A City as a Small Open Economy

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Abstract

Our representative city exports a commodity at "world" prices and imports food, also at "world" prices. In addition there is a local, endogenous good produced under constant returns to scale and a housing sector that is land-using. Land supply is increasing exogenously in a local land rent. Capital and labor are available at "world" prices. A representative resident is a "utility taker". Larger cities turn out to be denser cities (less land per resident) and this leads to equilibrium city size. We focus on labor productivity change in our city, on the impact of a local climatic amenity premium and on the introduction of a local public good.

1 Introduction

We explore the view of a town or city as a small open economy, in the textbook sense of "a small open economy". The city exports a commodity at "world" price p_x and imports say "food" in return, this latter also at "world" price, unity. By "world" we mean a system of many cities with our city in question "in the middle". Our city is then a price-taker for its export and import goods in this large system of cities. Capital is available a constant rental rate r and workers are available at wage w. In addition our representative city has a local or non-traded good (haircuts) produced with its price p_d endogenous to our city. A representative worker-household occupies a "house" with its price and size endogenous. Our model is roughly speaking a "trade model" version of Mills's (1967) monocentric city model. This latter had a sector producing export goods at "world" prices, with firms generally occupying very little or no land, a housing sector for residents, with each resident consuming positive amounts of land, and explicit commuting costs for a worker-household.[1] We assume that entry and exit of our homogeneous worker-households to the city is costless. Entry and exit flows become zero at an exogenous utility level set out in the large system of cities.[2] Our city is then a "utility taker" (we are employing here the "open city" assumption).

We abstract from the internal structure of our city. We postulate that additional land for residences for the city is only available at a rising price. This land supply schedule is exogenous to our city. It should be viewed as a reduced form schedule that derives from costly movement of households WITHIN the city. A desirable location in the city would be associated with a lower cost of "commuting" but a higher cost for a house-site. Central to an equilibrium for our city is the property that a larger equilibrium city has a denser arrangement of residents (less land per resident). Hence in terms of utility, a resident of a larger city is consuming less "housing" (land in our model) but more imported food and the local, non-traded good. Thus one can say that city-size in our model is determined by a representative resident's utility level rising to the exogenous level, given costless migration between cities. A representative worker is assumed to have the same skill and training *ex ante* as any other worker in our model. Production of the local and

[1] Mills's city had a simple internal structure, a center with employment and a residential annulus around the central employment area. This structure derived from earlier work of Alonso. Recent extension of the Mills model were pursued by Lucas and Rossi—Hansberg (2003). This new work introduced explicit positive externalities between nearby producers and a variety of new land-use patterns emerged. A quite different approach to modelling the internal structure of a city was pursued in Hartwick and Hartwick (1974).

[2] Eaton and Eckstein (1997) and Lucas (2004) each treat migration as turning on lifetime utility rather than an agent's current single period utility. This approach is both superior and considerably more complicated.

export goods takes place with constant returns to scale. Equilibrium size for our city turns on in migration of workers into and out of our city and on more costly housing. Later we introduce a local public good, also produced under constant returns to scale.

Our model allows us to address two issues: (a) two very similar workers, one in city i (large) and the other in city j (small) typically have distinctly different wages, the higher wage being earned in the larger city. We link the wage premium to an explicit productivity premium in our model, a productivity premium that emerges because a larger city has more urban-ness associated with it. (b) Cities with a climate amenity premium are typically larger and exhibit higher local costs of living (higher "housing" costs). We address these two topics with numerical simulation, solving our explicit model numerically with Matlab software. Like Mills, we remain somewhat agnostic about the mechanism of agglomeration for our city. We have production at a point or points in space and associated workers (residents) occupying space nearby. Implicit then is each worker-resident commuting to the workplace or places from her place of residence, and workplaces are not so distributed that commuting can be ruled out. One might say that some scale is implicit in local goods production and thus each worker does not work in her own factory and in doing so avoid all commuting costs. We abstract from transportation costs for getting the export good from its production site to outside the city as well as from transportation costs for getting the import good to places of direct consumption by households. We are also somewhat agnostic about why a similar worker in a larger city exhibits a productivity premium. We take this as a central empirical regularity that is related to city size and not necessarily to an explicit scale economy in the production of commodities in the city.[3] We leave open the mechanism that connects city size to a labor pro-

[3] Henderson (1974) had explicit scale economies in production figure centrally in his analysis of types and sizes of cities.

ductivity premium. Ciccone and Hall (1997), Glaeser and Mare (2001) and Glaeser and Gyourko (2005) document this effect and Glaeser and Mare observe that there is a lag between the time a worker arrives in a new city and the time her productivity has risen to the local average level. In our view this productivity fillip turns on an urban-ness factor, as with more intensive person to person interaction, rather than on an explicit scale factor linked to say a dominant industry, such as our export-good sector. There is a voluminous literature on so-called agglomeration economies which make it efficient for people to cluster in high densities in urban areas in seemingly productive arrangements. We do not pause at this time to delve into this valuable literature. We adopt a somewhat black-box approach: the productivity of labor in our export sector rises with city size. In our simulations we take the productivity shift as exogenous and proceed to back out the change in city-size that goes along with the shift in question. We note however that there are most probably industry composition effects at work here. Glaeser and Gyourko (2005) document that larger cities typically have higher proportions of residents with more years of schooling, including post-secondary education. In addition, larger cities have somewhat different mixes of types of industries and thus have somewhat different mixes of worker-residents. Each of these composition effects would make a basic "larger city, higher wage" effect somewhat harder to tease out of the data. However, there is no doubt about the ubiquitous nature of this "larger city, higher wage" effect for very similar workers (similar in terms of training and experience).

Thirdly, we introduce a local government sector. In the city, a government is assumed to be producing a pure Samuelson public good and the cost is funded by a benefit charge per household. This variant of our equilibrium city can be treated as a benchmark in the sense that one presumes that no city is in reality offering one or more pure public goods and is setting charges in accord with marginal benefits.

2 The Basic Model

Our city's export sector has a fixed coefficient technology:

$$K_x = a_K q_x$$
$$N_x = a_N q_x$$

for K_x and N_x inputs of capital and labor to the production of q_x of the export good. a_K and a_N are technical coefficients (constants).[4] Equilibrium in this sector requires that

$$p_x * q_x = rK_x + wN_x.$$

p_x is the exogenous "world" price for the export good. Alternatively, we have $p_x = ra_K + wa_N$.

The local good has substitutability in production, with a constant returns to scale production function $g(.)$,

$$q_d = g(K_d, N_d)$$

and equilibrium requires

$$\frac{g_{K_d}}{g_{N_d}} = \frac{r}{w}$$

and

$$p_d * q_d = rK_d + wN_d.$$

Food, q_c is imported at "world" price, UNITY. Exports earnings fund the cost of imports. Hence

$$p_x * q_x = q_c.$$

We can substitute above ("discard" $p_x * q_x = rK_x + wN_x$) to get

$$q_c = rK_x + wN_x.$$

[4] We take a fixed coefficient technology for the production of our export good because it is easy to incorporate a productivity change for labor.

The size of the city is open at this point. We introduce the open city assumption. The OPEN CITY assumption requires that the utility achieved by a resident in our city be at the exogenously given "world" level. That is,

$$u(\frac{q_c}{N_x+N_d}, \frac{q_d}{N_x+N_d}, \frac{L}{N_x+N_d}) = \overline{u}$$

where L is the total land occupied by our city and $\frac{L}{N_x+N_d}$ is "housing" per worker.[5] Given utility function, $u(.)$ for each worker-family in our city, we require that

$$\frac{u_{\frac{q_c}{N_x+N_d}}}{u_{\frac{q_d}{N_d+N_d}}} = \frac{1}{p_d}$$

where $u_{\frac{q_c}{N_x+N_d}}/u_{\frac{q_d}{N_d+N_d}}$ is the ratio of marginal utilities and $1/p_d$ is the ratio of price for a unit of q_c and q_d respectively. Demand for housing comes from the analogous relation

$$\frac{u_{\frac{q_c}{N_x+N_d}}}{u_{\frac{L}{N_d+N_d}}} = \frac{1}{p_L}.$$

Land for the city, in a textbook urban model, is typically assumed to be bidding at the spatial margin from agriculture, the latter available with exogenous price r_A. The essence of a city is that, given requirements of internal mobility, average land rent increases with L the amount of urban land "in use". Hence we have the land supply function

$$p_L = 0.1 * L^{0.75}.$$

To actually solve our model, we specify an explicit production function ($a_K = 0.6$ and $a_N = 0.195$, $q_x = K^{0.6}N^{0.4}$, $q_d = K^{0.4}N^{0.6}$) and a utility function ($u = [\frac{q_c}{N}]^{0.1}[\frac{q_d}{N}]^{0.2}[\frac{L}{N}]^{0.7}$). We set $r = 3$, and p_x =.2458/.1232.

[5]It is not unusual in urban economics to set out a model with housing simply as land per resident, as we are doing. It is also straight forward here to have housing locally produced with capital and land. We experimented with this in our simulation work but backed off because the solution algorithm in Matlab was not yielding useful outputs. To get our solutions we had to experiment with various vectors of initial conditions in order to finally get the algorithm to solve successfully.

We then have a ten equation system.[6] We can solve for q_x, q_c, q_d, p_d, K_x, N_x, K_d, p_L, w and L (we define $N_d = N - N_x$). (We report on a completed computer run using Matlab software in Appendix 1.) Output from our base run ($a_N = 0.195$) are reported in the first line of Table 1.

We then perturb a_N (change it to 0.190) and resolve with w satisfying

$$w' = (p_x - a_K r)/a'_N.$$

Outputs from this second run are reported in the second line of Table 1.

Table 1

	q_x	K_x	N_x	q_d	K_d	N_d	q_c	p_d	N
a_N	.1232	.0739	.0240	.2804	.2169	.3327	.2458	3.5074	.3567
a'_N	.1354	.0813	.0257	.3033	.2384	.3561	.2702	3.5640	.3818

con'd	w	L	p_L	Lp_L	q_c/N	q_d/N	L/N	Lp_L/N
a_N	1.0	4.1941	.2931	1.2292	.6891	.7861	11.758	3.4460
a'_N	1.027	4.4269	.3052	1.3511	.7077	.7944	11.595	3.5388

With labor more efficient ($a'_N < a_N$), the wage rises in the "new" city. That is The city has expanded both in terms of population and land area. The price of the domestic good is higher (produced with a higher wage for workers) and the representative household in the larger city is consuming somewhat more of q_c and q_d and less of L, "housing". Hence the city is larger but denser with labor more productive in the export sector. Housing cost per household has risen from 3.4460 to 3.5388, in spite of each household consuming somewhat less "housing". Roughly speaking the Law in operation is: larger cities are more costly on average to live in (land rents for residences are higher) and costless migration

[6] These problems were each somewhat tricky to get Matlab to solve. Solutions were sensitive to both initial values for the algorithm to work from as well as to parameter choices. We proceeded in two stages. We left parameter p_x out of the solution and set L and w at "convenient" exogenous values. We then solved the reduced problem. Given the solution, we (a) constructed an land supply function of the form $p_L = AL^B$ (i.e. selected values for A and B that satisfied the function) (b) "extracted" a value for p_x equal to q_c/q_x and (c) endogenized w. Then we proceeded to solve the un-reduced problem and it would give outputs for w, p_L, and L very close to those observed for the reduced problem. In this way we could get Matlab's algorithm to solve our relatively large non-linear systems.

of similar workers requires that average wages be higher in the larger cities. The thread tying these matters together is that for two similar workers, the one in the larger city is more productive and commands a higher wage as a consequence. In the words of Van Nieuwerburgh and Weill (2010): "House price differentials between metropolitan areas compensate for the income differential of the marginal, lowest ability household in the location, making that household indifferent between staying and moving to the next best metropolitan area. Households also live in smaller and more expensive quarters if they choose to work in higher income metropolitan areas. Lastly, higher income metropolitan areas have on average a larger housing stock and a larger workforce." (p. 1568). Van Nieuwerburgh and Weill part from our view somewhat and argue that a black-box sorting mechanism leads to on-average more productive workers residing in the on-average larger cities. We on the other hand follow Glaeser and Mare (2001) and argue that it is an urbanness factor that takes a new worker and makes her more productive when she is working and residing in a larger city. It would of course be of great interest to sort out what sort of mechanism is driving the empirical fact: larger cities have on average more productive workers.

We have the following "national" account for our city. Row entries sum to the value on the right and column entries sum to the value in the bottom row.

Table 1: Account Matrix

rK_x	wN_x		$= p_x q_x$ $(=q_c)$
rK_d	wN_d		$= p_d q_d$
		$Lp_L(L)$	$= Lp_L(L)$
$= rK$	$= wN$	$= Lp_L(L)$	

The right column sums to the flow value of product for our city and the bottom row sums to the flow value of inputs. All inputs are assumed to be owned locally. Recall that p_x, r and w are assumed to be parameters to our city. The supply schedule for urban land is parametric as is the system utility level, $\overline{u}$. This assumption on prices and on the exogenous utility level makes our model of a city a particular case of a small open economy.

In an earlier period, our export sector would be referred to as the "basic sector" of our urban economy and our local goods sector would be referred to as the "non-basic" sector. Our model is of course abstracting from the internal structure of a city. A standard approach here is to assume that the export sector is located in the center of the city (in the central business district of CBD) and that residential and other activity surrounds the CBD. Explicit commuting by residents to the place of work, at an explicit cost per trip, allows one to develop an internal structure of the city. We have not delved into the matter of a household's commuting cost or each household's detailed budget constraint to this point. We are assuming however that all inputs are owned locally in equal amounts by each worker-household. That is, income per household is assumed to be $[rK + wN + Lr(L)]/(N_x + N_d)$. Another simplification is that there is only one type of worker-household in our economy. All residents are homogeneous in our city. In addition, a considerably more complicated theory would have migration responding to lifetime utility achievable in various places and thus we are approximating lifetime utility with current single-period utility. Of course if a worker's year by year utility level does not change much over a lifetime then the single-period value becomes a very good proxy for the lifetime value.

The distinctive urban-ness of the above model is that city size is linked to the level of the local wage. There is much evidence that two very similar workers in two cities of different sizes will each get a distinctive wage, the higher wage associated with the larger city. Roughly speaking a worker in a larger city becomes a more productive worker because of subtle effects of interacting with other residents and ends up getting a higher wage as a result. We have illustrated this property with our model. The worker's utility level does not change under the pertubation (it is fixed in the system of cities) and so the higher wage must be "hammered" by various effects that rule out the worker getting onto a higher indifference curve. The obvious "hammering" has L/N decline under the wage increase. This appears to be a fundamental process of urban system equilibrium: larger cities have higher average wages for very similar workers but they are also on average more costly to live in (have higher average land values, essentially) and hence "housing" per worker is smaller in larger cities. Roughly speaking, as a city expands its population will be increasing faster than its spatial footprint is increasing. Observe also that we limit the productivity shift to one sector alone, namely that producing the city's export good. This approach means that other sectors are "stuck" with the local high wage that emerges when labor in the export sector experiences a productivity boost.

Baumol (1967) argued that productivity in a city rose steadily in some sectors but not in local government or in live entertainment. He inferred that wages would rise in these static sectors as wages rose in sectors experiencing technical progress and that local governments and organizations like symphony orchestras would be faced with rising costs, a "cost disease". Our model has productivity rise exogenously in the export sector and in so doing drive up wages throughout the city, somewhat the way Baumol argued things were in fact happening in the 1960's. We note however that once a real city is large and exhibits high wages, it will not be attractive to certain "low wage" activities. Hence a composition effect can be linked to cities with high wages.[7]

[7] Recently Baumol is arguing that the rapidly rising costs of education and health care provision are plagued by his "cost disease", essentially a failure to have labor productivity rise as fast as wages.

3 Climate Amenity Premium

There has been some divergence of opinion on local amenity superiority and local wage premia. For example how much of a local amenity premium, like more sunshine per year in a certain place, ends up in a local cost of living premium rather than a local wage premium? We can perturb our base case by adding an amenity premium to our city and examining changes in city size and other local economic magnitudes. With the wage rate, the rental rate on capital and the price of exports and imports exogenous and unchanging, we cannot observe any price effects of an amenity change. But we do see our city expand in both population and spatial footprint and we do see L/N decline. The rent on land rises with L. Hence most of the amenity premium is showing up in a higher local "cost of living" (higher "housing" cost or land rent cost) and also in some small changes in q_c/N, q_d/N and L/N. We take our original utility function as $Au(.)$, with $A = 1$ and then introduce a local climate amenity to say another similar city by setting $A = 3/2.93$. Hence $u'(.) = \frac{3.00}{2.93}u(.)$. With our base case, our parameter choice is as above and $A = 1$. We solved the ten equation system (see the Appendix for the model as a Matlab program) with results in the first line of Table 2. We then set $A = \frac{3.00}{2.93}$, indicating a locally superior climatic amenity. Our result are in the second line of Table 2.

Table 2.

	q_x	K_x	N_x	q_d	K_d	N_d	q_c	p_d	N
u	.1235	.0741	.0241	.2809	.2174	.3333	.2464	3.5088	.3574
u'	.1294	.0777	.0252	.2944	.2279	.3493	.2583	3.5088	.3745

con'd	w	L	p_L	Lp_L	q_c/N	q_d/N	L/N	Lp_L/N
u	1.0	4.2	.2934	1.2322	.6894	.7860	11.75	3.4477
u'	1.0	4.314	.2993	1.2913	.6897	.7861	11.52	3.4480

Observe that prices are unchanged. The representative household consumes marginally more of q_c and q_d and significantly less "housing", L in the city with the better climate. The city with the better climate is larger both in terms of population and land area, but exhibits somewhat of a higher density of residents. Local land rent for the city has increased to 1.2913 dollars from 1.2322 dollars, a rise of 4.8%. These results capture the idea that cities in sunnier places are more expensive to live in (have higher "housing" costs per resident) and the "free" extra sunshine is "priced" in a local land rent premium (higher local "cost of living").[8]

Our wage was not free to move in this experiment. We know however that city largeness leads to labor productivity increase: here our city has expanded by about 4.8% in terms of population (co-incidently the same value as land rent increase) and this, given our earlier calculations, should result in a wage rise of about 1.9%. Hence most of the climate amenity is accounted for in the land rent increase but a smaller amout is showing up indirectly in a local wage premium. Given a 1.9% increase in the wage and a 4.8% increase in population (labor force), the wage bill would be rising slightly more than the land rent increase, but nevertheless, the two increases would be similar. This is probably not a co-incidence. Given the exogenous utility constraint, an exogenous increase in the wage for a city will yield a larger city and much of the wage bill increase overall often ends up in the overall land rent increase for the city. We have explored this elsewhere in a model of a monocentric city.

4 A Local Public Good (Local Government)

There is a large stream of research on the provision of local government services and on various ways of raising revenue to finance the selected vector of supplies.[9] Certainly the issue of a worker migrating in response to a "fiscal dividend" looms large in the public finance research

[8]Haurin (1980) admits a climatic effect on production efficiency in a city as well as directly on a resident's utility level.

[9]On public goods provision in distinct locations, seminal papers are Flatters, Henderson and Mieszkowski (1974) and the follow-up paper by Hartwick (1980).

on cities.[10] Our model is easily extended to incorporate a local public good in the sense of Samuelson (1954). Samuelson set out a general model in which each citizen was paying for the local public good (flow q_g of government output consumed equally by each citizen) in accord with her marginal benefit. He was well aware that this solution to the government sector provision problem was very difficult to implement in practice. We will proceed the way Samuelson did and simply set the equilibrium out and leave the issue of implementation un-addressed at this time.[11]

In our computer solving we alter our export sector to a production function with substitutability. Thus we now have

$$q_x = h(K_x, N_x)$$

produced under constant returns to scale with inputs K_x and N_x. Production efficiency requires that

$$\frac{h_{K_x}}{h_{N_x}} = \frac{r}{w}$$

and

$$p_x = rK_x + wN_x.$$

Our government good q_g is taken to be produced with fixed coefficients in

$$q_g a_K = K_g$$
$$q_g a_N = N_g$$

and we have

$$p_g a_K = r$$
$$\text{and } p_g a_N = w.$$

[10]See for example Wildasin (2002).

[11]O'Sullivan (2009) has very good sections on public good provision for cities and alternatives to the Samuelson approach.

We expand the utility function of our representative worker-household to include the flow of government services, q_g. Hence we now have

That is,

$$u(\frac{q_c}{N}, \frac{q_d}{N}, \frac{L}{N}, q_g) = \overline{u}$$

with

$$\frac{u_{\frac{q_c}{N}}}{u_{\frac{q_d}{N}}} = \frac{1}{p_d},$$

and

$$\frac{u_{\frac{q_c}{N}}}{u_{\frac{L}{N}}} = \frac{1}{p_L},$$

and demand for q_g in

$$\frac{N u_{q_g}}{u_{\frac{q_c}{N}}} = \frac{p_g}{1},$$

for $N = N_x + N_d + N_g$. This latter is the Samuelson condition, expressing the sum of marginal benefits equal to p_g in the determination of equilibrium supply of q_g. u_g is the marginal benefit for one worker-household. Each household's equilibrium charge for "consuming" current product q_g is to be set at $\frac{u_{q_g}}{u_{\frac{q_c}{N}}}$. Then current revenue for the government is $q_g N \frac{u_{q_g}}{u_{\frac{q_c}{N}}}$. Current cost of producing q_g is of course, $q_g p_g$. Our system for equilibrium in our city above now has four additional equations in q_g, p_g, K_g, and N_g. There are 15 equations in q_x, q_c, q_d, p_d, K_x, N_x, K_d, N, q_g, p_g, K_g, N_g, w, p_L, and L. See Appendix 2 for the Matlab program for solving. Our parameters are $q_x = K_x^{0.55} N_x^{1-0.55}$, $q_d = K_d^{0.2} N_d^{1-0.2}$, $q_g * 0.6 = K_g$, $q_g * 0.195 = N_g$, $r = 1.5$, $u = \frac{q_c}{N}^{0.08} \frac{q_d}{N}^{0.2} q_g^{0.6} \frac{L}{N}^{0.12}$, p_x =.5157/.2073 and $\overline{u} = 3.0$. The equilibrium solution values for the model are

q_x	K_x	N_x	q_d	K_d	N_d	q_c	p_d	N	q_g
0.2073	0.1891	0.2320	0.7021	0.1032	1.1341	0.5158	1.8364	4.3896	15.5049

K_g	N_g	p_g	p_L	L	w
9.3030	3.0235	1.0951	0.1887	4.0996	1.0005

.

Our account matrix for this expanded model is

Table 2: Account Matrix

rK_x	wN_x		$= p_x q_x \ (=q_c)$
rK_d	wN_d		$= p_d q_d$
		$Lp_L(L)$	$= Lp_L(L)$
rK_g	wN_g		$= q_g p_g \ (=Nq_g \frac{u_{q_g}}{u_{\frac{q_c}{N}}})$
$= rK$	$= wN$	$= Lp_L(L)$	

A household is said to re-locate to community i in response to a local fiscal dividend when the move is motivated by a level of government services available in community i at a charge less than the benefits the mover is assigning to the government services in question. Our Samuelson framework has no gaps between a worker-household's benefits from consuming q_g and the charges being assessed. Hence there are no issues of fiscal dividends to consider here. When charges for local government services are based on the market value of a resident's home, it is easy to have a gap open up between assessed charges for q_g and individual dollar benefits. Then the analysis of fiscal dividends becomes of interest. We emphasize that re-location in our framework depends of one's overall utility level attainable in a place, not on one component of an individual's utility level.

5 Concluding Remarks

We have drawn together diverse strands of the urban economics literature into a unified framework, one that is a particular instance of the classic model of a small open economy. When the economy in question is a city rather than a nation, one must amend the basic trade model in certain ways in order to accommodate "standard assumptions" about cities. Of particular interest is utility-level "taking" by a worker free to select her place of work and residence. Utility level "taking" becomes a key part in the determination of equilibrium size for a city. In our view the complementary part is the fact that local land rent is rising with city size. We are able to proceed largely as if neither explicit scale economies in production, nor externalities between close-by firms are central to the question of equilibrium size for our city. We end up with a simple framework for exploring city size and local wage levels, a subject of considerable current exploration. Our framework also allows for an easy look at the question of city size and local climate amenities. And certainly novel in a small open economy model is our introduction of the production of a local pure public good. From the perspective of n equations in n unknowns, our models are fairly complicated but have the merit of being numerically solvable with Matlab.

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APPENDIX 1: Matlab Program for Labor Efficiency Perturbation

```
.............
function f=nlso(x)
% feb 10/13; Small, open economy CITY, housing is land and Land
supply
% depends on price, pL in pL=0.1*L^(0.75);
ax=0.6;r=3;ad=0.4;au1=0.1;au2=0.4;A=1.0;ubr=3.0;
ak=0.6;an=0.195;
%
px=.2458/.1232;
qx=x(1);
Kx=x(2);
Nx=x(3);
qd=x(4);
Kd=x(5);
pd=x(6);
```

```
qc=x(7);
N=x(8);
L=x(9);
w=x(10);
%
%
f(1)=qx*ak-Kx;
f(2)=qx*an-Nx;
f(3)=r*Kx+w*Nx-qc;
f(4)=qd-Kd^ad*(N-Nx)^(1-ad);
f(5)=(ad/(1-ad))*((N-Nx)/Kx)-(r/w);
f(6)=r*Kd+w*(N-Nx)-(pd*qd);
f(7)=au1*qd*pd-qc*au2;
f(8)=ubr-(qc/N)^au1*(qd/N)^au2*(L/N)^(1-au1-au2);
pL=0.1*L^(0.75);
f(9)=au1*L*pL-(1-au1-au2)*qc;
f(10)=((px-r*ak)/an)-w;
%
% ......................
% perturbation to labor efficiency:
% labor efficiency from 0.195 to 0.190, hold px constant, wage rises
% qx=0.1354 Kx=0.0813 Nx=0.0257 qd=0.3033 Kd=0.2384 pd=3.5640 qc=0.2702
% N=0.3818, larger population
% more L=4.4269 New wage=1.0270
%
% Monday Feb 11, success:
% x0=[1,.2,1,.2,1,.2,1,.2,1];
%
% Equation solved with wage exog at 1.0 and labor efficiency at 0.195
```

% qx=0.1232 Kx=0.0739 Nx=0.0240 qd=0.2804

% Kd=0.2169 pd=3.5074 qc=0.2458 N=0.3567

% L=4.1941

APPENDIX 2: Program WITH A LOCAL PUBLIC GOOD

....

```
function f=nlupg(x)
% feb 10/13; Small, open economy CITY, housing is land and Land
supply
% LOCAL PUBLIC GOOD AND SAMUELSON CHARGES
% land supply depends on price, pL
ax=0.55;r=1.5;ad=0.2;ag=0.7;au1=0.08;au2=0.2;au3=0.6;ubr=3.0;
ak=0.6;an=0.195;
% w=1.0;L=4.1;
px=.5157/.2073;
qx=x(1);
Kx=x(2);
Nx=x(3);
qd=x(4);
Kd=x(5);
pd=x(6);
qc=x(7);
N=x(8);
qg=x(9);
Kg=x(10);
Ng=x(11);
pg=x(12);
pL=x(13);
L=x(14);
w=x(15);
%
```

```
f(1)=qx-Kx^ax*Nx^(1-ax);
f(2)=qc-r*Kx-w*Nx;
f(3)=(ax*Nx)/((1-ax)*Kx)-(r/w);
f(4)=qd-Kd^ad*(N-Nx-Ng)^(1-ad);
f(5)=(ad/(1-ad))*((N-Nx-Ng)/Kx)-(r/w);
f(6)=r*Kd+w*(N-Nx-Ng)-(pd*qd);
f(7)=au1*qd*pd-qc*au2;
f(8)=ubr-(qc/N)^au1*(qd/N)^au2*qg^au3*(L/N)^(1-au1-au2-au3);
% pL=0.1*L^(0.75);
f(9)=au1*L*pL-(1-au1-au2-au3)*qc;
f(10)=qg*ak-Kg;
f(11)=qg*an-Ng;
f(12)=pg-an*w-ak*r;
f(13)=au1*qg*pg-N*au3*qc;
f(14)=pL-0.0655*L^(0.75);
f(15)=px-(qc/qx);
% Tues solved... ...
%
% qx=0.2073 0.1891 0.2320 qd=0.7023 0.1031 pd= 1.8357 qc= 0.5157
% N=4.3899
% qg= 15.5048 9.3029 3.0234 pg= 1.0950 pL= 0.1887
%
% wednesday:
% qx=0.2073 0.1891 0.2320 qd=0.7021 0.1032 pd=1.8364 qc=0.5158
% N=4.3896 qg=15.5049 9.3030 3.0235 pg=1.0951 pL=0.1887 L=4.0996
w=1.0005
```