Climate Change, Border Tax Adjustments, and the Green Paradox

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Abstract

A major problem in coordinating international efforts in combatting climate change is that some countries have incentives to free ride: they can capture the benefits without incurring the costs. In the context of international trade, the free riding countries enjoy an additional advantage: without imposing a carbon tax (or equivalent measures), the competitive positions of their firms will be enhanced. Concerns over the loss of competitive advantage due to free riding behaviour of trading partners have lead economists and policy makers to contemplate proposals for various forms of "border tax adjustments" (BTA), defined as differential taxation of traded goods that is motivated by differences in underlying carbon prices. In particular, they consider the possibility of setting a charge on imports equal to some notion of carbon tax 'not paid' abroad, and remitting tax on exports in similar fashion. BTA has been considered as a possible mechanism to combat "carbon leakage", defined as the phenomenon where the effort of abating countries are offset to some extent by increasing emissions in nonabating countries. Our paper presents a model of BTA in the context of fossil fuel extraction when a backstop technology is available. We study the impact of border tax adjustments (BTA) when owners of fossil fuel stocks adjust their extraction paths to maximize their wealth, and where the rate of interest is endogenously determined. Using this framework, we investigate the possibility of a Green-Paradox outcome when BTA is implemented. The term Green Paradox was coined by Sinn (2008), in his analysis of intertemporal supply behavior of extractive firms. It refers to the possibility that, due to intertemporal supply side adjustment, a policy measure intended to reduce CO_2 emissions might have the opposite effect. This phenomenon has been further explored in a series of papers investigating various channels through which a Green-Paradox outcome might arise. Our paper identifies a new channel: a policy measure intended to reduce carbon leakage may indirectly induce a change in the equilibrium interest rate which in turn increases extraction in the near future.

JEL-Classification:

Keywords: Resource extraction, international trade, general equilibrium.

1 Introduction

A major problem in coordinating international efforts in combatting climate change is that some countries have incentives to free ride: they can capture the benefits without incurring the costs. In the context of international trade, the free riding countries enjoy an additional advantage: without imposing a carbon tax (or equivalent measures), the competitive positions of their firms will be enhanced. Concerns over the loss of competitive advantage due to free riding behaviour of trading partners have lead economists and policy makers to contemplate proposals for various forms of "border tax adjustments" (BTA). Advocates for BTA include economists such as Stiglitz (2006) and Tirole (2009). Keen and Kotsogiannis (2011) define BTA as differential taxation of traded goods that is motivated by differences in underlying carbon prices. In particular, they consider the possibility of "setting a charge on imports equal to some notion of carbon tax 'not paid' abroad, and remitting tax on exports in similar fashion." (p. 2). BTA has been considered as a possible mechanism to combat "carbon leakage", defined by Eichner and Pethig (2011) as the phenomenon where "the effort of abating countries will be offset to some extent by increasing emissions in nonabating countries." (p. 767).

The focus of Keen and Kotsogiannis (2011) is on cooperative design. While the authors develop a general equilibrium model, they admit that "the fossil fuels whose use carbon emissions arise are not considered explicitly" because their interest is "not in their pricing" (p.4). Non-cooperative models involving trade and pollution have been developed by Chichilnisky (1994), Copeland and Taylor (1994, 1995, 2005), and Ishikawa and Kiyono (2006).[1] In particular, Copeland and Taylor (2005) focus on the relationship between global warming and international trade. Ishikawa and Kiyono (2006) use a static trade model to show that domestic control of emissions may not reduce aggregate emissions due to carbon leakages. While these papers offer significant insights on the relationship between pollution, trade, and carbon leakage, they did not consider the link between CO_2 emissions and the burning of fossil fuels which are extracted from finite reserves. Among the few analytical studies that deal with carbon leakage with an explicit recognition of intertemporal optimizing behavior of owners of depletable resources, Eichner and Pethig (2011) formulated a two-period model, while Grafton, Kompas, and Long (2010, 2012) analyzed a two-country model in continuous time. However, the rate of interest plays at

[1] In addition, the case of a small open economy was considered by Copeland (1994). Hoel (1996) addressed an important second best scenario.

most a minor role in these models. Eichner and Pethig (2011, p.771) assumed that the market rate of interest is zero, while Grafton, Kompas, and Long (2010, 2012) assume that utility is linear in the numeraire good, and thus the rate of interest is identical to the exogenous rate of utility discount.

The purpose of this paper is to study the impact of BTA in an international trade model where owners of finite resource stocks adjust their extraction paths to maximize their wealth, and where the rate of interest is endogenously determined. Using this framework, we investigate the possibility of an Green-Paradox outcome. The term Green Paradox was coined by Sinn (2008) who emphasizes the intertemporal supply response of resource owners. It refers to the possibility that, due to dynamic supply response, a policy measure intended to reduce CO_2 emissions might have the opposite effect. This phenomenon of adverse intertemporal adjustment has been further explored in a series of papers, including Hoel (2008,2011), Grafton, Kompas, and Long (2010, 2012), Ploeg and Withagen (2010), Gerlagh (2011), Eichner and Pethig (2011), Bretschger and Smulders (2012), among others. These papers investigated various channels through which Green-Paradox outcome might arise. Our paper identifies a new channel: a policy measure aimed at influencing the extraction path may indirectly induce a change in the equilibrium interest rate which in turn induces a change in the time path of extraction in the opposite direction. In particular, a border tax adjustment may result in more global emissions, or may bring climate chnage damages closer to the present.

We will use a simple two-period model which is sufficient to generate an equibrium interest rate response to policy changes. We assume that energy can be obtained from fossil fuels (an exhaustible resource) and from a clean renewable resource, which is a perfect substitute for the fossil fuels. It turns out that a Border Tax Adjustment scheme has an effect that is similar to an increase in the efficiency of producing the clean energy. Therefore we will at first consider a closed-economy model and investigate the possibilily of a Green Paradox outcome driven by an endogenous change in the equilibrium interest rate that arises from an increase in the efficiency of producing the clean energy. To focus on the role of the interest rate, our model is constructed such that the price of energy in terms of the final good is constant across the two periods.

After showing the possibility of a Green Paradox outcome due to interest rate response in a closed-economy model, we proceed to consider a two-country version of the model, in which one country imposes a BTA scheme. We show that a similar Green Paradox outcome also arise, due to the endogenous interest rate response to BTA,

2 The basic model: a closed economy

We employ a two period model of resource extraction. In order to focus on the role of the backstop technology, we assume that all markets are perfectly competitive. In any period $t, t \in \{1, 2\}$, a composite good Y_t is produced using energy, E_t, and a composite factor (capital), denoted by K_{y_t}, where the subscript denotes the use of the factor. Energy can be obtained from fossil fuel R_t (extracted from a resource stock S_t) and from renewable resources, such as wind, solar, or biofuels, denoted by x_t, and both are perfect substitutes such that $E_t = R_t + x_t$. The backstop technology is able to convert the composite good into renewable resources, and thus we have to distinguish between gross and net output. The gross output of the composite good is $Y_t = F(E_t, K_{y_t})$, and we assume that $F(E_t, K_{y_t})$ is a neoclassical production function and homogeneous of degree 1. Defining the energy intensity $e_t \equiv E_t/K_{y_t}$ we can rewrite production and marginal productivities as follows

$$Y_t = K_{y_t} f(e_t), F_{E_t} = f'(e_t), F_{K_{y_t}} = f(e_t) - e_t f'(e_t).$$

The composite good is the numeraire in our model. The backstop technology allows to convert the composite output into (renewable) energy, denoted by x. To produce one unit of x in period t one must use λ units of the composite good where λ is the technological parameter. Its inverse, $1/\lambda$ is a measure of the efficiency of the backstop technology. If both x and the fossil fuel are used by firms, λ is also the price of non-fossil fuels, in terms of the composite good. Consequently, the net output of the composite good, intended for consumption and denoted by Q is equal to

$$Q_t = Y_t - \lambda x_t = F(R_t + x_t, K_{y_t}) - \lambda x_t$$

We assume that the production function is of Cobb-Douglas type such that $f(e_t) = e_t^{\beta}$ with $0 < \beta < 1$. We are interested in the general equilibrium effects of the backstop technology on resource extraction, and we also want to take into account that resource extraction is subject to rising extraction costs. In our general equilibrium context, to extract the quantity R_t, the representative extracting firm must use the composite factor, and we denote by K_{R_t} the use of this factor in resource extraction. In order to model potential rising extraction costs, we suppose that, to extract any amount $R_2 > 0$, the amount of the composite factor required is dependent on how much was extracted in period 1. This assumption is meant to capture the salient features of fossil fuels: deposits are not homogeneous and lower-cost layers are extracted first. The input requirements for the two periods are given by

$$\begin{aligned} K_{R_1} &= \frac{1}{2} R_1^2, \\ K_{R_2} &= \theta R_1 R_2 + \frac{1}{2} R_2^2, \end{aligned} \tag{1}$$

where $\theta \in [0, 1]$ measures the strength of the increase in extraction costs. We close the supply side of model by the factor market clearing condition for the composite factor and the stock-flow condition that the sum of extractions over the two periods must not exceed the available stock S:

$$\begin{aligned} K_{y_t} + K_{R_t} &= K, \\ R_1 + R_2 &\leq S, \end{aligned}$$

where K is a constant in both periods. The demand side of the model is given by the behavior of a representative consumer who maximizes her intertemporal utility

$$U(c_1) + \frac{1}{1+\rho} U(c_2) \text{ subject to } c_1 + \frac{1}{1+i} c_2 = W_0$$

where W_0 denotes the initial wealth (the capitalized value of her income stream) and $\rho > 0$ denotes the rate of time preference. U is the per period utility with $U' > 0, U'' < 0$. In our model, the interest rate i is determined endogenously. To keep the model simple and keep other effects on the interest rate out of our model, we do not allow capital formation (K is fixed) or storing the consumption good which implies that $c_t = Q_t$. Utility maximization leads to

$$\frac{U'(Q_1)}{U'(Q_2)} = \frac{1+i}{1+\rho}. \tag{2}$$

The model can now be solved by scrutinizing the optimal behavior of resource owners and consumers over time. Clearly, dependent on parameter values, it is possible that in equilibrium resources are never extracted or the backstop technology is never used. In order to deal with realistic cases, we confine the analysis to the case in which both the resource and the backstop technology are used in both periods. Note carefully that this does not mean that the resource will be completely exhausted or not, so we have to distinguish these two cases.

In both cases, the final good producers use x_t as an input in both periods $t = 1, 2$, and consequently it must be the case that the marginal product of x_t is equated to its price, λ:

$$F_{E_t} \equiv f'(e_t) = \beta e_t^{\beta-1} = \lambda \text{ for } t = 1, 2 \implies e_t = \overline{e} = \left(\frac{\lambda}{\beta}\right)^{\frac{1}{\beta-1}}$$

Note that the existence of a backstop technology fixes the resource price and the energy intensity. Furthermore, a decrease in λ is equivalent to an increase in $\overline{e}$. The constant energy intensity also implies that the factor price of the composite factor, denoted by w, is identical in both periods:

$$w_t = f(e_t) - e_t f'(e_t) = (1-\beta)\overline{e}^{\beta} = w.$$

Since the resource is used simultaneously with the backstop technology, it must be the case that the final good producers are paying the same price for both energy inputs, i.e., $P_t = F_{E_t} \equiv f'(\overline{e}) = \lambda$. Given the Cobb-Douglas technology, the unit cost function for the composite good is

$$C(\lambda, w) = A\lambda^{\beta} w^{1-\beta}, \text{ where } A \equiv \frac{1}{\beta^{\beta}(1-\beta)^{1-\beta}}.$$

Since the backstop technology will be used in both periods, it works like a limit price for resource owners. Hence, it does not matter whether the resource industry is perfectly competitive or monopolistic. In general, each resource owner takes the price path $(P_1, P_2) = (\lambda, \lambda)$ and the path of the factor prices of the composite factor $(w_1, w_2) = (w, w)$ as given, and chooses R_1 and R_2 to maximize the discounted sum of profit

$$\lambda R_1 - wK_{R_1} + \frac{1}{1+i}\left[\lambda R_2 - wK_{R_2}\right] \text{ subject to (1)}, R_1 + R_2 \leq S \text{ and } R_t \geq 0.$$

Let $\eta \geq 0$ be the Lagrange multiplier associated with the constraint $S - R_1 - R_2 \geq 0$ and assume $R_1 > 0$ and $R_2 > 0$. The first-order conditions imply the Hotelling Rule

$$\lambda - wR_1 - \frac{\theta R_2}{1+i} w = \frac{1}{(1+i)} \left[\lambda - w\left(\theta R_1 + R_2\right)\right] = \eta. \tag{3}$$

This condition states that the present value of marginal profit from extraction in period 2 must be equal to the marginal profit from extraction in period 1 net of its impact on period 2 extraction cost.

Before we discuss the two different cases, complete exhaustion of the resource or no exhaustion, let us consider the determination of the interest rate in more detail. Let us define the input price ratio

$$\delta \equiv \frac{\lambda}{w} = \frac{\lambda}{(1-\beta)\left(\left(\frac{\lambda}{\beta}\right)^{\frac{1}{\beta-1}}\right)^{\beta}}, \frac{d\delta}{d\lambda} = \frac{1}{(1-\beta)w} > 0. \tag{4}$$

Thus an increase in the efficiency of the backstop technology (a fall in λ) can also be measured equivalently by a decrease in δ. We can also rewrite w to see the income effect technological progress has on the factor price of the composite factor,

$$w(\delta) = (1-\beta)^{(1-\beta)} \beta^{\beta} \delta^{-\beta}, \frac{dw}{d\delta} = -\beta \frac{w}{\delta}, \tag{5}$$

which clearly demonstrates that an increase in the backstop technology's productivity has a positive income effect the size of which depends on the importance of the composite factor as measured by β. Using $\overline{e} = (x_t + R_t)/K_{y_t}$, the net output in each period, $Q_t \equiv Y_t - \lambda x_t$, can be expressed as

$$\begin{aligned} Q_t &= [K - K_{Rt}]\, f\left(\overline{e}\right) - \lambda\left[\overline{e}\left(K - K_{R_t}\right) - R_t\right] = [K - K_{R_t}]\left[f\ \left(\overline{e}\right) - \overline{e} f'(\overline{e})\right] + f'(\overline{e}) R_t \\ &= \left(K - K_{R_t}\right) w + \lambda R_t = wK + \left(\lambda R_t - wK_{R_t}\right), \end{aligned} \tag{6}$$

This equation says that net output is equal to the payment to the composite factor plus the pure profit of the resource-extracting sector. We can now rewrite consumer behavior (2) as

$$\Omega \equiv (1+i) U'(w(K + \delta R_2 - K_{R_2})) - (1+\rho) U'(w(K + \delta R_1 - K_{R_1})) = 0 \tag{7}$$

Equation (7) determines the interest rate, and we are now interested how a change in the efficiency affects the interest rate. Suppose that resource extractions stay constant. Differentiation of (7) yields

$$\frac{di}{d\delta}|_{dR_1=dR_2=0} = \frac{(1+\rho)(w\delta R_1 - \beta Q_1)U''(Q_1) - (1+i)(w\delta R_2 - \beta Q_2)U''(Q_2)}{\delta U'(Q_2)} \tag{8}$$

This change gives the direct effect on i of a change in the efficiency of the backstop technology. The sign of the denominator is unambiguously positive, and the sign of the numerator depends on the relative resource extraction and the income effects across the two periods. If resource extractions do not change, an increase in the efficiency of the backstop technology (i.e. a fall in δ) will increase the interest rate if and only if

$$w\delta R_1 - \beta Q_1 > (w\delta R_2 - \beta Q_2)\left(\frac{1+i}{1+\rho}\right)\frac{U''(Q_2)}{U''(Q_1)}.$$

Suppose that $w\delta R_2 - \beta Q_2 < 0$. Then the above condition can be stated as

$$\frac{w\delta R_1 - \beta Q_1}{w\delta R_2 - \beta Q_2} < \left(\frac{1+i}{1+\rho}\right)\frac{U''(Q_2)}{U''(Q_1)}. \tag{9}$$

(If $w\delta R_2 - \beta Q_2 > 0$, then the above inequality must be reversed.) This condition can also be expressed as

$$\frac{\beta - \frac{\lambda R_1}{Q_1}}{\beta - \frac{\lambda R_2}{Q_2}} < \left(\frac{1+i}{1+\rho}\right)\frac{U''(Q_2)Q_2}{U''(Q_1)Q_1} \tag{10}$$

where $\frac{\lambda R_t}{Q_t}$ is the value share of fossil fuel input in national income, and β is the value share of energy in gross output.

Example: Suppose $U(Q_t) = \ln Q_t$. Assume $w\delta R_2 - \beta Q_2 < 0$. Then condition (10) becomes

$$\frac{\beta - \frac{\lambda R_1}{Q_1}}{\beta - \frac{\lambda R_2}{Q_2}} < \left(\frac{1+i}{1+\rho}\right)\left(\frac{Q_2^{-1}}{Q_1^{-1}}\right)$$

The initial consumer equilibrium, eq (7), gives

$$\frac{Q_2}{Q_1} = \frac{1+i}{1+\rho}$$

Then, at unchanged extractions, the rate of interest will rise as a result of a fall in δ iff

$$\frac{\beta - \frac{\lambda R_1}{Q_1}}{\beta - \frac{\lambda R_2}{Q_2}} < 1$$

i.e. iff $R_1/Q_1 > R_2/Q_2$. (Given the assumption that $\beta > \lambda R_2/Q_2$.) This condition implies the technical progress raises the national income in period 2 (at unchanged extractions) by a greater percentage than the national income in period 1.

This is, of course, only the direct effect which has not taken into account that resource extractions will respond to the change in δ and the change in the interest rate. We now scrutinize the optimal resource extraction plans and their dependence on the efficiency of the backstop technology and the interest rate. We will consider two cases of simultaneous use of fossil fuel and renewable energy in both periods. In the first case, the stock of resource is relatively large, so that exhaustion does not occur. The marginal resource rent is zero in that case. In the second case, the sum of extractions in the two periods is equal the initial resource stock S.

In what follows, we are going to prove the following:

Proposition 1 *In any equilibrium with simultaneous use of the backstop technology and resources and exhaustion, the effect of technological progress of the backstop technology on the extraction path is ambiguous. In case of exhaustion, technological progress may lead to a larger resource extraction in the first period. In the case of non-exhaustion, technological progress may also lead to a larger resource extraction in the first period and even to larger aggregate extraction. An increase in the endowment with the composite factor has similar implications.*

Thus, we show that a development which will clearly have a resource saving effect in a static environment may lead to opposite results in a dynamic context.

3 Non-exhaustion of the resource

If $R_1 > 0$ and $R_2 > 0$ and the resource is not exhausted, the marginal resource rent must be zero. This implies that, in the second period,

$$\lambda - w(\theta R_1 + R_2) = 0 \implies \theta R_1 + R_2 = \delta.$$

Note that the second period extraction depends on the first period extraction because a larger extraction in the first period increases the input requirement for extracting the resource in the second period. For the first period, the assumption that $R_1 + R_2 < S$ implies

$$\lambda - wR_1 - \frac{\theta R_2}{1+i}w = 0 \implies \delta - R_1 - \frac{\theta}{1+i}\left[\delta - \theta R_1\right] = 0$$

where we have used the second period optimal extraction. Solving for extractions, we obtain

$$R_1 = \delta\left(\frac{1+i-\theta}{1+i-\theta^2}\right), \tag{11}$$

$$R_2 = \delta\left(\frac{1+i-\theta-i\theta}{1+i-\theta^2}\right), \tag{12}$$

Notice that $R_1 = R_2 = \delta$ if $\theta = 0$, implying $Q_1 = Q_2$ and $i = \rho$ so that consumption levels in the two periods are equal. If $\theta = 1$, $R_2 = 0$. In what follows, we focus on the interesting case where $0 < \theta < 1$, and we find:

Lemma 1 *Assume* $0 < \theta < 1$. *A non-exhaustion equilibrium, with simultaneous use of the backstop technology and resources in both periods, exists if the following conditions are met*

$$S > \delta\left(\frac{2(1+i-\theta)-i\theta}{1+i-\theta^2}\right). \tag{13}$$

$$K > \max\left\{\frac{1}{2}R_1^2, \frac{1}{2}R_2^2 + \theta R_1 R_2\right\} \tag{14}$$

$$\frac{R_1}{K-K_{R_1}} \leq \bar{e} \text{ and } \frac{R_2}{K-K_{R_2}} \leq \bar{e} \tag{15}$$

where R_1 *and* R_2 *satisfy (11) and (12).*

Proof: This follows immediately from $R_1 + R_2 < S$.

Let us assume for a start that the interest rate stays constant. It can then easily be seen from eqs. (11) and (12) that technological progress of the backstop technology, that is, a decrease in δ, will lead to a decrease in both R_1 and R_2. So the good news is that, at constant interest rate, per period and aggregate resource use will decline with an increase in efficiency of the backstop technology. However, we already know from eq. (8)

that the interest rate will change with a change in the efficiency, and this effect will have repercussions on resource extractions. Differentiation of (11) and (12) yields

$$\frac{\partial R_1}{\partial i} = \frac{\delta\theta(1-\theta)}{(1+i-\theta^2)^2} > 0, \frac{\partial R_2}{\partial i} = -\frac{\delta\theta^2(1-\theta)}{(1+i-\theta^2)^2} < 0, \frac{\partial R_1}{\partial i} + \frac{\partial R_2}{\partial i} = \frac{\delta\theta(1-\theta)^2}{(1+i-\theta^2)^2} > 0,$$

and shows that an increase in the interest rate (i) will increase period 1 extraction, (ii) decrease period 2 extraction and (iii) increase aggregate extraction.

Let us now summarize the effects: for a *given* resource extraction plan, an increase in the efficiency of the backstop technology must imply an increase in the interest rate if eq. (9) is fulfilled. At the same time, both periods' extractions decline (by the same proportion) due to the better performance of the backstop technology, potentially scaling down the strength of the effect on the interest rate effect given constant resource extractions. Furthermore, an *increase in the interest rate* increases resource extraction in period 1 and decreases resource extraction in period 2, and increases aggregate extraction.

We will now demonstrate that the effects of technological progress are ambiguous even in our simple model. First, we consider how output levels compare in the two periods which has a clear implication for the interest rate.

Lemma 2 *In case of non-exhaustion, $Q_1 > Q_2$ and $i < \rho$.*

Proof: Due to (6), output levels are given by

$$\begin{aligned} Q_1 &= w\left(K + \delta R_1 - \frac{1}{2}R_1^2\right) \\ Q_2 &= w\left(K + \delta R_2 - \theta R_1 R_2 - \frac{1}{2}R_2^2\right). \end{aligned}$$

Let Π_1 and Π_2 denote the profit of the exhaustible resource sector, i.e. $\Pi_t = \lambda R_t - wK_{R_t}$. Then $Q_1 > Q_2$ iff $\Pi_1 > \Pi_2$.

Compare Q_1 and Q_2. Q_1 divided by w and net of K is equal to Π_1/w, which is

$$\delta\left(\delta\frac{1+i-\theta}{1+i-\theta^2}\right) - \frac{1}{2}\left(\delta\frac{1+i-\theta}{1+i-\theta^2}\right)^2 = \frac{\delta^2(1+i-\theta)\left(1+i+\theta-2\theta^2\right)}{2\left(1+i-\theta^2\right)^2}$$

Similarly, Q_2 divided by w and net of K is equal to Π_2/w, which is

$$\begin{aligned}
\delta\left(\delta\frac{1+i-\theta-i\theta}{1+i-\theta^2}\right) &- \theta\left(\delta\frac{1+i-\theta}{1+i-\theta^2}\right)\left(\delta\frac{1+i-\theta-i\theta}{1+i-\theta^2}\right) \\
-\frac{1}{2}\left(\delta\frac{1+i-\theta-i\theta}{1+i-\theta^2}\right)^2 &= \frac{(1+i)^2\delta^2(1-\theta)^2}{2\left(1+i-\theta^2\right)^2}.
\end{aligned}$$

Hence, $Q_1 > Q_2$ iff

$$\delta^2(1+i-\theta)\left(1+i+\theta-2\theta^2\right)-(1+i)^2\delta^2(1-\theta)^2=\delta^2\theta\left(2(1+i)^2-(2+i)^2\theta+2\theta^2\right).$$

Consider the function $\Psi(\theta)=\left(2(1+i)^2-(2+i)^2\theta+2\theta^2\right)$ with $\Psi'(\theta)=-(2+i)^2+4\theta<0$ so that Ψ has a minimum at $\theta=1$ in the relevant range. Since $\Psi(\theta=1)=i^2>0$, $Q_1>Q_2$. $Q_1>Q_2$ implies $i<\rho$ because of $U''<0$. □

We can now scrutinize the potential ambiguity of technological progress on the resource extraction path. We rewrite the equilibrium conditions as

$$\begin{aligned}
f(\cdot) &= (1+i-\theta^2)R_1-\delta(1+i-\theta)=0, \\
g(\cdot) &= (1+i-\theta^2)R_2-\delta(1+i)(1-\theta)=0, \\
h(\cdot) &= (1+i)U'\left[\underbrace{w\left(K+\delta R_2-\theta R_1R_2-\frac{1}{2}R_2^2\right)}_{Q_2}\right] \\
&- (1+\rho)U'\left[\underbrace{w\left(K+\delta R_1-\frac{1}{2}R_1^2\right)}_{Q_1}\right]=0.
\end{aligned}$$

Total differentiation yields

$$\underbrace{\begin{bmatrix} f_{R_1} & 0 & f_i \\ 0 & f_{R_1} & g_i \\ h_{R_1} & 0 & h_i \end{bmatrix}}_{=A} \begin{bmatrix} dR_1 \\ dR_2 \\ di \end{bmatrix} = \begin{bmatrix} -f_\delta \\ -g_\delta \\ -h_\delta \end{bmatrix} d\delta$$

where

$$f_{R_1} = g_{R_2} = 1 + i - \theta^2 > 0, f_i = R_1 - \delta < 0, g_i = R_2 - \delta(1-\theta) > 0,$$

because $R_1 < \delta$ and

$$R_2 = \delta \frac{(1+i)(1-\theta)}{1+i-\theta^2} > \delta(1-\theta) \Leftrightarrow 1 + i > 1 + i - \theta^2,$$

$$f_\delta = -(1 + i - \theta) < 0, g_\delta = -(1+i)(1-\theta) < 0,$$

$$h_{R_1} = -w\theta R_2(1+i)U''(Q_2) - w(\delta - R_1)(1+\rho)U''(Q_1) > 0$$

because $U'' < 0$,

$$h_{R_2} = w(\delta - \theta R_1 - R_2)(1+i)U''(Q_2) = 0$$

because $\theta R_1 + R_2 = \delta$,

$$h_i = U'(Q_2) > 0,$$

$$h_\delta = \left(wR_2 - \frac{\beta Q_2}{\delta}\right)(1+i)U''(Q_2) - \left(wR_1 - \frac{\beta Q_1}{\delta}\right)(1+\rho)U''(Q_1).$$

Note that h_δ is ambiguous in sign, and that its sign depends, *inter alia*, on the third derivative of the utility function. This is exactly the direct effect of a technology change on the interest rate (see (8)):

$$\frac{di}{d\delta}|_{dR_1=dR_2=0} = -\frac{h_\delta}{h_i}.$$

What is the intuition for this ambiguity? When δ declines, both Q_1 and Q_2 change in complex ways even for given resource extractions. First, both decline via the effect that resource extractions have become relative less productive. Second, the factor price of the composite factor increases, and the strength of this income effect depends on the importance of this factor for production as measured by β. The sign of

$$wR_t - \frac{\beta Q_t}{\delta} = \frac{\lambda R_t - \beta Q_t}{\delta}$$

is ambiguous for both periods and may even differ across both periods. From Cobb-Douglas, we know that the share of all energy input from gross production stays constant, that is, $\lambda E_t = \beta Y_t$, but $E_t > R_t$ and $Y_t > Q_t$ due to the backstop technology. In any case, this effect is positive (negative) if the income effect is weak (strong). At the same time, the marginal utilities change across both periods, and the strength of this effect in each periods depends on the second derivative of the utility function. However, it is not the absolute change which drives this effect but the relative strength as measured by the change of the second derivative, and this is the reason why the third derivative plays an important role as well.

Note that a sufficient condition for $\lambda R_t - \beta Q_t$ to be negative is that K is sufficiently large. To see this, we observe that

$$\frac{\lambda R_t}{Q_t} = \frac{\lambda R_t(i, \delta)}{Y_t - \lambda x_t}$$

where

$$\begin{aligned} Y_t - \lambda x_t &= (K - K_{R_t}) f(\overline{e}) - \lambda \left[E - R_t \right] \\ &= (K - K_{R_t}) f(\overline{e}) + \lambda R_t - \lambda \left(K - K_{R_t} \right) \overline{e} \\ &= (K - K_{R_t}) \left[f(\overline{e}) - \overline{e} f'(\overline{e}) \right] + \lambda R_t \end{aligned}$$

such that $\frac{\lambda R_t}{Q_t}$ decreases with K, and hence a sufficiently large endowment with the composite factor K implies $\frac{\lambda R_t}{Q_t} < \beta$.

Let us now turn to the equilibrium and its properties in more detail. Expanding the matrix A along the first row yields the determinant

$$\det(A) = f_{R_1}(f_{R_1} h_i - f_i h_{R_1}) > 0$$

which proves that the equilibrium is unique. Define

$$A_1 = \begin{bmatrix} -f_\delta & 0 & f_i \\ -g_\delta & f_{R_1} & g_i \\ -h_\delta & 0 & h_i \end{bmatrix}, A_2 = \begin{bmatrix} f_{R_1} & -f_\delta & f_i \\ 0 & -g_\delta & g_i \\ h_{R_1} & -h_\delta & h_i \end{bmatrix}, A_3 = \begin{bmatrix} f_{R_1} & 0 & -f_\delta \\ 0 & f_{R_1} & -g_\delta \\ h_{R_1} & 0 & -h_\delta \end{bmatrix}$$

so that the changes with δ are given by

$$\frac{dR_1}{d\delta} = \frac{\det(A_1)}{\det(A)}, \frac{dR_2}{d\delta} = \frac{\det(A_2)}{\det(A)}, \frac{di}{d\delta} = \frac{\det(A_3)}{\det(A)}$$

according to Cramer's Rule. Since $\det(A) > 0$, the signs of the changes are given by $\operatorname{sign}(dR_1/d\delta) = \operatorname{sign}(\det(A_1)), \operatorname{sign}(dR_2/d\delta) = \operatorname{sign}(\det(A_2)), \operatorname{sign}(di/d\delta) = \operatorname{sign}(\det(A_3))$ where the determinants

$$\begin{aligned} \det(A_1) &= f_{R_1}(-f_\delta h_i + f_i h_\delta), \\ \det(A_2) &= f_{R_1}(-g_\delta h_i + g_i h_\delta) + f_\delta(-g_i h_{R_1}) + f_i g_\delta h_{R_1}, \\ \det(A_3) &= f_{R_1}(-h_\delta f_{R_1} + h_{R_1} f_\delta) \end{aligned}$$

are all ambiguous in sign. In particular, if the direct effect on the interest rate for given resource extractions, measured by h_δ, and the responsiveness of first period resource extractions to the interest rate, measured by f_i, are **sufficiently strong**, first-period extraction will go up with technological progress, that is, $dR_1/d\delta < 0$. In this case, the interest rate effect is so strong that it overcompensates the second countervailing effect which is due to the increased productivity of the backstop technology. If $h_\delta > 0$, however, we also observe that $dR_2/d\delta > 0$ because the interest rate has the opposite effect on the response of second-period extractions to the interest rate ($g_i < 0$). Thus, while first period extraction may increase with technological progress, second period extraction will decrease with technological progress if $h_\delta > 0$. Since we consider the case of non-exhaustion, we may now also explore whether aggregate resource extraction (which is less than S) may increase with technological progress. Can it be the case that the aggregate $R_1 + R_2$ declines with δ? Consider $\det(A_1) + \det(A_2)$ which has the same sign as $d(R_1 + R_2)/d\delta$ and is ambiguous because

$$\det(A_1)+\det(A_2)=\underbrace{f_{R_1}h_\delta(f_i+g_i)}_{-\text{ if }h_\delta>0}+\underbrace{f_{R_1}(-f_\delta)h_i+f_{R_1}(-g_\delta)h_i+f_\delta(-g_ih_{R_1})+f_ig_\delta h_{R_1}}_{+}$$

as

$$f_i+g_i=R_1+R_2-\delta-\delta(1-\theta)=-\frac{\delta(1-\theta)^2\theta}{1+i-\theta^2}<0. \qquad (16)$$

Thus, we find that even aggregate resource extraction may increase with technological progress if h_δ is sufficiently large.

Now suppose K is large enough so that $\left(wR_2-\frac{\beta Q_2}{\delta}\right)<0$ and $\left(wR_1-\frac{\beta Q_1}{\delta}\right)<0$. Then $h_\delta>0$ if the absolute value of $U''(Q_1)$ is small relative to absolute value of $U''(Q_2)$, that is, if U''' is sufficiently positive.

We can also scrutinize the effect of an increase in the composite factor on the resource extraction path. Total differentiation of the equilibrium conditions yields

$$A\times\begin{bmatrix} dR_1\\ dR_2\\ di\end{bmatrix}=\begin{bmatrix}0\\0\\-h_K\end{bmatrix}dK$$

where the derivatives on the LHS are as before and

$$h_K=w\left((1+i)U''(Q_2)-(1+\rho)U''(Q_1)\right)$$

is ambiguous in sign and depends on the third derivative of the utility function as well. If $U'''\le 0$, $U''(Q_2)\le U''(Q_1)$ due to $Q_1>Q_2$, and since $i<\rho$, $h_K<0$ in this case. However, the sign is ambiguous if $U'''>0$. Define

$$C_1=\begin{bmatrix}0&0&f_i\\0&f_{R_1}&g_i\\-h_K&0&h_i\end{bmatrix},C_2=\begin{bmatrix}f_{R_1}&0&f_i\\0&0&g_i\\h_{R_1}&-h_K&h_i\end{bmatrix},C_3=\begin{bmatrix}f_{R_1}&0&0\\0&f_{R_1}&0\\h_{R_1}&0&-h_K\end{bmatrix}$$

so that the changes with K are given by $dR_1/dK=\det(C_1)/\det(A), dR_2/dK=\det(C_2)/\det(A), di/dK=\det(C_3)/\det(A)$. We find that

$$\begin{aligned}\det(C_1) &= f_{R_1} f_i h_K \\ \det(C_2) &= f_{R_1} g_i h_K \\ \det(C_3) &= -f_{R_1}^2 h_K\end{aligned}$$

are all ambiguous in sign and depend on the change of the interest rate with K: the interest rate increases with an increase in K if $h_K < 0$; in this case, resource extraction will increase in the first period and decrease in the second period. The intuition is as follows: if K increase, both Q_1 and Q_2 increase for given R_1 and R_2. It depends on the strength of the U''-effect whether the interest rate must increase or decrease to guarantee that $h(\cdot) = 0$ holds. The difference to technological progress is that a change in endowment depends on U''' only because K has the same marginal effect on Q_1 and Q_2.

Thus also an increase in the composite factor as an alternative to technological progress may lead the an increase in first-period extractions. Furthermore, we find that the change in aggregate extraction, the sign of which is given by

$$\det(C_1) = f_{R_1} h_K (f_i + g_i),$$

can be both negative or positive because $f_i + g_i$ is negative (see eq. (16)). Thus, an increase in the composite factor endowment increases aggregate resource extraction if $h_K < 0$, that is, if the interest rate increases with an increase in K.

4 Exhaustion of the resource

Let us turn to the case of exhaustion, with positive extraction in both periods. In this case, agregate extraction is fixed, but per period extraction will change with technological progress. According to eq. (3), the optimal resource extraction plan is given by

$$R_1 = \frac{S(1-\theta) + i\delta}{2(1-\theta) + i}, \qquad R_2 = S - R_1 = \frac{S(1+i-\theta) - i\delta}{2(1-\theta) + i}. \tag{17}$$

This happens if S is not too large (so that exhaustion will happen) and not too small (so that period 2 extraction is positive). We find:

Lemma 3 *An equilibrium with simultaneous use of the backstop technology and resources in both periods and exhaustion exists if*

$$\frac{i\delta}{1+i-\theta} < S \leq \delta\left(\frac{2(1+i-\theta)-i\theta}{1+i-\theta^2}\right). \tag{18}$$

Proof: For R_2 to be positive, we must have $S(1+i-\theta) > i\delta$. The other constraint simply follows from Lemma 1. □

Note that if $i\theta \geq 0$ and $\theta > 0$ condition (18) implies $S < 2\delta$. In the case of exhaustion, we have $R_1 > R_2$ provided that $i > 0$:

$$R_1 - R_2 = \frac{i(2\delta - S)}{2(1-\theta)+i}.$$

Again, we can show:

Lemma 4 *In case of exhaustion, $Q_1 > Q_2$ and $i < \rho$.*

Proof: Due to (6), output levels are given by as in Lemma 2. Compare Q_1 and Q_2. Q_1 divided by w and net of K is equal to

$$\frac{\delta^2(1+i-\theta)\left(1+i+\theta-2\theta^2\right)}{2\left(1+i-\theta^2\right)^2}$$

Similarly, Q_2 divided by w and net of K is equal to

$$\frac{\left((1+i)(S-3\delta)+(3+2i)\delta\theta-S\theta^2\right)\left((1+i)(-S+\delta)-\delta\theta+S\theta^2\right)}{2\left(1+i-\theta^2\right)^2}$$

Hence, $Q_1 > Q_2$ if

$$\chi(S) \equiv 2S\delta(-2(1+i)+(2+i)\theta)(1+i-\theta^2)+S^2(1+i-\theta^2)^2+2\delta^2(2(1+i)^2-(1+i)(3+i)\theta+\theta^3 > 0.$$

We find that

$$\chi''(S) = 2\left(1+i-\theta^2\right)^2 > 0$$

and thus χ has a minimum at

$$S = \delta \left(\frac{2(1+i-\theta) - i\theta}{1+i-\theta^2} \right)$$

which is the upper bound of the resource stock for the existence of an enhasution equilibrium (see Lemma 3). Furthermore,

$$\chi \left(S = \delta \left(\frac{2(1+i-\theta) - i\theta}{1+i-\theta^2} \right) \right) = \delta^2\theta \left(2(1+i)^2 - (2+i)^2\theta + 2\theta^2 \right) > 0$$

which completes the proof. □

For the case of exhaustion, we have to consider only the first period; opposite changes hold for the second period. Making use of (17) and $S < 2\delta$, we find that

$$\frac{\partial R_1}{\partial \delta} = \frac{i}{2(1-\theta)+i} > 0, \frac{\partial R_1}{\partial i} = \frac{(1-\theta)(2\delta - S)}{(2(1-\theta)+i)^2} > 0, \tag{19}$$

so these derivatives have the same signs as in the non-exhaustion case. First, we observe that, at constant interest rate, an increase in the efficiency of the backstop technology, that is, a decline in δ, will lead to less extraction in period 1 (and an increase in the extraction in the second period). This result differs from the the literature on a backstop technology where the backstop is not used in the current period, see e.g. Hoel (2008) and Gerlagh (2011). Hoel (2008) shows that lowering the cost of the (future) backstop will lead to more current extraction. However, eq. (19) gives only partial effects. To find the full effect of a decline in δ on R_1, we must take into account the adjustment of the equilibrium interest rate. For this purpose, we rewrite the equilibrium conditions as

$$\begin{aligned}
F(\cdot) &= (2(1-\theta)+i)R_1 - i\delta - (1-\theta)S = 0, \\
G(\cdot) &= R_1 + R_2 - S = 0, \\
H(\cdot) &= (1+i)U' \left[\underbrace{w\left(K + \delta R_2 - \theta R_1 R_2 - \frac{1}{2}R_2^2 \right)}_{Q_2} \right] \\
&- (1+\rho)U' \left[\underbrace{w\left(K + \delta R_1 - \frac{1}{2}R_1^2 \right)}_{Q_1} \right] = 0.
\end{aligned}$$

Total differentiation yields

$$\underbrace{\begin{bmatrix} F_{R_1} & 0 & F_i \\ 1 & 1 & 0 \\ H_{R_1} & H_{R_2} & H_i \end{bmatrix}}_{=B} \begin{bmatrix} dR_1 \\ dR_2 \\ di \end{bmatrix} = \begin{bmatrix} -F_\delta \\ 0 \\ -H_\delta \end{bmatrix} d\delta$$

where

$$F_{R_1} = 2(1-\theta) + i > 0, F_i = R_1 - \delta < 0, F_\delta = -i,$$

because $R_1 < \delta$,

$$H_{R_1} = -w\theta R_2(1+i)U''(Q_2) - w(\delta - R_1)(1+\rho)U''(Q_1) > 0$$

because $U'' < 0$,

$$H_{R_2} = w(\delta - \theta S)(1+i)U''(Q_2) < 0$$

because $U'' < 0$ and, due to the existence condition for exhaustion,

$$\theta S = \theta \frac{2(1+i-\theta) - i\theta}{1+i-\theta^2}\delta < \delta \Leftrightarrow 2\theta - \theta^2 < 1 \Leftrightarrow (1-\theta)^2 > 0,$$

$$H_i = U'(Q_2) > 0,$$

$$H_\delta = \left(wR_2 - \frac{\beta Q_2}{\delta}\right)(1+i)U''(Q_2) - \left(wR_1 - \frac{\beta Q_1}{\delta}\right)(1+\rho)U''(Q_1).$$

Note that H_δ is ambiguous in sign also in the case of exhaustion. Expanding along the first row yields the determinant

$$\det(B) = F_{R_1}H_i + F_i(H_{R_2} - H_{R_1}) > 0$$

which proves that the equilibrium is unique. Since $R_2 = R_1 - S$, we may confine the comparative statics results to the effects of a change in δ on R_1 and i. Define

$$B_1 = \begin{bmatrix} -F_\delta & 0 & F_i \\ 0 & 1 & 0 \\ -H_\delta & H_{R_2} & H_i \end{bmatrix}, B_3 = \begin{bmatrix} F_{R_1} & 0 & -F_\delta \\ 1 & 1 & 0 \\ H_{R_1} & H_{R_2} & -H_\delta \end{bmatrix}$$

so that the signs of the changes are given by $\text{sign}\,(dR_1/d\delta) = \text{sign}\,(\det(B_1))\,, \text{sign}\,(di/d\delta) = \text{sign}\,(\det(B_3))$ due to $\det(B) > 0$. We find that

$$\begin{aligned} \det(B_1) &= -F_\delta H_i + F_i H_\delta \\ \det(B_3) &= F_{R_1}(-H_\delta) - F_\delta(H_{R_2} - H_{R_1}) \end{aligned}$$

are all ambiguous in sign, and thus we have the similar effects as for the case of no exhaustion. The interest effect depends on the size of the income effect and the third derivative of the utility function which measures the relative change of marginal utilities. If the interest rate increases with technological progress and this effect is sufficiently strong, first period extraction will increase and second period extraction will decrease.

We know that the interest rate decreases with technological progress $((di/d\delta) < 0)$ if $H_\delta > 0$. Now suppose that the endowment with the composite factor K is large enough so that $\left(wR_2 - \frac{\beta Q_2}{\delta}\right) < 0$ and $\left(wR_1 - \frac{\beta Q_1}{\delta}\right) < 0$. Then $H_\delta > 0$, if the absolute value of $U''(Q_1)$ is small relative to absolute value of $U''(Q_2)$, that is if U''' is sufficiently positive. In this case $(H_\delta > 0)$, we find a Green Paradox $((dR_1/d\delta < 0)$.

We can also scrutinize the effect of an increase in the composite factor on the resource extraction path. Total differentiation of the equilibrium conditions yields

$$D_1 = \begin{bmatrix} 0 & 0 & F_i \\ 0 & 1 & 0 \\ -H_K & H_{R_2} & H_i \end{bmatrix}, D_3 = \begin{bmatrix} F_{R_1} & 0 & 0 \\ 1 & 1 & 0 \\ H_{R_1} & H_{R_2} & -H_K \end{bmatrix}$$

so that the changes with K are given by $dR_1/dK = \det(D_1)/\det(B), di/dK = \det(D_3)/\det(B)$ according to Cramer's Rule. We find:

$$\det(D_1) = f_i h_K, \tag{20}$$

$$\det(D_3) = -f_{R_1}^2 H_K \tag{21}$$

are all ambiguous in sign and depend on the change of the interest rate with K, and thus also these effect depend in the same way on the third derivative of the utility function. If an increase in the endowment of the composite factor makes the interest rate rise, first period extraction will increase.

5 Trading equilibrium

Now, suppose that the world consists of two countries, Home (H) and Foreign (F). Let K^H and K^F be the endowments of the composite factor in H and F. Assume that F owns the resource stocks.

Let the subscript I denote the equilibrium values in the integrated world equilibrium. Then, for $t = 1, 2$,

$$\frac{x_t^I + R_t^I}{K^H + K^F - K_{Rt}^I} = \bar{e} \tag{22}$$

Suppose the composite factor is internationally mobile. Then obviously the integrated equilibrium can be replicated by trade and movement of the composite factor.

5.1 Replication of the integrated equilibrium without international factor mobility

It is useful to consider first a special case where the movement of the composite factor is not necessary for the replication of the integrated equilibrium. Assume that, for both $t = 1, 2$,

$$K^F > K_{Rt}^I \tag{23}$$

and

$$\frac{R_t^I}{K^F - K_{Rt}^I} > \bar{e} \tag{24}$$

It follows from (22) and (24) that for both $t = 1, 2$

$$\frac{x_t^I}{K^H} < \bar{e} \tag{25}$$

Then trade can take place as follows: F allocates K_{Rt}^I of its composite factor to the extractive sector, produces R_t^I, and exports a fraction α_t of R_t^I, such that

$$\frac{x_t^I + \alpha_t R_t^I}{K^H} = \overline{e} \tag{26}$$

i.e., the quantity of fossil fuel exported to H in period 1 is

$$\alpha_t R_t^I = \overline{e} K^H - x_t^I \tag{27}$$

It is easy to verified that

$$\frac{(1-\alpha_t) R_t^I}{K^F - K_{Rt}^I} = \overline{e} \tag{28}$$

Thus F's export revenue in period t is

$$\lambda \alpha_t R_t^I = \lambda \left(\overline{e} K^H - x_t^I \right)$$

On the other hand, from the Appendix, under logarithmic utility, the saving of the resource-rich consumer is

$$s^r = \frac{(1+i)(\pi_1 + \overline{w}) - (1+\rho)(\pi_2 + \overline{w})}{(2+\rho)(1+i)}$$

Recall that a country's trade surplus equals its saving (excess of income over expenditure). Thus in period 1, F's trade surplus (in the replicated integrated equilibrium) is

$$TB_1 = \left(\frac{(1+i)(\pi_1^I + \overline{w}) - (1+\rho)(\pi_2^I + \overline{w})}{(2+\rho)(1+i)} \right) K^F$$

Then F's net import of the consumption good in period 1 is

$$M_1^F = TB_1 - \lambda \alpha_t R_t^I$$

Remark: The net output in period t available for consumption by H is

$$Q_t^H = K^H f \left(\frac{x_t^I + \alpha_t R_t^I}{K_H} \right) - \lambda x_t^I - \lambda \alpha_t R_t^I$$

It is convenient to define

$$K_{td}^H = \overline{e} \alpha_t R_t^I \tag{29}$$

$$K^H_{tc} = \overline{e} x^I_t \tag{30}$$

Then $K^H_{td} + K^H_{tc} = K^H$. We can partition the firms in H into two subsets: a **"clean subset"** that uses only clean energy and a "**dirty subset**" that uses only imported fossil fuels. Let us called the final consumption good "green consumption good" if its production does not use fossil fuels, in opposition to the "brown consumption good." Composite factor employment in the clean subset is L^H_{tc} and in the dirty subset is L^H_{td}. Clearly,

$$K^H f\left(\frac{x^I_t + \alpha_t R^I_t}{K_H}\right) = K^H_{td} f\left(\frac{\alpha_t R^I_t}{K^H_{td}}\right) + K_{tc} f\left(\frac{x^H_t}{K^H_{tc}}\right).$$

Note that instead of importing the fossil fuels, the dirty subset can move its composite factor K^H_{td} to F and uses the fossil fuel there.Then no fossil fuel is used in H.

5.2 Modeling BTA

Suppose that starting from a potential trade equilibrium that replicates the world integrated equilibrium, the Home country decides to impose a tax on carbon input: any producer in H that uses fossil fuel as an input must pay a tax on the carbon content.

In H, producers of the final goods are indifferent between using the fossil-fuel and the non-fossil fuel iff

$$\lambda = p_{Rt} + T_t$$

where T_t is the tax per unit of carbon (we equate one unit of fossil fuel with one unit of carbon, by the choice of units). Here p_{Rt} is the price that manufacturers pay to resource owners, and T_t is the tax they pay to the government. If producers in H use only renewable energy (and thus do not buy fossil fuels), then

$$\lambda \leq p_{Rt} + T_t$$

Assume that the composite factor is internationally mobile. Then the imposition of T_t in H has no effect resource price: those factories that previously use fossil fuel just simply move to F (bringing the composite factor with them). There will no longer be export of fossil fuels. The price of fossil fuel in F will be $p_{Rt} = \lambda$ (because the wage rate is the same in both countries and is equal to $\overline{w}$, where $\overline{w} = (1-\beta)\left(\overline{e}\right)^{\beta}$, as in the initial benchmark integrated equilibrium).

It follows that the carbon tax imposed by H on the use of carbon by factories in H has no effect whatsoever on emissions. The tax revenue will be zero. Consumers in H now earn income by sending a fraction of their composite factor abroad. The amount sent is K_{td}^H, where K_{td}^H is defined by equation (29). They repatriate their foreign earnings by importing an amount $\overline{w}K_{td}^H$ of "brown" consumption good produced in F. The only effect of the carbon tax is the "de-industrialization" of H.

We suppose that the renewable resource can only be produced in H and is not tradable. Then K_{tc}^H $(= \overline{e}x_t^I)$will remain in H, and factories that remain in H produce the composite good without using the fossil fuel. The energy intensity remains the same in both countries,

$$\overline{e} = \left(\frac{\lambda}{\beta}\right)^{\frac{1}{\beta-1}}.$$

We have thus demonstrated that a carbon tax imposed by H will have no effect on emissions and on world output. Only the location of the dirty factories changes. Then factories in H will specialize in the green consumption good, while factories in F will specialize in the brown consumption good.

To have any effect on emissions, H therefore must impose a border tax adjustment (BTA) in the form of tax on the carbon content of the brown consumption good imported from F (by the owners of the composite input who send K_{td}^H to F.) Suppose these owners earn in F the factor income $\overline{w}$ per unit of composite factor, but must consume in H. The repatriation of their income (in the form of importing into H the brown consumption good produced in F using the dirty input) is then subject to BTA. Assume that the BTA takes the form of ice-berg cost: in order to bring back to H one unit of the brown good, $\tau > 1$ units must be shipped. (One may assume that the amount $\tau - 1$ either disappears as ice-berg transport cost, or is surrendered to the government of H as BTA.[2]) For composite factor owners who are H's nationals to be indifferent between their factor allocation in H or in F, it must be the case that the price (in F) of a brown good is $1/\tau$.

Then in the new equilibrium, the price of fossil fuel in both countries is $p_{Rt} = \lambda - T_t$. Without loss of generality, write $T_t = \gamma_t \lambda$, where $0 < \gamma_t < 1$. Since the factor price (in terms of the green consumption good) is still at $\overline{w}$, while fossil fuel price has fallen to $\lambda(1-\gamma)$, this implies that the unit cost of production of the consumption good in F is

[2]The tax revenue is assumed to be distributed in a lump sum fashion to H's nationals. This has no distortionary effect on demand (because of identical homothetic preferences).

lower:

$$C(p_{Rt}, \overline{w}) = A\left(\lambda(1-\gamma_t)\right)^{\beta}(\overline{w})^{1-\beta} < A(\lambda)^{\beta}(\overline{w})^{1-\beta} = C(\lambda, \overline{w}) = 1$$

This lower production cost in F does not drive H's producers out of business, because, in order to level the playing field, any import of the brown consumption good into H will be taxed under the BTA scheme, such that producers in F no longer have an advantage over producers in H, i.e.

$$C(p_{Rt}, \overline{w}) = \frac{1}{\tau}$$

It follows that the BTA factor τ that levels the playing field is related to the carbon tax γ by the relationship

$$\tau = \frac{1}{(1-\gamma)^{\beta}} \tag{31}$$

Under this scheme of BTA, owners of the composite factor in H are indifferent between allocating their factor to a green consumption good factory in H or a brown consumption good factory in F.

What is the effect of the above BTA on the pattern of production and resource extraction?

To answer this question, we must note that with the BTA, the relative input price in F, $\lambda(1-\gamma)/\overline{w}$, is **lower** than the relative input price in H, $\lambda/\overline{w}$, and at the same time, the relative price of brown good to green good is $1/\tau < 1$. The energy intensity in F will be different from that in in H. Let us denote the former by e^F and the latter by $\overline{e}$. Then the FOC for the brown firm in F is

$$\frac{1}{\tau}f'(e^F) = \lambda(1-\gamma) \implies e^F = \left(\frac{\tau\lambda(1-\gamma)}{\beta}\right)^{\frac{1}{\beta-1}} = \left(\frac{\lambda(1-\gamma)^{1-\beta}}{\beta}\right)^{\frac{1}{\beta-1}} = \overline{e}(1-\gamma)^{-1} > \overline{e}$$

Thus F's energy intensity increases as a result of the BTA. (It is already a kind of Green-Paradox result.)

In H, it remains true that $f'(e) = \lambda$, i.e. $e^H = \overline{e}$.

5.3 Effect of BTA on emissions: is there a Green Paradox?

Let us consider the case of non-exhaustion in the original intergrated equilibrium. Then extractions in the two periods are

$$R_1 = \delta \left(\frac{1 + i - \theta}{1 + i - \theta^2} \right),$$

$$R_2 = \delta \left(\frac{1 + i - \theta - i\theta}{1 + i - \theta^2} \right)$$

Assume K^F is bigger than the amount required for extraction, and all firms in the extractive industry employs the composite factor from the pool K^F. The remaining $K^F - K_{Rt}$ are employed in the final good sector. Assume that

$$\frac{R_t}{K^F - K_{Rt}} > \overline{e}$$

Then if there is no carbon tax, F will export a fraction of R_t to H. Denote this fraction by α_t where α_t is defined by

$$\frac{(1 - \alpha_t)R_t}{K^F - K_{Rt}} = \overline{e}$$

In H, the equilibrium amount of renewable energy produced is x_t, which is defined by

$$\frac{\alpha_t R_t + x_t}{K^H} = \overline{e}$$

Without loss of generality, assume that in H the production of the final good is carried out in two types of factories: brown factories, using $\alpha_t R_t$ units of fossil fuel and K_{td} units of the composite factor, where K_{td} satisfies

$$\frac{\alpha_t R_t}{K_{td}} = \overline{e}$$

and green factories, using x_t units of reneable energy and K_{tc} units of the composite factor, where K_{tc} satisfies

$$\frac{x_t}{K_{tc}} = \overline{e}$$

(It can be verified that $K_{td} + K_{tc} = K^H$).

After H introduces the **carbon tax** (but without BTA on the final consumption good imported from F), the equilibrium world output and extraction will be unchanged: K_{td} is simply sent to F to work there, earning the same factor price $\overline{w}$. The energy intensity remains the same in both countries.

With both the carbon tax and the BTA, the energy intensity in F will be different from that in H. In H, the energy intensity is still at $\overline{e}$. Recall that $\overline{e}$ is defined by $f'(\overline{e}) = \lambda$, i.e,

$$\overline{e} = \left(\frac{\lambda}{\beta}\right)^{\frac{1}{\beta-1}}$$

and $\overline{w}$ is defined by $\overline{w} = f(\overline{e}) - \overline{e}f'(\overline{e})$, implying that

$$\left(\frac{\overline{w}}{1-\beta}\right)^{\frac{\beta-1}{\beta}} = \frac{\lambda}{\beta} \tag{32}$$

In F, the following relationships must hold

$$\frac{1}{\tau}f'(e^F) = \lambda(1-\gamma)$$

and

$$\frac{1}{\tau}\left[f(e^F) - e^F f'(e^F)\right] = \overline{w}$$

Thus

$$\frac{\beta}{\tau}\left(e^F\right)^{\beta-1} = \lambda(1-\gamma) \tag{33}$$

and

$$\frac{1}{\tau}\left[(1-\beta)\left(e^F\right)^{\beta}\right] = \overline{w} \tag{34}$$

Equation (34) yields

$$e^F = \left(\frac{\tau\overline{w}}{1-\beta}\right)^{\frac{1}{\beta}} \tag{35}$$

Substituting (35) into (33), we obtain

$$\frac{\beta}{\tau}\left(\frac{\tau\overline{w}}{1-\beta}\right)^{\frac{\beta-1}{\beta}} = \lambda(1-\gamma) \tag{36}$$

Using (36) and (32) we can solve for τ

$$\tau = (1-\gamma)^{-\beta} \tag{37}$$

This confirms our earlier derivation of τ using the cost function approach, see eq (31).

Under BTA the equilibrium price of the fossil fuel in boh H and F is $\lambda(1-\gamma)$, hence the Hotelling Rule gives

$$\lambda(1-\gamma) - \overline{w}R_1 - \frac{\theta R_2}{1+i}\overline{w} = \frac{1}{(1+i)}\left[\lambda(1-\gamma) - \overline{w}\left(\theta R_1 + R_2\right)\right]$$

Thus, in the case of non-exhaustion, under the carbon tax γ together with the BTA, the extractions in the two periods are

$$R_1^* = \delta(1-\gamma)\left(\frac{1+i-\theta}{1+i-\theta^2}\right), \tag{38}$$

$$R_2^* = \delta(1-\gamma)\left(\frac{1+i-\theta-i\theta}{1+i-\theta^2}\right). \tag{39}$$

So the good news is that, *at constant interest rate*, extractions will fall relative to the case without BTA. However, we should take into account the fact that the equilibrium rate of interest responds to the change in extractions and production and consumption patterns.

The net output of green consumption good in H in period t is

$$q_t^H \equiv K_{tc}^H f(\overline{e}) - \lambda x_t$$

where K_{tc}^H is home composite factor employed in green factories (since all the brown factories are located in F). Since $f'(\overline{e}) = \lambda$ and

$$\frac{x_t}{K_{tc}^H} = \overline{e}$$

it follows that

$$q_t^H = K_{tc}^H\left[f(\overline{e}) - \overline{e}f'(\overline{e})\right] = \overline{w}K_{tc}^H$$

Recall that the quantity of home composite factor employed in brown factories in F is

$K_{td}^H = K^H - K_{tc}^H$. Thus, H's net national income in period t is

$$Z_t^H = \overline{w}K_{tc}^H + \overline{w}K_{td}^H = \overline{w}K^H.$$

H's factor owners repatriate their earning by bringing back brown consumption goods produced in F (subject to BTA). Let us ignore for the moment the tax revenue of H (i.e., suppose the border tax collected is not distributed to H's nationals; this assumption is harmless if we do comparative statics starting from an original situation of a tax rate of zero). Then the budget constraint of H's national is

$$C_1^H + \frac{C_2^H}{1+i} = \overline{w}K^H + \frac{\overline{w}K^H}{1+i} \tag{40}$$

This constraint is exactly the same as in the integrated economy case (except the equilibrium interest rate may be different). If the rate of interest i is lower than the rate of utility discount ρ, H will consume more than its national income in period 1, i.e. it borrows from F. It is important to note that in eq (40), C_t^H are measured in terms of the green comsumption good, whose price is unity. The actual consumption would also include brown goods produced in F, but their "brown-ness" has been purged by the payment of the BTA.

Turning to F, its national income in period 1 is

$$Z_1^F = \overline{w}K^F + \Pi_1$$

where Π_1 is the period 1 profit of the extractive firms,

$$\Pi_1 = \lambda(1-\gamma)R_1 - \overline{w}K_{R1}$$

While profit is lower than in the (no-tax) integrated economy scenario, consumers in F has the benefit of consuming their brown consumption good, whose price is $1/\tau < 1$.

Let us define

$$\widehat{\delta} = \frac{\lambda(1-\gamma)}{\overline{w}}$$

Then world's income in period 1, divided by $\overline{w}$, is

$$\widehat{Q}_1 \equiv K + \widehat{\delta} R_1^* - \frac{1}{2} (R_1^*)^2$$

and, in period 2,

$$\widehat{Q}_2 \equiv K + \widehat{\delta} R_2^* - \frac{1}{2} (R_2^*)^2 - \theta R_1^* R_2^*$$

where R_1^* and R_2^* are given by eqs, (38) and (39). Assuming that preferences are logarithmic, it can be shown (see the Appendix) that equilibrium requires that

$$\Omega = (1+\rho)\widehat{Q}_2 - (1+i)\widehat{Q}_1 = 0 \tag{41}$$

Recall that the BTA reduces $\widehat{\delta}$ from δ to $\delta(1-\gamma)$. Applying to equation (41) the same analysis as that used for the closed economy case, we conclude that a BTA can increase R_1.

Finally, for the case of exhaustion, a similar analysis applies, showing that R_1 may increase in response to BTA, provided that the rate of interest rises sufficiently.

6 Concluding remarks

We have shown that border tax adjustments, advocated by some economists as a means of inducing non-cooperative countries to reduce green-house gas emissions, may have the opposite effect. The mechanism we identify is the (hitherto unexplored) effect of BTA on the interest rate. Under certain conditions, both under exhaustion and under non-exhaustion, BTA will raise the interest rate, causing first period extraction to rise. This brings the climate change damages closer to the present. Second period extraction will fall in the case of exhaustion, where cumulative extraction is equal to the stock S. In the case of non-exhaustion, it is possible that cumulative extraction rises in response to BTA, provided that the rate of interest increases sufficiently.

Appendix

A.1 Consumption demand and savings under logarithmic utility function

Suppose that each individual owns a unit of composite factor. Let $K^p = (1-\eta)K$ denote the number of individuals who do not own the resource. We call them the "resource-poor individuals". (They are nationals of H.) Then $K - K^p$ individuals own the resource. We call them the "rich individuals". (They are nationals of F.)

The income profile of a resource-poor individual is $w_1 = w_2 = \overline{w}$. At the rate of interest i, the resource-poor individual solves:

$$\max \ln c_1^p + \frac{1}{1+\rho} \ln c_2^p$$

subject to

$$c_1^p = w_1 - s^p$$

$$c_2^p = w_2 + (1+i)s^p$$

where s^p is his saving (if s^p is negative, it represents borrowing). Let $W_0^p = w_1 + w_2/(1+i)$. Then the necessary conditions yields

$$c_2^p = \left(\frac{1+i}{1+\rho}\right) c_1^p$$

Thus

$$c_1^p + \frac{1}{1+i}\left(\frac{1+i}{1+\rho}\right) c_1^p = W_0^p$$

ie

$$c_1^p \left[\frac{2+\rho}{1+\rho}\right] = W_0^p = \overline{w}\left[\frac{2+i}{1+i}\right]$$

$$c_1^p = \overline{w}\left[\frac{2+i}{1+i}\left(\frac{1+\rho}{2+\rho}\right)\right]$$

and the net saving of the resource-poor individual is

$$s^p = \overline{w} - c_1^p = \overline{w}\left[\frac{(1+i)(2+\rho) - (2+i)(1+\rho)}{(1+i)(2+\rho)}\right] = \overline{w}\left(\frac{i-\rho}{(1+i)(2+\rho)}\right) < 0$$

Thus resource-poor individuals borrow in period 1 if $i < \rho$.

Let π_t be the dividend payment (expressed in terms of the green consumption good) to a representative resource owner:

$$\pi_t = \frac{\Pi_t}{K - K^p}$$

The resource- rich individual's income in period t is $w_t + \pi_t$. At the rate of interest i, the resource-rich individual who lives in F solves:

$$\max \ln c_1^r + \frac{1}{1+\rho} \ln c_2^r$$

Since the representative resource-rich individual (who live in F) consumes the brown consumption good, whose price is $1/\tau < 1$, the budget constraint is

$$\frac{1}{\tau} c_1^r = (w_1 + \pi_1) - s^r$$

$$\frac{1}{\tau} c_2^r = (w_2 + \pi_2) + (1+i)s^r$$

where s^r is his saving (if s^r is negative, it represents borrowing). Let

$$W_0^r = (\overline{w} + \pi_1) + \frac{1}{1+i}(\overline{w} + \pi_2)$$

Then the necessary conditions yields

$$c_2^r = \left(\frac{1+i}{1+\rho}\right) c_1^r$$

Thus

$$c_1^r + \frac{1}{1+i}\left(\frac{1+i}{1+\rho}\right) c_1^r = \tau W_0^r$$

ie

$$c_1^r \left[\frac{2+\rho}{1+\rho}\right] = \tau W_0^r$$

So

$$c_1^r = \frac{1+\rho}{2+\rho} \tau W_0^r = \frac{1+\rho}{2+\rho}\left(\tau(\overline{w} + \pi_1) + \frac{1}{1+i}\tau(\overline{w} + \pi_2)\right)$$

Thus the saving of the rich individual is

$$\begin{aligned}
s^r &= (\overline{w}+\pi_1) - \frac{1}{\tau}c_1^r \\
&= \frac{1}{2+\rho}(\overline{w}+\pi_1) - \frac{1+\rho}{2+\rho}\left(\frac{1}{1+i}\right)(\overline{w}+\pi_2) \\
&= \frac{(i-\rho)\overline{w}+(1+i)(\pi_1)-(1+\rho)\pi_2}{(2+\rho)(1+i)} \\
&= \frac{(1+i)(\pi_1+\overline{w})-(1+\rho)(\pi_2+\overline{w})}{(2+\rho)(1+i)}
\end{aligned}$$

Finally, we must verify that equilibrium, the interest rate i must be such that the aggregate savings of the rich is equal to the aggregate borrowing of the poor:

$$s^r(K-K^P)+s^pK^p=0$$

Now

$$s^r(K-K^P) = \frac{(i-\rho)\overline{w}(K-K^P)}{(2+\rho)(1+i)} + \frac{(1+i)\Pi_1}{(2+\rho)(1+i)} - \frac{(1+\rho)\Pi_2}{(2+\rho)(1+i)}$$

$$s^pK^p = K^p\overline{w}\left(\frac{i-\rho}{(1+i)(2+\rho)}\right)$$

So

$$\begin{aligned}
s^r(K-K^P)+s^pK^p &= \frac{(i-\rho)\overline{w}K+(1+i)\Pi_1-(1+\rho)\Pi_2}{(2+\rho)(1+i)} \\
&= \frac{(1+i)\left[\overline{w}L+\Pi_1\right]-(1+\rho)\left[\overline{w}L+\Pi_2\right]}{(2+\rho)(1+i)}
\end{aligned}$$

which is equal zero iff

$$\frac{(1+i)}{(1+\rho)} = \frac{\overline{w}K+\Pi_2}{\overline{w}K+\Pi_1}$$

This is satisfied by setting i according to eq $(1+\rho)Q_2-(1+i)Q_1=0$. (Recall that $Q_t \equiv \overline{w}K+\Pi_t$.)

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