 3.63%, the average size 22%, and the average duration 3.6 years. For GDP disasters, the disaster probability is 3.69%, the average size 21%, and the average duration 3.5 years.[12]

[12] Specifically, Barro and Ursúa (2008) measure disaster moments as follows. Suppose there are two states, normalcy and disaster. The disaster probability measures the likelihood with which the economy shifts from normalcy to disaster in a given year. The number of disaster years is the number of years in the interval between peak and trough for each disaster event. The number of normalcy years is the total number of years in the sample minus the number of disaster years. Finally, the disaster probability is the ratio of the number of disasters over the number of normalcy years.

Figure 3 : Empirical Cumulative Distribution Functions of Unemployment, Output, Consumption, Dividends, as well as the Equity Premium and the Conditional Stock Market Volatility (Both in Annual Percent) Simulated from the Model's Stationary Distribution

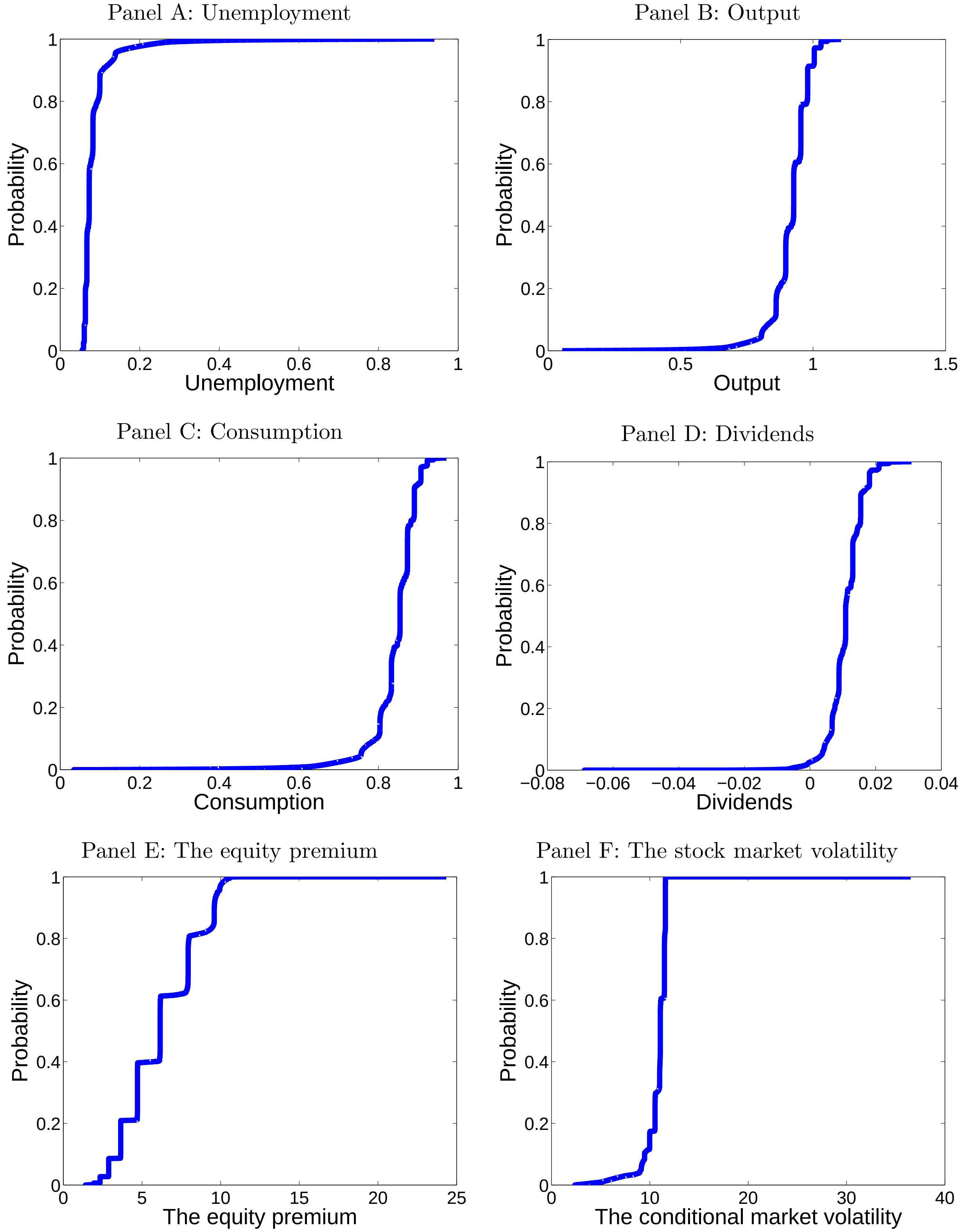


Table 5 : Moments of Economic Disasters

The data moments are from Barro and Ursúa (2008). The model moments are from 1,000 simulations, each with 1,644 monthly observations. We time-aggregate these monthly observations of consumption and output into 137 annual observations. On each artificial sample, we apply Barro and Ursúa's peak-to-trough method to identify economic crises, defined as cumulative fractional declines in per capita consumption or GDP of at least 10%. We report the averages as well as the 5 and 95 percentiles across the simulations. The p-values are the percentages with which a given model moment is higher than its data moment. The disaster probabilities and average size are all in percent, and the average duration is in terms of years.

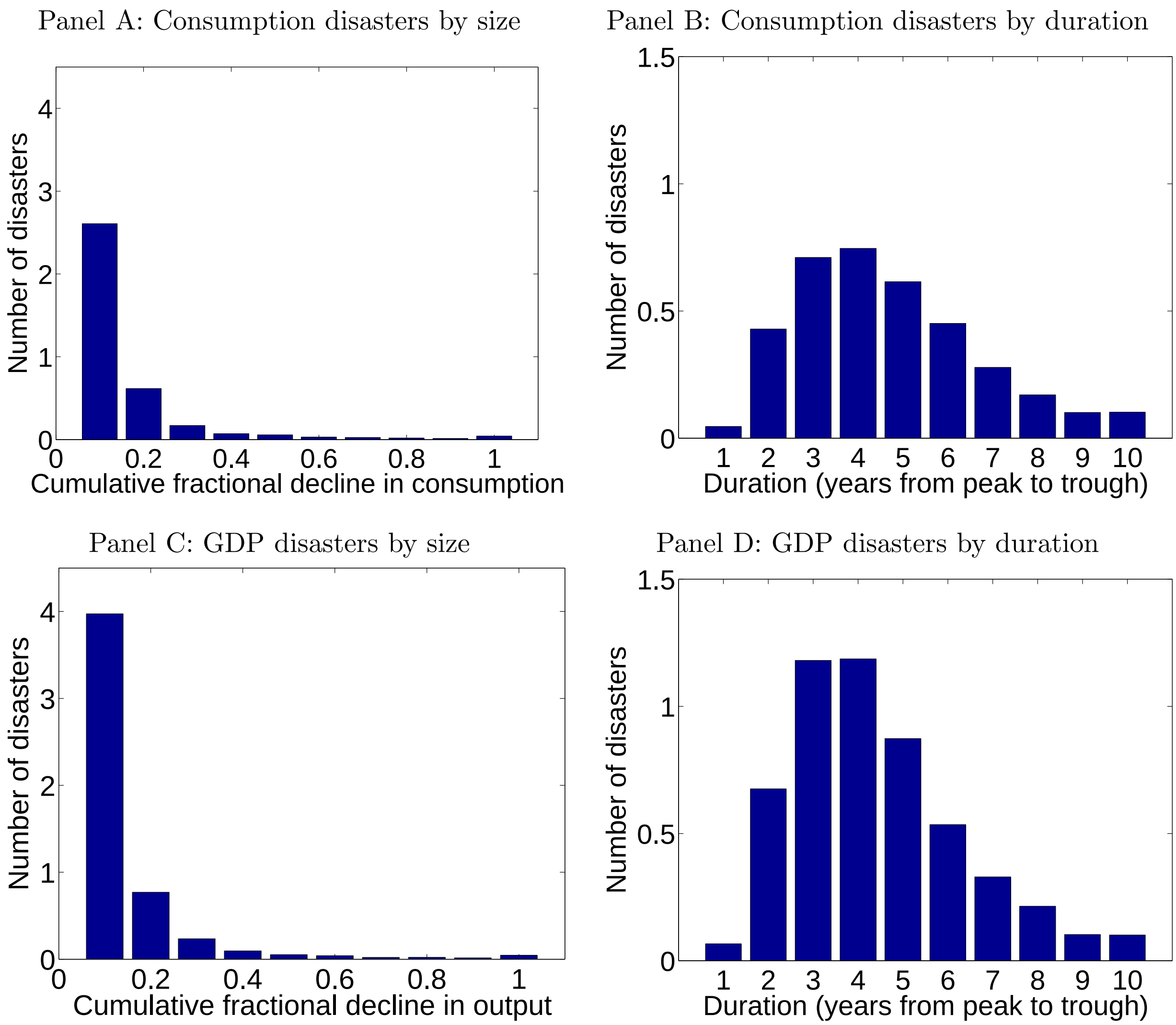
	Data	Model			
		Mean	5%	95%	p-value
Panel A: Consumption disasters					
Probability	3.63	3.08	0.75	6.42	0.30
Average size	22	20.21	11.49	39.37	0.25
Average duration	3.6	4.81	3.00	7.00	0.83
Panel B: GDP disasters					
Probability	3.69	4.66	1.58	8.29	0.30
Average size	21	19.12	12.34	34.05	0.28
Average duration	3.5	4.51	3.25	6.18	0.84

To quantify the magnitude of the disasters in the model, we repeatedly simulate 1,000 artificial samples from the model's stationary distribution. Each sample has 1,644 months (137 years) to match the average sample size in Barro and Ursúa (2008). On each sample, we time-aggregate the monthly observations of consumption and output into annual observations. We then apply Barro and Ursúa's measurement, and report the cross-simulation averages and the 5 and 95 percentiles for the disaster probability, size, and duration for both consumption and GDP (output) disasters.

Table 5 reports the detailed results. For consumption disasters, Panel A shows that the disaster probability and the average disaster size are 3.08% and 20.21% in the model, which are close to 3.63% and 22% in the data, respectively. The average duration is 4.81 years, which is longer than 3.6 years in the data. The cross-simulation standard deviation of the average duration is 1.71 years, meaning that the data duration is within one standard deviation from the model's estimate.

From Panel B of Table 5, the average size of GDP disasters in the model, 19.12%, is close to that in the data, 21%. However, the disaster probability of 4.66% is somewhat higher than 3.69% in the data. The cross-simulation standard deviation of this probability is 2.01%, meaning that the probability in the data is within one standard deviation from the model. In addition, the average duration of the GDP disasters in the model is 4.51 years, which is longer than 3.5 years in the data.

Figure 4 : Distributions of Consumption and GDP Disasters by Size and Duration



The cross-simulation standard deviation of the duration is 0.88 years, meaning that the duration in the data is slightly more than one standard deviation from the model.

Figure 4 reports the frequency distributions of consumption and GDP disasters by size and duration averaged across 1,000 simulations (each with 137 years) from the model. We observe that the size and duration distributions for consumption and GDP disasters in the model display roughly similar patterns as those in the data (see Barro and Ursúa's (2008) Figures 1 and 2). In particular, the size distributions seem to follow a power-law density per Barro and Jin (2011).

5.2 Disasters with Log Utility

To see the economic mechanisms underlying the endogenous disasters, we first show that a simplified economy with log utility displays disaster dynamics that are similar to those in the benchmark economy. As such, although important for asset prices, recursive preferences are not important for disasters in macroeconomic quantities, a finding that echoes Tallarini (2000).[13] We then conduct comparative statics on the log-utility model to obtain intuition behind the disaster dynamics.

5.2.1 Quantitative Results with Log Utility

With log utility, the stochastic discount factor becomes $M_{t+1} = \beta(C_t/C_{t+1})$, and the equity premium is largely nonexistent. To make the log-utility model comparable with the benchmark economy, we recalibrate $\beta = e^{-.00716}$ so that the discount rate is around 8.6% per annum, which is the average discount rate in the benchmark model. Except for the preference parameters, all the other parameters remain identical to those in benchmark economy with recursive preferences.

Figure 5 reports the empirical cumulative distribution functions for key quantities from the log-utility model. The simulation design is identical to that in Figure 3. We observe that the log-utility model displays similar disaster dynamics as in the benchmark model. From Panel A, unemployment is positively skewed with a long right tail. The mean unemployment rate is 8.41%, the median is 7.27%, and the skewness is 7.63. The 2.5 percentile is close to the median, 5.9%, but the 97.5 percentile is far away, 18.93%. Overall, these statistics are fairly close to those in the benchmark model. The remaining panels in Figure 5 report that output, consumption, and dividends all have long left tails. With small probabilities, the economy falls off a cliff even with log utility.

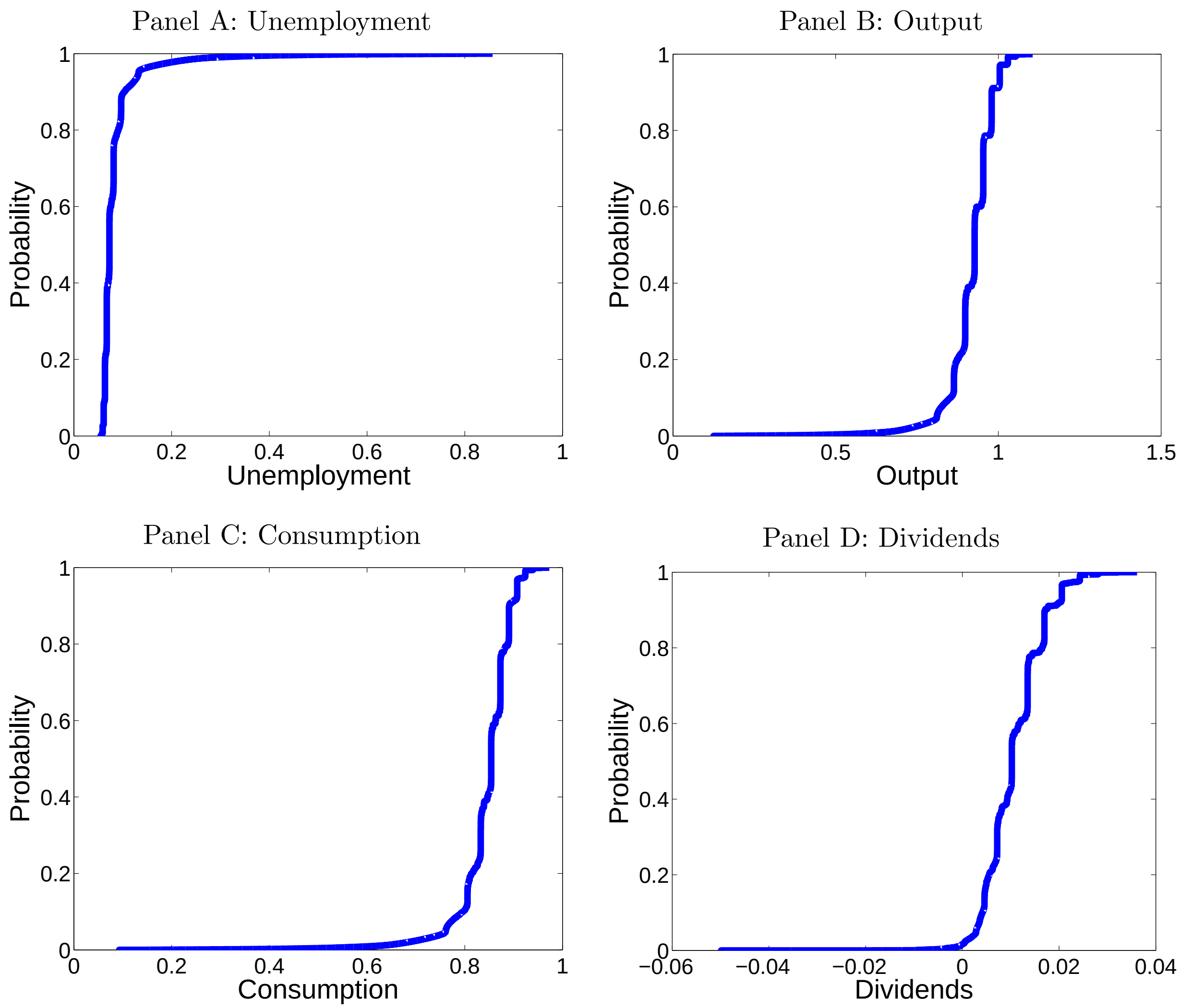
Table 6 show that the disaster moments in the log-utility model are also close to those in the benchmark model. Using the same simulation design as in Table 5, we find that for consumption disasters, the probability is 2.82%, the average size is 20.25%, and the average duration is 4.95 years in the log-utility model. For GDP disasters, the probability is 4.32%, the average size is 19.17, and the average duration is 4.62 years. All these statistics are close to those in the benchmark model.[14]

The last four columns of Table 6 reports four comparative statics on the log-utility model. The

[13] Appendix D shows further that recursive preferences are also largely irrelevant for labor market volatilities.

[14] We have also studied the disaster dynamics in a linear utility model, in which the stochastic discount factor is given by $M_{t+1} = \beta$. We find that the volatilities of consumption growth and output growth are somewhat higher than those in the benchmark model (which are in turn somewhat higher than those in the log-utility model). However, the disaster moments are again fairly close to those in the other two models. We do not focus on the linear utility model because consumption can be negative even at the annual frequency after time-aggregation from monthly data.

Figure 5 : Empirical Cumulative Distribution Functions of Unemployment, Output, Consumption, and Dividends Simulated from the Log-Utility Model's Stationary Distribution



experimental design is similar to that in Table 4. We see that the low-b economy shows no disaster risks. The consumption disaster probability is only 0.40%, and the GDP disaster probability 0.65%. The average magnitudes of the consumption and GDP disasters are also substantially lower at 11.60% and 13.27%, respectively. The low-b economy also takes longer to accumulate a given magnitude of declines in consumption and GDP. Intuitively, with small profits, wages are inelastic to productivity. When productivity is very low, wages remain at a relatively high level, shrinking the small profits even further, stifling job creation flows. In contrast, with large profits, wages are more sensitive to shocks to productivity. When employment falls, wages drop as well, providing incentives for the firm to hire to offset large job destruction flows. As such, disaster risks are minimized.

In the second experiment, reducing the separation rate from 5% to 3.5% per month makes the

Table 6 : Comparative Statics on Disaster Dynamics

"Benchmark" denotes the benchmark economy with recursive preferences. "Log-utility" denotes the log-utility model with β recalibrated to $e^{-.00716}$. Except for the preference parameters, all the other parameters in the log-utility model are identical to those in the benchmark model. The four remain columns report four comparative static experiments based on the log-utility model: (i) $b = .4$ is for the value of unemployment activities set to .4; (ii) $s = .035$ is for the job separation rate set to 0.035; (iii) $\kappa_t = .813$ is for the proportional unit costs of vacancy $\kappa_0 = .813$ and the fixed unit costs $\kappa_1 = 0$, in which .813 is the average κ_t in the simulations from the log-utility model; and (iv) $\iota = .9$ is for the elasticity of the matching function set to .9. In each experiment, all the other parameters are identical to those in the log-utility model.

	Benchmark	Log-utility	$b = .4$	$s = .035$	$\kappa_t = .813$	$\iota = .9$
	Panel A: Consumption disasters					
Probability	3.08	2.82	0.40	1.30	1.84	3.87
Size	20.21	20.25	11.60	15.17	16.80	20.25
Duration	4.81	4.95	6.10	5.48	5.17	4.85
	Panel B: GDP disasters					
Probability	4.66	4.32	0.65	2.16	3.28	5.62
Size	19.12	19.17	13.27	15.39	16.52	19.86
Duration	4.51	4.62	5.79	5.11	4.76	4.54

disasters less extreme and less frequent. The consumption and GDP disaster probabilities reduce to 1.30% and 2.16%, respectively, which are more than halved relative to those in the log-utility model. The average magnitudes of the disasters are also smaller: 15.17% and 15.39%. These results are very intuitive. Because jobs are destroyed only at a lower rate, the economy can create enough jobs to shore up aggregate employment and to reduce disaster risks.

The next-to-last column in Table 6 shows that without fixed matching costs, disasters are less frequent and less severe. The consumption disaster probability drops to 1.84% from 2.82% in the log-utility model, and the GDP disaster probability drops to 3.28% from 4.32%. The average magnitudes of the disasters are also reduced somewhat to 16.80% and 16.52%, respectively.

Finally, the last column shows that reducing the elasticity of the matching function, ι, from 1.25 to .9 increases disaster risks. The consumption disaster probability rises to 3.87%, and the GDP disaster probability to 5.62%. However, the magnitude of the consumption disasters remains unchanged at 20.25%, and that of the GDP disasters grows only slightly to 19.86%. Intuitively, a lower elasticity in the matching function means that the labor market is more frictional in matching vacancies with unemployed workers to form new hires. Because job creation flows are hampered, whereas job destruction flows continue to be large, the economy slips into disasters more frequently.

5.2.2 Additional Intuition

To further illustrate the impact of small profits and matching frictions on the disaster dynamics, Figure 6 plots the vacancy filling rate, $q(\theta_t)$, and the marginal costs of hiring, $\kappa_0/q(\theta_t) + \kappa_1$, for the small profits model ($b = 0.85$) and for the large profits model ($b = 0.4$), both with log utility. Each panel has three lines that correspond to three different values of labor productivity.

In addition to the magnitude and the elasticity of wages, the other key determinant of the firm's hiring decisions is the marginal costs of hiring, $\kappa_0/q(\theta_t) + \kappa_1$. Small profits also work through the downward rigidity in the marginal costs. Panel A of Figure 6 shows that when productivity is very low, the vacancy filling rate, $q(\theta_t)$, is close to one. Intuitively, the labor market is populated by a large number of unemployed workers competing for a few vacancies. Filling a vacancy with an unemployed worker is easy, and the vacancy filling rate stays close to one, with no room to increase further. Accordingly, the marginal costs of hiring are close to the constant $\kappa_0 + \kappa_1$, with no room to drop, giving rise to the downward rigidity (Panel B). The rigid marginal costs in turn suppress the firm's incentives of hiring, smothering job creation flows, deepening recessions that occasionally turn into disasters. Arising from the intrinsic nature of the matching process, this downward rigidity subsists even without fixed matching costs ($\kappa_1 = 0$). By putting the constant κ_1 into the marginal costs of hiring, the fixed costs further restrict the ability of the marginal costs to decline, fortifying the downward rigidity. This mechanism explains why removing the fixed costs makes disasters less frequent and less severe in the log-utility model (see Table 6).

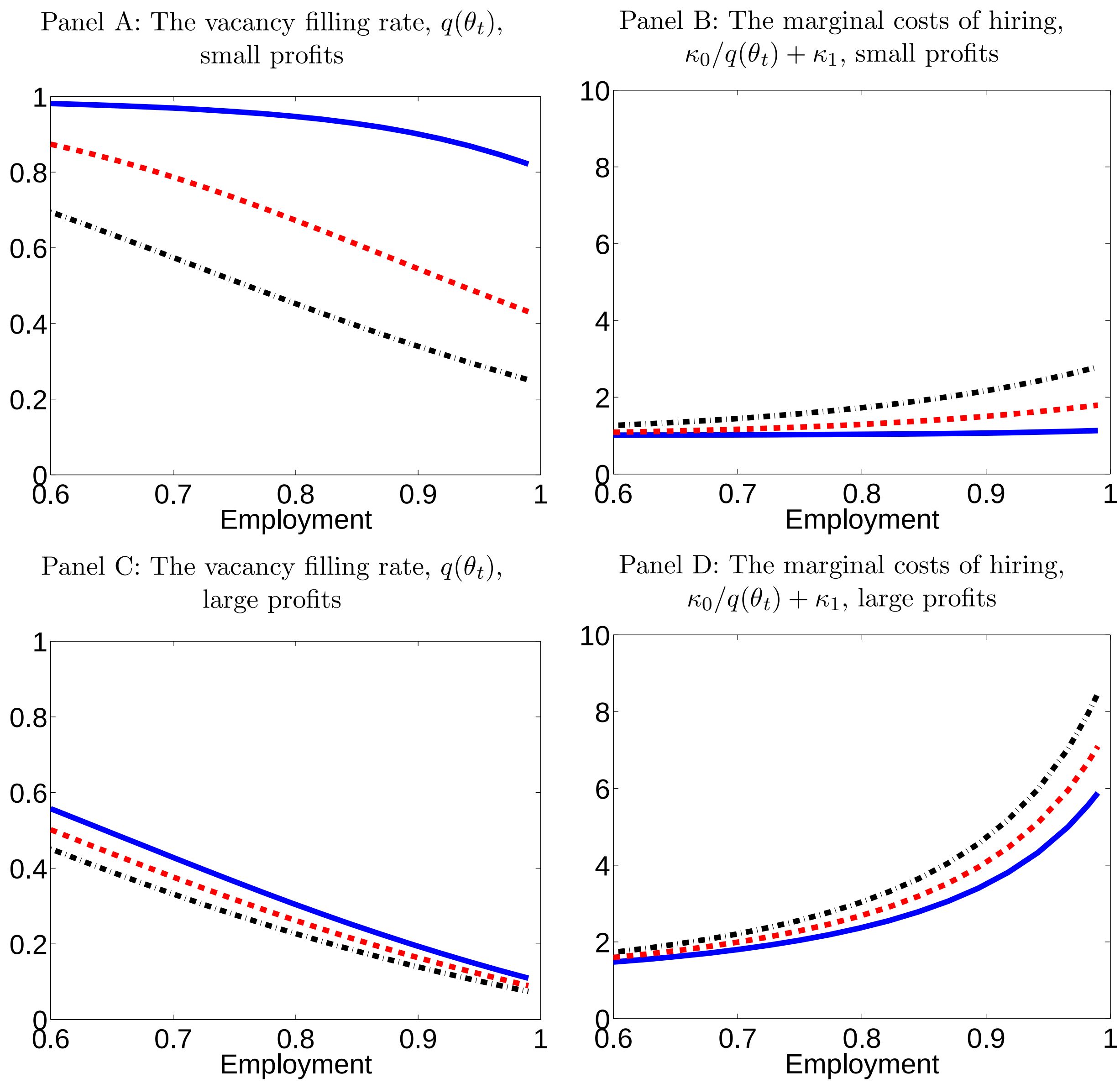
In contrast, the downward rigidity in the marginal costs is largely absent with large profits. Panel C shows that $q(\theta_t)$ is quite sensitive to changes in employment when profits are large. Intuitively, with large profits, vacancies are plentiful even when productivity is low. The labor market is then populated by a fair number of both vacancies and unemployed workers. As such, as employment falls, $q(\theta_t)$ keeps climbing, reducing the marginal costs of hiring (Panel D). The falling marginal costs simulate job creation flows, preventing the economy from slipping into disasters.

6 Conclusion

We take a first stab at embedding the standard Diamond-Mortensen-Pissarides search model of the labor market into an equilibrium asset pricing framework. We find that labor market frictions are important for equilibrium asset prices. Quantitatively, the model reproduces a realistic equity pre-

Figure 6 : Labor Market Properties in the Log-Utility Model, Small versus Large Profits

Let $x_i, i = 1, 2, \ldots, 15$ denote the i^{th} point on the x grid in an ascending order. In each panel, the blue solid line is for $x = x_3$, the red dashed line for $x = x_8$, and the black dashdot line for $x = x_{13}$.



mium, a high stock market volatility, and a low interest rate volatility. The equity premium is also countercyclical, and is forecastable by labor market tightness, a prediction that we confirm in the data. Intriguingly, three key ingredients in the model (small profits, large job flows, and matching frictions) combine to create endogenously rare disasters as in Rietz (1988) and Barro (2006).

As a first stab in embedding labor market frictions into equilibrium asset pricing, we have tried to keep the model parsimonious. In particular, we do not claim that the baseline search model "explains" the equity premium puzzle. Nevertheless, the rich dynamics displayed even in this baseline model, many of which are conducive to interpreting the equity premium, yet are entirely absent from standard production economies, suggest that labor market frictions might be essential for equilibrium asset pricing. Likewise, we do not interpret the absence of financial frictions in our model as saying that financial frictions are not important for disasters. Rather, we interpret our findings as saying that one should not ignore labor market frictions when trying to understand disasters.

Several directions are possible for future work. First, the relatively high value of unemployment activities is not the only mechanism that can reproduce realistic labor market dynamics. One can explore asset pricing implications of, for example, alternative wage bargaining games that are important for labor market dynamics (e.g., Hall and Milgrom (2008)). Second, more generally, one can introduce endogenous labor supply and endogenous capital accumulation into our framework to develop a unified equilibrium framework for both asset prices and business cycles. Third, one can introduce financial frictions such as defaultable bonds into the model to study the interaction between labor market frictions and financial frictions in endogenizing disasters. Finally, with the equity premium in sight in production economies, one can introduce firm heterogeneity to develop an equilibrium framework for the cross-section of expected returns.

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A The Stock Return Equation

We prove equation (13) following the same logic in Liu, Whited, and Zhang (2009) in the context of the q-theory of investment. Rewrite the equity value maximization problem as:

$$S_t = \max_{V_{t+\triangle t}, N_{t+\triangle t+1}} E_t \left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t} \left[\begin{array}{c} X_{t+\triangle t} N_{t+\triangle t} - W_{t+\triangle t} N_{t+\triangle t} - \kappa_{t+\triangle t} V_{t+\triangle t} \\ -\mu_{t+\triangle t} \left[N_{t+\triangle t+1} - (1-s) N_{t+\triangle t} \right. \\ \left. -V_{t+\triangle t} q(\theta_{t+\triangle t}) \right] + \lambda_{t+\triangle t} q(\theta_{t+\triangle t}) V_{t+\triangle t} \end{array} \right] \right], \tag{A.1}$$

in which μ_t is the Lagrange multiplier on the employment accumulation equation, and λ_t is the Lagrange multiplier on the occasionally binding constraint on job creation. The first-order conditions with respect to V_t and N_{t+1} in maximizing the market value of equity are given by, respectively:

$$\mu_t = \frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t, \tag{A.2}$$

$$\mu_t = E_t \left[M_{t+1} \left[X_{t+1} - W_{t+1} + (1-s) \mu_{t+1} \right] \right]. \tag{A.3}$$

The Kuhn-Tucker conditions are given by equation (12). Define dividends as $D_t \equiv X_t N_t - W_t N_t - \kappa_t V_t$ and the ex-dividend equity value as $P_t \equiv S_t - D_t$. Expanding S_t yields:

$$\begin{aligned} & P_t + X_t N_t - W_t N_t - \kappa_t V_t = S_t = X_t N_t - W_t N_t - \kappa_t V_t + \lambda_t q(\theta_t) V_t \\ & \quad -\mu_t \left[N_{t+1} - (1-s) N_t - V_t q(\theta_t) \right] + E_t M_{t+1} \left[X_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa_{t+1} V_{t+1} \right. \\ & \quad \left. -\mu_{t+1} \left[N_{t+2} - (1-s) N_{t+1} - V_{t+1} q(\theta_{t+1}) \right] + \lambda_{t+1} q(\theta_{t+1}) V_{t+1} \right] + \ldots \end{aligned} \tag{A.4}$$

Recursively substituting equations (A.2) and (A.3) yields:

$$P_t + X_t N_t - W_t N_t - \kappa_t V_t = X_t N_t - W_t N_t + \mu_t (1-s) N_t. \tag{A.5}$$

Using equation (A.2) to simplify further:

$$P_t = \kappa_t V_t + \mu_t (1-s) N_t = \mu_t [(1-s) N_t + q(\theta_t) V_t] + \lambda_t q(\theta_t) V_t = \mu_t N_{t+1}, \tag{A.6}$$

in which the last equality follows from equation (12).

To show equation (13), we expand the stock returns:

$$\begin{aligned} R_{t+1} &= \frac{S_{t+1}}{S_t - D_t} = \frac{\mu_{t+1} N_{t+2} + X_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa_{t+1} V_{t+1}}{\mu_t N_{t+1}} \\ &= \frac{X_{t+1} - W_{t+1} - \kappa_{t+1} V_{t+1}/N_{t+1} + \mu_{t+1} \left[(1-s) + q(\theta_{t+1}) V_{t+1}/N_{t+1} \right]}{\mu_t} \\ &= \frac{X_{t+1} - W_{t+1} + (1-s) \mu_{t+1}}{\mu_t} + \frac{\mu_{t+1} q(\theta_{t+1}) V_{t+1} - \kappa_{t+1} V_{t+1}}{\mu_t N_{t+1}} \\ &= \frac{X_{t+1} - W_{t+1} + (1-s) \mu_{t+1}}{\mu_t}, \end{aligned} \tag{A.7}$$

in which the last equality follows because the Kuhn-Tucker conditions imply:

$$\mu_{t+1} q(\theta_{t+1}) V_{t+1} - \kappa_{t+1} V_{t+1} = -\lambda_{t+1} q(\theta_{t+1}) V_{t+1} = 0. \quad \blacksquare \tag{A.8}$$

B Wage Determination under Nash Bargaining

Let $\eta \in (0,1)$ denote the relative bargaining weight of the worker, J_{Nt} the marginal value of an employed worker to the representative family, J_{Ut} the marginal value of an unemployed worker to the representative family, ϕ_t the marginal utility of the representative family, S_{Nt} the marginal value of an employed worker to the representative firm, and S_{Vt} the marginal value of an unfilled vacancy to the representative firm. A worker-firm match turns an unemployed worker into an employed worker for the representative household as well as an unfilled vacancy into a filled vacancy (an employed worker) for the firm. As such, we can define the total surplus from the Nash bargain as:

$$\Lambda_t \equiv \frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt}. \tag{B.1}$$

The wage equation (17) is determined via the Nash worker-firm bargain:

$$\max_{\{W_t\}} \quad \left(\frac{J_{Nt} - J_{Ut}}{\phi_t}\right)^{\eta} (S_{Nt} - S_{Vt})^{1-\eta}, \tag{B.2}$$

The outcome of maximizing equation (B.2) is the surplus-sharing rule:

$$\frac{J_{Nt} - J_{Ut}}{\phi_t} = \eta \Lambda_t = \eta \left(\frac{J_{Nt} - J_{Ut}}{\phi_t} + S_{Nt} - S_{Vt}\right). \tag{B.3}$$

As such, the worker receives a fraction of η of the total surplus from the wage bargain. In what follows, we derive the wage equation (17) from the sharing rule in equation (B.3).

B.1 Workers

To derive J_{Nt} and J_{Ut}, we need to specify the details of the representative household's problem. Tradeable assets consist of risky shares and a risk-free bond. Let R_{t+1}^f denote the risk-free interest rate, which is known at the beginning of period t, Π_t the household's financial wealth, χ_t the fraction of the household's wealth invested in the risky shares, $R_{t+1}^{\Pi} \equiv \chi_t R_{t+1} + (1-\chi_t) R_{t+1}^f$ the return on wealth, and T_t the taxes raised by the government. The household's budget constraint is given by:

$$\frac{\Pi_{t+1}}{R_{t+1}^{\Pi}} = \Pi_t - C_t + W_t N_t + U_t b - T_t. \tag{B.4}$$

Note that the household's dividends income, D_t, is included in the current financial wealth, Π_t.

Let ϕ_t denote the Lagrange multiplier for the household's budget constraint (B.4). The household's maximization problem is given by:

$$J_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left[E_t \left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} - \phi_t \left(\frac{\Pi_{t+1}}{R_{t+1}^{\Pi}} - \Pi_t + C_t - W_t N_t - U_t b + T_t\right), \tag{B.5}$$

The first-order condition for consumption yields:

$$\phi_t = (1-\beta) C_t^{-\frac{1}{\psi}} \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left[E_t \left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1}, \tag{B.6}$$

which gives the marginal utility of consumption.

Recalling $N_{t+1} = (1-s)N_t + f(\theta_t)U_t$ and $U_{t+1} = sN_t + (1-f(\theta_t))U_t$, we differentiate J_t in equation (B.5) with respect to N_t:

$$\begin{aligned} J_{Nt} &= \phi_t W_t + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\times \frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \end{aligned} \tag{B.7}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Nt}}{\phi_t} = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}}\left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[(1-s)J_{Nt+1} + sJ_{Ut+1}]\right]. \tag{B.8}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned} \frac{J_{Nt}}{\phi_t} &= W_t + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\ &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right]. \end{aligned} \tag{B.9}$$

Similarly, differentiating J_t in equation (B.5) with respect to U_t yields:

$$\begin{aligned} J_{Ut} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\times \frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \end{aligned} \tag{B.10}$$

Dividing both sides by ϕ_t:

$$\frac{J_{Ut}}{\phi_t} = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}}\left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}[f(\theta_t)J_{Nt+1} + (1-f(\theta_t))J_{Ut+1}]\right]. \tag{B.11}$$

Dividing and multiplying by ϕ_{t+1}:

$$
\begin{aligned}
\frac{J_{Ut}}{\phi_t} &= b + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] \\
&= b + E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right]. \qquad \text{(B.12)}
\end{aligned}
$$

B.2 The Firm

We start by rewriting the infinite-horizon value-maximization problem of the firm recursively as:

$$S_t = X_t N_t - W_t N_t - \kappa_t V_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \qquad \text{(B.13)}$$

subject to $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$. The first-order condition with respect to V_t says:

$$S_{Vt} = -\kappa_t + \lambda_t q(\theta_t) + E_t[M_{t+1} S_{Nt+1} q(\theta_t)] = 0. \qquad \text{(B.14)}$$

Equivalently,

$$\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t = E_t[M_{t+1} S_{Nt+1}]. \qquad \text{(B.15)}$$

In addition, differentiating S_t with respect to N_t yields:

$$S_{Nt} = X_t - W_t + (1-s)E_t[M_{t+1} S_{Nt+1}]. \qquad \text{(B.16)}$$

Combining the last two equations yields the intertemporal job creation condition in equation (11).

B.3 The Wage Equation

From equations (B.9), (B.12), and (B.16), the total surplus of the worker-firm relationship is:

$$
\begin{aligned}
\Lambda_t &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{J_{Nt+1}}{\phi_{t+1}} + s\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] - b \\
&\quad - E_t\left[M_{t+1}\left[f(\theta_t)\frac{J_{Nt+1}}{\phi_{t+1}} + (1-f(\theta_t))\frac{J_{Ut+1}}{\phi_{t+1}}\right]\right] + X_t - W_t + (1-s)E_t[M_{t+1} S_{Nt+1}] \\
&= X_t - b + (1-s)E_t\left[M_{t+1}\left(\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}} + S_{Nt+1}\right)\right] - f(\theta_t)E_t\left[M_{t+1}\frac{J_{Nt+1} - J_{Ut+1}}{\phi_{t+1}}\right] \\
&= X_t - b + (1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] - \eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right], \qquad \text{(B.17)}
\end{aligned}
$$

in which the last equality follows from the definition of Λ_t and the surplus sharing rule (B.3).

The surplus sharing rule implies $S_{Nt} = (1-\eta)\Lambda_t$, which, combined with equation (B.16), yields:

$$(1-\eta)\Lambda_t = X_t - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right]. \qquad \text{(B.18)}$$

Combining equations (B.17) and (B.18) yields:

$$\begin{aligned}
X_t - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] &= (1-\eta)(X_t - b) + (1-\eta)(1-s)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\
&\quad -(1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\
X_t - W_t &= (1-\eta)(X_t - b) - (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right] \\
W_t &= \eta X_t + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}\Lambda_{t+1}\right].
\end{aligned}$$

Using equations (B.3) and (B.15) to simplify further:

$$\begin{aligned}
W_t &= \eta X_t + (1-\eta)b + \eta f(\theta_t)E_t\left[M_{t+1}S_{Nt+1}\right] && \text{(B.19)} \\
&= \eta X_t + (1-\eta)b + \eta f(\theta_t)\left[\frac{\kappa_0}{q(\theta_t)} + \kappa_1 - \lambda_t\right]. && \text{(B.20)}
\end{aligned}$$

Using the Kuhn-Tucker conditions, when $V_t > 0$, then $\lambda_t = 0$, and equation (B.20) reduces to the wage equation (17) because $f(\theta_t) = \theta_t q(\theta_t)$. On the other hand, when the nonnegativity constraint is binding, $\lambda_t > 0$, but $V_t = 0$ means $\theta_t = 0$ and $f(\theta_t) = 0$. Equation (B.20) reduces to $W_t = \eta X_t + (1-\eta)b$. Because $\theta_t = 0$, the wage equation (17) continues to hold.

C Details of the Globally Nonlinear Solution Algorithm

In the numerical algorithm, the two functional equations (19) and (21) should be expressed only in terms of two state variables N_t and x_t. As noted, we exploit a convenient mapping from the conditional expectation function, $\mathcal{E}_t$, to policy and multiplier functions to eliminate the need to parameterize the multiplier separately. After obtaining $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) = \kappa_0/\left(\mathcal{E}_t - \kappa_1\right)$. If $\tilde{q}(\theta_t) < 1$, the nonnegativity constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\cdot)$ defined in equation (3), and $V_t = \theta_t(1 - N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the nonnegativity constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$. We then perform the following set of substitutions:

$$\begin{aligned}
U_t &= 1 - N_t && \text{(C.1)} \\
N_{t+1} &= (1-s)N_t + q(\theta_t)V(N_t, x_t) && \text{(C.2)} \\
x_{t+1} &= \rho x_t + \sigma\epsilon_{t+1} && \text{(C.3)} \\
C(N_t, x_t) &= \exp(x_t)N_t - \left[\kappa_0 + \kappa_1 q(\theta_t)\right]V(N_t, x_t) && \text{(C.4)} \\
M_{t+1} &= \beta\left[\frac{C(N_{t+1}, x_{t+1})}{C(N_t, x_t)}\right]^{-\frac{1}{\psi}}\left[\frac{J(N_{t+1}, x_{t+1})}{E_t[J(N_{t+1}, x_{t+1})^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}, && \text{(C.5)}
\end{aligned}$$

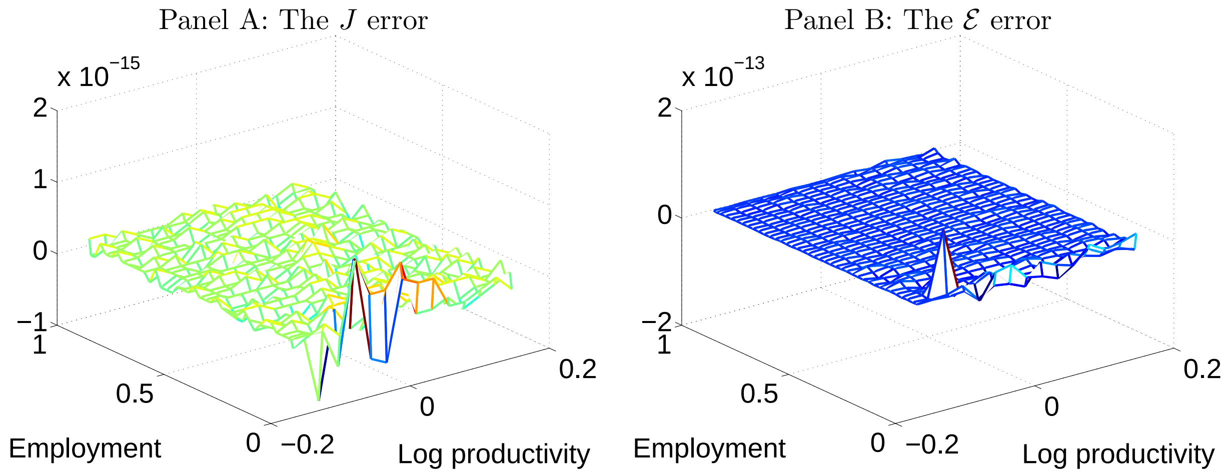
and

$$W_t = \eta\left[\exp(x_t) + \left[\kappa_0 + \kappa_1 q(\theta_t)\right]\theta_t\right] + (1-\eta)b. \quad \text{(C.6)}$$

We approximate the x_t process in equation (6) based on the discrete state space method of Rouwenhorst (1995) with 15 grid points.[15] This grid is large enough to cover the values of x_t within four unconditional standard deviations from its unconditional mean of zero. We set the minimum value of N_t to be 0.0365 and the maximum value to be 0.99. This range is large enough

[15] Kopecky and Suen (2010) show that the Rouwenhorst (1995) method is more reliable and accurate than other methods in approximating highly persistent first-order autoregressive processes.

Figure C.1 : Errors in the J and $\mathcal{E}$ Functional Equations



so that N_t never hits one of the boundaries in simulations. We use cubic splines with 40 basis functions on the N space to approximate $J(N_t, x_t)$ and $\mathcal{E}(N_t, x_t)$ on each grid point of x_t. We use extensively the approximation took kit in the CompEcon Toolbox in Matlab of Miranda and Fackler (2002). To obtain an initial guess for the projection algorithm, we use the social planner's solution via value function iteration. Solving the nonlinear model takes a lot of care, otherwise the projection algorithm would not converge. (Unlike the value function, iterating on the first-order conditions is not a contraction mapping.) The idea of homotopy continuation methods (e.g., Judd (1998, p. 179)) is used extensively to ensure convergence for a wide range of parameter values.[16]

Figure C.1 reports the error in the J functional equation (19), defined as $J(N_t, x_t)^{1-\frac{1}{\psi}} - (1-\beta)C(N_t, x_t)^{1-\frac{1}{\psi}} - \beta\left(E_t\left[J(N_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}$, and the error in the $\mathcal{E}$ functional equation (21), defined as $\mathcal{E}(N_t, x_t) - E_t\left[M_{t+1}\left[X_{t+1} - W_{t+1} + (1-s)\left(\kappa_0/q(\theta_{t+1}) + \kappa_1 - \lambda(N_{t+1}, x_{t+1})\right)\right]\right]$. These errors, in the magnitude no higher than 10^{-13}, are extremely small. As such, our nonlinear algorithm does an accurate job in characterizing the competitive equilibrium in the search economy.

D Labor Market Moments

We first report a standard set of moments, as in Shimer (2005), using an updated sample. As noted, we obtain seasonally adjusted monthly unemployment (thousands of persons 16 years of age and older) from BLS, and seasonally adjusted help wanted advertising index from the Conference Board. The sample is from January 1951 to June 2006. We take quarterly averages of the monthly series to obtain 222 quarterly observations. The average labor productivity is seasonally adjusted real average output per person in the nonfarm business sector from BLS.

[16]In practice, when we solve the model with a new set of parameters, we set the lower bound of N_t to be 0.4 to alleviate the burden of nonlinearity on the solver. After obtaining the model's solution, we then apply homotopy to gradually reduce the lower bound to 0.0365 or whatever level that N_t never hits in simulations. Time-wise, with all the trial and error that comes with homotopy, solving the model with a parametrization that admits strong nonlinearity can take almost a week. Indeed, we have encountered several specifications and parametrizations of the model for which the projection solver has failed to converge at all.

Hagedorn and Manovskii (2008) report all variables in log deviations from the HP-trend with a smoothing parameter of 1,600. In contrast, we detrend all variables as the HP-filtered cyclical component of proportional deviations from the mean with the same smoothing parameter. We do not take logs because vacancies can be zero in the model's simulations when the nonnegativity constraint on vacancy is binding. In the data, the two detrending methods yield quantitatively similar results, which are in turn close to Hagedorn and Manovskii's. In particular, from Panel A of Table D.1, the standard deviation of the V/U ratio is 0.26. The V/U ratio is procyclical with a positive correlation of 0.30 with labor productivity. Finally, vacancy and unemployment have a negative correlation of -0.91, indicating a downward-sloping Beveridge curve.

To evaluate the model's fit with the labor market moments, we simulate 1,000 artificial samples from the benchmark economy, each with 666 months. We take the quarterly averages of the monthly unemployment, U, vacancy, V, and labor productivity, X, to obtain 222 quarterly observations for each series. We then apply the exactly same procedures as in Panel A on the artificial data, and report the cross-simulation averages (as well as standard deviations) for the model moments. From Panel B, the standard deviations of U and V in the model are 0.15 and 0.12, respectively, which are close to those in the data. The model implies a standard deviation of 0.17 for the V/U ratio, which is lower than 0.26 in the data. The model also generates a Beveridge curve with a negative U-V correlation of -0.57, but its magnitude is lower than -0.91 in the data. The correlation between the V/U ratio and labor productivity is 0.99, which is higher than 0.30 in the data.

In Panels C and D, we repeat the same analysis as in Panel B but on the log-utility model and the linear utility model, respectively. The labor market moments from these two alternative models are quantitatively close to those in the benchmark model with recursive utility. As such, although important for asset prices, recursive preferences are largely irrelevant for labor market moments.

Table D.1 : Labor Market Moments

In Panel A, seasonally adjusted monthly unemployment (U, thousands of persons 16 years of age and older) is from the Bureau of Labor Statistics. The seasonally adjusted help wanted advertising index, V, is from the Conference Board. The series are monthly from January 1951 to June 2006 (666 months). Both U and V are converted to 222 quarterly averages of monthly series, and $\theta = V/U$. The average labor productivity, X, is seasonally adjusted real average output per person in the nonfarm business sector from the Bureau of Labor Statistics. All variables are in HP-filtered proportional deviations from the mean with a smoothing parameter of 1,600. In Panels B to D, we simulate 1,000 artificial samples from the respective model, with 666 monthly observations in each sample. We take the quarterly averages of monthly U, V, and X to convert to 222 quarterly observations. We implement the exactly same empirical procedures as in Panel A on these quarterly series, and report the cross-simulation averages and standard deviations for all the model moments.

	U	V	θ	X		U	V	θ	X
Panel A: Data									
Standard deviation	0.12	0.13	0.26	0.01					
Quarterly autocorrelation	0.90	0.92	0.89	0.76					
Correlation matrix		−0.91	−0.80	−0.22	U				
			0.87	0.39	V				
				0.30	θ				
	Cross-simulation mean					Cross-simulation standard deviation			
Panel B: The benchmark model with recursive preferences									
Standard deviation	0.15	0.12	0.17	0.02		0.08	0.02	0.03	0.00
Quarterly autocorrelation	0.84	0.69	0.79	0.78		0.06	0.06	0.04	0.04
Correlation matrix		−0.57	−0.65	−0.64	U		0.12	0.15	0.15
			0.95	0.97	V			0.04	0.02
				0.99	θ				0.02
Panel C: The log-utility model									
Standard deviation	0.14	0.11	0.16	0.02		0.08	0.02	0.03	0.00
Quarterly autocorrelation	0.85	0.70	0.80	0.77		0.06	0.05	0.04	0.04
Correlation matrix		−0.59	−0.67	−0.63	U		0.12	0.14	0.16
			0.95	0.98	V			0.03	0.01
				0.99	θ				0.02
Panel D: The linear utility model									
Standard deviation	0.18	0.12	0.16	0.02		0.14	0.03	0.03	0.00
Quarterly autocorrelation	0.81	0.62	0.77	0.77		0.06	0.08	0.04	0.04
Correlation matrix		−0.52	−0.63	−0.63	U		0.17	0.17	0.16
			0.92	0.92	V			0.08	0.07
				1.00	θ				0.00