How Much Would You Pay to Resolve Long Run Risk?

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Bansal (2007): *Several key features of asset markets are puzzling. Among others, these include the level of equity premium, asset price volatility, and the large cross-sectional differences in average returns across equity portfolios such as value and growth portfolios. In bond and foreign exchange markets, the violations of the expectations hypothesis and the ensuing return predictability is quantitatively difficult to explain. What risks and investor concerns can provide a unified explanation for these asset market facts? One potential explanation of all these anomalies is provided by the* **long-run risks (LRR) model**

Bansal and Yaron (2004) - **1100+ cites on Google Scholar

A sample of the LRR literature

Piazzesi and Schneider (2007): bond risk premia & term structure

Hansen, Heaton and Li (2008): evidence for LR components in consumption growth

Colacito and Croce (2011): int'l risk sharing & exchange rate volatility

Chen (JF, forthcoming): credit spread and leverage puzzles in corporate sector

Collin-Dufresne, Johannes and Lochster (2012): parameter learning

Beeler and Campbell (2012): critique

How do we judge success?

The puzzles are *quantitative*. Calibration is key

Mehra-Prescott lesson: Need not only to match moments, but to do so with parameter values that make sense, in terms of fitting other 'evidence' - both market-based and introspection

Attention is paid to elasticity of intertemporal substitution (EIS) and to the degree of relative risk aversion (RRA)

But *there is more to preference than EIS and RRA*

Utility implies that the temporal resolution of risk matters and a quantitative assessment of how much it matters should be part of the calibration process

OUTLINE OF TALK

1. Preferences: Epstein-Zin utility and the timing of resolution

2. Endowment: Long-run risks (simple form)

3. Thought experiment & model's quantitative implications

4. Discussion & perspective

Rules of the Game and Lexicon

Treat the representative agent as real
- there is no sensible aggregation theorem rationalizing a fictional agent

'Introspection' = intuitively plausible. You can imagine yourself and/or others behaving in this way

Example re risk aversion: How much would you pay to avoid the fair gamble $75,000 \pm \epsilon$? If $RRA = 10$, then

ϵ	250	2500	25,000	40,000
pay	4	410	21,000!!	38,000!!!

Epstein-Zin ('CES') Utility

- Continuation utilities $\{U_t\}$ defined at each node in an infinite horizon multistage lottery/tree

- Utilties are defined recursively by

$$\begin{cases} U_t = \left\{(1-\beta)\, c_t^{\rho} + \beta\left[E_t(U_{t+1}^{\alpha})\right]^{\rho/\alpha}\right\}^{1/\rho} & \rho \neq 0 \\ \log U_t = (1-\beta)\log c_t + \beta \log\left[\left[E_t(U_{t+1}^{\alpha})\right]^{1/\alpha}\right] & \rho = 0 \end{cases}$$

  The expected additive utility model is the special case $\alpha = \rho$

- Each U_t is linearly homogeneous; representation by constant perpetuities

Disciplining parameters?

$RRA = 1 - \alpha \neq 1 - \rho = EIS^{-1}$ "separation"
Attention is paid to magnitudes of RRA & EIS
Typically, take $RRA > EIS^{-1}$

But there is another aspect of preference introduced via separation ($\alpha \neq 0$) that is widely cited as important in itself for matching asset market data, but that has yet to be considered quantitatively

In other words, α *models more than risk aversion*

Temporal resolution *(Kreps-Porteus, 1978)*

- Compound lotteries cannot be reduced - the temporal resolution of consumption risk matters at the level of utility

- $c_2 = c_3 = ...$ either high or low depending on outcome of a coin toss

  May care whether the coin is tossed at $t = 0$ or at $t = 1$

- Early resolution preferred iff $RRA > EIS^{-1}$,

  which is the case in BY, where $RRA, \ EIS > 1$

- Increasing $1 - \alpha$ increases risk aversion AND changes preference for early resolution

Long Run Risk (special case)

- $\log c_{t+1} - \log c_t = m + x_t + W_{c,t+1}$

  $x_{t+1} = a x_t + W_{x,t+1}, \quad 0 < a < 1$

  W's indep innovations, $N\left(0, \sigma_c^2\right)$ and $N\left(0, \sigma_x^2\right)$, $\sigma_x << \sigma_c$

- $\frac{Var\left(\Sigma_1^T \log(c_t/c_{t-1})\right)}{T} \overset{T}{\nearrow}$: larger volatility at longer horizons (constant in T if $a = 0$)

- $E_t\left[\Sigma_{\tau=t}^{\infty} \beta^t \log\left(c_{\tau+1}/c_\tau\right)\right] = \frac{m}{1-\beta} + \frac{\beta}{1-\beta a} x_t$: shocks to x_t can have large effect on expected growth rate for LR even if σ_x is small

Why this consumption (endowment) process?

1. Arguably difficult to reject empirically

2. C-CAPM discount factor or MRS ($\alpha = \rho$): $\pi_{t,t+1} = \beta(c_{t+1}/c_t)^{\rho-1}$
   But consumption growth rates are 'too smooth' to match risk premia

3. Current shocks have implications also for LR growth And when $\alpha \neq \rho = 0$, the discount factor is

$$\pi_{t,t+1} = \beta \left[\frac{c_{t+1}}{c_t}\right]^{\rho-1} \left[\frac{U_{t+1}}{(E_t U_{t+1}^{\alpha})^{1/\alpha}}\right]^{\alpha-\rho}$$

   Continuation utility term embodies concern with LR consumption growth. Small shocks today can have a big effect because they influence the entire future

4. Key is that LR *risks are not resolved until much later and are consequently treated differently than are current risks.* But is the differential treatment required to match asset returns data plausible? This is a quantitative question

Not aware of any *other* market evidence or of any evidence from psychology that helps with quantitative assessment. Therefore, we suggest thought experiment & introspection as a guide (treating representative agent as real)

Thought experiments re risk aversion: Kandel and Stambaugh (1990, 1991), Rabin (2000): "how much would you pay to avoid this hypothetical gamble?"

Here: "what fraction of your consumption stream would you give up in order for all risk to resolve next period (month)?"

Thought experiment

You are given the LRR consumption process. In particular, the riskiness of consumption resolves only gradually over time (c_t and x_t are realized only at time t). You are offered the option of having all risk resolved at time 1. The cost is that you would have to relinquish the fraction π of both current consumption and of the consumption that is realized subsequently for every later period. What is the maximum value π^* for which you would be willing to accept this offer? Call π^* the *timing premium*

$$((1-\pi^*)\,c, \text{early}) \;\sim\; (c,\ \text{late})$$

'Compensating variation' of the change to early resolution of the given risk

SOLUTION: Closed form iff $EIS = 1$. Numerical otherwise

Utility U_0: Solve backwards

Compare with U_0^*, the utility for the case where all risk is resolved at time 1 (in one month). Then

$$\left(U_1^*\left(c_1, c_2, ...\right)\right)^{1/\rho} = (1-\beta)\left[c_1^\rho + \beta c_2^\rho + \beta^2 c_3^\rho + ...\right]$$

Monte Carlo simulation to compute sample realizations of U_1^*, then compute

$$U_0^* = \left\{(1-\beta)\, c_0^\rho + \beta\left[E_0([U_1^*]^\alpha)\right]^{\rho/\alpha}\right\}^{1/\rho}$$

$EIS = 1 \Longrightarrow$

$$\log(1-\pi^*) = \log\frac{U_0}{U_0^*}$$
$$= \tfrac{1}{2}\alpha[\sigma_c^2 + \tfrac{1}{(1-\beta a)^2}\sigma_x^2] \cdot \tfrac{\beta^2}{1-\beta^2}$$

Timing premium is positive if $\alpha < 0$, and then increasing in $1-\alpha$, β, σ_c, σ_x and a

Also increasing in EIS; BY take $EIS = 1.5 > 1$

Parameter Values and Premia

	BY (2004)	BY'	Hansen (07)
σ_c	.0075	.0075	.0054
σ_x	.0003	.0003	.0005
a	.9790	.9790	.9800
β	.998	.998	.998
RRA	10 or 7.5	10 (7.5)	2
EIS	1.5	1	1
Timing premium π^*	24.9% (19.6%)	22.4% (16.7%)	6.6%
Risk premium $\overline{\pi}$	46.1% (37.1%)	40.7% (31.7%)	13.5%

DISCUSSION

- Stochastic volatility as in Bansal, Kiku and Yaron (2012)?
  - work in progress

- Why pay a premium?

- How related to total welfare cost of risk (Lucas, 1987)?

- What about a different endowment? more general risk preferences?

- Is it preference for early resolution or aversion to persistence that is key?

Why pay a premium?

At issue is consumption risk, not income or return risk

Anxiety? But arguably would lead to violation of dynamic consistency and thus is not a valid interpretation of recursive utility
(Grant-Kajii-Polak (2000), Caplin-Leahy (2001), Epstein (2008))

Hidden unmodeled planning problem (Kreps & Porteus, 1978): How to make this concrete/quantitative? Can it make plausible a timing premium of 20%?

How related to total cost of risk?

Are the timing premia large only because the cost of risk is large?
Or are they large also in relative terms?

Lucas-style calculation: Consider the deterministic consumption process $\overline{c} = (\overline{c}_t)$ where $\overline{c}_0 = c_0$ and $\overline{c}_{t+1}/\overline{c}_t = E_0\left(c_{t+1}/c_t\right)$:

$$\log(\overline{c}_{t+1}/\overline{c}_t) = m + a^t x_0 + \frac{1}{2}\left[\sigma_c^2 + (\Sigma_{i=0}^{t-1} a^{2i})\sigma_x^2\right]$$

Compute $\overline{U}_0$

Take $EIS = 1$

$\overline{U}_0 > U_0^* > U_0$ if $\alpha < 0$

$\log\left(U_0/U_0^*\right) = \log(1-\pi^*)$, $\log\left(U_0/\overline{U}_0\right) = \log(1-\overline{\pi})$

RESULT: Independently of how one calibrates LRR,

$$\frac{\pi^*}{\overline{\pi}} \geq \tfrac{1}{2}\frac{RRA-1}{RRA}$$

Another criterion for judging RRA

Different Endowment Process?

Table 2: Timing Premia for IID Growth Rate ($\beta = .998$, $\sigma_C = .00007$)

$RRA \setminus EIS$	1.5	1	.2	.1
10	9.6%	7.9%	1.1%	0.0%
7.5	6.9%	5.6%	0.4%	−0.5%
5	4.3%	3.4%	0.0%	−0.8%
2	1.2%	0.8%	−0.9%	−1.1%
1	1.0%	0.0%	−1.0%	−1.2%

Premia smaller than for LRR process

Low values for EIS argued by Hall (1988), Campbell (2003)

Look for ‘patterns’

(RRA, EIS)	$(10, 1.5)$	$(2, 1.5)$	$(10, 0.2)$	$(2, 0.1)$
$RRA - EIS^{-1}$	28/3	4/3	5	−8
$RRA * EIS$	15	3	2	.2
π^*	9.6%	1.2%	1.1%	−1.1%

π^* is monotonic in $RRA * EIS$ but not in $RRA - EIS^{-1}$

Premium small for $(10, 0.2)$, a specification favored by Campbell

Preference for late resolution is small even when $RRA - EIS^{-1}$ is significantly different from zero

Add Risk Premia for IID Growth Rate ($\pi^*/\overline{\pi}$)

$RRA \setminus EIS$	1.5	1	.2	.1
10	9.6/20.2	7.9/16.1	1.1/4.7	0.0/2.5
7.5	6.9/15.5	5.6/12.2	0.4/3.5	−0.5/1.8
5	4.3/10.6	3.4/8.2	0.0/2.3	−0.8/1.2
2	1.2/4.5	0.8/3.2	−0.9/0.9	−1.1/0.5
1	1.0/3.1	0.0/1.4	−1.0/0.6	−1.2/0.3

More general risk preferences?

Define utility by: $U_t = \left\{(1-\beta)\, c_t^{\rho} + \beta\, [\mu(U_{t+1} \mid I_t)]^{\rho}\right\}^{1/\rho}$

$\mu(\cdot)$ models risk preferences

Above we took $\mu(U_{t+1} \mid I_t) = (E_t\,(U_{t+1})^{\alpha})^{1/\alpha}$

Some have adopted non-EU models for μ in order to accommodate: (i) Allais Paradox; (ii) known limitations of the expected utility functional form
E.g. Epstein-Zin (2001), Zin-Routledge (JF, 2010), Bonomo et al (RFS, 2011)

Roughly and under suitable assumptions, modeling a "given level of risk aversion" by non-EU vs EU preferences **increases** timing premia

Timing or persistence?

Persistence also seems to be relevant in the LRR model

What is the meaning of aversion to persistence? Take IID growth rates

Three scenarios: Fix c_0 and for every $t > 0$, $\log(c_t/c_{t-1})$ is either high or low depending on outcomes of coin toss(es)

A. toss coin at each t to determine $\log(c_t/c_{t-1})$

B. toss all coins at 1

C. one toss at 1 determines all

For homothetic additive expected utility:
toss at each $t \sim$ toss all at $1 \sim$ toss one coin at 1
[the second indifference is arguably counterintuitive]

For EZ utility (with $RRA > 1 = EIS$):
toss at each $t \prec$ toss all at 1 - *preference for early resolution* AND

one toss at $1 \prec$ toss all at 1 - *aversion to persistence*

These two properties are conceptually distinct, but they come together in EZ utility

Open question: We have *assumed* that timing is the crucial ingredient of preference in LRR? How can one be sure?

CONCLUDE

- *"Don't Bansal and Yaron show that the timing premium implicit in asset markets is large? Why bother with introspection?"*
  The same could be said about risk aversion in the equity premium context. Want a model that is consistent with a broad range of 'facts.' Our objective is to remind readers that this principle has not been applied as consistently as it should

- Not taking a stand on what levels of timing preference are plausible.
  Our objective is to inject the subject into the discussion of the quantitative properties of LRR & related models

- Disaster models (Reitz (1988), Barro (2006,9)) also use Epstein-Zin utility
  Timing premia to be computed

- Another competing model is the external habits model (Campbell-Cochrane, 1999). Plausibility of the specified habit process has been judged solely by how it matches asset market data

- It may be difficult to find other market evidence about the timing premium and about the habit formation process. But these should not become “free parameters”