

Time Series and Cross-Sectional Properties of Equity Market Liquidity with Applications to the Financial Crisis.

Jeffrey R. Russell¹

University of Chicago, Booth School of Business

April 30, 2013

Preliminary draft

Abstract: Financial market liquidity varies over time and is cross-sectionally correlated. Despite a growing literature suggesting that liquidity impacts asset prices and the importance of co-movement in liquidity to investors holding diversified portfolios, relatively little is understood about the economic sources of this co-movement. Most economic theory is built around risk factors faced by market makers trading individual assets. This paper proposes the idea that commonality or co-movement in liquidity comes from co-movement in the asset specific liquidity risk factors. We show that a factor structure in the risk factors implies a factor structure for liquidity. If the common risk factor is not directly observable (such as asymmetric information) we show that the factor structure implies that observable risk factors can be constructed by taking cross-sectional averages of the asset specific liquidity risk variables that are often used to proxy for the unobserved. Estimates of the factor models on a sample of S&P100 stocks allows us to identify which common risk factors are important in determining co-movement in liquidity. We find that inventory risk due to common volatility shocks, market wide asymmetric information and drying up of liquidity suppliers are responsible for a large part of the co-movement in liquidity during the financial crisis. Interestingly, counterparty risk appears to affect liquidity, but does not imply broad co-movement in liquidity.

¹ Xiao Qiao provided excellent research assistance. The author would like to thank Lubos Pastor for helpful discussion.

Section 1. Introduction

Financial market liquidity varies over time. This is true for individual assets and for aggregate measures implying co-movement (or commonality) in liquidity among different assets. For example, we find that bid ask spreads on S&P100 stocks increased by an average factor of 5 relative to their usual size during the peak of the financial crisis in the summer of 2008. These large swings and co-movement in liquidity are important to investors who hold portfolios since idiosyncratic variation in liquidity will tend to average and only the common component will be important in determining transaction costs when buying or selling diversified portfolios. Recent evidence suggests that liquidity affects asset prices so co-movement in liquidity has implications for co-movement in asset prices. Despite the magnitude of the variation in liquidity, very little is understood about the sources of this variation and even less is understood about the nature of the co-movement in liquidity across assets. This paper adds to the literature on co-movement in liquidity by proposing an econometric structure to model the cross-sectional and time series properties of liquidity.

With a few recent exceptions (for example Brunnermeier and Pedersen (2009)) market microstructure theory has focused on determinants of liquidity for individual assets. Early studies include Stoll (1978) who considers inventory effects on liquidity and Glosten and Milgrom (1985), Admati and Pfleiderer (1988) and Kyle (1985) who consider asymmetric information effects on liquidity. We conjecture that a large part of the reason that liquidity co-moves is due to co-movement in the determinants of individual asset liquidity such as the inventory and asymmetric information components. We show that if the determinants of individual asset liquidity follow a factor structure (inducing co-movement in the determinants) then this implies a factor structure for liquidity. The co-movement in liquidity is determined by the product of the sensitivities of an asset to the risk determinants and the exposure of the determinants to the common market wide factor. Furthermore, we show that the liquidity factors can be recovered by taking cross sectional averages of the individual asset liquidity determinants.

We apply the methodology to a panel data set of daily liquidity measures for the stocks in the S&P100 for a three year span that includes the financial crisis. Theoretical market microstructure models predict that high volatility and infrequent trading imply high inventory costs to market makers and therefore less liquid markets. Frequent trades may be indicative of informed trading and therefore implies less liquid markets, possibly counteracting the inventory affect. We construct liquidity factors based on volatility and number of trades. In addition we consider two other factors that seem relevant during the financial crisis. We include the TED spread (the difference between rates charged on overnight loans to banks and the three month t-bill) to reflect a counter party risk in market making during the crisis. We also include the number of quote updates to capture the possibility that some market makers may have simply stopped trading during the crisis. Fewer market makers imply less competition for order flow and potentially less liquid markets.

Our methodology provides a unique insight into the sources of co-movement of liquidity. We find that for a typical stock in the S&P100, about 50% of the day-to-day variation in the liquidity can be explained by a 4 factor model. We find that virtually all the stocks in the S&P100 become less liquid when market wide volatility is high suggesting that a large reason for co-movement is due to market wide swings in volatility effecting inventory costs. We find that nearly all assets' liquidity decline when the frequency of trade is high suggesting that co-movement informed trading leads to co-movement in liquidity. We find that nearly all assets' liquidity decline when there are fewer liquidity suppliers suggesting that there are periods of time when there are fewer market participants supplying liquidity market wide. Finally, we find that many assets' liquidity tends to decline when the TED spread increases.

Some of the factors are highly persistent in our sample which implies that illiquidity episodes can be highly persistent lasting more than a year. This finding is consistent with simple non-parametric estimates. We propose a new method we call factor implied impulse response functions to get a better understanding of how movements in the factors translate into co-movement in liquidity. We are able to quantify the expected change in the cross sectional of liquidity following shocks to each of these factors. We find that for our sample that includes the financial crisis, shocks can introduce changes in the cross sectional of liquidity that are highly persistent lasting up to a year. Moderate sized shocks to the factors can increase the expected cost of trading by 10 or 15 percent and the shocks for volatility, trading frequency, and liquidity supply tend to have an unambiguous affect causing virtually all stocks to move in the same direction.

Our work is related to a growing body of empirical market microstructure studies. The fact that liquidity co-moves was established in early work by Chordia, Roll and Subrahmanyam (2000) and Hasbrouck and Seppi (2001). Chordia et al. (2000) show that liquidity for individual assets is correlated with market wide and industry specific liquidity measures as well as a common volatility factor. Hasbrouck and Seppi (2001) demonstrate that there is co-movement in illiquidity by examining principal components. Since then, the literature has moved in several directions. First, several more recent papers have analyzed co-movement in liquidity using aggregate measures of liquidity in a vector autoregression. Studies include Chordia, Roll and Subrahmanyam (2001) who examine the role of trading activity, interest rates, and deterministic effects, Chordia, Sarkar, and Subrahmanyam (2005) who study co-movement in liquidity across stock and bond markets.

Another branch of literature has examined the cross sectional pricing implications of time varying aggregate liquidity. Pastor and Stambaugh (2001) provide evidence that a market wide liquidity measure is priced and more recently, Korajczyk and Sadka (2008) extract a principal component for liquidity and also find some evidence that liquidity affects asset prices. Bao, Pan, and Wang (2011) find similar pricing implications in the bond market.

Yet another branch of literature use detailed market maker level inventory data to study how market maker inventories affect spreads both at the asset level and market wide level as in Comerton-Forde, Hendershott, Jones, Moulton, and Seasholes (2010). Domoqita, Hansch, and Wang (2005) use simulation methods to show that correlated order flows can drive in liquidity.

Our methodology can be viewed as a generalization of the ideas in the existing literature and our approach nests many of the ideas in this literature. Like by Chordia, Roll and Subrahmanyam (2000) and Hasbrouck and Seppi (2001) we consider a factor approach to modeling liquidity. Unlike these papers, however, we propose that the commonality in liquidity is derived from commonality in determinants of asset liquidity. This allows us to place economic structure on the dynamics of liquidity in ways that simply finding evidence of co-movement cannot. Additionally, the existence of common factors does not imply liquidity co-moves (in the same direction), it simply means there is a common factor that could make liquidity large on some assets and small on others. Indeed, we find that beyond the first principal component, we get high mixed signs on the factor loadings.

The second branch of literature using vector autoregression focuses on aggregate measures of liquidity, these studies also have little direct evidence about the fundamental question of why and how much individual asset liquidity tends to move together. The cross section of estimated factor loading on a given common factor provide direct measures of co-movement in liquidity. Analysis of the dynamics of these common factors provides quantitative measures of the degree and persistence of co-movement in liquidity that is not yet understood in the literature.

Section 2. A Factor Structure

Classic market micro structure theory predicts that market makers receive compensation for risk taking. The two sources of risk are inventory (Stoll (1978)) and asymmetric information (Glosten and Milgrom (1985)). The former is risk induced due to holding an (undiversified) position until it can be unwound. The latter is the risk of trading against better informed agents presenting an adverse selection problem.

Consider a liquidity measure L_{it} for asset i on day t . The liquidity measure could be average bid ask spread for the day, the effective bid ask spread, or some other measure of liquidity. We allow liquidity depends on two types of variables. The first is a set of time invariant characteristics such as price discreteness, or the exchange where the asset trades. The second are dynamic variables that capture dynamic sources of risk for market makers for that day such as inventory risk or adverse selection. We refer to the first set as asset characteristics and the second as asset risks (for market makers). Our goal here is to set up a model for the determinants of liquidity for a given asset i on day t that depends on the idiosyncratic, time invariant component and a time varying risk that induces time variation in the liquidity. In order to allow for common co-movement in liquidity we allow the dynamic risk factors for liquidity of specific assets to correlated in the cross section. This is accomplished by specifying that individual asset risk as a function of a common market wide risk and an idiosyncratic component. We denote the time invariant characteristics by α_i . We denote the k -dimensional vector of asset i specific liquidity risks time t by R_{it} .

While it is common in factor models to entertain unobserved components, our goal is to learn about the economic liquidity risks shared across different assets so we adopt the approach taken in empirical market microstructure and use directly observable risk factors or proxies. The liquidity for asset i on day t is assumed (as in most empirical studies) to depend linearly on the asset specific characteristics and the risks:

$$(1) L_{it} = \alpha_i + \lambda_i' R_{it} + \varepsilon_{it}$$

Where the ε_{it} are mean zero and independent across assets and λ_i is a conforming vector of parameters. A special case of the model is $\lambda_i = \lambda$ so that all assets have the same sensitivity to each risk. Dependence in the risks and therefore co-movement in liquidity is induced by a common factor. We denote the common factor for risk j by R_t^j . We specify that asset risk j take the following form $R_{it}^j = \gamma_i^j + \beta_i^j R_t^j + \eta_{it}^j$ where η_{it}^j uncorrelated across different assets and is uncorrelated with R_t^j . Stacking the k factors into a vector R_t and the idiosyncratic errors into a vector η_{it} we get:

$$(2) R_{it} = \gamma_i + \beta_i R_t + \eta_{it}$$

where β_i is a conforming $k \times k$ diagonal matrix. The co-movement between risk R_{it} and are determined by β_i , β_j , and the variance covariance matrix of R_t denoted by Ω_R . The covariance matrix for the risks for asset i and asset j is given by $\beta_i \Omega_R \beta_j'$. As long as the diagonal of the β matrices are not zero there will now be a non-zero correlation across the risks for different assets. This non-zero correlation in the risks across assets will generate co-movement in liquidity across different assets. Moreover, the liquidity for individual assets now follows a factor model. Substituting (2) into (1) we get

$$(3) L_{it} = \alpha_i^* + \lambda_i^{*'} R_t + \varepsilon_{it}^*$$

where $\alpha_i^* = (\alpha_i + \lambda_i' \gamma_i)$, $\lambda_i^* = \lambda_i' \beta_i$, and $\varepsilon_{it}^* = (\lambda_i' \eta_{it} + \varepsilon_{it})$. This representation gives a simple explanation for the observed phenomenon of commonality of co-movement in liquidity recently observed in the literature. Furthermore, this factor structure suggests an econometric modeling approach that captures co-movement in liquidity. Since our goal here is to understand co-movement our concern is in estimating the factor structure in (3). Once this model has been estimated the co-movement is given by the dynamics of the factor and the corresponding factor loadings $\lambda_i^{*'}$.

Since η_{it} and ε_{it} are uncorrelated with R_t , the factor loadings λ_i^* can be estimated by standard least squares if R_t is observable. The resulting estimate is the produce of the sensitivity of liquidity to the risk and how correlated the risk is with the factor.

Often, the risk factors used in empirical work are not actual risk factors, but rather variables that are thought to be correlated with the risk factor. For example, information asymmetries are not directly observed, but the frequency of trade may be used as a proxy. Kyle (1985) shows that competition among informed traders to capitalize off their information will lead to bursts in trading activity. This suggests using the frequency or number of trades as a proxy for informed trading as is often done in empirical market microstructure. Consider a simple case where R_{it} and hence R_t are scalar (there is just one risk). In the structure of our factor model, we can think of R_t as the true market wide source of risk of trading

with better informed agents, but of course it is unobserved. We can think of R_{it} as the noisy observed proxy for the risk of trading with informed agents such as the number of trades per day. This noisy correlation is captured in equation (2). Now, taking cross sectional averages of the observed noisy signal yields from (2):

$$(4) \quad \bar{R}_{it} = \bar{\gamma}_i + \bar{\beta}_i R_i + \bar{\eta}_{it}$$

A law of large numbers says that the last term will go to zero so that in large samples $\bar{R}_{it} = \bar{\gamma}_i + \bar{\beta}_i R_i$ and is (nearly) perfectly correlated with the unobserved risk.

In this case, we don't observe R_i directly, but this suggests that as long as the cross sectional average of β is not equal to zero, we can replace the regression in equation (3) with the following feasible regression:

$$(5) \quad L_{it} = \tilde{\alpha}_i + \tilde{\lambda}_i^* \bar{R}_i + \varepsilon_{it}$$

where now $\tilde{\alpha}_i = (\alpha_i + \lambda_i \bar{\gamma}_i)$ and $\tilde{\lambda}_i^* = \lambda_i^* \beta_i \bar{\beta}$

Hence if the common factors are not directly observed, we can still estimate the factor model for liquidity by using cross-sectional averages of the proxy variables used for each asset. Now, the resulting slope estimates λ_i^* are proportional to the sensitivity of liquidity to the risk. Furthermore, for practical purposes the unobserved risk factor R_i is now essentially observable up to a proportion. Importantly, since our goal is to study the sources of co-movement in liquidity across assets (and through time) it is sufficient to know the λ_i^* and not its individual components. However, ratios of the estimated values $\lambda_i^* / \lambda_j^* = \lambda_i / \lambda_j$ so relative comparisons can still be made across assets.

One last possibility is that the common risk factor R_i is observed, but not R_{it} . This might be the case, for example, where we have an aggregate measure of financial distress, but we don't observe measures for individual assets. This suggests that we again can estimate the equation (3) with the same interpretation of the estimated coefficients.

Section 3. Data

Our empirical work requires a panel data set of liquidity measures and common factors for the liquidity risk. The common factors can be directly observed or constructed as cross sectional averages of asset specific liquidity risks. Our methodology does not specify a specific liquidity measure and there are many ways of measuring liquidity. In our empirical work we use bid ask spreads as our measure of liquidity. We consider both directly observable and constructed common factors.

In principal, our analysis could be performed at any frequency. Hasbrouck and Seppi (2001) use 15 minute intraday time intervals. Tarun Chordia, Richard Roll Avandhar Subrahmanyam (2000) use daily data. We also focus our analysis on daily data for two reasons. First, for the common investor the exact timing of trades is not important so transitory illiquidity within day can be avoided by simply not trading at illiquid times. Illiquidity that last a day or more is a more serious constraint. Second, there are numerous intraday patterns deterministic patterns in many liquidity and other liquidity risk factors that induce non-stationarity. While there are econometric methods to deal with these features of intraday it will unnecessarily complicate our analysis without an obvious gain. Nevertheless, our general approach could be applied (with appropriate adjustments) to intraday data, or weekly data or longer frequencies.

Even though the analysis will be at the daily frequency, many of the measures constructed require aggregates of intraday trades and quotes. We use trades and quotes data from TAQ for the period of January 3, 2007 through December 31, 2009 a 3 year period that encompasses the financial crisis and contains 756 trading days. Modern high frequency data sets are extremely large. We select all stocks from the S&P100. This is a group of stocks that are reasonably large in terms of market capitalization so they would be relevant to many portfolio investment strategies, but not so many stocks as to become computationally unmanageable.

The S&P100 contains stocks from both the NYSE and NASD. During the time period for our data, NYSE was using a hybrid system with both centrally located specialists as well as a new electronic market called ARCA. We consider trades and quotes from both the floor as well as ARCA and NASD. The data is filtered to remove bid, ask, or transaction prices of zero. Any bid ask spread (calculated as a percent of the midpoint) that exceeds 10% is excluded. We exclude days that fall just before or after the major holidays of July 4, Thanksgiving, Easter, Christmas, and New Years Day.

For a given stock, we match the prevailing bid and ask prices with each trade in the sample. Suppressing the time subscript, we construct the bid ask spread measured as a percent at the time of each trade. For asset i , we have $spread_i = (bid_i - ask_i) / midpoint_i$ where $midpoint_i = (bid_i + ask_i) / 2$. It is natural to use the bid ask spread divided by the midpoint so as to capture the cost per dollar traded. For each day and each asset we construct the daily average bid ask spread to use as our liquidity measure. Since the data are constructed in trade time the result is a trade weighted average bid ask spread for each asset.

Section 4. Liquidity Risks and Common Factors

The classic market microstructure theory of (Stoll (1978)) (Glosten and Milgrom (1985)) predict two types of liquidity risks for market makers; inventory costs and adverse selection costs respectively. Inventory costs reflect the risk of taking an undiversified temporary position in a single asset that will be held for an uncertain amount of time before the position can be unwound with a trader on the other side of the market. Two typical variables used to capture this risk at the asset specific level are number or frequency of trades and volatility of the asset's price. Lower frequency of trades results in higher risk to market makers as the expected time to unwind the position is inversely related to the frequency of trade. The more volatility the stock price, the more risk to the market maker as prices are expected to move by a larger amount over any time interval when volatility is higher.

Adverse selection is the risk of trading with better informed traders. Capturing this risk requires some knowledge of when the likelihood of trading with a better informed agent is high and when it is low. Clearly this is not easily observable. Proxies used in empirical research include the number of analysts following the asset (larger number of analysts predicts less information asymmetries), the probability of informed trading, a measure proposed by Easley et. al. (1996), or the frequency of trading proposed by Kyle (1985). In a world with more than one informed agent, Kyle's model predicts that bursts in trading activity should be associated with higher numbers of informed agents present as the informed agents as informed agents race to trade before other informed agents reveal the private information through their trades.

Our discussion of the liquidity risk factors in section 2. derived the common liquidity factors for observed and unobserved common factors. We will make use of both types of common factors in our analysis. The PIN numbers of Easley et. al (1996) are difficult to construct on modern sized data sets so we leave these measures to a future study. The number of analysts will not vary much in the time series dimension so this is not a natural factor for our analysis. We focus on two variables, the number of trades and the volatility. The number of trades should be positively related to liquidity risk under the adverse selection theory and negatively correlated with liquidity risk under the inventory theory. Volatility should be negatively correlated with liquidity under the inventory theory.

We construct a number of trades common factor for each day. For each asset in the S&P100 we construct the time series of the number of trades on each day. For each day, we then take the cross sectional average to get the average number of trades per asset on day t , we denote this by the variable $Trades_t$. Note that ideally we would construct this risk factor by averaging over all assets, not just those in the S&P100, but that is not feasible due to the scope of the computing problem. For the volatility risk, we can use the VIX as the average. This measure spans a greater number of stocks than are in the S&P100. We denote this variable by VIX_t .

We construct one additional factor that is not directly related to classic microstructure theory, but is interesting in the context of the financial crisis. Metrick and Gorton (2010) and Nagel (2012) suggest that market turmoil and tightening funding constraints during the financial crisis reduced the number of market makers willing to supply liquidity and hence reduced competition among market makers leading to less liquid markets. In modern electronic markets it is difficult to define a market maker (unlike the days of the specialist driven market) and in fact many market participants may play both the role of liquidity demander and supplier at different times. We do not get to directly observe the number of market makers participating in our TAQ data.

Liquidity is supplied to the market by the posting of depth at bid and ask prices. A quote is updated every time that there is a change in the limit order book for a given market. Intuitively, the more updates there are on a given day, the larger the likely number of liquidity suppliers there are participating in posting depth on the bid and ask. For each asset in the S&P100 we construct the total number of quote updates. The theory of evaporating liquidity suppliers suggests that the larger the number of quote updates for a given stock, on a given day, the larger the number of liquidity suppliers are present in the market and therefore the more liquid the market. We construct the common factor then by again taking the cross sectional average number of quote updates for each day and we denote this variable by $Quotes_t$.

The financial crisis is characterized by counterparty risk inducing systemic risk throughout financial markets. This counterparty risk is likely reflected in the compensation for providing liquidity. This leads us to a fourth common factor that is directly observed. The TED spread is the difference between the rate the banks charge other banks for overnight loans and the yield on a 3 month government treasury. The difference between the rates is driven by the risk of default by a commercial bank or counterparty risk. We denote the TED spread on day t by TED_t .

Finally, Brunnermeier and Pedersen (2009) provide a novel theory of interplay between liquidity and asset prices. In their model, the ability to supply liquidity depends on margin requirements and margin requirements depend liquidity. Under certain conditions this implies a spiral effect where liquidity can suddenly dry up and that liquidity should move with market prices. In our context this suggests including a common factor of returns on a broad based index. Here we use returns on the S&P500 and denote it by SPY_t .

To summarize, our empirical work will use the following 5 factors.

Common Factors constructed from cross-sectional averages

$Trades_t$	Cross-sectional average number of trades per day
VIX_t	The VIX index
$Quotes_t$	Cross-sectional average number of quote updates per day

Common Factors that are observed

TED_t	The TED spread
SPY_t	The daily return on the S&P500

Section 5. Results

Our liquidity measure for each stock is the average bid ask spread for a given day t is denoted by $Spread_{it}$. In this section we begin with some summary statistics of our liquidity measures. After examining our summary statistics we next estimate the common factor models for liquidity using the factors defined in the last section and finally we examine the factor model's implications about co-movement in liquidity.

Section 5.1 Summary statistics

We begin with some summary statistics for the spread. First, we construct the unconditional correlation estimates for the daily spreads. In doing this, we sort the assets by 12 industry classifications. So diagonal blocks will correspond to assets in the same industry. We might imagine that correlations within industry may be stronger than across industries. Figure 1 presents a graphical correlation across assets for the log of the daily spread measures. The table to the right defines the industry blocks along the diagonal where 1 (NoDur) is the upper left and 12 (Other) is the lower right box.

Clearly, the correlation in daily log spreads is very high. The average correlation is .5 with a maximum of .97 and a minimum of -.3. Interestingly, there is evidence of industry specific correlation, most notably, block 11 (second from the last) is the finance industry which exhibits strong correlations.

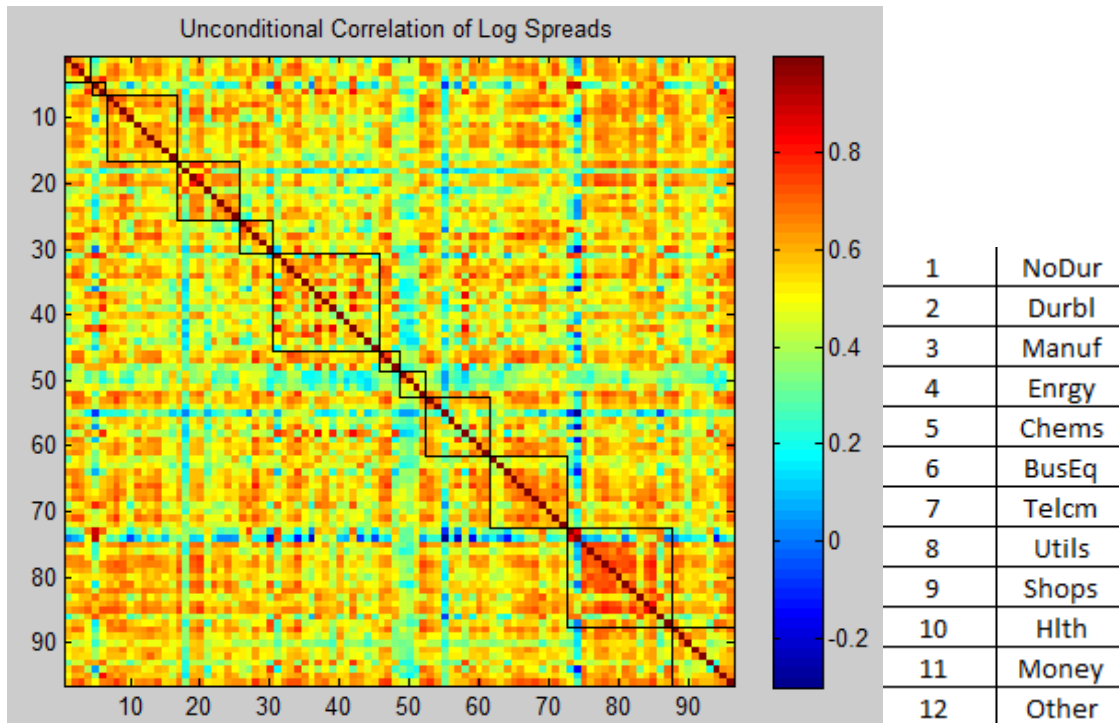


Figure 1. Unconditional correlations of the logarithmic spreads. The assets are sorted into 12 industry groups so that block diagonal elements of the matrix correspond to assets in the same industry. The blocks are in the order of the list to the right.

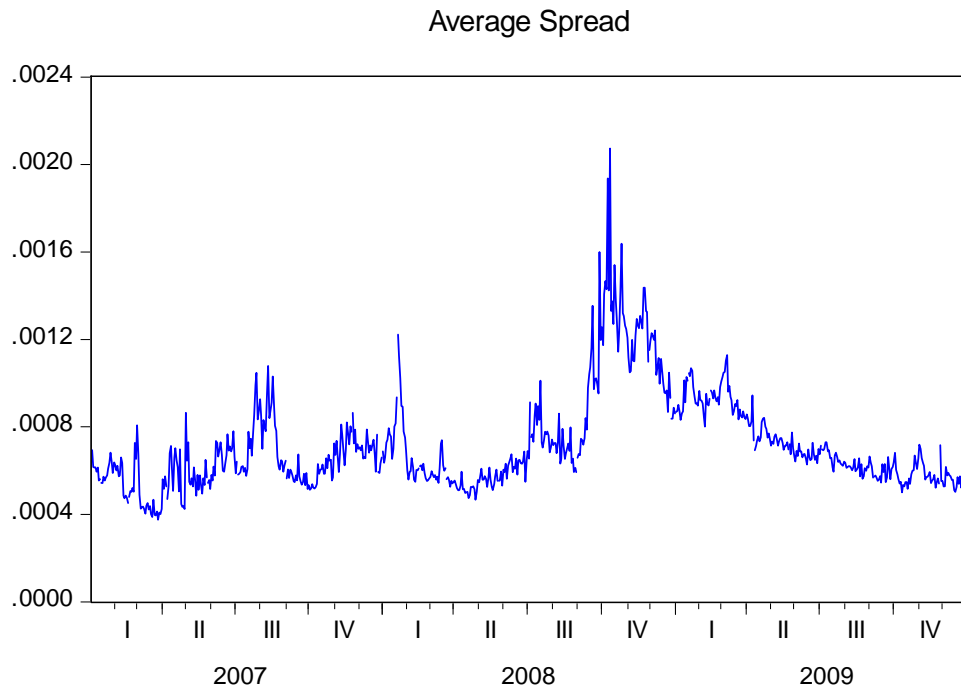


Figure 2. Time series plot of the cross sectional average of the spreads for each day.

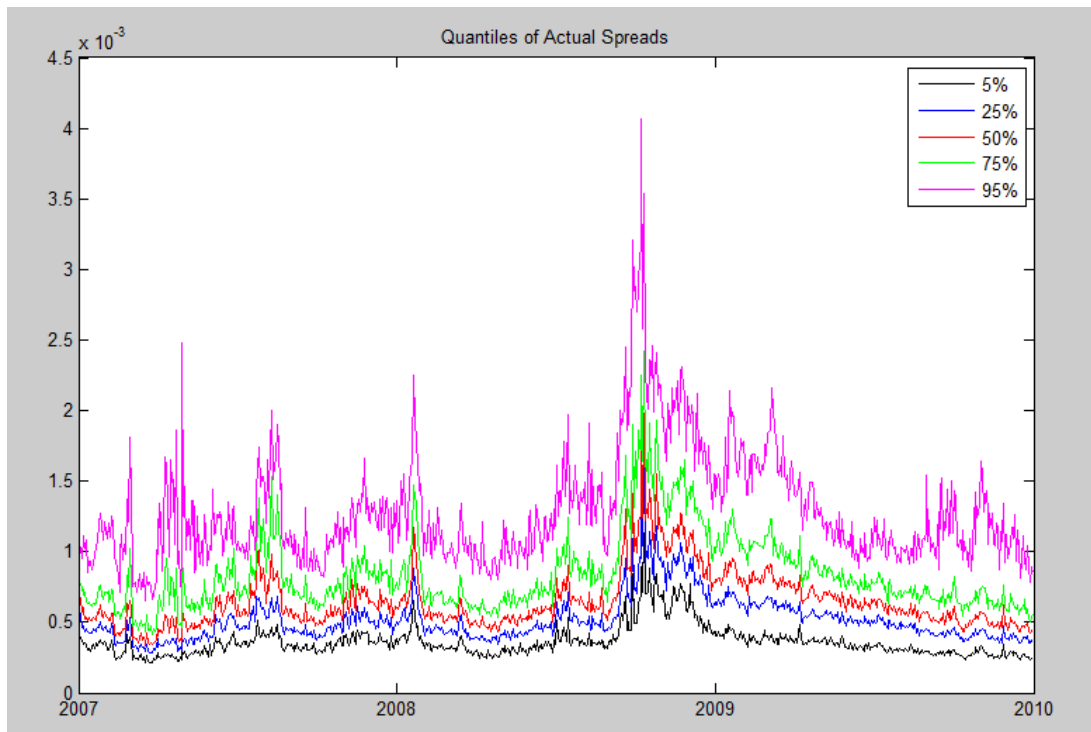


Figure 3. Time series plot of the quantiles of spreads for each day.

Next, we consider the time series variation in the spreads. For each day, we compute the cross sectional average of the spread measure as well as quantiles. Figure 2 presents the time series plot of the average spread. This average (or similar measures) are often the variable of interest in the liquidity literature using vector autoregressions. The average spread peaks October 10, 2008 at 20 basis points. This is about 5 times the low points in our ample of about 4 basis points. The average spread is highly persistent as it takes almost a year for the spike in October of 2008 to return to its typical level. Despite the high persistence, the null of a unit root is rejected at the 4% level for a standard augmented Dickey Fuller test. It seems natural to think of this series as stationary over a three year period, although clearly over very long time horizons stationarity may not be satisfied as changes in market structure (such as discreteness) can induce non-stationary behavior.

The average spread is clearly time varying. A different question is whether all stocks tend to be co-moving or rather a subset. To get a better idea of the nature of the overall co-movement of spreads, we plot the quantiles of the spreads for each day in our sample. These are presented in a time series plot in figure 3. Clearly, all quantiles of the distribution move together indicating broad co-movement in the spreads. Interestingly, we can see that the distribution of spreads tends to be more spread out when spreads are large than when they are small. This can be seen by looking at the widening vertical separation of the quantiles during periods of high overall spreads.

Before we estimate the factor models it is interesting to examine the principal components of the panel data set of spreads. Figure 4 presents the scree plot for eigenvalues of the variance covariance matrix.

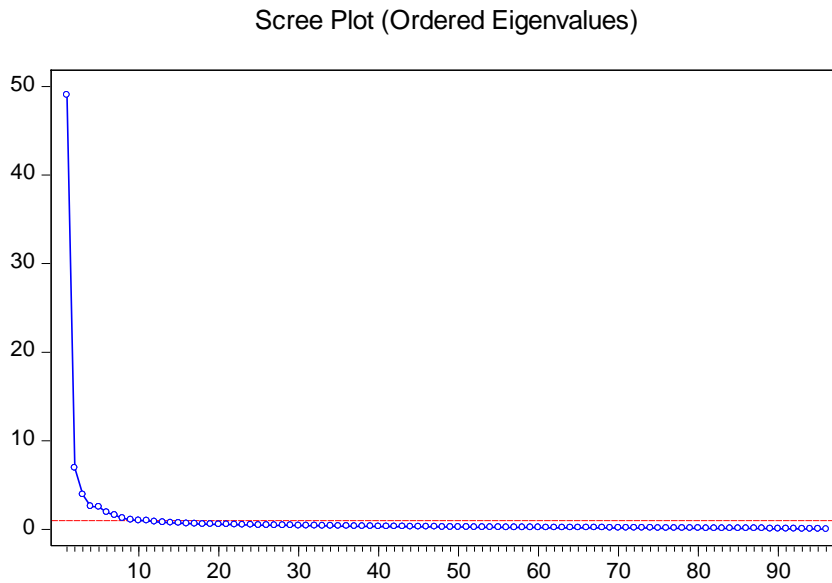


Figure 4. Scree plot of eigenvalues associated with the panel data set of daily spread measures.

The first principal component is able to capture 50% of the variance while the second, third, and fourth capture 7% 4% and 3% respectively. The red line is the 5% cutoff for significance of the eigenvalue. There is not much significance after the fourth component. Consistent with the earlier studies, we find strong evidence of co-movement implied by the principal components.

Section 5.2 Estimation of Factor models

In this section we present the results for the estimated factor models using the daily average spread. Since the spread must be positive and is right skewed it is natural to take the logarithmic transform. Hence in our work modeling work we use $L_{it} = \ln(\text{Spread}_{it})^2$.

For each stock we estimate the factor loadings on each factor. We take logs of all the common factors except for the S&P500 return series and estimate by least squared the factor loadings. We begin by estimating a 4 factor model with the log of Trades_t , VIX_t , Quotes_t , and TED_t as the four factors. Specifically for asset i , we run the following least squared regression:

$$(6) \log(\text{Spread}_{it}) = \alpha_{it} + \lambda_{i1} \log(\text{TED}_t) + \lambda_{i2} \log(\text{VIX}_t) + \lambda_{i3} \log(\text{Trades}_t) + \lambda_{i4} \log(\text{Quotes}_t)$$

In the end, we have 100 sets of factor loadings for each factor. We summarize the model estimates in figures 5 and 6. Figure 5 contains histograms of the estimated factor loadings for each factor. Figure 6 contains histograms of the t-stats for the individual factor loadings. While not presented here, Newey-West t-stats look very similar. We find that the about 70% of the slope estimates for the TED spread are positive. Since a larger value of the spreads means markets are less liquid this implies a negative relationship between the TED spread and liquidity. This makes sense since higher counterparty risk should be associated with more risk to market makers and hence less liquid markets. In figure 6 we can see that the majority of the factor loadings are statistically significant, however it is a little puzzling that some of the factor loadings are significantly negative which is counter to our prior beliefs regarding the factor.

The VIX factor has virtually all factor loadings positive and significant. This is consistent with a common factor in volatility which is already well known. The results for VIX are consistent with the idea that asset volatility is highly and positively cross-sectionally correlated. Since high volatility is associated with increased inventory liquidity risk we see that spreads tend to widen when the VIX is large. The fact that virtually all the factor loadings are positive suggests that this factor could induce strong co-movements in liquidity.

The vast majority of the factor loadings for the number of trades common factor are also positive and significant. This indicates that higher transaction rates at the market wide level tend to be associated with wider spreads and less liquid markets. The frequency of trades plays a role in both the adverse selection models and the inventory models of liquidity risk. Adverse selection says that high trading rates are associated with informed trading and hence spreads should be wider while high trading rates are associated with lower inventory risk and smaller spreads in the inventory models. The slope estimates here indicate that the higher trading rates in the market may be associated with more informed traders.

² We also considered using the spread without taking logs. Most of the conclusions in the section are not very sensitive to the logarithmic transformation, however we did occasionally generate negative forecasts for spreads which is inconsistent with the data.

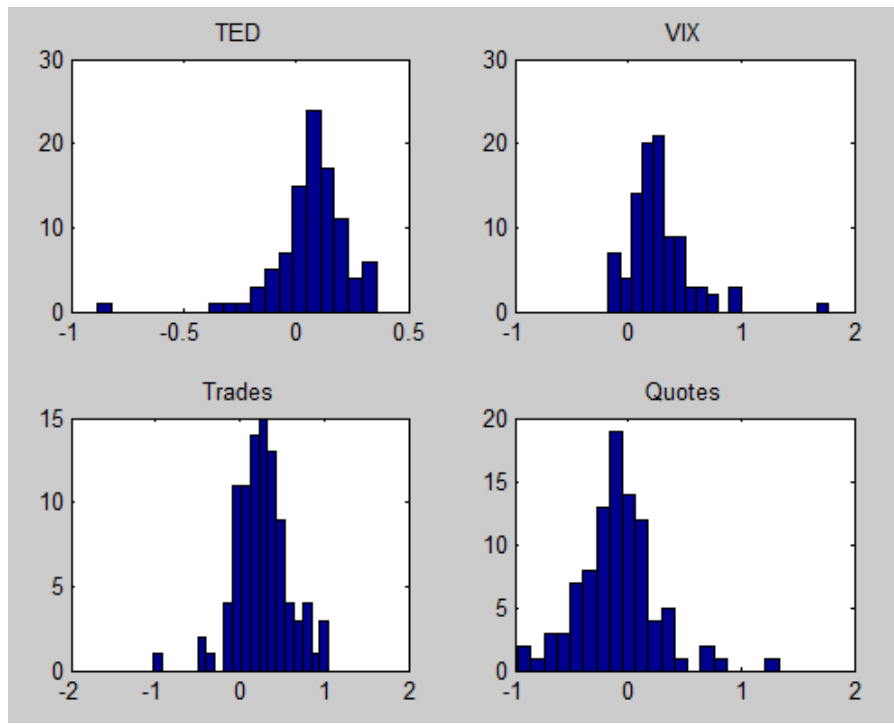


Figure 5. This figure presents the estimated factor loadings for (logarithmic) TED spread, the VIX index, the average number of trades, and the average number of quotes.

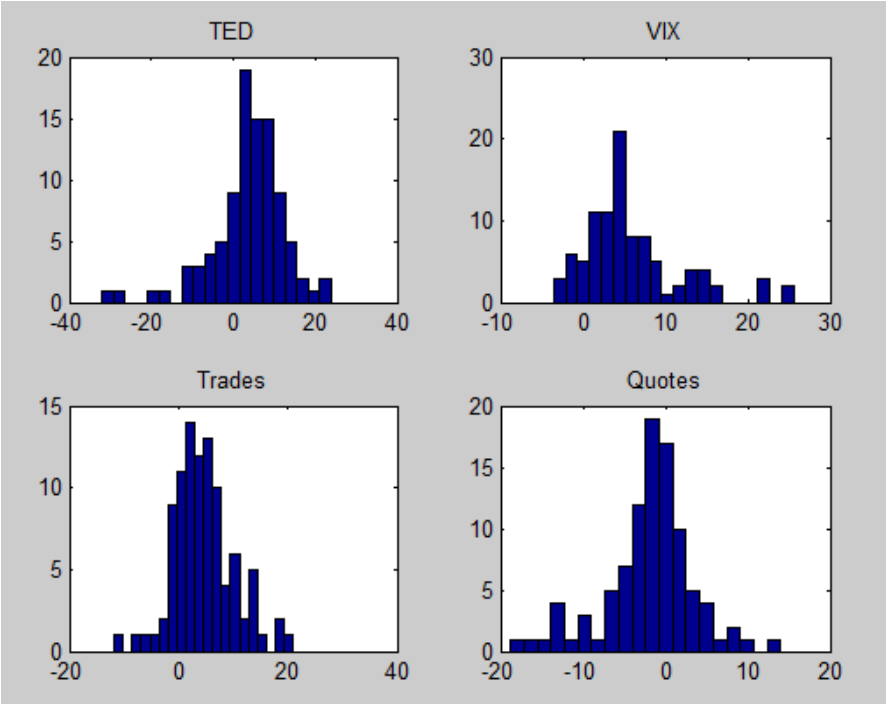


Figure 6. This figure presents the t-statistics for the estimated factor loadings for (logarithmic) TED spread, the VIX index, the average number of trades, and the average number of quotes.

Again, since the vast majority of slope coefficients are positive this suggests that a large part of co-movement in liquidity may be driven by market wide informed trading. While it may at first appear odd that informed trading is market wide, more thought suggests this may be reasonable. For example, a policy announcement by the Federal Reserve would likely have an impact on most stock values. Traders that are better and more quickly able to interpret the announcements effect would have an informational advantage in trading. Hence we should, perhaps, not be surprised to find evidence of market wide information asymmetries.

The number of quotes common factor is somewhat mixed, but the majority of factor loadings are negative. This indicates that, overall, as liquidity providers enter the market so that quote activity increases, spreads tend to go down and individual assets become more liquid. Of course, the flip side of this is that as liquidity providers leave the market, the spread goes up and liquidity goes down on individual assets. This result is consistent with the conjectures that liquidity providers may have been forced to leave the market due to increased liquidity funding or margin requirements suggested by Metrick and Gorton (2010) and Nagel (2012).

An interesting question to ask is how much of the variation in liquidity can be explained by the 4 factor model? Figure 7 plots a histogram of the R-squareds from the estimated models. The average R-squared is 45%, the minimum is 15% and the maximum is 78%. Some of the assets with low R-squareds include companies with well-known idiosyncratic behavior during this time period. For example, both Bank of America and Citigroup both have low R-squareds and both had large idiosyncratic components over this time period.

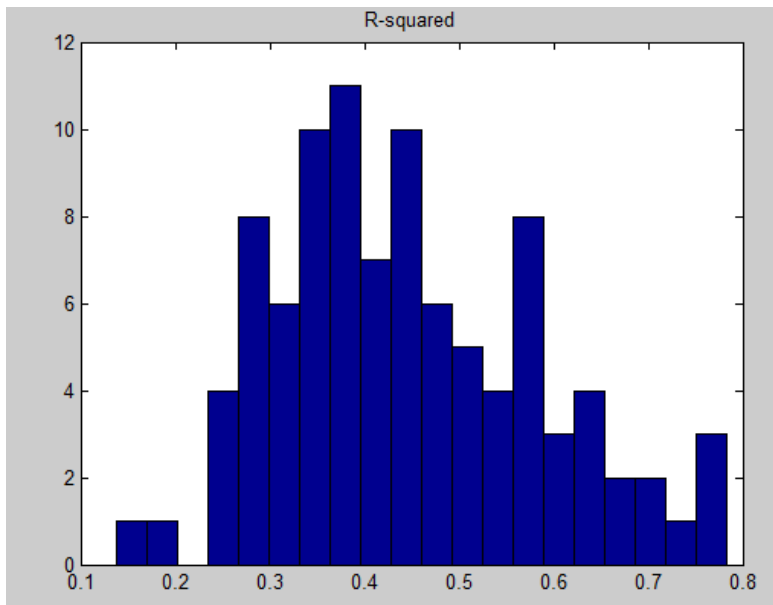


Figure 7. Histogram of the R-squareds for the estimated factor models.

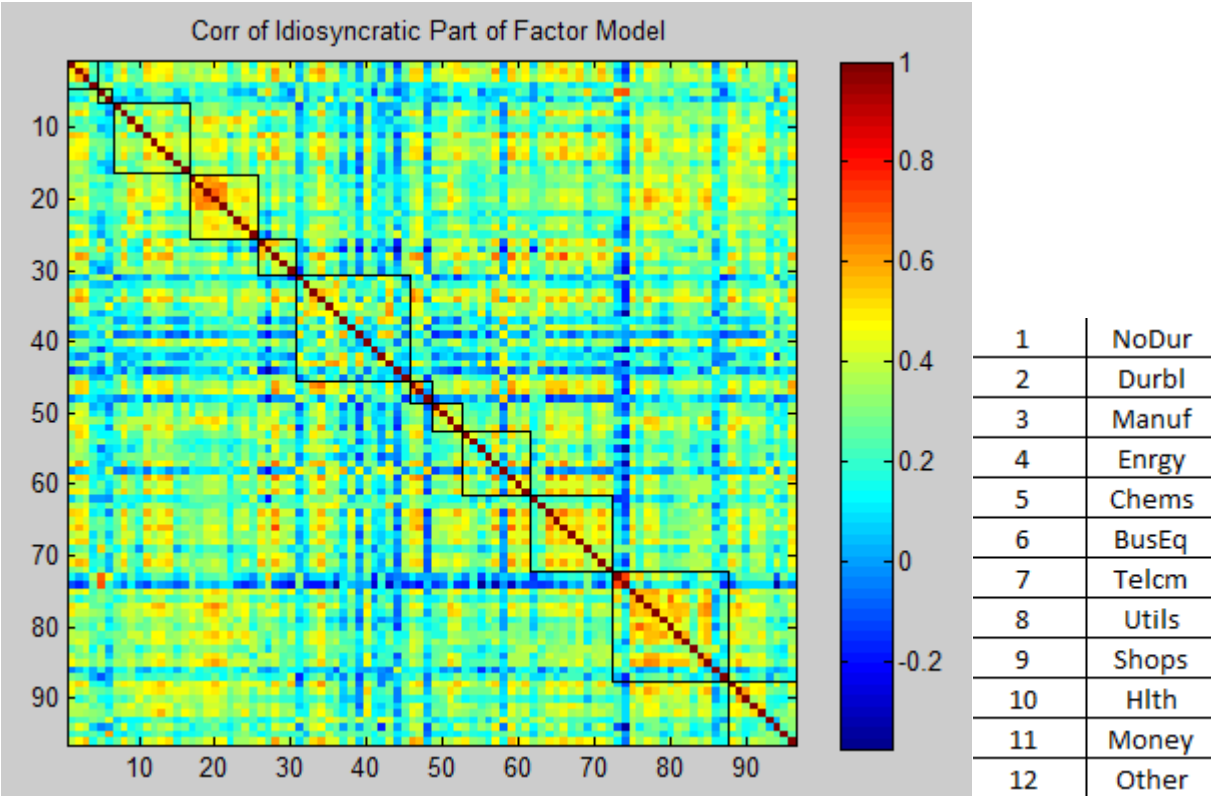


Figure 8. Correlation of the idiosyncratic errors of the factor model.

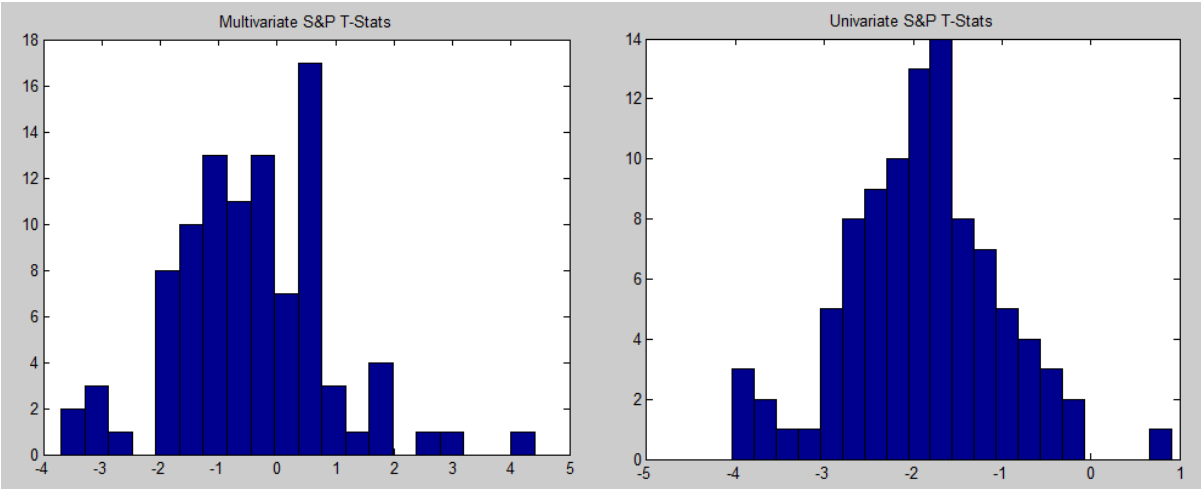


Figure 9. t-stats for the slope coefficient estimates on the S&P500 return factor in a 5 factor model. The other four factors are the same as discussed above.

Clearly these large idiosyncratic components will not be captured well by a factor structure. Finally, figure 8 presents a correlation plot for the idiosyncratic errors. If the 4 factor structure were correct, we would expect to find small correlation levels. Indeed, the correlations are much smaller and often near zero. A notable exception is that there is clearly still some remaining correlation within certain sectors such as the financial sector.

We next perform a test of the Brunnermeier and Pedersen (2009) theory that predicts that falling price levels increase pressure on margin requirements and therefore increases the cost of making markets which implies less liquid markets. Hence their theory predicts a negative relationship between spreads and the return or negative factor loadings. Hendershott, Johnes, Moulton, and Seasholes (2010) use a specialized data set with information on individual market makers balance sheets and find that individual market makers tend to widen spreads when positions are large or after realizing large losses, consistent with Brunnermeier and Pedersen (2009).

It is interesting to see if these affects extend beyond a single market maker and are pervasive. We estimate the factor loading in two ways. First, we add the S&P500 return to the 4 factor model discussed above and second we estimate a one factor model with only S&P500 return as the common factor. The results are presented in figure 9 with the multifactor estimates on the left and the one factor model on the right. Here, we only present the t-stats for the estimated coefficients on the S&P500 factor. For both sets of results, there is a slight tendency for the coefficients to be negative, however, the overwhelming majority are not significant. The results are slightly more favorable for their margin requirement hypothesis for the one factor model. However, the well-known leverage effect implies that falling prices tend to be associated with higher volatility so it is possible that the one factor model looks more favorable is due to this correlation. When volatility is included the results significance is much less. Hence we

find weak evidence of margin requirements effecting liquidity. At a minimum, it is of second order importance.

Section 5.3. Implications of the model.

In this section we assess the importance of each of the factors in the co-movement and overall movement in liquidity. We begin by examining the dynamics in liquidity that are implied by the factor model. Figure 10 presents the quantiles of the spread implied by the predicted values from the factor model. That is, in each period, we predict the spread using the values of the factors and the factor loading for each asset. We then compute the quantiles of these predictions for each day. Since the expected outcomes will exhibit less variability than actual outcomes this figure cannot be directly compared to figure 3. Nevertheless, the general movements in the quantiles and strong co-movement in the distribution is very similar to that of figure 3. The overall movement of the predicted values matches closely the overall pattern of the empirical quantiles. This suggests that the 4 factor model does a good job of capturing co-movement in liquidity.

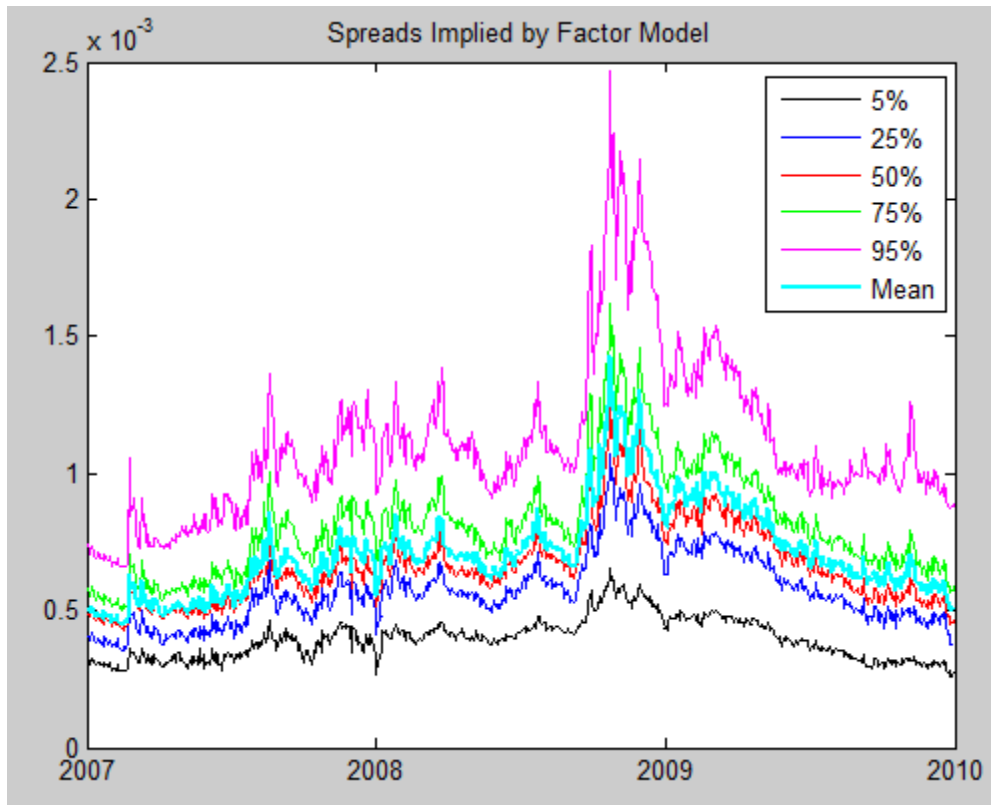


Figure 10. Implied quantiles (and mean) of the spreads from the 4 factor model depend on the factor loading and the magnitude of changes to that factor.

Figure 10 suggests the model captures the movement well, but it is interesting to assess how each of the separate factors affects co-movement of liquidity. The degree of co-movement induced by a factor will depend on the factor loading. If the factor loadings are not of the same sign then this could induce movement in opposite directions. If the factor loadings are mostly of the same sign, then this will induce co-movement in the same direction. In order to understand the implications of a change in one factor to future liquidity we propose the following methodology. First, we fit a vector autoregression to the factors. We then compute the impulse response function for each of the factors given a one-standard deviation shock to one of the factors. Since the spreads on an individual asset are linear functions of the factors, we can trace out the impulse response of the spread of an asset by simply multiplying that asset's factor loadings times the vector of impulse responses. In this way, we can infer how liquidity co-moves given a shock to any one or more than one of the factors. We are not aware

of any previous use of this methodology and we therefore term this a factor implied impulse response function.

We begin by fitting vector autoregression to the 4 factor model. The model estimates are presented in table 1. The residual diagnostics look very good for this model and we pass a portmanteau test for the null that all the first 10 (cross) autocorrelations are zero. Since the dynamics of liquidity will be driven by the dynamics of these 4 factors we begin by looking at the impulse response functions for these 4 factors.

Figure 11 presents the impulse response function for the factors out through 200 periods, almost one year. We give a one standard deviation shock to each series. While the series are contemporaneously correlated, we are interested in the marginal effect of a shock to one factor, so we only shock one variable at time zero with no other contemporaneous affects (we do not use a Choleski decomposition here). The red lines are 95% asymptotic confidence bounds. All of the factors respond positively and persistently to their own shocks. The TED spread is only affected by shocks to itself. The VIX is mildly positively affected by shocks to the TED spread, but not affected by the number of trades or quotes. The number of quotes increases following increases to the VIX. The number of trades responds positively to the VIX and negatively to the number of quotes.

We now examine how the spreads respond to shocks to each of the four factors. We consider a two standard deviation of the error term shock to a given factor and create the implied shock to the spreads given the estimated factor loadings and the impulse response function of the factors. Specifically, we can write

$$(7) L_{it} = \alpha_i + \lambda_i' R_t + \varepsilon_{it} = \alpha_i + \lambda_i' \sum_{m=1}^{\infty} \psi_m v_{t-m} + \varepsilon_{it}$$

where ψ_i are the matrices of the infinite moving average representation for the VAR model of the factors R_t and v_t is the vector of errors for the VAR. The ψ_i define the impulse response for the VAR and $\lambda_i' \psi_{t-m}$ define the impulse response for asset i , to a shock to the innovation to the factor model m periods ago. Specifically, $dL_{it} = \lambda_i' \psi_{t-m} dv_{t-m}$. Notice that even if the idiosyncratic error are temporally correlated that should not affect the interpretation of the impulse response function because the idiosyncratic errors are uncorrelated with the factors. The co-movement of the spreads can now be understood in terms of co-movement of the factors. We call this a factor implied impulse response function. Unlike the existing methods of analyzing co-movement in factors, this took provides a direct way of assessing what the common factors are that make liquidity move together.

Since the spread is in logs the units of the vertical axis can be interpreted as the percent increase in transaction cost. A number like .1 means it is 10% more expensive to transact. The result is a set of 100 impulse response functions all generated from a shock to a given factor. We summarize these results graphically by plotting the percentiles of the expected change in the spreads. Note that this is not the same thing as the percentiles of the distribution of the changes since we are ranking expected changes only, this is simply a way of presenting 100 impulse response functions in a parsimonious way. The upper left corner of figure 12 shows the response of the spreads to a shock to the TED spread. The average effect of a two standard deviation increase in the TED spread on asset spreads is to make them wider, however, the effects are mild and some spreads actually decrease. The overall effect is some increase in the spread although, but not co-movement in the same direction.

Increasing the number of trades has the largest average impact. Initially the effect of a two standard deviation increase in the number of trades factor results in a 10% increase in the cost of trade. The effects are nearly all in the same direction as all quantiles with the exception of the 5% increase. The effects also last a long time spanning nearly 150 days.

	Endogenous Variable			
	LOG(TED)	LOG(VIX)	LOG(QUOTES)	LOG(TRADES)
LOG(TED(-1))	1.129	0.042	0.059	0.130
St. Err	-0.037	-0.036	-0.086	-0.091
t-stats	[30.1797]	[1.16159]	[0.69018]	[1.43850]
LOG(TED(-2))	-0.314	-0.128	-0.323	-0.365
St. Err	-0.056	-0.054	-0.128	-0.135
t-stats	[-5.63421]	[-2.38205]	[-2.53014]	[-2.70136]
LOG(TED(-3))	0.183	0.091	0.255	0.238
St. Err	-0.038	-0.036	-0.086	-0.091
t-stats	[4.85633]	[2.49190]	[2.95925]	[2.60553]
LOG(VIX(-1))	0.012	0.779	0.512	0.477
St. Err	-0.041	-0.039	-0.093	-0.098
t-stats	[0.29986]	[19.9592]	[5.52537]	[4.86720]
LOG(VIX(-2))	0.009	0.089	-0.118	-0.074
St. Err	-0.050	-0.048	-0.115	-0.122
t-stats	[0.18354]	[1.84874]	[-1.03021]	[-0.60981]
LOG(VIX(-3))	-0.039	0.092	-0.239	-0.245
St. Err	-0.040	-0.039	-0.092	-0.098
t-stats	[-0.95367]	[2.35437]	[-2.58125]	[-2.50153]
LOG(QUOTES(-1))	-0.031	0.009	0.380	-0.248
St. Err	-0.042	-0.040	-0.095	-0.101
t-stats	[-0.73057]	[0.23231]	[3.98426]	[-2.45276]
LOG(QUOTES(-2))	-0.026	0.007	0.143	-0.048
St. Err	-0.048	-0.046	-0.110	-0.116
t-stats	[-0.53861]	[0.14774]	[1.29777]	[-0.41520]
LOG(QUOTES(-3))	0.058	-0.002	0.318	0.194
St. Err	-0.042	-0.040	-0.095	-0.100
t-stats	[1.40041]	[-0.05922]	[3.35150]	[1.93205]
LOG(TRADES(-1))	0.069	-0.007	0.212	0.815
St. Err	-0.039	-0.038	-0.090	-0.095
t-stats	[1.75409]	[-0.17535]	[2.36319]	[8.59049]
LOG(TRADES(-2))	0.007	0.024	-0.055	0.136
St. Err	-0.045	-0.044	-0.104	-0.110
t-stats	[0.16124]	[0.55926]	[-0.53417]	[1.23665]
LOG(TRADES(-3))	-0.062	-0.002	-0.197	-0.065
St. Err	-0.039	-0.038	-0.090	-0.095
t-stats	[-1.57795]	[-0.06120]	[-2.20158]	[-0.68327]
C	-0.121	-0.207	1.912	1.992
St. Err	-0.128	-0.123	-0.292	-0.309
t-stats	[-0.94749]	[-1.68898]	[6.55534]	[6.45288]

Table 1. This table presents estimates for the VAR(3) model for the risk factors.

Response to Nonfactorized One S.D. Innovations ± 2 S.E.

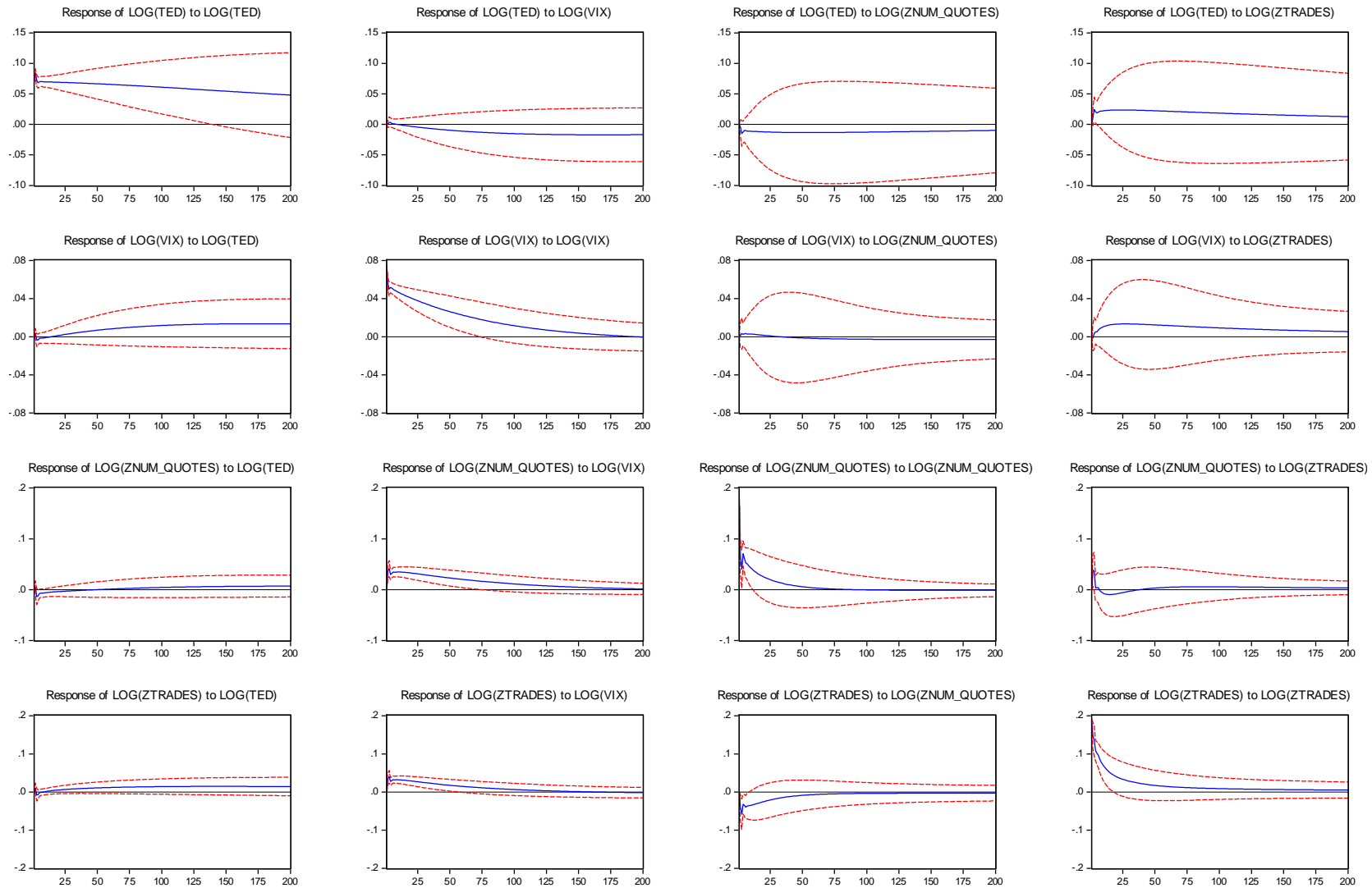


Figure 11. Impulse response functions from VAR(3) model for the 4 factors TED, VIX, Quotes, and Trades.

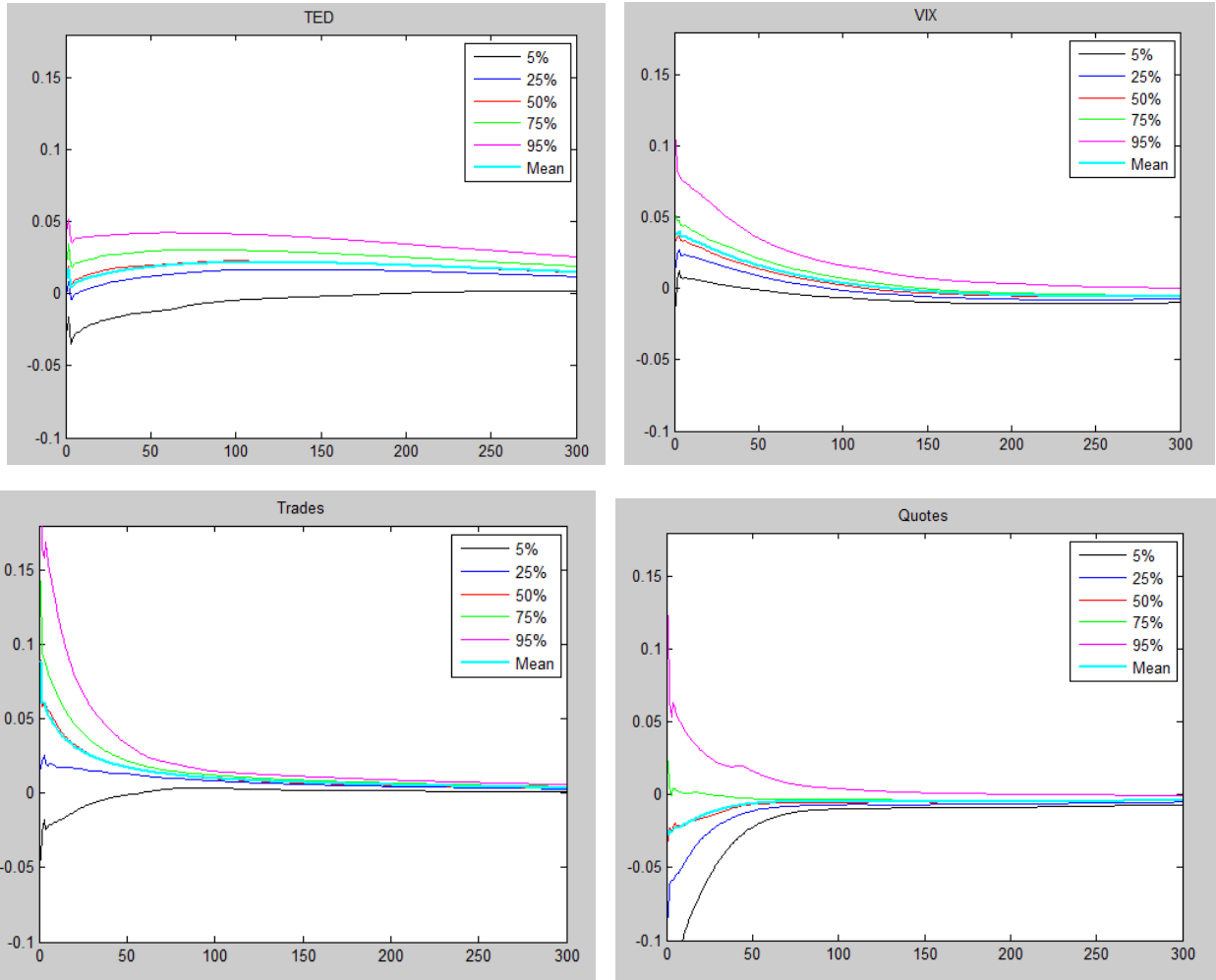


Figure 12. This figure plots the implied quantiles of the spread given a two standard deviation shock to a factor. The upper left corner is the response to a shock to TED. The upper right corner is the response to a shock to the VIX. The lower left is the response to a shock to trades and the lower right is the response to a shock to quotes.

The VIX has a more substantial and obvious effect on future liquidity. We see that a two standard deviation shock to the VIX yields a stronger effect and is substantially in the same direction, namely an increase in transaction cost. The average increase in transaction costs in the short run is around 4%, but all quantiles increase following an increase in the VIX. The effect of the shock are long lived in the sense that the average has not mean reverted 150 days after the shock.

An increase in the participation of liquidity providers as measured by the number of quotes results in a mostly negative impact on the spreads. Only the 95th percentile has a mild positive response. This is consistent with market wide reduction in the supply of liquidity causing co-movement in liquidity.

These results suggest that the reason that we see co-movement in liquidity is due to comovement or market wide informed trading inducing asymmetric information costs, co-movement in volatility inducing co-movement in inventory risk and periods of liquidity suppliers moving out of the market. Given our knowledge of co-movement in volatility it is probably not surprising that co-movements in volatility and therefore inventory costs are a strong determinant of co-movement in liquidity. The degree of market wide informed trading we believe is less obvious and perhaps novel.

Section 6. Conclusions

Partly as a result of the financial crisis, there has been a growing interest in quantifying and understanding the nature of co-movement in liquidity. This paper starts with the idea that co-movement in liquidity should be driven by co-movement in the risks that drive liquidity in individual assets. To accommodate this co-movement in liquidity risks in individual assets we propose a factor structure. We show that if the liquidity risks for individual assets follows a factor structure then this implies a factor structure for liquidity. Our method can accommodate both observed factors and the case where for each asset we only observe a variable that is correlated with the common risk. We show that in this case, cross sectional averages of the variables can be used to uncover the common risk and subsequently used to estimate the factor model for liquidity.

We apply our methodology to a sample of S&P100 stocks spanning a three year period that includes the financial crisis. We use both observable factors and factors created from cross-sectional averages designed to capture a common volatility component, liquidity supply factor, and trading rates. We also include the observable risk factor for counterparty risk as captured by the TED spread.

We find that inventory risk due to common volatility shocks, market wide asymmetric information and drying up of liquidity suppliers are responsible for a large part of the co-movement in liquidity during the financial crisis. Interestingly, counterparty risk appears to affect liquidity, but does not imply broad co-movement in liquidity. These effects can be quantified using our method of factor implied impulse response functions. A two standard deviation shock can let to in excess of a 5% average change in liquidity. The upper quantiles of these effects can exceed 15%. We find mild evidence to support the Brunnermeier and Pedersen (2009) theory that predicts that falling price levels increase pressure on

margin requirements and therefore increases the cost of making markets which implies less liquid markets.

References

- Admati, A. and P. Pfleiderer, 1988, "A Theory of Intraday Patterns: Volume and Price Variability" *The Review of Financial Studies* Vol. 1, No. 1, (Spring, 1988), pp. 3-40
- Bao, J., J. Pan., and J. Wang, 2011 . The illiquidity of corporate bonds. *The Journal of Finance*, 66(3), 911-946.
- Brunnermeier and Pedersen, 2009, "Market Liquidity and Funding Liquidity", *The Review of Financial Studies*, 22, 2201-2238.
- Chordia, Roll, and Subrahmanyam, 2000, Commonality in Liquidity, *Journal of Financial Economics* 56 28.
- Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam. "Market liquidity and trading activity." *The Journal of Finance* 56.2 (2001): 501-530.
- Chordia, Tarun, Asani Sarkar, and Avanidhar Subrahmanyam. "An empirical analysis of stock and bond market liquidity." *Review of Financial Studies* 18.1 (2005): 85-129.
- Comerton-Forde, Carole, Terrence Hendershott, Charles M. Jones, Pamela C. Moulton, and Mark S. Seasholes. "Time Variation in Liquidity: The Role of Market-Maker Inventories and Revenues." *The Journal of Finance* 65, no. 1 (2010): 295-331.
- Easley, D., N.Kiefer, M. O'hara, and J. Paperman. "Liquidity, information, and infrequently traded stocks." *The Journal of Finance* 51, no. 4 (1996): 1405-1436.
- Glosten and Milgrom, 1985, "Bid and Ask Transaction Prices in a Specialist Market with Heterogeneously Informed Traders" *Journal of Financial Economics*, 14, 71-100
- Hasbrouck and Seppi, 2001, Common factors in Prices, Orders, and Liquidity, *Journal of Financial Economics*, 3, pp, 383-411.
- Korajczyk, Robert A., and Ronnie Sadka. "Pricing the commonality across alternative measures of liquidity." *Journal of Financial Economics* 87.1 (2008): 45-72.
- Kyle, 1985, "Continuous Auctions and Insider Trading" *Econometrica*, Vol. 53, No. 6. pp. 1315-1336
- Metrick, A. and G. Gorton, 2010, "Haircuts" NBER working paper.
- Nagel, S. 2012, "Evaporating liquidity." *Review of Financial Studies* 25, no. 7: 2005-2039.