Coarse Reputation and Entrepreneurial Risk Choice*

Pierre-Yves Yanni†

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Abstract

I analyze a model of anonymous credit markets with adverse selection and moral hazard in which investors finance entrepreneurs and only observe whether they have defaulted in the past. There is a dynamic complementarity in risk choice: incentives to choose the safe project over the risky project in the current period are stronger if other entrepreneurs ran safe projects in the past and if they are expected to run safe projects in the future. This results in multiple equilibria that depend not only on agents' expectations but also on history. I show that reputation concerns can induce entrepreneurs to be too conservative. In this case the economy converges to an inefficient equilibrium in which entrepreneurs choose safe projects over risky ones despite their lower return. A transition to the socially optimal risky equilibrium can be implemented in a way that balances the budget and is Pareto improving.

JEL Classification: C73, L14

Keywords: Stigma of Failure, Entrepreneurship, Risk-Taking, Reputation, Credit Markets

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†ESG UQAM; email: yanni.pierre-yves@uqam.ca

1 Introduction

Attitudes towards failure are often invoked to explain differences in entrepreneurial activity between countries or regions, for example between Europe and the US:

> *"America has proved to be more entrepreneurial than Europe in large part because it has embraced a culture of 'failing forward' as a common tech-industry phrase puts it: in Germany bankruptcy can end your business career whereas in Silicon Valley it is almost a badge of honour."*
>
> The Economist, April 14th 2011

Indeed, European entrepreneurs are more concerned about failure than their American counterparts. According to the European Commission, in 2007, 48% of Europeans agreed with the statement "You should not start a business if it might fail", compared with just 19% in the United States. Given that failure is part of the entrepreneurial process, this certainly hampers entrepreneurship, especially innovative projects. Since attitudes towards failure seem to be play a major role in entrepreneurship, it is important to understand how they are determined, why they can differ across regions or countries and what it implies in terms of public policy.

In this paper I propose an explanation for this difference in attitude towards failure based on an endogenous stigma of failure that can dissuade investors to finance failed entrepreneurs. I show how, despite identical fundamentals, an economy can converge to two different stationary equilibria: one in which entrepreneurs run risky projects and failure is not heavily penalized and one in which entrepreneurs run only safe projects and are excluded from financial markets if they fail. Furthermore, I analyze how entrepreneurs' current risk choices depend on other entrepreneurs' past and future risk choices. This sheds light on equilibrium selection and on their persistence. The latter seems of particular importance since the stigmatization of failure is not a new phenomenon in Europe, as this communication by the European Commission (1998) exemplifies:

> *"In Europe a serious social stigma is attached to bankruptcy. In the USA bankruptcy laws allow entrepreneurs who fail to start again relatively quickly and failure is considered to be part of a learning process. In Europe those who go bankrupt tend to be considered as "losers". They face great difficulty in obtaining finance for a new venture."*

In the model, the driving force behind this endogenous stigma of failure is that past business failure can result from bad luck or from a lack of entrepreneurial talent but that investors cannot distinguish between them. This a problem for an investor deciding whether to finance an entrepreneur whose previous venture went bankrupt: good entrepreneurs who were unlucky should be given a second chance while those who lack entrepreneurial skills should not be refinanced.

Information about the entire history of successes and failures of the entrepreneur might help in the decision process. However, in practice, complete histories are not available. Credit scores, when they exist, only imperfectly account for a borrower's credit history and are strongly affected by defaults. Furthermore, private credit bureaus and public registers report mostly negative information such as defaults and arrears. In any case, it is also easier and cheaper for a creditor to only check if a borrower went bankrupt. For example, in the US, nearly all bankruptcy courts have an automated line which allows a bankruptcy search by case number, name, or social security number. Similarly, in some countries creditors can ask borrowers to provide a certificate of non-bankruptcy.

In the model, limited access to credit histories and predominance of negative information are represented by a simple information structure: the entrepreneur who never defaulted has a *clean record* whereas a default is sanctioned by a (permanent) *dirty record*. This tractable structure is used to analyze entrepreneurial risk-taking in a dynamic model of anonymous credit markets with adverse selection and moral hazard.

Every period, long-lived entrepreneurs borrow money from different investors to run either a safe or a risky project. A safe project run by a good entrepreneur never fails while a risky project succeeds only with probability $p < 1$. Bad entrepreneurs always fail, no matter what type of project they undertake. Investors do not observe entrepreneurs' types or their project choices but they observe their records, which are *clean* if an entrepreneur never defaulted and *dirty* otherwise.

Records are used by investors to infer the probability of being repaid, which happens only when an entrepreneur's project is successful. This probability is a function of the ratio of good to bad entrepreneurs with a given record. The lower the ratio, the higher the probability an investor finances a bad entrepreneur who will fail and not repay her debt. Since bad entrepreneurs always fail while good entrepreneurs succeed at least with positive probability, the pool of dirty records contains a lower ratio of good relative to bad entrepreneurs than the pool of clean records. Thus, investors charge a lower cost to clean records than to dirty records, or even refuse to finance the latter when the pool of entrepreneurs with dirty records contains too few good entrepreneurs. As a result, good entrepreneurs face an intertemporal trade-off between current and future profits when they choose a project. The risky project offers a higher expected return than the safe one but also fails with positive probability, which results in the entrepreneur having a dirty record. Dirty records incur higher borrowing costs (or are not financed), which decreases future expected profits.

Entrepreneurs' risk choices are interconnected because borrowing costs for clean and dirty records depend on all of the entrepreneurs' decisions. When they run safe projects, good entrepreneurs are always successful and only bad entrepreneurs fail, which means that the pool of clean records improves (the ratio of good to bad entrepreneurs increases) while the pool of dirty records deteriorates. As a result borrowing costs decrease for clean records but increase for dirty records. This in turn increases the value of a clean record and gives entrepreneurs stronger incentives to choose safe projects. When they run risky projects, good entrepreneurs fail with positive probability. The pool of dirty records improves while the pool of clean records deteriorates, which means that borrowing costs decrease for dirty records and increase for clean records. Consequently, a clean record has a lower value and incentives to run risky projects are stronger.

Not only is there a complementarity in project choice in the current period but there is also a dynamic complementarity with all entrepreneurs' past and future project choices. If good entrepreneurs ran safe projects in the past, fewer good entrepreneurs failed and hold dirty records today. This implies that the ratio of good to bad entrepreneurs is high for clean records and low for dirty records. As a result, the latter are either excluded from financial markets or pay a high premium compared to the former, which increases the value of a clean record. Then, good entrepreneurs have stronger incentives to run safe rather than risky projects in order to keep their clean record. Analogously, if good entrepreneurs are expected to run safe projects in the future, borrowing costs will become lower for clean records and higher for dirty records (until dirty records are not financed anymore), which increases the current and future value of a clean record and induces good entrepreneurs to prefer safe projects. For a range of fundamentals, this complementarity in project choice either pushes the economy towards a safe or towards a risky stationary equilibrium. The two equilibria differ in terms of welfare, which implies that entrepreneurs' decisions can be suboptimal.

The inefficiency arises because entrepreneurs' individual decisions have no impact on the quality of the pools of clean and dirty records and hence do not take into account the effect of their risk choice on future borrowing costs. However, when they run risky projects, good entrepreneurs with clean records fail with positive probability, which lowers future borrowing costs for dirty records and increases them for clean records. Conversely, when they run safe projects, good entrepreneurs never fail, which lowers future borrowing costs for clean records and increases them for dirty records.

The main result of this paper is that, for some fundamentals, the economy can be in a safe equilibrium although the risky equilibrium would be optimal. In this case, entrepreneurs all choose to run safe projects because risky projects present a positive probability of losing their clean records and thus not being financed in the future. However, the social surplus would be higher if they all ran risky projects. In this situation, reputation is bad: eliminating records, and hence reputation concerns, would lead to a Pareto improvement since all entrepreneurs would then run risky projects. I show that a social planner endowed with the same information as investors can fix this market failure by taxing entrepreneurs' revenues and subsidizing investment in failed entrepreneurs in a way that balances the budget and is Pareto improving. By allowing failed entrepreneurs to start new projects, this policy encourages risk taking. Moreover, once good entrepreneurs run risky projects they also fail with positive probability. Then, a dirty record does not signal solely a lack of entrepreneurial ability but can also result from bad luck. Therefore, the stigma attached to a failure diminishes and the economy transitions to the Pareto-superior risky equilibrium.

The remainder of this paper is organized as follows. Following the literature review, Section 2 introduces the model. Equilibria are described in Section 3. Section 4 explores the welfare properties of the model and discusses the policy implications. Section 5 concludes.

1.1 Related Literature

There is a large literature on reputation in debt markets, pioneered by Diamond (1989), that shows that reputational concerns can deter excessive risk-taking arising from limited liability. In this type of model, if the risky project is optimal, borrowers have no incentive to develop and maintain a reputation for choosing safe projects, which implies that they run risky projects and there is no inefficiency. In my setting, entrepreneurs want to appear talented to investors since financing a bad entrepreneur leads to a loss for investors. Despite the moral hazard problem resulting from limited liability, the economy can be in an equilibrium in which entrepreneurs choose safe projects over risky ones when the latter would be socially optimal.

Several papers analyze the effects of asymmetric information on entrepreneurial finance.

In Gromb and Scharfstein (2002), agents become either entrepreneurs or intrapreneurs (projects are financed by the firm). Firms use information about failed intrapreneurs to redeploy them into other jobs while entrepreneurs look for jobs in an imperfectly informed labor market. This results in equilibria with high and low entrepreneurship which sometimes coexist. Schumacher, Kowalik and Gerling (2010) show that if banks cannot perfectly observe the risk of previous projects, there exists an inefficient equilibrium in which entrepreneurs might undertake only low-risk projects because they have no access to finance after a failure.

The mechanism in my model is closest to Landier (2006). In his framework, failure is due either to bad luck or to a lack of talent and investors do not distinguish whether an entrepreneur starts a new venture because the previous one failed or to pursue a more promising project. If the cost of capital for new project is high (low), few (many) entrepreneurs abandon projects. This implies that the pool of failed entrepreneurs contains few (many) good entrepreneurs and in turn justifies the high (low) cost of capital. I use a similar idea to build a fully dynamic model of anonymous credit markets which, in contrast to these papers, has the feature that in equilibrium entrepreneurs' project choices depend on other entrepreneurs' choices in the current period and also on their past and future project choices. This implies that a suboptimal equilibrium is not only due to agents' beliefs but is inherited from previous generations.

Tirole (1996) develops a theory of collective reputation in which an agent's current incentives depend not only on her past behavior but also on the past behavior of the group to which she belongs. Similarly, in my model, a good entrepreneur's project choice depends on her record and on borrowing costs. The former reflects her own past behavior while the latter is determined by other entrepreneurs' past behaviors. Finally, in the spirit of Ely and Valimaki (2003), this paper provides an example in which reputation concerns can induce agents to take suboptimal actions.

2 The Model

2.1 Environment

There is a continuum of long-lived good and bad entrepreneurs, with respective fractions π and $1-\pi$. Entrepreneurs survive with probability δ to the next period. Those who die are replaced by entrepreneurs of the same type such that the distribution of good and bad entrepreneurs remains the same. Since they do not have access to any storage technology, at the beginning of every period entrepreneurs need to borrow one unit of capital from short-lived investors to finance projects. When they are financed, good entrepreneurs implement either a safe project that returns Y^s with certainty or a risky project that delivers Y^r with probability p and 0 otherwise. The risky project has a higher expected return than the safe one and both have positive net value:

$$pY^r - 1 > Y^s - 1 > 0 \tag{A1}$$

Projects run by bad entrepreneurs have a negative net value $qY^b - 1 < 0$, $Y^b \geq Y^r$[1]. For simplicity and without loss of generality, I assume that $q = 0$ in the remainder of the paper.[2]

Entrepreneurs' types, project choices and project outcomes are private information. Investors only observe entrepreneurs' records, which are either *clean* if an entrepreneur never defaulted or *dirty* otherwise. Both entrepreneurs and investors are risk neutral[3]. Investors compete with each others to finance entrepreneurs, which means that they just break even in equilibrium. For simplicity investors' outside investment opportunities have zero return.

Since project outcomes are private information for entrepreneurs, equity contracts are infeasible and only debt contracts are used. In case of failure, the entrepreneur defaults on the loan and the project is liquidated. I assume that investors commit to use a liquidation technology that destroys the entire output of the project when an entrepreneur defaults. This is a standard way to induce truth-telling by borrowers: when the output of the project is higher than the value of debt it is always in entrepreneurs' interest to repay the lender.

[1] If the return of the bad entrepreneur's project was lower than the return of a good entrepreneur's project, i.e. $Y^b < Y^r$, investors would be able to screen bad entrepreneurs (in some cases). I focus on reputation effects by imposing $Y^b \geq Y^r$.

[2] Bad entrepreneurs are indifferent whether they get financed or not when $q = 0$. One can think of q positive but arbitrarily small to make bad entrepreneurs strictly better off (ex-ante) when financed.

[3] The main result of the paper, that there exists equilibria in which entrepreneurs do not take enough risk, would hold for a wider range of parameters if entrepreneurs were risk-averse. However, this would introduce risk-sharing issues and would render the model less tractable.

2.2 Timing

The timing in each period is as follows:

1. $1-\delta$ new entrepreneurs are born to replace those who died at the end of the previous period. $(1-\delta)\,\pi$ are good entrepreneurs and $(1-\delta)\,(1-\pi)$ are bad ones and all start with a clean record.

2. Entrepreneurs are matched with new investors who only observe their record, which is c (clean) if she never defaulted and d (dirty) if she defaulted at least once in the past. Each investor decides whether she wants to lend one unit to the entrepreneur she is matched with.

3. Safe and risky projects run by good entrepreneurs succeed with probability 1 and p, respectively. Projects run by bad entrepreneurs always fail.

4. If a project succeeds the entrepreneur repays the investor and consumes the remaining cash flow. If the project fails, the entrepreneur defaults and the project is liquidated. An entrepreneur's record remains clean only if she always repaid investors.

5. Entrepreneurs die with probability $1-\delta$.

2.3 Aggregate Variables

Agents' decisions depend on the distribution of good and bad entrepreneurs with clean and dirty records. However, one variable, the fraction of good entrepreneurs with clean records, is sufficient to summarize the distribution of entrepreneurs.

Since bad entrepreneurs always fail, only $(1-\delta)(1-\pi)$ newborn bad entrepreneurs have clean records and the remaining $\delta(1-\pi)$ bad entrepreneurs have dirty records. These fractions remain constant over time, which allows to focus solely on good entrepreneurs. The distribution of good entrepreneurs is determined in equilibrium and depends on the type of projects good entrepreneurs ran in the past. The fraction of good entrepreneurs with clean records μ is then a sufficient variable to keep track of the distribution of entrepreneurs since the remaining $\pi-\mu$ good entrepreneurs have dirty records.

Figure 1: Aggregate Variables

2.4 Investor's Problem

Every period there is a new continuum of investors[4] competing to lend one unit to entrepreneurs. Investors form beliefs about entrepreneurs' types and project choices based on their records $\phi \in \{c, d\}$ and the fraction of good entrepreneurs with clean records μ. The lending market is competitive, which means that investors take the interest factor $R(\mu, \phi)$ as given and decide whether to lend to an entrepreneur with record ϕ.

[4]Investors could live for more than one period as long as the same entrepreneur and investor do not meet more than once.

2.5 Entrepreneur's Problem

Assuming she gets financed, the expected lifetime utility of a good entrepreneur with a clean record is

$$V(\mu, c) = \max \left\{ \begin{array}{c} \overbrace{Y^s - R(\mu, c) + \delta V(\mu', c)}^{\text{Safe project}}, \\ \underbrace{p(Y^r - R(\mu, c) + \delta V(\mu', c)) + (1-p)\,\delta V(\mu', d)}_{\text{Risky project}} \end{array} \right\} \tag{1}$$

Individually, entrepreneurs have no effect on the law of motion $\mu'(\mu)$ for the fraction of good entrepreneurs with clean records and hence take it as given. If a good entrepreneur runs a safe project, she gets $Y^s - R(\mu, c)$ and, if she survives, enters the following period with a clean record. If she chooses the risky project, she is successful with probability p, in which case she repays $R(\mu, c)$, consumes $Y^r - R(\mu, c)$ and enters the next period with a clean record. With probability $1 - p$, she defaults, gets nothing in the current period and enters the next period with a dirty record.

When financed, an entrepreneur with a dirty record has expected lifetime utility

$$V(\mu, d) = \max\{Y^s - R(\mu, d), p(Y^r - R(\mu, d))\} + \delta V(\mu', d)$$

Since dirty records never become clean, entrepreneurs with dirty records have no reputation motive. They face a static choice and always choose the project with the highest expected return, which, by assumption (A1), is the risky one:

$$V(\mu, d) = p(Y^r - R(\mu, d)) + \delta V(\mu', d) \tag{2}$$

3 Equilibrium

In this section, I first describe how borrowing costs are determined. Then I introduce the two stationary equilibria and the conditions under which they coexist. Finally, I characterize the set of threshold equilibria and discuss the dynamics of the model.

3.1 Pricing

There is free entry in the lending market, which implies that interest rates are pinned down by zero-profit conditions and depend on beliefs about the type of entrepreneur (good or bad) and the type of project (safe or risky).

The probability of being a good entrepreneur given a clean record and a fraction μ of good entrepreneurs with clean records (see Figure 1) is

$$\Pr\left(G|\,\mu,c\right)=\frac{\mu}{\underbrace{\mu+\left(1-\delta\right)\left(1-\pi\right)}_{\text{entrepreneurs with clean records}}}$$

while for dirty records it is

$$\Pr\left(G|\,\mu,d\right)=\frac{\pi-\mu}{\pi-\mu+\delta\left(1-\pi\right)}$$

In games with strategic complementarities such as this one, a mixed strategy equilibrium would not survive an infinitesimal perturbation in the mass of agents playing a given strategy. Therefore, I focus on pure strategy equilibria. Investors' beliefs about project choices are given by the function $\hat{\alpha}\left(\mu,\phi\right)\in\{s,r\}$. If the entrepreneur is expected to run a safe project ($\hat{\alpha}=s$), the debt is always repaid when the project is run by a good entrepreneur. Given that the cost of a project is 1, the expected profit is

$$E\left[\Pi\left(\mu,\phi,s\right)\right]=\Pr\left(G|\,\mu,\phi\right)R^{s}\left(\mu,\phi\right)-1$$

If the entrepreneur is expected to run a risky project ($\hat{\alpha}=r$), the debt is repaid only when the project is successful, which happens with probability p for a good entrepreneur:

$$E\left[\Pi\left(\mu,\phi,r\right)\right]=\Pr\left(G|\,\mu,\phi\right)pR^{r}\left(\mu,\phi\right)-1$$

If entrepreneurs with clean records are expected to run safe projects, the zero-profit condition $E\left[\Pi\left(\mu,c,s\right)\right]=0$ implies that

$$R^{s}\left(\mu,c\right)=\frac{\mu+\left(1-\delta\right)\left(1-\pi\right)}{\mu}=1+\frac{\left(1-\delta\right)\left(1-\pi\right)}{\mu}\tag{3}$$

The interest rate is given by the ratio of bad entrepreneurs over good entrepreneurs, which means that μ good entrepreneurs with clean records pay a risk premium $\frac{(1-\delta)(1-\pi)}{\mu}$ that finances $(1-\delta)(1-\pi)$ bad entrepreneurs with clean records. If entrepreneurs with clean records are expected to run risky projects, $E[\Pi(\mu, c, r)] = 0$ means that the interest factor is multiplied by $\frac{1}{p}$ since the project is only successful with probability p:

$$R^r(\mu, c) = \frac{R^s(\mu, c)}{p} \tag{4}$$

Thus, for clean records, the interest factor is

$$R(\mu, c) = \begin{cases} R^s(\mu, c) & \text{if} \quad \hat{\alpha}(\mu, \phi) = s \\ R^r(\mu, c) & \text{if} \quad \hat{\alpha}(\mu, \phi) = r \end{cases}$$

For the remainder of the paper, I assume that

$$Y^s > \frac{1}{\pi} \tag{A2}$$

This is a sufficient condition for entrepreneurs with clean record to always be financed. When good entrepreneurs are expected to run safe projects,

$$Y^s > \frac{1}{\pi} \geq R^s(\mu, c) = 1 + \frac{(1-\delta)(1-\pi)}{\mu}$$

where the second inequality follows from the fact that the fraction of good entrepreneur with clean records is at least as large as the fraction of newborn good entrepreneurs, i.e. $\mu \geq (1-\delta)\pi$. This, in turn, implies that when a risky project is successful, the return is also sufficient to repay the investor:

$$Y^r > \frac{Y^s}{p} > \frac{R^s(\mu, c)}{p} = R^r(\mu, c)$$

The first inequality comes from the fact that the expected return of the risky project is higher than the safe one (assumption (A1)).

As explained above, entrepreneurs with dirty records always run risky projects. Thus only $E[\Pi(\mu, d, r)] = 0$ is relevant. For mathematical convenience, the interest factor $R(\mu, d)$ is set equal to the return of the risky project Y^r when the entrepreneur is not financed, such that the period payoff is equal to zero (see (2)):

$$R(\mu, d) = \min\left\{\frac{1}{p}\left(1 + \frac{\delta(1-\pi)}{\pi - \mu}\right), Y^r\right\} \tag{5}$$

The cost of financing $\delta(1-\pi)$ bad entrepreneurs with dirty records is born by the $\pi - \mu$ good entrepreneurs that have dirty records.

3.2 Stationary Equilibria

Hereafter I discuss the two stationary equilibria of this model and show that for some parameters they both exist. The formal definition of an equilibrium is postponed to Subsection 3.4.

3.2.1 Safe Equilibrium

When good entrepreneurs with clean records run safe projects, aggregate dynamics are described by the following figure:

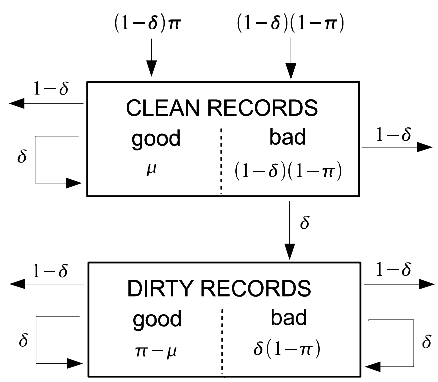


Figure 2: Flows with Safe Project

Every period entrepreneurs die with probability $1-\delta$ and are replaced by $(1-\delta)\,\pi$ new good entrepreneurs and $(1-\delta)\,(1-\pi)$ new bad entrepreneurs. Since bad entrepreneurs always fail, they have a clean record only during the first period of their lives and a dirty record thereafter. Good entrepreneurs with clean records never fail and hence keep their clean records unless they die.

The safe law of motion for good entrepreneurs with clean record is

$$\mu' = \delta\mu + (1-\delta)\,\pi \tag{6}$$

The fraction of good entrepreneurs with clean records (μ') is given by the previous period's fraction that survived and the new good entrepreneurs. The fixed point of the safe law of motion (6), $\mu^s = \pi$, corresponds to the safe stationary equilibrium, in which all good entrepreneurs have a clean record.

From the entrepreneur's problem (1), the expected lifetime utility of a good entrepreneur in the safe equilibrium is

$$V\left(\mu^s, c\right) = \frac{Y^s - R^s\left(\mu^s, c\right)}{1-\delta} \tag{7}$$

Only bad entrepreneurs default, which implies that a dirty record unambiguously signals a bad type and failed entrepreneurs are thus not financed:

$$R(\mu^s, d) = Y^r \text{ and } V(\mu^s, d) = 0$$

A safe equilibrium exists if there is no profitable deviation for investors and entrepreneurs. Investors cannot charge less since they just break even by charging $R^s(\mu^s, c)$ and would not finance any entrepreneur if they charged more than $R^s(\mu^s, c)$. For a good entrepreneur, a one-shot deviation from running safe projects results in a higher expected period payoff $p(Y^r - R^s(\mu^s, c))$ but the project would fail with probability $1-p$. Thus, for a safe equilibrium to exist, $(p, \delta, \pi, Y^s, Y^r)$ are such that

$$\underbrace{pY^r - Y^s}_{\text{Return differential}} + \underbrace{(1-p) R^s(\mu^s, c)}_{\text{Limited liability}} \leq (1-p)\,\delta \underbrace{\Delta V(\mu^s)}_{\text{Value of a clean record}} \tag{8}$$

where $R^s(\mu^s, c)$ is given by (3) and the value of a clean record is

$$\Delta V(\mu^s) \equiv V(\mu^s, c) - V(\mu^s, d) = V(\mu^s, c)$$

The left-hand side of (8) is the expected benefit of choosing the risky rather than the safe project and consists in a higher return and a lower expected cost for a given interest factor. The right-hand side is the expected cost: the risky project increases the probability of losing a clean record from 0 to $1-p$, in which case the discounted value of a clean record is lost.

3.2.2 Risky Equilibrium

When good entrepreneurs with clean records run risky projects they succeed with probability p and keep their clean records if they survive (with probability δ). Otherwise, with probability $1-p$ the project fails and their record stays dirty for the rest of their lives.

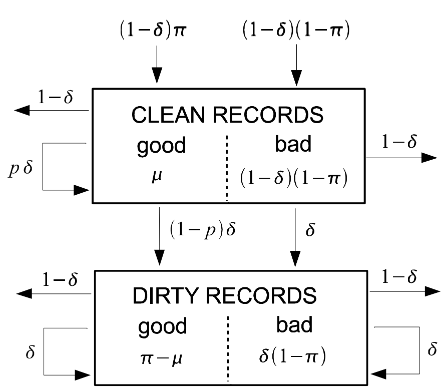


Figure 3: Flows with Risky Project

The risky law of motion is

$$\mu' = p\delta\mu + (1-\delta)\,\pi \tag{9}$$

and its fixed point is $\mu^r = \frac{(1-\delta)\pi}{1-p\delta} < \mu^s = \pi$.

For the remainder of the paper I assume that fundamentals are such that dirty records create a positive surplus in the risky equilibrium:

$$(\pi - \mu^r)\,(pY^r - 1) - \delta\,(1-\pi) > 0 \tag{A3}$$

This assumption guarantees that dirty records are financed in the risky equilibrium ($R\,(\mu^r, d) < Y^r$), which is the case of interest. It is also a necessary condition for an inefficient safe equilibrium to exist[5] (see Proposition 4).

Given Assumption (A3), all entrepreneurs are financed in the risky equilibrium. Good entrepreneurs with dirty records run risky projects every period and have expected lifetime utility

$$V\,(\mu^r, d) = \frac{p\,(Y^r - R\,(\mu^r, d))}{1-\delta}$$

When running a risky project, entrepreneurs with clean records are successful with probability p, in which case they get $Y^r - R\,(\mu^r, c)$ in the current period and keep their clean record. Otherwise they get zero in the current period and hold a dirty record for the rest of their lives

$$V\,(\mu^r, c) = p\,(Y^r - R^r\,(\mu^r, c)) + p\delta V\,(\mu^r, c) + (1-p)\,\delta V\,(\mu^r, d) \tag{10}$$

A risky equilibrium exists if a one-shot deviation to the safe project is not profitable. That means, if $(p, \delta, \pi, Y^s, Y^r)$ are such that

$$(1-p)\,\delta \underbrace{\Delta V\,(\mu^r)}_{\text{Value of a clean record}} \leq \underbrace{pY^r - Y^s}_{\text{Return differential}} + \underbrace{(1-p)\,R^r\,(\mu^r, c)}_{\text{Limited liability}} \tag{11}$$

where $R^r\,(\mu^r, c)$ is given by (4) and the value of a clean record corresponds to the difference in expected borrowing costs between a dirty and a clean record discounted by a good entrepreneur's probabilities of success and survival:

$$\Delta V\,(\mu^r) \equiv V\,(\mu^r, c) - V\,(\mu^r, d) = \frac{p\,(R\,(\mu^r, d) - R^r\,(\mu^r, c))}{1-p\delta}$$

with $R\,(\mu^r, d)$ defined in (5).

[5] If (A3) is not satisfied, I obtain the standard result that due to limited liability entrepreneurs take too much risk compared to the social optimum.

3.3 Multiplicity of Equilibria

Using the necessary conditions for existence of a safe (8) and risky (11) equilibrium, the plane (pY^r, Y^s) can be divided in different zones. If the difference in expected return between the risky and the safe project is high enough only the risky equilibrium exists while if it low enough, only the safe equilibrium exists. Otherwise, depending on entrepreneurs' beliefs and the initial fraction of good entrepreneurs with clean records μ_0 there can be a safe or a risky equilibrium for the same fundamentals.

Proposition 1 *For any $p, \delta, \pi \in (0,1)$, there are Y^s and Y^r such that the safe and the risky equilibrium both exist.*

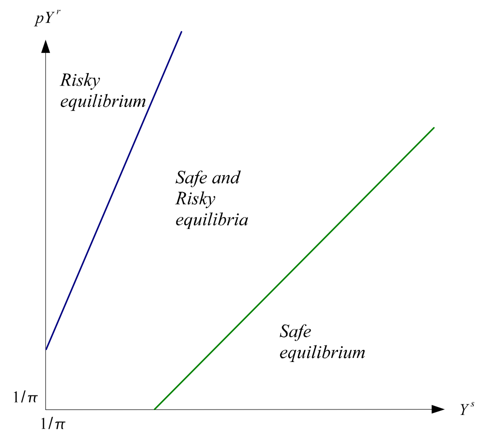


Figure 4: Multiplicity

Proof. Pick $pY^r - Y^s$ and Y^s such that

$$\begin{aligned} & (1-p)\,\delta\frac{p\left(R\left(\mu^r,d\right)-R^r\left(\mu^r,c\right)\right)}{1-p\delta}-(1-p)\,R^r\left(\mu^r,c\right) \\ \leq\ & pY^r - Y^s \\ \leq\ & (1-p)\,\delta\frac{Y^s-R^s\left(\mu^s,c\right)}{1-\delta}-(1-p)\,R^s\left(\mu^s,c\right) \end{aligned} \quad (12)$$

Then (8) and (11) are satisfied. ■

The multiplicity of equilibria results from the higher value of a clean record and the lower interest factor in the safe than in the risky equilibrium. If entrepreneurs run safe projects, the pool of clean records contains a high fraction of good entrepreneurs, which translates into low borrowing costs for clean records. The pool of dirty records, on the other hand, contains only bad entrepreneurs and thus dirty records are not financed. The value of a clean record is then high and the moral hazard problem stemming from limited liability is not severe. This provides strong incentives to choose safe projects.

If entrepreneurs run risky rather than safe projects, the pool of clean records is of lesser quality but the pool of dirty records contains enough good entrepreneurs for them to be financed (by (A3)). Thus the difference in borrowing cost between a clean and a dirty record is smaller. The value of a clean record is then lower and the interest factor is higher, which induces entrepreneurs to prefer the risky to the safe project.

For the remainder of the paper, I assume that fundamentals are such that both stationary equilibria exist. That means,

$$p, \delta, \pi, Y^s, Y^r \text{ are such that (12) is satisfied} \tag{A4}$$

Below, I analyze equilibria out of the steady state and discuss the conditions under which the economy converges to each of the stationary equilibria.

3.4 Threshold Equilibrium

I focus on symmetric threshold equilibria in which good entrepreneurs with clean records run risky projects as long as $\mu < \mu^*$ and safe projects when $\mu \geq \mu^*$[6]. As shown below, given that other entrepreneurs use such a strategy it is also optimal for any entrepreneur to use a threshold strategy.

Definition 2 (Threshold Equilibrium) *A threshold equilibrium is a μ^* such that given $R(\mu, c)$, $R(\mu, d)$ and the equilibrium law of motion*

$$\mu'(\mu) = \begin{cases} p\delta\mu + (1-\delta)\pi & \text{if} \quad \mu < \mu^* \\ \delta\mu + (1-\delta)\pi & \text{if} \quad \mu \geq \mu^* \end{cases} \tag{13}$$

1. *good entrepreneurs with clean records maximize their lifetime utility (1) by running risky projects for all $\mu < \mu^*$ and safe projects for $\mu \geq \mu^*$*

2. *investors maximize expected profit by financing entrepreneurs whenever $E[\Pi(\mu, \phi, \hat{\alpha}(\mu, \phi))] \geq 0$*

3. *the zero-profit conditions $E[\Pi(\mu, c, s)] = 0$ and $E[\Pi(\mu, c, r)] = 0$ are satisfied respectively for $\mu \geq \mu^*$ and $\mu < \mu^*$ and $E[\Pi(\mu, d, r)] = 0$ holds when dirty records are financed*

4. *beliefs are correct: $\hat{\alpha}(\mu, \phi) = \begin{cases} s & \text{for} \quad \mu \geq \mu^* \\ r & \text{for} \quad \mu < \mu^* \end{cases}$*

[6] At μ^* entrepreneurs are indifferent between the risky and safe project. The type of project they choose has no influence on the results since μ^* has measure zero on the real line $[(1-\delta)\pi, \pi]$.

3.5 Cost of Borrowing

Entrepreneurs' decisions are determined by current and future interest factors, which depend on the evolution of the fraction of good entrepreneurs with clean records. As the fraction of good entrepreneurs with clean records μ increases, the pool of entrepreneurs with clean records improves in the sense that the ratio of bad over good entrepreneurs $\frac{(1-\delta)(1-\pi)}{\mu}$ decreases. As a result, the probability of drawing a good entrepreneur from the pool of entrepreneurs with clean records increases in μ, which implies that the interest factor for clean records, $R(\mu, c)$, decreases in μ. Given the equilibrium law of motion (13), the interest factor jumps downward at μ^* when clean records start running safe projects (and repay their debt with probability one if they are good entrepreneurs).

Similarly, when μ increases the ratio of bad over good entrepreneurs with dirty records $\frac{\delta(1-\pi)}{\pi-\mu}$ increases and the probability of drawing a good entrepreneur from the pool of entrepreneur with dirty records decreases. Thus the interest factor for dirty records $R(\mu, d)$ increases in μ until dirty records are not financed anymore ($R(\mu, d) = Y^r$). This implies that the interest premium between a dirty and a clean record, $R(\mu, d) - R(\mu, c)$, increases in the proportion of good entrepreneurs with clean records.

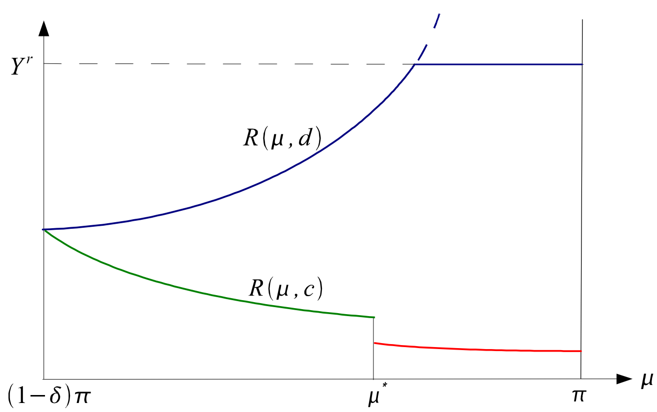


Figure 5: Borrowing Costs

3.6 Equilibrium Characterization

In order to characterize the set of threshold equilibria, I analyze what happens in two extreme cases: when good entrepreneurs with clean records run safe projects forever and when they run risky projects forever. This allows me to define the lower and the upper bounds on the set of equilibria.

Firstly, I consider the case when all good entrepreneurs with clean record run safe projects forever. Using entrepreneurs' problems (1) and (2) and rewriting the safe law of motion (6)

$$\mu_t = \delta^t \mu_0 + \left(1 - \delta^t\right) \mu^s \tag{14}$$

the difference in payoff between running a safe and a risky payoff in the current period and safe projects forever after is defined as

$$\begin{aligned} \Lambda^s(\mu_0) \equiv & - \underbrace{(pY^r - Y^s)}_{\textit{Return Differential}} - \underbrace{(1-p) R^s(\mu_0, c)}_{\textit{Limited Liability}} \\ & + (1-p)\delta \underbrace{\sum_{t=1}^{\infty} \delta^{t-1} \left(Y^s - R^s(\mu_t, c) - p\left(Y^r - R(\mu_t, d)\right)\right)}_{\textit{Value of a Clean Record (when running safe projects forever)}} \end{aligned}$$

where μ_t is given by (14).

Starting from a fraction μ_0 of good entrepreneurs with clean records and given that all good entrepreneurs with clean records run safe projects forever, a one-shot deviation to a risky project in the current period is not profitable if $\Lambda^s(\mu_0) \geq 0$. Since $R^s(\cdot, c)$ and $R(\cdot, d)$, the interest factors to entrepreneurs with clean records who run safe projects and to entrepreneurs with dirty records, are respectively decreasing and increasing in the fraction of good entrepreneurs with clean records, Λ^s is increasing. This implies that if $\Lambda^s(\mu_0) \geq 0$ there are no profitable deviation from the safe project for any $\tilde{\mu}_0 \geq \mu_0$. Notice that from assumption (A4), fundamentals are such that a safe equilibrium exists, which can be rewritten as

$$\Lambda^s(\mu^s) = -(pY^r - Y^s) - (1-p) R^s(\mu^s, c) + (1-p)\delta \frac{Y^s - R^s(\mu^s, c)}{1-\delta} \geq 0$$

Secondly, I consider the case in which all good entrepreneurs with clean record run risky projects forever. Using entrepreneurs' problems (1) and (2) and rewriting the risky law of motion (9)

$$\mu_t = (p\delta)^t \mu_0 + \left(1 - (p\delta)^t\right) \mu^r \tag{15}$$

the difference in payoff between running a safe and a risky project in the current period and risky projects forever after is

$$\begin{aligned} \Lambda^r(\mu_0) \equiv & -(pY^r - Y^s) - (1-p) R^r(\mu_0, c) \\ & + (1-p)\delta \sum_{t=1}^{\infty} (p\delta)^{t-1} \left(p\left(Y^r - R^r(\mu_t, c)\right) - p\left(Y^r - R(\mu_t, d)\right)\right) \end{aligned}$$

where μ_t is given by (15). Starting from a fraction μ_0 of good entrepreneurs with clean records and given that all good entrepreneurs with clean records run risky projects forever, a one-shot deviation to a safe project in the current period is not profitable if $\Lambda^r(\mu_0) \leq 0$. By the argument made above for Λ^s, Λ^r is also increasing, which implies that if $\Lambda^r(\mu_0) \leq 0$ there are no profitable deviation from the risky project for any $\tilde{\mu}_0 \leq \mu_0$. From condition (A4), fundamentals are such that a risky equilibrium exists:

$$\Lambda^r(\mu^r) = -(pY^r - Y^s) - (1-p) R^r(\mu^r, c) + (1-p)\delta \frac{p(R(\mu^r, d) - R^r(\mu^r, c))}{1 - p\delta} \leq 0$$

In a threshold equilibrium, $\Lambda^r(\mu_0) \leq 0$ for all $\mu_0 < \mu^*$ and $\Lambda^s(\mu_0) \geq 0$ for all $\mu_0 > \mu^*$:

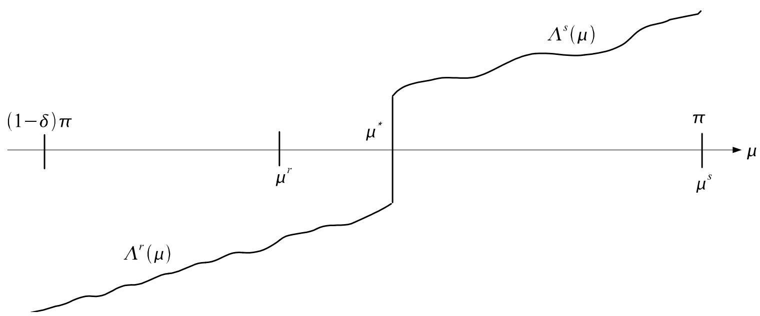
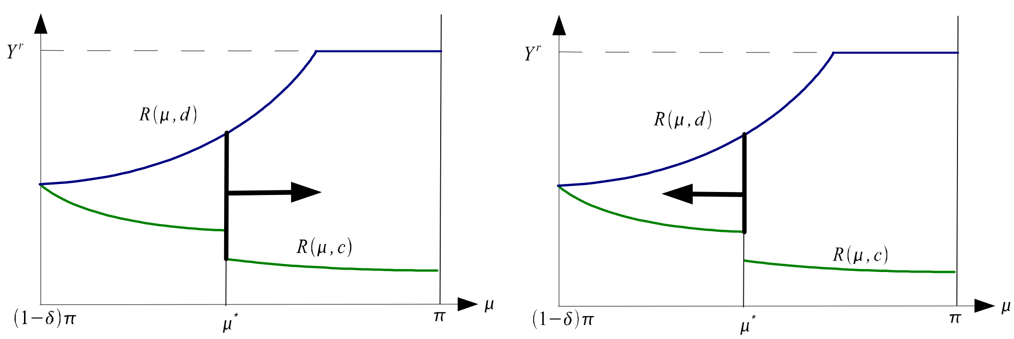


Figure 6: Threshold Equilibrium

The jump $\Lambda^s(\mu^*) - \Lambda^r(\mu^*)$ at μ^* is due to the difference in borrowing costs, $R^s(\mu^*, c)$ and $R^r(\mu^*, c)$, and in value of a clean record when everyone runs safe rather than risky projects. The figures below illustrates these two effects.



Figures 7 and 8: Interest Rate Differentials with Safe and Risky Projects

The figure on the left depicts what happens when all entrepreneurs with clean records run safe projects forever. The thick vertical line represents the interest rate differential at μ^*. The arrow indicates that the fraction of good entrepreneurs with clean records increases, which leads to increasing cost differences between dirty and clean records. The figure on the right shows the opposite case, when all entrepreneurs with dirty records run risky projects forever.

As for stationary equilibria, other entrepreneurs' project choices affect individual risk choices through two channels. First, for a given fraction of good entrepreneurs with clean records μ_0 the interest factor is higher when investors expect that risky projects (figure on the right) rather than safe ones (figure on the left) are chosen. Other things being equal, risky projects are more attractive when the interest factor is higher because the debt is repaid only if the project is successful, which arises with a lower probability with risky than safe projects. Second, the fraction of good entrepreneurs with clean records increases when they run safe projects, which leads to increasing interest rate premia and, once $R(\mu, d) = Y^r$, no financing for dirty records. The value of a clean record is then higher than when good entrepreneurs with clean records run risky projects and the interest rate premium decreases over time.

There is a complementarity in project choice: if all other entrepreneurs with clean records run safe rather than risky projects, incentives to choose a safe project in the current period are higher. The current interest factor is lower if entrepreneurs with clean records are expected to run safe projects, which attenuates the moral hazard problem. By running safe projects, good entrepreneurs do not fail and get a dirty record, which implies that the pool of clean records improves in quality while the pool of dirty records deteriorates. As a result, future interest rates decrease for clean records and increase for dirty records, which increases the value of a clean record. The alleviation of the moral hazard problem and the increase in the value of a clean record provide stronger incentives to run safe projects.

Since $\Lambda^s(\mu^*) > 0 > \Lambda^r(\mu^*)$ there exists a continuum of threshold equilibria. To understand why, consider a potential equilibrium threshold $\mu^* + \varepsilon$, with $\varepsilon > 0$ small. Since $R^{\cdot}(\cdot, \cdot)$ is continuous in the fraction of good entrepreneurs with clean records, Λ^s and Λ^r inherit the same property. Thus, at μ^{**}, Λ^s and Λ^r are just slightly higher and

$$\Lambda^s(\mu^* + \varepsilon) > 0 > \Lambda^r(\mu^* + \varepsilon)$$

It is still optimal to run safe (risky) projects when other entrepreneurs also run safe (risky) projects, which implies that there exists a threshold equilibrium at $\mu^* + \varepsilon$. The same reasoning applies for ε negative and small. The figure below illustrates the continuum of threshold equilibria between $\underline{\mu}$ and $\overline{\mu}$ and Proposition 3 characterizes it.

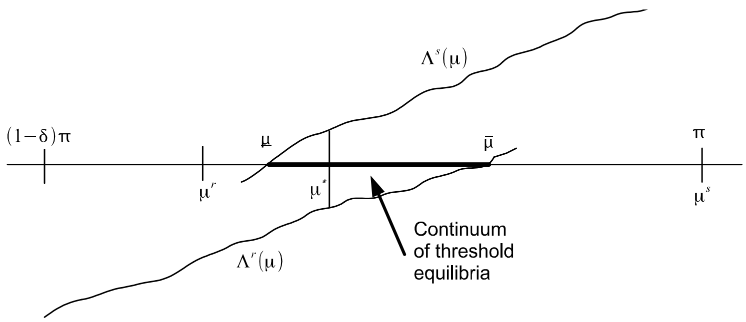


Figure 9: Continuum of Threshold Equilibria

Proposition 3 *There exists a continuum of threshold equilibria* $\mu^* \in \left[\underline{\mu}, \overline{\mu}\right] \subset [\mu^r, \mu^s]$, *where* $\underline{\mu}$ *and* $\overline{\mu}$ *are defined by*

$$\Lambda^r\left(\overline{\mu}\right)\left(\overline{\mu} - \mu^s\right) = 0 \tag{16}$$

and

$$\Lambda^s\left(\underline{\mu}\right)\left(\underline{\mu} - \mu^r\right) = 0 \tag{17}$$

Proof. From (A4), $\Lambda^r(\mu^r) \leq 0$ and $\Lambda^s(\mu^s) \geq 0$. Since Λ^r and Λ^s are increasing and continuous, (16) and (17) imply that

- either $\Lambda^r(\overline{\mu}) = 0$ for some $\overline{\mu} \leq \mu^s$ or $\Lambda^r(\mu^s) < 0$ and $\overline{\mu} = \mu^s$
- either $\Lambda^s\left(\underline{\mu}\right) = 0$ for some $\underline{\mu} \geq \mu^r$ or $\Lambda^s(\mu^r) > 0$ and $\underline{\mu} = \mu^r$

To show that all $\mu^* \in \left[\underline{\mu}, \overline{\mu}\right]$ are threshold equilibria, I demonstrate that given that all other clean records choose the risky project below μ^* and the safe project above μ^*, it is optimal for any clean record to also do so.

For any $\mu_0 < \mu^*$,

$$\Lambda^r(\mu_0) < \Lambda^r(\mu^*) \leq \Lambda^r(\overline{\mu}) \leq 0$$

The inequalities follow from Λ^r increasing and non-positive at $\overline{\mu}$. Incentives to choose the risky project decrease as $\mu_t \to \mu^r$. Thus, if the risky project is chosen at some point, it is also chosen forever after when everyone runs risky projects forever.

Similarly, for any $\mu_0 > \mu^*$,

$$\Lambda^s(\mu_0) > \Lambda^s(\mu^*) \geq \Lambda^s\left(\underline{\mu}\right) \geq 0$$

The inequalities follow from Λ^s increasing and non-negative at $\underline{\mu}$. Incentives to choose the safe project increase as $\mu_t \to \mu^s$. Thus, if the safe project is chosen at μ_0 it is chosen for all μ_t, $t > 0$. ■

3.7 Equilibrium Dynamics

By Proposition 3, there exists a continuum of equilibria μ^* above μ^r:

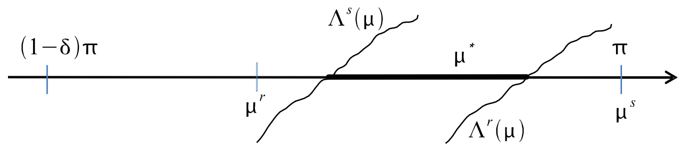


Figure 10: Continuum above μ^r

In these equilibria, the economy converges to the safe stationary equilibrium if the initial fraction of good entrepreneurs with clean records is higher than the equilibrium threshold and to the risky stationary equilibrium otherwise:

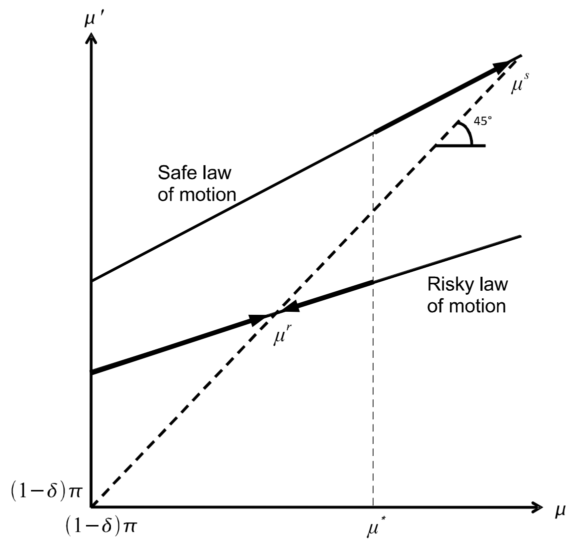


Figure 11: Threshold above μ^r

In addition, when $\underline{\mu} = \mu^r$ there also exist threshold equilibria below μ^r for some fundamentals (see Proposition 8 in Appendix):

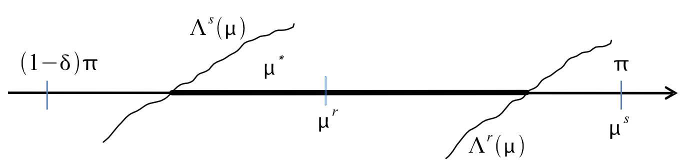


Figure 12: Continuum below μ^r

If $\mu^* < \mu^r$, the economy converges to the safe stationary equilibrium for any initial fraction of good entrepreneurs with clean records. In this case good entrepreneurs with clean records run risky projects for any $\mu_0 < \mu^*$ and the economy converges to the risky stationary equilibrium μ^r. However, in finite time it reaches $\mu_t > \mu^*$, at which point the value of a clean record becomes too high to take the risk of losing it. Then, good entrepreneurs run safe projects forever and the economy converge to the safe stationary equilibrium μ^s.

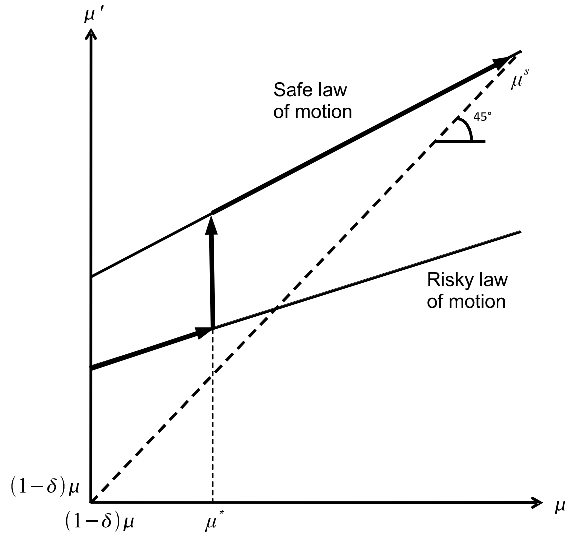


Figure 13: Dynamics below μ^r

4 Welfare

In a world with perfect information, only good entrepreneurs would be financed and the risky project would always be preferred to the safe. With imperfect information, good and bad entrepreneurs are sorted using their records. The safe project sorts good entrepreneurs from bad ones more efficiently than the risky one since good entrepreneurs are always successful when they run the safe project, which is not the case with the risky project. However, there is also a loss since the risky project is more productive than the safe one.

Stationary equilibria can easily be compared in terms of welfare. In the safe equilibrium, the total period surplus is equal to the surplus of successful projects, $Y^s - 1$, produced by $\mu^s = \pi$ good entrepreneurs net of the cost of financing $(1-\delta)(1-\pi)$ projects run by bad entrepreneurs (who have a clean record):

$$S^s = \underbrace{\pi \left(Y^s - 1\right)}_{\text{Net revenue from good entrepreneurs}} - \underbrace{(1-\delta)(1-\pi)}_{\text{Cost of bad entrepreneurs}} \tag{18}$$

In the risky equilibrium, the expected return of a project run by a good entrepreneur is higher, $pY^r > Y^s$ but all $1-\pi$ bad entrepreneurs are financed

$$S^r = \pi \left(pY^r - 1\right) - (1-\pi) \tag{19}$$

Thus, total revenue is higher in the risky than in the safe equilibrium since all good entrepreneurs run risky projects instead of safe ones. However, the total cost is also higher since all bad entrepreneurs are financed instead of only the new bad entrepreneurs.

The cost of financing bad entrepreneurs is fully internalized by the market since borrowing costs reflect the quality of the pools of clean and dirty records and dirty records are only financed when it is efficient to do so. Indeed, (18) and (19) can be rewritten

$$S^s = \mu^s \left(Y^s - R^s\left(\mu^s, c\right)\right)$$

and

$$S^r = \mu^r p \left(Y^r - R^r\left(\mu^r, c\right)\right) + \left(\pi - \mu^r\right) p \left(Y^r - R\left(\mu^r, d\right)\right)$$

However, entrepreneurs' choices are not always welfare maximizing for two reasons. First, due to limited liability, entrepreneurs do not bear the full cost of the risks they take and hence might choose risky projects when safe ones are socially optimal. Second, entrepreneurs' individual decisions have no effect on the quality of the pools of clean and dirty records and hence on future borrowing costs. Nevertheless, collectively, entrepreneurs improve the pool of clean records and deteriorate the pool of dirty records when they run safe projects. This generates a decrease in borrowing costs for clean records and an increase in cost for dirty records.

4.1 Welfare Comparisons across Equilibria

In general, welfare comparisons between stationary equilibria are ambiguous if transition costs are taken into account. Specifically, if the economy is in the risky equilibrium but the period surplus is higher in the safe equilibrium, it does not follow that a transition to the safe equilibrium is desirable. The reason is that if good entrepreneurs with clean records run safe rather than risky projects, they produce a smaller output ($Y^s < pY^r$) and the fraction of good entrepreneurs with clean records only gradually increases from μ^r to μ^s. Hence, if the probability of survival δ, which corresponds to the discount rate, is small enough, the current generation of entrepreneurs is made worse off by a transition to the safe equilibrium. Then, welfare depends on the weights given to future generations by the social planner. If the weights on future generations are high enough the transition is worthwhile, otherwise the decrease in utility of current generations is not compensated by future generations' increase in utility.

Fortunately, a clear welfare statement can be made when the economy is in the safe equilibrium but the period surplus would be higher in the risky equilibrium ($S^r > S^s$). Then the benefit of sorting entrepreneurs, namely not financing $\delta(1-\pi)$ bad entrepreneurs with dirty records, is smaller than the opportunity cost of running safe instead of risky projects:

$$\delta(1-\pi) < \pi(pY^r - Y^s) \tag{20}$$

This implies that there is no social value in sorting entrepreneurs using their records. In this case, good entrepreneurs with clean records should run risky projects and, as soon as the pool of dirty records contains enough good entrepreneurs, dirty records should be financed as well. No information about entrepreneurs' types is necessary to achieve the period surplus of the risky equilibrium since everyone is financed. Thus, the surplus is also higher during the transition to the risky equilibrium than in the safe equilibrium. For all $\mu \in [\mu^r, \mu^s]$

$$\begin{aligned} S^r(\mu) &= \mu(pY^r - 1) - (1-\delta)(1-\pi) \\ &\quad + \max\{(\pi - \mu)(pY^r - 1) - \delta(1-\pi), 0\} \geq S^r > S^s \end{aligned} \tag{21}$$

As a result, when (20) is satisfied, the safe equilibrium is dynamically inefficient. Not only does the risky equilibrium Pareto dominate the safe equilibrium but a transition from the safe to the risky equilibrium is Pareto-improving.

4.2 Inefficient Safe Equilibrium

Entrepreneurs with clean records do not internalize the fact that, by failing, they deteriorate the pool of clean records but also improve the pool of dirty records. Thus the economy can be trapped in the safe equilibrium although the risky one would produce a higher social surplus.

When fundamentals are such that the period surplus is higher in the risky than in the safe equilibrium, good entrepreneurs are also better off in the risky equilibrium. The expected lifetime utility of a good entrepreneur with a clean record in the safe equilibrium (7) can be rewritten

$$V(\mu^s, c) = \frac{1}{1-\delta} \frac{S^s}{\pi}$$

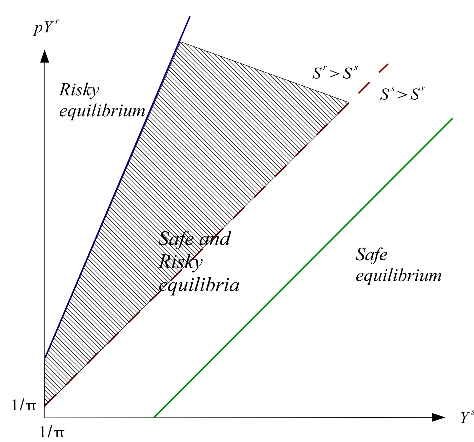
Every period the surplus S^s is shared between π good entrepreneurs who live on average $\frac{1}{1-\delta}$ periods. Similarly, in the risky equilibrium, the expected lifetime utility of a good entrepreneur with a clean record (10) is

$$V(\mu^r, c) = \frac{1}{1-\delta} \frac{S^r}{\pi}$$

As long as a good entrepreneur keeps her clean record, she gets a higher share of the period surplus than if she has a dirty record since she benefits from lower borrowing costs. However, over her lifetime an entrepreneur's expected average period payoff is $\frac{S^r}{\pi}$.

Thus, if the surplus created in the risky equilibrium is higher than in the safe one, the risky equilibrium also Pareto-dominates the safe one since the lifetime utility of a good entrepreneur with a clean record is higher: $V(\mu^r, c) > V(\mu^s, c)$. The following proposition establishes that the economy can be in a safe equilibrium that is dynamically inefficient:

Proposition 4 *For any $p, \delta, \pi \in (0,1)$, there are Y^s and Y^r such that there exists a safe equilibrium that is dynamically inefficient*



Proof. Since $R^s(\mu^s, c)$ does not depend on Y^s and Y^r (and is finite), one can pick $pY^r - Y^s$ and Y^r such that

$$\frac{\delta(1-\pi)}{\pi} < pY^r - Y^s \leq (1-p)\,\delta\frac{Y^s - R^s(\mu^s, c)}{1-\delta} - (1-p)\,R^s(\mu^s, c)$$

The weak inequality guarantees the existence of a safe equilibrium (8) and the strict inequality implies that $S^r > S^s$ (see (20)). If all entrepreneurs run risky projects and share the surplus between successful entrepreneurs (π good entrepreneurs successful with probability p), all good entrepreneurs are strictly better off since their expected lifetime utility is

$$\sum_{t=0}^{\infty}\delta^t p\frac{S^r\left((p\delta)^t\mu_0 + \left(1-(p\delta)^t\right)\mu^r\right)}{p\pi} \geq \frac{p\frac{S^r}{p\pi}}{1-\delta} > \frac{\frac{S^s}{\pi}}{1-\delta} = V(\mu^s, c)$$

for any $\mu_0 \in [\mu^r, \mu^s]$. The inequalities follow from (21). ■

4.3 Coordination Problem or Free-Riding?

The necessary condition for existence of a safe equilibrium (8) states that at μ^s no clean record unilaterally chooses the risky projects when everyone else runs safe projects. In some cases, this inefficient safe equilibrium is due to a coordination problem: if all good entrepreneurs with clean records decided to run risky projects forever no entrepreneur would want to continue running safe projects and the economy would converge to the risky equilibrium. This happens when, starting from the safe equilibrium, a one shot deviation from the risky to the safe project in the current period is not profitable given that all clean records run risky projects forever: $\Lambda^r(\mu^s) \leq 0$. Stated differently, the upper bound on the set of threshold equilibria is $\overline{\mu} = \mu^s$:

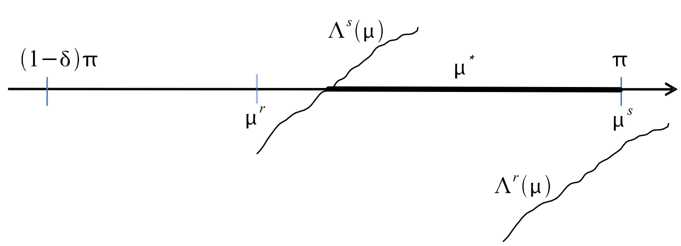


Figure 15: Coordination Problem

However, it can also be the case that solving the coordination problem is not enough to convince entrepreneurs to choose risky projects. In other words, the net benefit of choosing the safe project over the risky project in the current period is positive even if everyone else runs risky projects forever: $\Lambda^r(\mu^s) > 0$. This corresponds to

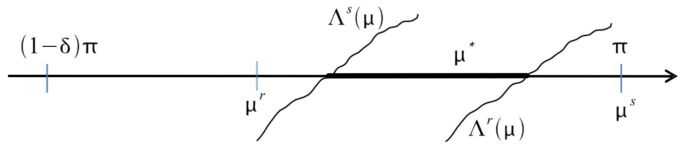


Figure 16: Free-Riding Problem

In this case there is no threshold equilibrium $\mu^* = \mu^s$. Thus, at μ^s, even if all entrepreneurs could coordinate on running risky projects forever, they would be better off deviating unilaterally and choosing the safe project instead. All entrepreneurs would be better off ex-ante if the economy transitions to μ^r but the high initial value of a clean record results in a free-riding problem: despite the fact that everyone would be better off running risky projects, nobody wants to incurs the risk of losing their clean records.

4.4 Bad Reputation

In an environment with limited liability, reputation concerns are useful to deter excessive risk-taking. However, the inefficient safe equilibrium described in Proposition 4 also results from the fact that entrepreneurs care about their reputations. Indeed, if the economy is in the safe equilibrium but the surplus would be higher in the risky equilibrium (condition (20) is satisfied), suppressing records would lead to a Pareto improvement.

In this case, reputation is bad: if investors could not observe entrepreneurs' records, this inefficient safe equilibrium would not arise. Without records, entrepreneurs always run risky projects. There would be a pooled equilibrium in which investors finance all entrepreneurs at cost $R^p = \frac{1}{p}\left(1 + \frac{1-\pi}{\pi}\right)$ and break even in expectation[7]. All good entrepreneurs would be better off since their expected lifetime utilities would be

$$V^p = \frac{p\left(Y^r - R^p\right)}{1-\delta} = \frac{\frac{S^r}{\pi}}{1-\delta} > \frac{\frac{S^s}{\pi}}{1-\delta} = \frac{Y^s - R\left(\mu^s, c\right)}{1-\delta} = V\left(\mu^s, c\right)$$

The policy of taxes and subsidies I describe in the next section is based on the idea of reducing the value of a clean record to zero to eliminate reputation concerns and induce entrepreneurs to take on risk.

4.5 Government Intervention

If the economy is stuck in an inefficient safe equilibrium the government can tax returns and subsidize investment in dirty records to induce a transition to the risky equilibrium.

[7] From assumption (A2), $Y^r > R^p$.

This intervention balances the budget and is a Pareto improvement, including during the transition from the safe to the risky equilibrium.

The social surplus being larger when entrepreneurs choose the risky project, resources are redistributed such that borrowing costs are the same for dirty and clean records. This implies that clean records have no value and entrepreneurs choose projects solely according to their period return. Although clean records subsidize dirty records they also take advantage of this policy when they fail, which arises with probability $1-p$ every period.

From the first period onwards successful entrepreneurs pay a tax

$$T(\mu_t) = \frac{\pi - \mu_{t+1}}{\pi} \frac{R(\mu_{t+1}, d) - R^r(\mu_{t+1}, c)}{p}$$

where $\mu_t = (p\delta)^t \mu^s + \left(1 - (p\delta)^t\right) \mu^r$. From the second period onwards the proceeds from the tax are used to finance the subsidy for dirty records:

$$F(\mu_t) = \frac{\pi - \mu_t}{\pi - \mu_t + \delta(1-\pi)} \left(R(\mu_{t+1}, d) - R^r(\mu_{t+1}, c)\right)$$

The subsidy is such that investors finance entrepreneurs with dirty records at the same cost $R^r(\mu_t, c)$ as clean records. Since a clean record has no value, this system of taxes and subsidies induces clean records to take on risk, such that the economy moves towards μ^r. Good entrepreneurs are better off and investors are not worse off. Moreover, this policy is self financed.

Proposition 5 *If the economy is in an inefficient safe equilibrium, there exists a Pareto improving and budget balanced policy that implements a transition to the risky equilibrium*

Proof. With the system of taxes and subsidies described above investors break even when they lend to dirty records at rate $R^r(\mu_t, c)$:

$$\Pr(G|\mu_t, d)\, pR^r(\mu_t, c) + F(\mu_t) - 1 = 0$$

In period t, π good entrepreneurs succeed with probability p and pay a tax $T(\mu_t)$. This finances subsidies to dirty records, $\pi - \mu_{t+1}$ good entrepreneurs and $\delta(1-\pi)$ bad entrepreneurs in period $t+1$:

$$\pi p T(\mu_t) = \left(\pi - \mu_{t+1} + \delta(1-\pi)\right) F\left(\mu_{t+1}\right)$$

Finally, the expected lifetime utility for a good entrepreneur is

$$\begin{aligned}
\sum_{t=0}^{\infty} \delta^t \left(p\left(Y^r - T(\mu_t) - R^r(\mu_t, c)\right)\right) &= \sum_{t=0}^{\infty} \delta^t \left(pY^r - \frac{1}{\pi} + \frac{(1-\delta)(1-\pi)\left(\mu_t - \mu_{t+1}\right)}{\mu_{t+1}\mu_t}\right) \\
&> \frac{pY^r - \frac{1}{\pi}}{1-\delta} = \frac{\frac{S^r}{\pi}}{1-\delta} > \frac{\frac{S^s}{\pi}}{1-\delta} = \frac{Y^s - R(\mu^s, c)}{1-\delta}
\end{aligned}$$

which implies that entrepreneurs with clean records are also better off during the transition.

■

5 Conclusion

The model proposed in this paper differs from the standard literature on reputation in two ways. First, entrepreneurs try to maintain a reputation for talent as opposed to a reputation for choosing a certain type of project. Second, investors have only access to coarse information (i.e. the record) about borrowers instead of observing the entire history of repayment. Several new insights on the role of reputation in risk-taking are developed in this framework.

Multiple equilibria arise in my model for a wide range of parameters, which offers an explanation for large variations in attitudes towards failure and entrepreneurial risk-taking reported among economies with similar fundamentals. Moreover, this model exhibits a dynamic complementarity in project choice: entrepreneurs are more likely to implement safe/risky projects if others have been running safe/risky projects in the past. This offers a potential explanation for the persistence of the difference in entrepreneurial climate between the US and Europe. It also sheds light on why a transition from a traditional (safe) to an innovative (risky) industry can be difficult even if technical knowledge and good institutions are present.

The main result of the paper is that the economy can be in an inefficient equilibrium in which entrepreneurs run safe projects because a failure results in exclusion from financial markets. I describe a budget-balanced policy that consists in taxing successful projects to subsidize investment in failed entrepreneurs, which leads to a Pareto improvement. By allowing failed entrepreneurs to start new projects, this policy removes hurdles that discourage risk-taking. The implication of this result is that economic policies that seek to improve entrepreneurs' probability of success by mitigating their fear of failure might be counterproductive. To remove the stigma of failure and provide incentives to take on risk, good entrepreneurs must sometimes fail.

More work is needed to understand the effects of coarse reputation in markets with adverse selection. An avenue for further research is to study how financial markets work when investors' information is somewhere between the two extremes that are clean/dirty records and the entire history of debt repayment.

6 Appendix

The following lemmas will be used to prove Proposition 8 below.

Lemma 6 *For a given μ^*, if $\mu > \hat{\mu}$, $\mu' > \hat{\mu}'$.*

Proof. Using (13),

$$\mu' - \hat{\mu}' = \left\{ \begin{array}{lll} \delta(\mu - \hat{\mu}) & \text{if} & \mu > \hat{\mu} \geq \mu^* \\ \delta(\mu - p\hat{\mu}) & \text{if} & \mu \geq \mu^* > \hat{\mu} \\ p\delta(\mu - \hat{\mu}) & \text{if} & \mu^* > \mu > \hat{\mu} \end{array} \right\} > 0$$

■

Lemma 7 $\Delta V(\mu_0)$ *strictly increases in* μ_0

Proof. If project choices are the same at all t for two sequences $\{\mu_t\}_{t=0}^{\infty}$ and $\{\hat{\mu}_t\}_{t=0}^{\infty}$ with $\mu_0 > \hat{\mu}_0$, the value of a clean record will be higher with $\{\mu_t\}_{t=0}^{\infty}$ than $\{\hat{\mu}_t\}_{t=0}^{\infty}$ since the expected revenues are the same and the cost differential $R(\mu_t, d) - R(\mu_t, c)$ is higher for $\{\mu_t\}_{t=0}^{\infty}$ by Lemma 6. If the optimal project choice is not identical for all t, define $\tilde{V}(\mu_0, c)$ as the expected lifetime utility that would be obtained by picking the sequence of projects that is optimal for $\{\hat{\mu}_t\}_{t=0}^{\infty}$. Then,

$$\begin{aligned} \Delta V(\mu_0) &= V(\mu_0, c) - V(\mu_0, d) \geq \tilde{V}(\mu_0, c) - V(\mu_0, d) \\ &> V(\hat{\mu}_0, c) - V(\hat{\mu}_0, d) = \Delta V(\hat{\mu}_0) \end{aligned}$$

The weak inequality follows from $V(\mu_0, c) \geq \tilde{V}(\mu_0, c)$ since the sequence of project chosen is not necessarily optimal under $\{\mu_t\}_{t=0}^{\infty}$. The strict inequality results from the cost differential being strictly higher with $\{\mu_t\}_{t=0}^{\infty}$ than with $\{\hat{\mu}_t\}_{t=0}^{\infty}$. ■

In order to establish the next proposition, I define Λ^{rs} as the net benefit of running a safe instead of a risky project in the current period, knowing that all good entrepreneurs with clean records run risky projects in the current period and safe ones forever after:

$$\Lambda^{rs}(\mu_0) \equiv -(pY^r - Y^s) - (1-p) R^r(\mu_0, c) + (1-p)\delta \sum_{t=0}^{\infty} \delta^t \left(Y^s - R^s(\mu_t, c) - p\left(Y^r - R(\mu_t, d)\right)\right)$$

where $\mu_t = \delta^t (p\delta\mu_0 + (1-\delta)\pi) + \left(1 - \delta^t\right)\mu^s$

Proposition 8 *If $\Lambda^s(\mu^r) > 0$ and $\Lambda^{rs}((1-\delta)\pi) < 0$, the set of equilibria is $\mu^* \in \left[\underline{\underline{\mu}}, \overline{\overline{\mu}}\right] \cup [\mu^r, \overline{\mu}]$ where $\underline{\underline{\mu}}, \overline{\overline{\mu}} \in [(1-\delta)\pi, \mu_R]$ such that*

$$\Lambda^{rs}\left(\overline{\overline{\mu}}\right)\left(\overline{\overline{\mu}} - \mu^r\right) = 0$$

and

$$\Lambda^s \left(\underline{\underline{\mu}}\right) \left(\underline{\underline{\mu}} - (1-\delta)\,\pi\right) = 0$$

and $\overline{\mu}$ *is defined as in Proposition 3.*

Proof. Hereafter I prove that $\mu^* \in \left[\underline{\underline{\mu}}, \overline{\overline{\mu}}\right]$ are threshold equilibria. (For $\mu^* \in [\mu^r, \overline{\mu}]$ see proof of Proposition 3)

For $\mu_0 < \mu^*$,

$$\begin{aligned} & -\left(pY^r - Y^s\right) - (1-p)\,R^r\left(\mu_0, c\right) + (1-p)\,\delta\Delta V\left(p\delta\mu_t + (1-\delta)\,\pi\right) \\ < \quad & -\left(pY^r - Y^s\right) - (1-p)\,R^r\left(\mu_0, c\right) + (1-p)\,\delta\Delta V\left(p\delta\mu^* + (1-\delta)\,\pi\right) \\ = \quad & \Lambda^{rs}\left(\mu^*\right) \leq \Lambda^{rs}\left(\overline{\overline{\mu}}\right) \leq 0 \end{aligned}$$

where $\Delta V\left(\mu\right)$ is the value of a clean record given that project choices are optimal and other entrepreneurs run risky project below μ^* and safe ones for $\mu \geq \mu^*$. The first inequality results from Lemma 7. The equality follows from $p\delta\mu^* + (1-\delta)\,\pi \geq \mu^*$ for $\mu^* \leq \mu^r$, so that it is indeed optimal to run safe projects after the current period. The other inequalities follow from Λ^{rs} increasing and $\Lambda^{rs}\left(\overline{\overline{\mu}}\right) < 0$ if $\overline{\overline{\mu}} = \mu^r$ and $\Lambda^{rs}\left(\overline{\overline{\mu}}\right) = 0$ otherwise.

For $\mu_t > \mu^*$

$$\Lambda^s\left(\mu_t\right) \geq \Lambda^s\left(\mu^*\right) \geq \Lambda^s\left(\underline{\underline{\mu}}\right) \geq 0$$

The first two inequalities follow from Λ^s increasing. The last one comes from $\Lambda^s\left(\underline{\underline{\mu}}\right) > 0$ if $\underline{\underline{\mu}} = (1-\delta)\,\pi$ and $\Lambda^s\left(\underline{\underline{\mu}}\right) = 0$ otherwise. ∎

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