A Staggered Pricing Approach to Modeling Speculative Storage: Implications For Commodity Price Dynamics

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Abstract

This paper modifies the speculative storage model by embedding staggered prices features into the structural model of Deaton and Laroque (1996). In an attempt to replicate the stylized facts of the observed commodity price dynamics, we add an additional source of intertemporal linkage to Deaton and Laroque (1996), namely intermediate good inventories speculators. The introduction of this type of friction into the model is motivated by its ability to increase price stickiness which gives rise to an increased persistence in the first and higher conditional moments of commodity prices. By incorporating intermediate risk neutral speculators and a final bundler with a staggered pricing rule in the spirit of Calvo (1983) into the storage model, we are able to capture the high degree of serial correlation, as well as skewness, kurtosis and conditional heteroskedasticity, observed in the actual data. The structural parameters of both Deaton and Laroque (1996) and our modified models are estimated by pseudo maximum likelihood using the prices for 21 commodities. Simulated data are then employed to assess the effects of our staggered price approach on the time series properties of commodity prices. Our results lend empirical support to the possibility of staggered prices.

1 Introduction

The last decade has witnessed a surge in commodity prices and a widespread financialization of commodity products. The upward movements and the increased volatility of the commodity prices has been largely attributed to strong demand by China and other emerging markets as well as massive capital flows into the commodity markets by institutional investors, portfolio managers and speculators. While the importance of commodity price movements for the economic policy and investors' sentiment has generated a substantial research interest, the behavior and the determination of commodity prices is not yet fully understood. The main objective of this paper is to develop a structural model of commodity price determination that reflects the empirical properties (high persistence and conditional heteroskedasticity) of commodity prices. In order to achieve this goal and to gain further understanding into the fundamental factors that drive the

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observed behavior of commodity prices, we modify the structure of the speculative storage model from one where prices adjust almost instantaneously to harvest shocks to a setup where they change slowly and infrequently. More specifically, we depart from the assumption that market prices are determined in a perfectly competitive environment and extend the basic speculative storage model by explicitly introducing intermediate goods speculators with a staggered pricing rule. One appealing aspect of this approach is its ability to mimic some important characteristics of the actual commodity prices such as high persistence, positive skewness, excess kurtosis and conditional heteroskedasticity, which can be generated even in the absence of correlated harvest shocks. Another advantage of our proposed approach is the possibility of conducting policy analysis by tracing the dynamic effects of a harvest shock on commodity prices over time.

The speculative storage model for commodity prices can be dated back to Gustafson (1958) who defines a set of optimal storage rules that state how much grain should be carried over into the next period given the current year supply. Moreover, by introducing intertemporal storage arbitrage and supply shocks, Gustafson (1958) incorporates rational expectations which is further elaborated in Muth (1961). Samuelson (1971) develops a model for commodities which determines the behavior of the prices as the solution to a stochastic dynamic programming problem. Furthermore, Beck (1993) builds upon the work by Muth (1961) and provides a theoretical basis for treating the variance as serially correlated when commodities are storable which suggests that commodity prices may exhibit conditional heteroscedasticity. The presence of storage is instrumental in ensuring that the price variance in one period directly affects inventory variance which in turn is transmitted to next period's price variation. Williams and Wright (1991) provide a comprehensive discussion of the basic storage model and its extensions, and summarize the time series properties of storable commodities. Williams and Wright (1991) put an emphasis on the complex non-linear storage behavior resulting from the fact that aggregate storage cannot be negative.

Deaton and Laroque (1992, 1995, 1996) develop a partial equilibrium structural model of commodity price determination and apply numerical methods to test and estimate the model parameters, confronting for the first time the storage model with the documented behavior of actual prices. Their analysis suggests that the introduction of speculative inventories and serially correlated supply shocks do not appear to generate sufficient persistence in commodity prices although they prove to be successful in replicating the significant volatility, skewness and kurtosis observed in the actual data.

More recently, many authors have tried to modify the storage model in order to accommodate the persistence of commodity prices. Chambers and Bailey (1996) relax the *iid* assumption on harvest shocks, and study the price fluctuations of storable commodities, assuming that shocks are either time dependent or that the model exhibits periodic disturbances. Ng and Ruge-Murcia (2000) incorporate additional features into the storage model in an attempt to generate the a higher degree of persistence in commodity prices. In particular, Ng and Ruge-Murcia (2000) allow for serially correlated shocks assuming that harvest follows a first-order moving average (MA(1)) process. They also examine the ability of production lags and heteroskedastic supply shocks, multi-period forward contracts and a convenience yield to generate an increased persistence in commodity prices. Cafiero, Bobenrieth, Bobenrieth, and Wright (2011) establish that the competitive storage model can give rise to high levels of serial correlation observed in commodity prices if it is estimated by using more precise numerical methods such as a finer grid of points. Moreover, estimates for seven commodities supported the specification of the speculative storage model with positive constant marginal costs and no deterioration, which is in line with Gustafson (1958).

Furthermore, Cafiero, Bobenrieth, Bobenrieth, and Wright (2011) use a maximum likelihood framework to estimate the storage model with stock-outs, which is extended to include unbounded harvests and free disposal. Their results produce more accurate small sample estimates of the structural parameters of the model compared to the previous studies based on the pseudo-maximum likelihood procedure. Miao and Funke (2011) add shocks to the trends of output and demand. Evans and Guthrie (2007) include transaction cost frictions into the speculative storage model. One important finding that emerges from their analysis is that these frictions tend to have explanatory power for the dynamic behavior of spot and futures commodity prices. In a competitive equilibrium framework, the model of Evans and Guthrie (2007) is able to capture the serial correlation and GARCH characteristics of commodity prices. Finally, Arseneau and Leduc (2012) embed the speculative storage model into a general equilibrium framework. Their main result is that the interaction between storage and interest rates in general equilibrium increases the impact of competitive storage on commodity prices and leads to higher persistence than the one observed in the storage model with fixed interest rate.

In spite of this extensive literature and numerous studies that attempt to understand and replicate the dynamic patterns of commodity prices, reproducing all their salient properties, such as high persistence, skewness, excess kurtosis and conditional heteroskedasticity, within a well-articulated structural model proved to be a challenging task. In this paper, we address issues regarding the commodity price dynamics in a unified fashion by embedding a staggered pricing mechanism into the speculative storage model. While Arseneau and Leduc (2012) also suggest to "introduce staggered price setting on the part of the final goods producing firm" in a general equilibrium framework as a possible extension for future research, our paper is the first to implement this approach and assess the properties of the model-generated commodity prices against the observed data.

Moreover, in an attempt to depart from the assumption of perfect competition at both the production and storage activity, Newbery (1984), Williams and Wright (1991), and McLaren (1999) investigate the effects of market power on the storage behavior. Our model differs from their work along the dimension that the final bundler does not store the good and the storage is only done by intermediate risk neutral speculators. The final bundler only bundles intermediate prices in order to set the final price. Finally, Mitraille and Thille (2009) examine the market power in production with competitive storage by analyzing the effects that competitive storage has on the behavior of a monopolist. Using his market power, the monopolist can influence speculative activity by manipulating prices and consequently affect the distribution of prices. One of the findings of Mitraille and Thille (2009) is that stockouts occur less frequently under monopoly.

The focus of this paper is on the ability of the storage model with staggered prices to account for the empirical features of commodity prices. The main impact of staggered prices in our model is to dampen the movements in prices as well as the market power of intermediate speculators to affect prices. This leads to gradual adjustments and persistent responses of prices following a harvest shock. In addition to generating sufficient persistence in commodity prices, the staggered pricing approach allows us to match other important moments in the unconditional and conditional distributions of the commodity prices.

Nominal price rigidity is widely used in the context of dynamic general equilibrium models. There are two widely used nominal price rigidity specifications in this literature. On one hand, the partial adjustment model developed by Calvo (1983), Rotemberg (1987), and Rotemberg (1996) allows for only a randomly chosen fraction of firms to adjust their prices according to some constant hazard rate in any given period. On the other hand, the staggered price setting rule adopted by Taylor (1980) and Blanchard and Fisher (1989) permits all firms to optimize their prices after a fixed number of periods N.

In this paper, we assume that the pricing decisions are staggered as in Calvo (1983) and use a similar modeling framework as the one developed in McCandless (2008). Our results confirm the importance of staggered prices for commodity price dynamics and suggest that the staggered pricing mechanism appears to be consistent with the behavior of the actual data. Moreover, we show how our model can be used to analyze the response of commodity prices to harvest shocks which provides a framework for economic and policy evaluation.

The remainder of the paper is organized as follows. The competitive storage model with staggered prices as well as the statistical characterizations of this model are presented in section 2. Section 3 studies the practical implications of our staggered prices speculative storage model using simulated data. Section 4 contains a brief description of the data and the estimation method used in the paper and discusses the main empirical results. Section 5 concludes.

2 Competitive Storage Model with Staggered Prices

The rational expectations model determines the optimal inventory decisions by risk- neutral speculators. The basic version of the model developed by Deaton and Laroque (1992, 1995, 1996)[1] incorporates competitive storage into the consumer demand and supply dynamics and establishes the concept of stationary rational expectations equilibrium (SREE). The model with serial correlation in harvest shocks is tested by Ng and Ruge-Murcia (2000). In their paper, Ng and Ruge-Murcia (2000) consider an MA(1) specification for the model harvest shocks. Our model complements and extends the original DL model by embedding staggered price setting into the speculative storage model. Regarding the harvest shock specification, we consider both (i) *iid* harvest shocks and (ii) MA(1) harvests shocks.

Our modified model has three types of commodity market participants: final consumers, intermediate risk neutral speculators and the bundler[2] who bundles the commodities in order to set the final price. In the absence of storage, the behavior of final consumers is characterized by a linear inverse demand function:

$$p_t = P(z_t) = a + bz_t,$$

[1] For brevity, we denote the basic speculative storage model of Deaton and Laroque by DL.

[2] In the literature, it is common to use the term "monopolist" instead of the term "bundler" that we use in this paper. The reason that we prefer the latter is the following: in the staggered pricing literature, the final good producer maximizes his profit by setting the price. In this paper, we do not consider any profit maximization and any type of price setting for the final good producer. Instead, we use the final good prices set as in (2.6).

where a and $b < 0$ are parameters to be estimated and z_t denotes the harvest in period t.

Let the harvest z_t be given by:

$$z_t = \bar{z} + u_t,$$

where $\bar{z}$ is constant (perfectly inelastic) and u_t is a random disturbance term which is assumed either to be *iid* or to follow an MA(1) process:

$$u_t = e_t + \rho e_{t-1},$$

where e_t is $iid(0, \sigma^2)$. If $\rho = 0$, we have the case of *iid* shocks as in DL, and when $\rho > 0$, we have MA(1) shocks as in Ng and Ruge-Murcia (2000). In this paper we investigate both cases and show that when we add staggered prices, the case for $\rho = 0$ gives better results compared to the case of non-staggered prices and $\rho > 0$.

Intermediate risk neutral speculators or inventory holders know the current year harvest and demand the commodity to transfer to the next period. They will do so whenever they expect to make a profit above the storage and interest cost. The depreciation rate of storage is denoted by δ. A simple form of proportional deterioration is considered which means that if in period t the speculators store I units of the commodity, they have at their disposal $(1 - \delta)I$ units at the beginning of the next period. Moreover, speculators have to pay a real interest rate on the value of their storage. Let r represents the constant exogenous real interest rate. The sum of harvest and inherited inventories is referred to as the amount on hand, x_t, which is given by

$$x_t = (1 - \delta)I_{t-1} + z_t.$$

The relationship between the amount of storage and its net profit can be summarized by:

$$\begin{cases} I_t > 0 \text{ if } (1-\delta)/(1+r)\mathbb{E}_t[p_{t+1}] = p_t, \\ I_t = 0 \text{ otherwise,} \end{cases}$$

where $\mathbb{E}_t$ denotes the expectation given the information at time t.

The condition for non-negative inventories is the crucial source of non-linearity in the model. This specification does not allow the market participants to borrow commodities that have not yet been grown. In addition, intermediate speculators benefit from a market power that reflects their ability to affect the price. In this framework, we assume that there is a continuum of intermediate speculators (of unit mass indexed by $k \in [0, 1]$) and final big players in the market. Final players will collect all the commodities from intermediate speculators and bundle intermediate speculators' prices into the final price in order to sell the commodity to consumers. Since we only have few big players in the market, they are best described by an oligopolistic environment. In reality, the price level of many commodities is influenced either through the formation of cartels by producers or through government intervention by imposing export control agreements or keeping strategic stock reserves. Although some of those cartels brake up in the long run, as discussed in Gilbert (1987), all of them have a strong influence on commodity prices, at least in the short-run. Hence, the introduction of these final big players who bundle prices will generate persistence in commodity prices over consecutive periods.

For simplicity, we assume that there exists a bundler who bundles all intermediate speculators' prices into a single one. Each period t, a fraction $1-\gamma$ $(0 < 1-\gamma < 1)$ of the speculators are able to exploit their market power and get to reset the prices of their commodities $P_t^*(k)$. The rest, who did not benefit from their market power to affect prices, keep their prices at the same level as the last period: $P_t^*(k) = P_{t-1}^*(k)$. Given this staggered pricing rule, along with the assumptions that speculators are risk neutral and have rational expectations, intermediate speculators' current and expected future prices must satisfy:

$$P_t^*(k) = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t[P_{t+1}^*(k)] + \gamma P_t^*(k)\right\}. \tag{2.1}$$

The first term in the brackets represents the price if the harvest is sold to consumers in period t and no inventories are carried over to the next period. The second term is known as the intertemporal Euler equation. This is the value of one unit stored if $1-\gamma$ of the speculators benefit form their market power to affect the price. This, in turn, happens if the speculators expect to cover their costs (after depreciation) from buying the commodity at time t. Since the current period bundler prices are not yet determined, it is important to stress that speculators, who do not reset their prices, use their own current prices and not the market ones in order to determine $P_t^*(k)$ in (2.1).

Finally, the bundler will bundle all intermediate prices together according to the following pricing rule (see McCandless (2008)):

$$P_t^{1-\psi} = \gamma P_{t-1}^{1-\psi} + (1-\gamma)P_t^*(k)^{1-\psi},$$

where P_t denotes the bundler final price of the good. The parameter ψ denotes the gross markup of the intermediate goods speculators and $P_t^*(k)$ represents the price for intermediate goods speculators who can set their prices. Since all intermediate goods speculators who can fix their prices are assumed to have the same markup over the same marginal costs, $P_t^*(k)$ is the same for all intermediate risk neutral speculators who adjust their prices. Prices for intermediate speculators who cannot set their prices are the same as the previous period prices denoted by P_{t-1}.

In order to simplify the bundler's pricing rule, we use the log-linearized version of this equation so that the final price becomes:

$$\tilde{P}_t = \gamma\tilde{P}_{t-1} + (1-\gamma)\tilde{P}_t^*(k), \tag{2.2}$$

where $\tilde{P}_t$ and $\tilde{P}_t^*$ denote the logarithm of P_t and P_t^*, respectively.

After completing the description of our model, we elaborate on some important implications of equation (2.1). As implied by this equation, the intermediate risk neutral speculators' price follows a non-linear first-order Markov process with a kink at the price above which we do not have inventories. In the case of *iid* shocks, the kink is determined by

$$\hat{p} = (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}p(z) + \gamma\hat{p}.$$

This implies that

$$\hat{p} = \frac{1-\delta}{1+r}\mathbb{E}p(z) \tag{2.3}$$

which coincides with the kink given in DL.

However, as in Chambers and Bailey (1996), the price kink $\hat{p}$ in the case of correlated harvests shocks is no longer constant and varies with the current harvest. This is due to the fact that with serially correlated harvest shocks, speculators form their price forecasts using all the information contained in the current shock.

Under some regularity conditions, most notably $r + \delta > 0$ and that z has a compact support, DL establish the existence of a solution to (2.1) when $\gamma = 0$ and shocks are independent. Indeed, to show the existence of the demand function for non-independent shocks, it is enough to prove the independent case conditioning on time t. In our case, we proceed by following a similar approach to proving that such an equilibrium exists. Assume that the demand x_t always lies in a subset $\mathbb{X} = [\underline{z}, +\infty)$ of the real numbers and that the harvest shock z_t belongs to a compact set $\mathbb{Z} = [\underline{z}, \bar{z}]$.

Definition 2.1 *Assume that $\gamma \in [0, 1)$. A staggered stationary rational expectation equilibrium (SSREE) is a price function $f : \mathbb{X} \times \mathbb{Z} \to \mathbb{R}$ which satisfies the following equation:*

$$p_t = f(x_t, z_t) = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t f(z_{t+1} + (1-\delta)I_t, z_{t+1}) + \gamma f(x_t, z_t)\right\}$$

where

$$I_t = x_t - p^{-1}(p_t) = x_t - p^{-1}(f(x_t, z_t)). \tag{2.4}$$

This defines the price function:

$$P_t^*(k) = f(x_t, z_t),$$

where $f(x_t, z_t)$ is the unique, monotone decreasing in its first argument, solution to the functional equation. Since this price function is non-linear, numerical techniques similar to the ones adopted by DL and Michaelides and Ng (2000) are used to solve for $f(x_t, z_t)$:

$$f(x_t, z_t) = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t f((z_{t+1} + (1-\delta)I_t), z_{t+1}) + \gamma f(x_t, z_t)\right\}.$$

In the case when the shocks are independent, we can remove the time subscript and the shocks in f.

When $\gamma = 0$ and shocks are *iid*, we have the same model as considered by DL. Hence, the equilibrium is simply called SREE. In the following theorem we show that the staggered stationary rational expectation equilibrium (SSREE) coincide with the stationary rational expectation equilibrium (SREE) derived from the basic DL speculative storage model.

Theorem 2.1 *If shocks are iid, then SSREE=SREE.*

Proof See Appendix A for a proof. ■

Remark 2.1 *Theorem 2.1 shows that $p_t = P_t^*$. This allows us to use all of the results for the process p_t, that are available in the literature, for the process P_t^*.*

We next show that the final demand for the bundler in our staggered speculative model is different form the one in DL. It proves useful to compare the price processes in the speculative storage model with and without staggered prices for the market participants who can reset their prices. In the basic speculative storage model of DL, the market participants cannot hold negative inventories. If prices are expected to increase or decrease by less than the cost of carrying the commodity from one period to another, inventories are zero. If inventories are positive, the expected price next period is equal to the current price plus the storage costs. The final price of the commodity in the basic speculative storage model satisfies

$$p_t = \max\left\{p(x_t), \frac{1-\delta}{1+r}\mathbb{E}_t p_{t+1}\right\}.$$

Hence,

$$\begin{cases} p_t = \frac{1-\delta}{1+r}\mathbb{E}_t p_{t+1} & \text{if } I_t > 0; \\ p_t = p(x_t) & \text{if } I_t = 0. \end{cases}$$

However, as stated in the description of our speculative storage model with staggered prices, the intermediate risk neutral speculators price function satisfies

$$P_t^* = \max\left\{p(x_t), (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t P_{t+1}^* + \gamma P_t^*\right\}$$

In this case,

$$\begin{cases} P_t^* = (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}_t P_{t+1}^* + \gamma P_t^*, & \text{if } I_t > 0; \\ P_t^* = p(x_t) & \text{if } I_t = 0. \end{cases} \tag{2.5}$$

It can be easily seen from (2.5) that prices for intermediate risk neutral speculators who can adjust them satisfy the same equation as the one that speculators face in the basic storage model of DL. This result can be illustrated by comparing Figures 1 and 2. Figure 1 presents the demand functions for the speculative storage model with correlated harvests shocks. Since the harvests shocks are discretized using 10 values as in DL, Figure 1 plots 10 demand functions with 10 different kinks. In contrast, Figure 2 plots the demand function for intermediate risk neutral speculators in the speculative storage model with staggered prices and *iid* harvest shock.

Since the final price process in the speculative storage model with staggered prices is given by

$$\tilde{P}_t = \gamma \tilde{P}_{t-1} + (1-\gamma)\tilde{P}_t^*(k), \tag{2.6}$$

one can infer that the demand of the bundler (the final demand) will be different from the demand presented by DL in the basic speculative storage model. We expect the final demand for speculative storage model with staggered prices to be in between the DL demand and the regular market demand. Moreover, we expect this demand to be more inelastic than the one derived from the basic speculative storage model. This is more consistent with the commodity elasticities estimated from actual data.

2.1 Statistical characterization

Under the assumption of *iid* harvests shocks, the final log-price process satisfies equation (2.6). The bundler price can then be written as

$$P_t = {P_{t-1}}^{\gamma}\ {P_t^*}^{1-\gamma}. \tag{2.7}$$

The persistence of commodity prices is then simply an outcome of the staggered prices which is extensively discussed in the literature on staggered pricing. Here, we provide and alternative explanation. From the logarithmic form of the relation (2.7), we have by induction that

$$\tilde{P}_{t+1} = (1-\gamma)\sum_{i=0}^{t} \gamma^i \tilde{P}^*_{t+1-i}$$

which in turn yields

$$P_{t+1} = \left(\Pi_{i=0}^{t} {P^*_{t+1-i}}^{\gamma^i}\right)^{1-\gamma}.$$

This shows that P_{t+1} shares many overlapping terms with P_t which gives rise to high persistence.

Next, we show that the final prices of the bundler exhibit conditional heteroskedasticity which is another salient characteristic of the observed commodity prices. Note that from (2.7), we have

$$\mathbb{E}_{t-1}(P_t^2) = P_{t-1}^{2\gamma}\mathbb{E}_{t-1}(P_t^{*2(1-\gamma)}) \tag{2.8}$$

and

$$(\mathbb{E}_{t-1}P_t)^2 = P_{t-1}^{2\gamma}(\mathbb{E}_{t-1}({P_t^*}^{1-\gamma}))^2. \tag{2.9}$$

Combining (2.8) and (2.9) and assuming that the shocks are *iid*, the conditional variance of final prices is given by

$$\mathrm{Var}_{t-1}(P_t) = P_{t-1}^{2\gamma}\left[\mathbb{E}\left(f(z+(1-\delta)I_{t-1})^{2(1-\gamma)}\right) - (\mathbb{E}(f(z+(1-\delta)I_{t-1})^{1-\gamma})^2\right]. \tag{2.10}$$

In the absence of inventories in the previous period, $I_{t-1} = 0$, the variance reduces to

$$\mathrm{Var}_{t-1}(P_t) = P_{t-1}^{2\gamma} var\left(f(z)^{1-\gamma}\right) \tag{2.11}$$

From (2.10) and (2.11), we can see that the variance is time-varying and, as a result, the final commodity prices derived from our model exhibit conditional heteroskedasticity. In addition, it is worth noting that the variance also depends on the value of γ.

The form of the conditional variance in (2.11) bears strong resemblance to the form of conditional heteroskedaticity in interest rate models. As in Brenner, Harjes, and Kroner (1996), the Euler-Maruyama discrete time approximation to the continuous time model of the spot interest rate is given by:

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + \epsilon_t$$

$$\mathrm{Var}_{t-1}(\epsilon_t) = \psi^2 r_{t-1}^{2\gamma}, \tag{2.12}$$

where ψ^2 denotes a scale factor and the parameter γ allows the volatility of interest rates to depend on the level of the process. Similarly to this model, higher values of the parameter γ in equation (2.11) indicate that the volatility of commodity prices is more sensitive to their past level which generates volatility clustering.

3 Model Comparisons Using Simulated Data

In this section we examine the statistical properties of the simulated data from our commodity price model with staggered pricing. In order to assess the qualitative and quantitative implications of our model, we compare it to the basic speculative storage model of DL and the modified version of the speculative model of Ng and Ruge-Murcia (2000). The model of Ng and Ruge-Murcia (2000) extends the DL model by adding serially correlated harvest shocks that follow an MA(1) process, as well as gestation lags, heteroskedastic supply shocks, multi-period forward contracts and convenience yields.

In our simulations, we calibrate the models using the parameter values estimated by Deaton and Laroque (1996) for a set of 12 commodities. These parameters (a, b, δ), presented in Table 1, are the same as the parameters used by Ng and Ruge-Murcia (2000). The data are simulated using *iid* harvest shocks or MA(1) harvest shocks with a MA parameter $\rho = 0.8$. We denote our speculative storage model with staggered prices by ADG.

Table 2 presents the results for the first-order autocorrelation of the simulated prices from the different models. The first column of Table 2 reports the autocorrelations from the actual data used in Deaton and Laroque (1996), the second column shows the results of the basic DL model ($\rho = 0$) and the third column contains the results obtained using DL model with MA(1) shocks ($\rho = 0.8$). The highest autocorrelation for the simulated prices from the DL model is for Maize (0.413 for the basic DL model and 0.644 for the specification with MA(1) harvest shocks). For all other commodities, the serial correlation in the simulated prices is well below the persistence in the actual prices.

The last two columns of Table 2 report the results for our model. For all commodities, the autocorrelation coefficients of the simulated prices based on the ADG model are much higher than those of the DL model specifications and are very close to the autocorrelations obtained from real data. Once we account for staggered pricing, the additional effect of serially correlated harvest shocks is minimal.

Furthermore, Table 3 lends additional support for our ADG model with staggered prices. In this table, we compare the autocorrelation coefficients for the model by Ng and Ruge-Murcia (2000) with gestation lags, overlapping contracts and convenience yields to those computed from our ADG model in column 4 and 5 of Table 2.

Ng and Ruge-Murcia (2000) add gestations lags to DL's basic model in an attempt to reduce the number of periods where the intertemporal price link between periods with and without production is severed. Consequently, this increases the serial correlation in prices. For this purpose, Ng and Ruge-Murcia (2000) assume that there are odd and even periods and that harvest takes place in the even periods. Hence, the random disturbance term of the harvest process has a variance that could differ if the period is odd (σ_1) or even (σ_2). The highest autocorrelations are reached for a value of $\frac{\sigma_2}{\sigma_1} = 1.8$. This model is denoted by GS. The results from the GS specification are reported in column 2 of Table 3.

Ng and Ruge-Murcia (2000) also show, in contrast to the earlier literature on storage where contracts are absent and stockholders are free to roll-over their inventories, that a model with overlapping contracts can partially explain the high serial correlation in prices. Column 3, denoted by OV in Table 3 reports the corresponding autocorrelation coefficients.

Finally, Ng and Ruge-Murcia (2000) add a convenience yield to the DL model. Since inventory holders might derive a convenience from holding inventories, Ng and Ruge-Murcia (2000) introduce both a speculative and a convenience motive for inventory holding. Hence, since the convenience yield partially compensate inventory holders for the expected loss when the basis is below carrying charges, their model with convenience yield generates a smaller number of stock-outs and, as a result, the demand for inventories for convenience purposes will strengthen the intertemporal link resulting in a higher persistence of prices. Results for $c = 50$ are reported in column 4 of Table 3. The model is denoted by CY.

Overall, the results in Table 3 suggest that the different specifications of Ng and Ruge-Murcia (2000) cannot generate autocorrelation coefficients greater than 0.640 and they are below the autocorrelation coefficients from our ADG model and the actual data across all commodities.

4 Empirical Application

4.1 Data

The data set employed in this empirical application is different from the one used by DL and Ng and Ruge-Murcia (2000). Our commodity prices are obtained from the Commodity Research Bureau. Prices are available at daily frequency for the period March 1983 – July 2008. The data set consists of prices for 21 commodities from 6 commodity groups: energy (heating oil), foodstuffs (cocoa, coffee, orange juice, sugar), grains and oilseeds (canola, corn, oats, soybeans, soybean oil, wheat), industrials (cotton, lumber), livestock and meats (cattle feeder, cattle live, hogs lean) and metals (copper, gold, palladium, platinum, silver). The trading characteristics of these commodities are summarized in Table 4. The choice of this particular set of commodities is dictated by data availability.

The spot price is approximated by the price of the nearest futures contract. Monthly commodity price series are constructed from daily data by averaging the daily prices in the corresponding month. The real commodity prices are obtained by deflating the nominal spot prices by the CPI (seasonally adjusted) index obtained from the Bureau of Labor Statistics (BLS). Each deflated price series is then further normalized by dividing by the sample average. By performing this additional normalization, we will have a historical mean of one which allow us to conduct easier comparisons of estimated parameters across various price series. Columns 4, 7 and 10 of Table 8 report the autocorrelation, skewness and kurtosis sample coefficients of the real prices for these 21 commodities.

Next, we repeat the experiment performed in the previous section and show that for our data set, ADG with $\rho = 0$ fits significantly better the actual data, not only in terms of persistence but also in terms of skewness and kurtosis. Table 6 compares the statistical properties of simulated price series obtained from two specifications of our ADG model. The first is ADG with serially correlated harvest shocks and no staggered prices ($\rho = 0.8$, $\gamma = 0$). This is the same as the DL model with correlated harvest shocks. The second one is ADG with no serial correlation in the harvest shocks and staggered prices ($\rho = 0$, $\gamma = 0.8$). In order to simulate those prices, we have chosen a set of parameters (presented in Table 5) which is very close to what we estimate from actual monthly data (see the next section). The entries in Table 6 represent averages from 100 samples of simulated price series of length 310 observations. By comparing the results in Table 6 to those from real data (columns 4, 7 and 10 of Table 8), one can easily see that the ADG model performs very well in matching the autocorrelation, skewness and excess kurtosis of the observed commodity prices.

To visualize the differences between the two models, Figure 3 plots simulated prices generated by our ADG model with $i.i.d$ harvest shocks and by the speculative storage model with correlated harvests shocks, using the same set of parameters described in Table 5. It is clear from the graph that our staggered price model generates more persistent data with some volatility clustering, i.e., conditional heteroskedasticity. Finally, we trace the dynamic responses of the simulated commodity prices following a negative harvest shock which are plotted in Figure 5. The gradual adjustment of the commodity prices from the ADG model stands in sharp contrast with the stronger but short-lived impact of the harvest shock on commodity prices in the DL model.

4.2 Estimation method

The estimation method adopted in this paper is pseudo maximum likelihood. This estimation method was also used by DL (1995, 1996) and Cafiero, Bobenrieth, Bobenrieth, and Wright (2011) although our implementation of the pseudo-likelihood estimation differs from theirs. The pseudo-likelihood function is maximized with respect to the vector of structural parameters $\theta = (a, b, \delta)'$ by assuming iid harvests shocks and fixing $\gamma = 0.75$ for the ADG model and $\gamma = 0$ for the DL model. Gourieroux, Monfort, and Trognon (1984) established the consistency and asymptotic normality of the pseudo-maximum likelihood estimates.

We next describe briefly the implementation of the pseudo-maximum likelihood estimation used in this paper. As shown in DL, in the case for $i.i.d$ harvest shocks, the price is only a function of the amount in hand. When the amount in hand is less than a critical level x^*, there is no storage and prices are equal to what they should be when nothing is stored. In this case,

$$p_{t+1} = f(z_{t+1})$$

and

$$E(p_{t+1}|p_t) = Ef(z), \text{ for } p_t \geq \hat{p}.$$

From (2.3), we have

$$Ef(z) = \frac{1+r}{1-\delta}\hat{p}.$$

Hence,

$$E(p_{t+1}|p_t) = \frac{1+r}{1-\delta}\hat{p}, \text{ for } p_t \geq \hat{p} \tag{4.1}$$

$$E(p_{t+1}|p_t) = \frac{1+r}{1-\delta}p_t, \text{ for } p_t \leq \hat{p}. \tag{4.2}$$

Combining equations (4.1) and (4.2) gives a linear equation for the conditional expectation of prices:

$$E(p_{t+1}|p_t) = \frac{1+r}{1-\delta}\min[p_t, \hat{p}]. \tag{4.3}$$

From (4.3), we can write

$$p_{t+1} = \frac{1+r}{1-\delta}\min[p_t, \hat{p}] + e_t, \tag{4.4}$$

where e_t is an error term. Assuming that $e_t = p_{t+1} - \frac{1+r}{1-\delta}\min[p_t, \hat{p}]$ is normally distributed with mean 0 and variance σ^2, the pseudo-likelihood function has the form

$$2\log L = \sum_{t=1}^{T-1} \log\, l_t = -(T-1)\log(2\pi\sigma^2) - \sum_{t=1}^{T-1}\left(\frac{{e_t}^2}{\sigma^2}\right). \tag{4.5}$$

Note that minimizing the pseudo-likelihood function in equation (4.5) is equivalent to minimizing the sum of squared errors

$$\sum_{t=1}^{T-1} {e_t}^2. \tag{4.6}$$

Therefore, the vector of parameters $\theta = (a, b, \delta)'$ can be estimated by least squares.

It is worth noting that the prices that we use in minimizing (4.6) represent the prices of intermediate risk neutral speculators, not the final prices that are given by the data set described in the paper. Hence, we first retrieve the prices of intermediate risk neutral speculators from the final prices given by the time series of commodity prices using the equation:

$$P_t^* = \left(\frac{P_t}{{P_{t-1}}^{\gamma}}\right)^{\frac{1}{1-\gamma}}. \tag{4.7}$$

Finally, the asymptotic variance-covariance matrix of the parameters is computed as

$$\hat{H}^{-1}\hat{M}\hat{H}^{-1} \tag{4.8}$$

where $\hat{H}$ denotes a consistent estimate of the Hessian $H = E\left[\frac{\partial^2 \log\, l_t(\theta_0)}{\partial\theta\partial\theta'}\right]$ and $\hat{M}$ is a consistent estimate of the outer product of the of scores $M = E\left[\frac{\partial \log l_t(\theta_0)}{\partial\theta}\frac{\partial \log l_t(\theta_0)}{\partial\theta}'\right]$. The matrices $\hat{H}$ and $\hat{M}$ are obtained by taking the numerical derivatives of the pseudo log-likelihood, $\log L$ and its components $\log l_t$, evaluated at the points estimates of the vector of parameters θ, denoted by $\hat{\theta}$ (see DL and Cafiero, Bobenrieth, Bobenrieth, and Wright (2011)).

4.3 Estimation results for DL and ADG models

Table 7, reports the parameter estimates of $\theta = (a, b, \delta)'$ for the DL and ADG models with *iid* harvest shocks. The standard errors of the estimated parameters are presented in parentheses. The parameters were estimated directly by minimizing (4.6). The prices used for estimation of

ADG model parameters are intermediate risk neutral speculators prices computed from real data as described in equation (4.7). The parameters estimates were obtained using MATLAB, where the function $f(x)$ is approximated using cubic splines and 100 grid points for x. This function was calculated using an iterative procedure, starting with an initial value $f_0(x) = \max[p(x_t), 0]$. As in DL, the interest rate r is not estimated in the model. Instead, it is fixed at 5 percent per annum or $r = (1.05)^{\frac{1}{12}} - 1$ per month. Moreover, harvest shocks z are discretized using a discrete approximation to standard normal, where z can take only one of the following 10 values: $(\pm 1.755, \pm 1.045, \pm 0.677, \pm 0.386, \pm 0.126)$. Each of these values can occur with equal probability of 0.1. For the ADG model the staggered price parameter γ, is set to be equal to 0.75, as it is common in the literature; see McCandless (2008). As described in the data section, the prices have been normalized to have a sample mean of 1. The parameter estimates for b for both models satisfy the constraint $b < 0$. Estimates for the depreciation rates δ are reasonable for both models. For most of the cases, standard errors of the estimated parameters a and b are smaller for the ADG model. In the case of δ, the standard errors for estimates of δ from the ADG model are smaller for eight commodities.

After estimating the parameters, we simulate price series for each commodity, using parameter estimates from Table 7, and compute the autocorrelation, skewness and kurtosis for each series. Similarly to our previous results, Table 8 shows that incorporating staggered prices into the speculative storage model does increase the first-order autocorrelation of the prices and makes it very close to the sample autocorrelation of the actual data. More specifically, the autocorrelations from our ADG model vary from 0.90 to 0.95 as opposed to the DL model where the highest possible autocorrelation is 0.65. Moreover, a comparison of the skewness and kurtosis coefficients generated by the DL and ADG models reveals the advantages of our ADG model in the matching the higher order moments of the data. By contrast, the DL model tends to generate data with skewness and kurtosis that exceed substantially their sample values obtained from real data.

5 Conclusion

The main objective of this paper is to propose a model which is able to reproduce the statistical characteristics of real commodity prices. Our modified speculative storage model embeds a staggered price feature into the DL storage model. The staggered pricing rule is incorporated by introducing intermediate good speculators and a final goods bundler. We examine the empirical relevance of the structural modification by comparing our model performance with several models in the literature, namely DL and the extended DL version of Ng and Ruge-Murcia (2000). Our analysis suggests that the proposed model outperforms the existing models along several dimensions such as matching the autocorrelation, skewness and kurtosis sample coefficients of the observed commodity prices. We also estimate the vector of structural parameters for the ADG and DL model with uncorrelated harvest shocks using a cross-section of 21 commodity prices at monthly frequency. Finally, we simulate price series of for each commodity using the parameter estimates and compare the statistical properties generated from both models to the ones from actual data. Our results lend strong support to the staggered pricing feature of the modified speculative storage model of commodity price determination.

A Appendix: Proof of Theorem 2.1

Proof of Theorem 2.1 First we state the assumptions for the theorem.

A.1 $r + \delta > 0$.

A.2 The harvest shocks z belongs to a compact set $\mathbb{Z} = [\underline{z}, \bar{z}]$.

A.3 The function $p^{-1} : (q_0, q_1) \to \mathbb{R}$ is continuous and strictly decreasing such that

$$\lim_{q \to q_0} p^{-1}(q) = +\infty.$$

Furthermore, we have that $\underline{z} \in p^{-1}(p_0, p_1)$ and $p(\underline{z}) \in \mathbb{R}_+ \setminus \{0\}$

Following Deaton and Laroque (1992), for any function g on the set $\mathbb{X} = [\underline{z}, +\infty)$ we introduce a function G on $\mathbb{Y} = \{(q,x)|x \in \mathbb{X}\,,\, p(x) \leq q < q_1\}$ which has the form

$$G(q,x) = (1-\gamma)\frac{1-\delta}{1+r}\mathbb{E}g(z + (1-\delta)(x - p^{-1}(q))) + \gamma q. \tag{A.1}$$

If $\gamma = 0$, then G is the same as in Deaton and Laroque (1992). Let G^{DL} denote the function when $\gamma = 0$:

$$G^{DL}(q,x) = \frac{1-\delta}{1+r}\mathbb{E}g(z + (1-\delta)(x - p^{-1}(q))).$$

It can be seen that $G = (1-\gamma)G^{DL} + \gamma p$.

Theorem 2.1 aims to find a function f such that

$$f(x) = \max\{G(f(x), x), p(x)\} \quad , \forall x \in \mathbb{X}, \tag{A.2}$$

where also $f = g$. To prove the theorem, we use the following lemma.

Lemma A.1 *For a given g, the unique solution $f : \mathbb{X} \to \mathbb{R}$ to* (A.2) *equals f^{DL}, where f^{DL} is the unique solution to the same problem when $\gamma = 0$.*

Proof For each x, $f(x)$ is the solution to the following equation for q

$$\max\{G(q,x) - q, p(x) - q\} = 0. \tag{A.3}$$

It can be seen that

$$G(q,x) - q = (1-\gamma)G^{DL}(q,x) + \gamma q - q = (1-\gamma)(G^{DL}(q,x) - q).$$

Thus, the solution q is a solution to

$$\max\{(1-\gamma)(G^{DL}(q,x) - q), p(x) - q\} = 0. \tag{A.4}$$

But this is equivalent to solving[3]

$$\max\{G^{DL}(q,x) - q, p(x) - q\} = 0, \tag{A.5}$$

which gives the desired result. ■

[3]For a positive number θ and two real numbers a, b, we have that $\max\{a,b\} = 0 \Leftrightarrow \max\{\theta a, b\} = 0$.

This lemma shows that for any g, there is a unique f which is the solution to (A.2). Therefore, we can introduce an operator $\mathbb{T}$ and denote f with $\mathbb{T}g$.

Proof of Theorem 2.1 From lemma above one can see that $\mathbb{T}$ is the same as the operator introduced in Deaton and Laroque (1992). It is shown in Deaton and Laroque (1992) that $\mathbb{T}$ is an operator from the set of non-increasing and continuous functions on $\mathbb{X}$ to itself and has a unique fixed point f, i.e., $f = \mathbb{T}f$. It then follows that this unique fixed point is the unique SSREE or SREE. This completes the proof of Theorem 2.1. ■

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Table 1: Parameter estimates from the DL (1996) model.

Commodity	a	b	δ
Cocoa	0.162	-0.221	0.116
Coffee	0.263	-0.158	0.139
Copper	0.545	-0.326	0.069
Cotton	0.642	-0.312	0.169
Jute	0.572	-0.356	0.096
Maize	0.635	-0.636	0.059
Palm oil	0.461	-0.429	0.058
Rice	0.598	-0.336	0.147
Sugar	0.643	-0.626	0.177
Tea	0.479	-0.211	0.123
Tin	0.256	-0.170	0.148
Wheat	0.723	-0.394	0.130

Table 2: Comparing autocorrelations for DL and ADG models based on 5000 observations.

Commodity	Actual	DL	DL	ADG	ADG
		$\rho = 0$	$\rho = 0.8$	$\rho = 0$	$\rho = 0.8$
		$\gamma = 0$	$\gamma = 0$	$\gamma = 0.8$	$\gamma = 0.8$
Cocoa	0.834	0.352	0.609	0.7715	0.8446
Coffee	0.804	0.219	0.576	0.7811	0.8501
Copper	0.838	0.335	0.619	0.8918	0.9074
Cotton	0.884	0.173	0.564	0.8626	0.9053
Jute	0.713	0.289	0.589	0.8817	0.9072
Maize	0.756	0.413	0.644	0.9246	0.9180
Palm oil	0.730	0.397	0.637	0.9079	0.9050
Rice	0.829	0.237	0.579	0.8700	0.9078
Sugar	0.621	0.266	0.583	0.8860	0.9184
Tea	0.778	0.213	0.571	0.8332	0.8893
Tin	0.895	0.238	0.567	0.7547	0.8462
Wheat	0.863	0.250	0.602	0.8834	0.9198

Table 3: Comparing autocorrelations for Ng and Ruge-Murcia and ADG models based on 5000 observations.

Commodity	Actual	GL	OV	CY	ADG	ADG
					$\rho = 0$	$\rho = 0.8$
					$\gamma = 0.8$	$\gamma = 0.8$
Cocoa	0.834	0.511	0.462	0.522	0.7715	0.8446
Coffee	0.804	0.433	0.385	0.530	0.7811	0.8501
Copper	0.838	0.526	0.394	0.608	0.8918	0.9074
Cotton	0.884	0.365	0.337	0.473	0.8626	0.9053
Jute	0.713	0.486	0.365	0.545	0.8817	0.9072
Maize	0.756	0.620	0.418	0.623	0.9246	0.9180
Palm oil	0.730	0.640	0.438	0.625	0.9079	0.9050
Rice	0.829	0.398	0.334	0.475	0.8700	0.9078
Sugar	0.621	0.427	0.370	0.424	0.8860	0.9184
Tea	0.778	0.428	0.302	0.509	0.8332	0.8893
Tin	0.895	0.428	0.355	0.472	0.7547	0.8462
Wheat	0.863	0.411	0.368	0.505	0.8834	0.9198

Table 4: Commodity price data.

Description	Exchange	Contract size	Contract month
Foodstuffs			
CC : Cocoa/Ivory Coast	NYBOT	10 metric tons	H,K,N,U,Z
KC : Coffee (C)/ Columbian	NYBOT	37,500 lbs.	H,K,N,U,Z
JO : Orange Juice, Frozen Concentr.	NYBOT	15,000 lbs.	F,H,K,N,U,X
SB : Sugar No.11/World raw	NYBOT	112,000 lbs.	H,K,N,V
Grains and Oilseeds			
WC : Canola/ No.1	WCE	20 tonnes	F,H,K,N,X
C : Corn/No.2 Yellow	CBOT	5,000 bu.	F,H,K,N,U,X,Z
O : Oats/N0.2 White heavy	CBOT	5,000 bu.	H,K,N,U,Z
S : Soybean/No.1 Yellow	CBOT	5,000 bu.	F,H,K,N,Q,U,X
BO : Soybean Oil/Crude	CBOT	60,000 lb.	F,H,K,N,Q,U,V,Z
W : Wheat/No.2 Soft red	CBOT	5,000 bu.	H,K,N,U,Z
Industrials			
CT : Cotton/1-1/16”	NYBOT	50,000 lbs	H,K,N,V,Z
LB : Lumber/Spruce-Pine Fir 2×4	CME	110,000 brd.feet	F,H,K,N,U,X
Livestock and Meats			
FC : Cattle, feeder/Average	CME	50,000 lbs.	F,H,J,K,Q,U,V,X
LC : Cattle, Live/Choice Average	CME	40,000 lbs.	G,J,M,Q,V,Z
LH : Hogs, Lean/Average Iowa/S M	CME	40,000 lbs.	G,J,M,N,Q,V,Z
Metals			
HG : Copper High Grade/Scrap N0.2	NYMEX	25,000 lbs.	H,K,N,U,Z
GC : Gold	NYMEX	100 troy ounces	G,J,M,Q,V,Z
PA : Palladium	NYMEX	100 troy ounces	H,M,U,Z
PL : Platinum	NYMEX	100 troy ounces	F,J,N,V
SI : Silver	NYMEX	5,000 troy ounces	H,K,N,U,Z
Energy			
HO : Heating Oil N0.2/Fuel Oil	NYMEX	42,000 gallons	F-Z

Notes: This table provides a brief description about each commodity. The first column presents the symbol description and the second one lists the futures exchange where the commodity is traded. In this table, CBOT refers to Chicago Board of Trade, CME: Chicago Mercantile Exchange, NYBOT: New York Board of Trade, NYMEX: New York Mercantile Exchange and WCE:Winnipeg Commodity Exchange. The third column states the contract size and the last column provides the contract months denoted by: F = January, G = February, H = March, J = April, K = May, M = June, N= July, Q = August, U = September, V = October, X = November and Z = December.

Table 5: True values of the parameters.

r	a	b	δ
0.004	2	-1.6	0.04

Notes: The annual interest rate is $r = 0.05$. The monthly interest rate is $r = (1 + 0.05)^{\frac{1}{12}} - 1 = 0.004$.

Table 6: Statistical properties of different model specifications.

	Autocorrelation	Skewness	Kurtosis
DL : $\rho = 0.8$, $\gamma = 0$	0.2070	0.0812	2.4542
ADG: $\rho = 0$, $\gamma = 0.8$	0.9015	0.5472	2.9825

Table 7: Parameters estimation for DL and ADG models using real data.

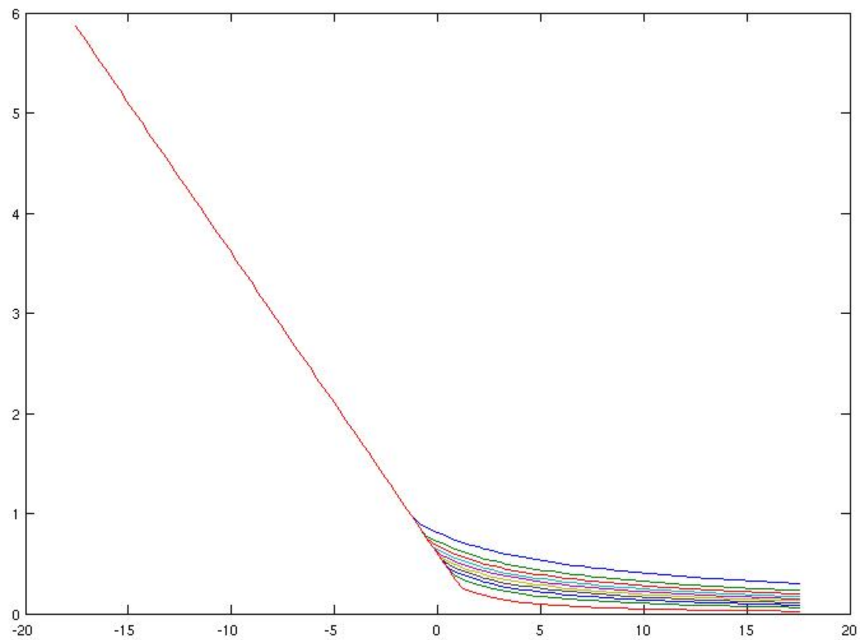
Commodity		ADG			DL	
		$\rho = 0$ $\gamma = 0.75$			$\rho = 0$ $\gamma = 0$	
	a	b	δ	a	b	δ
LH	0.9693 (0.00267485)	-0.9811 (0.00241258)	0.0334 (0.00012046)	0.9732 (0.00114623)	-0.9817 (0.00012098)	0.0024 (0.00001413)
HG	1.9301 (0.01873690)	-1.5638 (0.01530050)	0.0124 (0.00015553)	1.6598 (0.02033708)	-1.4031 (0.01740406)	0.0241 (0.00023405)
GC	0.9054 (0.00028589)	-0.9414 (0.00024988)	0.0278 (0.00001264)	1.0497 (0.01485146)	-1.0315 (0.01338691)	0.0197 (0.00024565)
PA	3.0295 (0.05267478)	-2.0509 (0.03988282)	0.0067 (0.00021581)	2.7953 (0.01823549)	-1.8787 (0.01369867)	0.0086 (0.00008094)
SI	1.4917 (0.02370854)	-1.3033 (0.02072760)	0.0080 (0.00000118)	1.9675 (0.02112379)	-1.5887 (0.01748892)	0.0171 (0.00018337)
HO	1.1648 (0.00306166)	-1.1014 (0.00280745)	0.0533 (0.00015461)	1.3195 (0.02653088)	-1.1967 (0.02384119)	0.0207 (0.00000177)
PL	1.0531 (0.00066787)	-1.0334 (0.00060818)	0.0328 (0.00002288)	1.1843 (0.03155363)	-1.1149 (0.02877205)	0.0210 (0.00026949)
LC	0.7786 (0.00023067)	-0.8627 (0.00016605)	0.0185 (0.00001040)	1.0015 (–)	-1.0010 (–)	0.0007 (–)
FC	0.7521 (0.00103824)	-0.8503 (0.00088733)	0.0365 (0.00008026)	1.0000 (–)	-1.0000 (–)	0.0100 (–)
LB	0.8680 (0.00102521)	-0.9186 (0.00094904)	0.1050 (0.00012762)	1.0205 (0.00007980)	-1.0137 (0.00000001)	0.0079 (0.00007253)
CT	0.9967 (0.00453644)	-0.9262 (0.00355042)	0.0171 (0.00015688)	1.0937 (0.00206247)	-1.0585 (0.00204623)	0.0020 (0.00000013)
W	0.8821 (0.00142986)	-0.9265 (0.00129811)	0.0481 (0.00011040)	1.0000 (0.01313210)	-1.0000 (0.01111692)	0.0100 (0.00013595)
BO	1.5471 (0.00540174)	-1.3355 (0.00459034)	0.0176 (0.00005635)	1.3589 (0.05319227)	-1.2198 (0.04721169)	0.0204 (0.00028297)
S	1.4117 (0.00007510)	-1.2528 (0.00000008)	0.0233 (0.00006970)	1.3539 (0.07537322)	-1.2187 (0.06698975)	0.0172 (0.00028485)
O	1.0103 (0.00036977)	-1.0148 (0.00000094)	0.0983 (0.00023317)	1.4373 (0.04881743)	-1.2710 (0.04259895)	0.0179 (0.00026130)
C	1.3226 (0.00367095)	-1.1985 (0.00336091)	0.0175 (0.00000024)	1.3237 (0.02017312)	-1.1986 (0.01756584)	0.0182 (0.00024423)
WC	1.6471 (0.00006383)	-1.2917 (0.00000006)	0.0210 (0.00005942)	1.4049 (0.08080268)	-1.2501 (0.07141127)	0.0203 (0.00029564)
SB	1.0341 (0.00279250)	-1.0217 (0.00258141)	0.0808 (0.00020549)	1.1540 (0.00931699)	-1.0952 (0.00798571)	0.0188 (0.00019093)
JO	1.3238 (0.00416203)	-1.1979 (0.00358495)	0.0215 (0.00009565)	1.3828 (0.03874838)	-1.2344 (0.03386892)	0.0172 (0.00026795)
CC	1.5095 (0.00287009)	-1.2471 (0.00000194)	0.0196 (0.00214677)	1.6089 (0.00013042)	-1.3673 (0.00007169)	0.0172 (0.00006582)
KC	1.6982 (0.00668678)	-1.3671 (0.00559456)	0.0442 (0.00011081)	1.8541 (0.03656370)	-1.5164 (0.03018406)	0.0159 (0.00023617)

Table 8: Autocorrelation, skewness and kurtosis sample coefficients for DL model, ADG model and actual prices.

	Autocorrelation			Skewness			Kurtosis		
Com.	DL	ADG	Actual	DL	ADG	Actual	DL	ADG	Actual
LH	0.7466	0.9463	0.9490	2.9123	1.0093	1.9148	18.0205	4.7111	7.6227
HG	0.5727	0.9528	0.7851	2.4964	0.8209	0.1366	11.4807	4.4793	2.5026
GC	0.5994	0.9526	0.9500	2.7145	0.7242	0.9853	13.2330	3.4368	3.2961
PA	0.6522	0.9339	0.9733	2.2615	0.1504	1.0003	11.8432	6.4273	3.8992
SI	0.6121	0.9574	0.9819	2.4196	0.9812	1.0834	12.0536	4.6086	3.7172
HO	0.6007	0.9310	0.9645	2.6510	0.8237	1.5128	13.2914	3.7523	5.6524
PL	0.6349	0.9472	0.9712	2.5958	0.8752	0.8487	12.8258	4.0017	3.2908
LC	0.7492	0.9608	0.9761	3.4853	1.3592	1.1516	24.2319	6.1742	3.7711
FC	0.6864	0.9452	0.9575	2.6711	1.0244	0.7974	15.0148	4.4451	3.2124
LB	0.6695	0.9081	0.9772	3.0403	0.7704	1.0459	17.1675	3.8517	3.0920
CT	0.7561	0.9551	0.9782	3.0019	0.9585	1.0575	20.6083	4.2793	3.7121
W	0.6805	0.9331	0.9786	2.8683	0.9516	1.2950	16.0840	4.3436	4.5457
BO	0.6060	0.9536	0.9589	2.6390	0.8399	3.1996	14.1652	3.8178	18.2842
S	0.6421	0.9476	0.9670	2.6478	0.9688	0.2641	13.7187	4.3921	2.6438
O	0.6573	0.9069	0.9899	2.6839	0.7634	2.5725	13.7029	3.5360	8.6288
C	0.5950	0.9519	0.9614	2.3832	0.9259	1.9882	11.3284	4.1751	8.4544
WC	0.5810	0.9439	0.9670	2.5615	0.7407	1.4421	12.6143	3.8446	4.8070
SB	0.6235	0.9136	0.9559	2.5485	0.7395	1.0304	12.9326	3.4713	3.8913
JO	0.6284	0.9518	0.9647	2.5630	0.7427	1.9227	13.1539	3.7723	8.9140
CC	0.6222	0.9482	0.9769	2.4819	0.9412	0.8473	12.5143	4.0651	2.8533
KC	0.6449	0.9312	0.9750	2.3543	0.6474	1.9410	11.8994	3.6723	7.8921

Notes: All simulations are based on a sample size of 5000 observations.

Figure 1: Demand functions for correlated shocks.



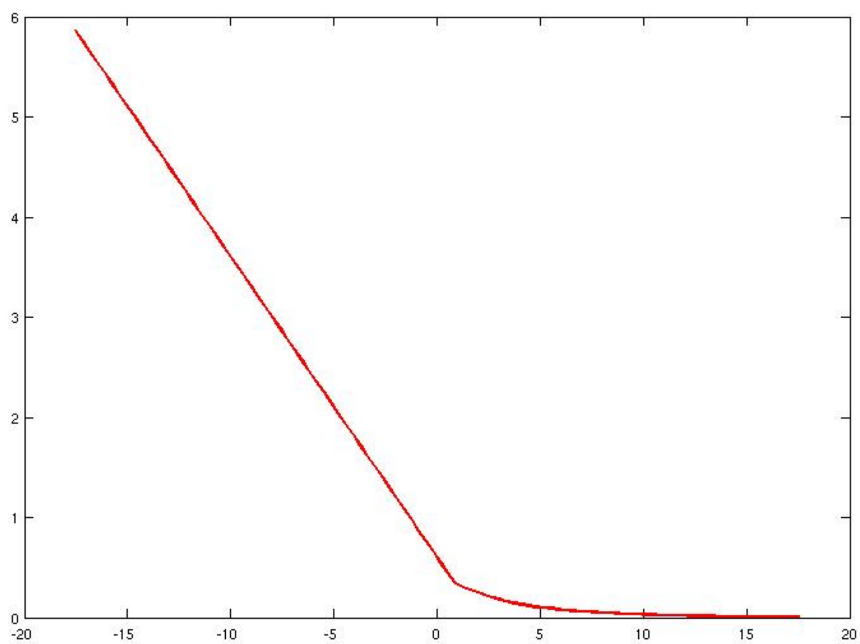
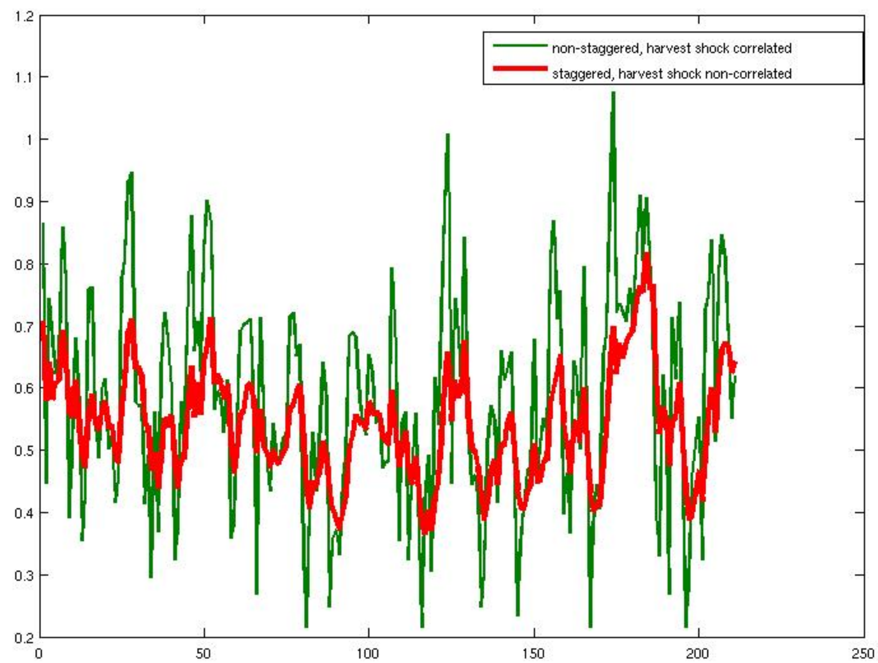


Figure 2: Demand functions for staggered prices.

Figure 3: Simulated data from models with staggered pricing and non-staggered pricing.



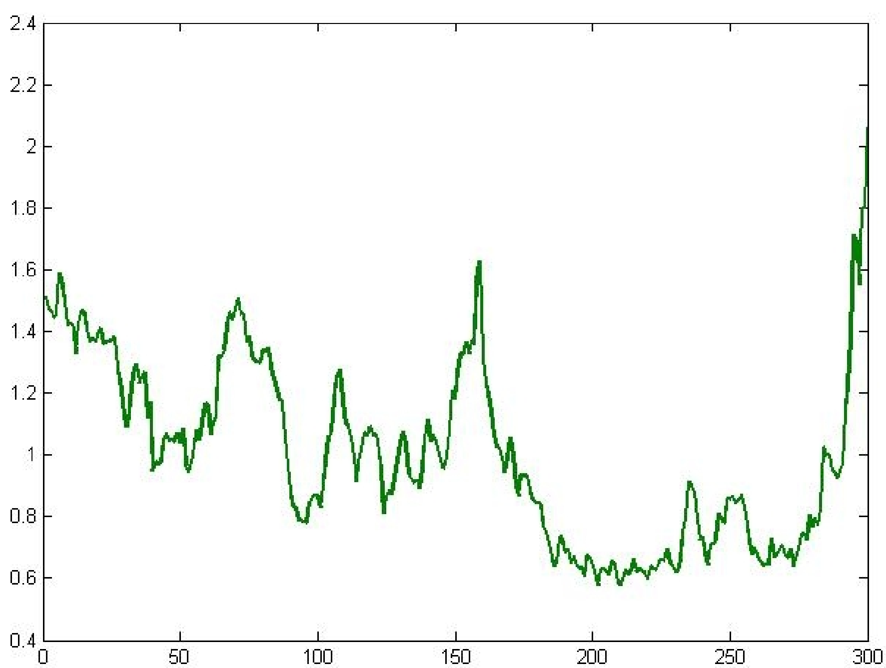


Figure 4: Actual prices for wheat.

Figure 5: Impulse response function based on simulated data.