# Figures of Speech in Strategic Communication*

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**Abstract**

We provide a class of tractable communication games where each sender type chooses a possibly truth-distorting figure of speech, which the receiver interprets before choosing an action. Because language is inherently vague, a figure of speech mapping determines informativeness of communication: Exaggeration results in better information transmission than understatement. A sender's figure of speech optimal choice trades off the distribution of actions he wants to induce and the conformity of the figure of speech to his ideal for accuracy. There can be between one and five equilibria, all of them fully separating, yet only partially revealing. Their informativeness is unambiguously ranked. At most two of these equilibria, the less informative ones, are ironic. The other (between one and three) equilibria are straight-talking, either exaggerating or understated. We find that a receiver may prefer a sender who is either more dissimilar to him, less honest or less competent. He may also prefer to communicate in a vaguer language. Finally, we study the limit of the equilibria as either (i) the language vagueness vanishes or (ii) the ideal for accuracy vanishes.

*Keywords:* Figures of Speech, Language, Strategic information transmission, Language vagueness, Noisy Communication, Ideal for Accuracy, Lying Costs.

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# 1 Introduction

Game theorists have devoted considerable attention to the study of situations where a Sender tries to influence the actions of a Receiver in a certain by communicating with him.[1] An important element in such situation is that the Sender tends to use language in ways that exhibit certain patterns. A researcher applying for a grant might exaggerate how good his project is and how much money he will need to undertake it. A person writing a letter of recommendation may exaggerate how good he really thinks the person he is writing the letter for is, and perhaps understate his weaknesses. In both of these instances, if the Receiver understands the Sender's behavior, he might be able to performs the rescaling that is needed to recover the truth behind the words. In some cases, the Sender may resort to irony, using words that have a meaning that is the opposite of what he actually means and in some of these cases, the Receiver may actually the true meaning of the words.

In our model, the Sender and the Receiver have a natural language that one could interpret as a reference point. It could be for example the litteral meaning of words, the one that can be found in the dictionnary. We analyze relations between the three following objects. First, the conflict of interest. Second, the words the sender chooses in equilibrium, i.e. the way he chooses to speak. Third, the amount of information that is transmitted in equilibrium. Several papers in the abundant existing literature provide important insights on the relation between the conflict of interest and information transmission but have relatively little to say about the choice of words (e.g. Crawford and Sobel, 1982). Others analyze the relation between the conflict and the words sent in equilibrium, but have relatively little to say about information transmission (Kartik, Ottaviani and Squintani, 2007; Kartik, 2009; Chen 2011). Our contribution in this paper is to provide a framwork that establishes links between the three.

One can think of many complicated ways in which people choose words. But there are recognizable patterns, which will sometimes be considered as lies and sometimes as mere figures of speech. It is remakable that the same patterns are found in most languages and cultures, even though some cultures use some of these patterns more

[1] Starting with Crawford and Sobel (1982), see Sobel (2013) for a recent survey.

than others.[2] In our model, we will focus on three such patterns, or departures from the conventional literal meaning of words.

The first one, exaggeration or overstatement, is found in a variety of shades, from an outright lie to a simple figure of speech: a hyperbola. The second one, understatement, can also be considered as a lie or as a figure of speech: a litotes or a euphemism. Al Gore, former vide president of the United States of America, has been for example accused of having

> "exaggerated his past support for Roe v. Wade, [...] inflated his experience as a farmer, [...] overstated his Army service in Vietnam and understated his youthful experimentation with marijuana."[3]

In his most famous exaggeration, Al Gore declared in March 1999 to the media

> "During my service in the United States Congress I took the initiative in creating the Internet."[4]

The third type language distortion pattern that we consider is irony, defined as the usage of words or messages to mean the opposite of their litteral conventional meaning. An interesting example occured in Belarus in recent years.

> "Young Belarusians have adopted a novel strategy to protest their frustration at the humorless and iron-fisted regime of Alexander Lukashenko: they have started clapping. Organized by way of social media [...], the flash-mob rallies began last month as a peaceful means of working around draconian laws that prohibit unsanctioned public gatherings. At first, a few hundred met up in the capital's Oktyabr Square and then fanned out into the city, breaking into spontaneous fits of clapping on sidewalks and street corners, much like sports fans celebrating a win on the way home. Their ranks have since swollen to several thousand. Lukashenko's thugs, however, saw nothing but a threat to public order. When scores

[2] For example, Anglo English uses irony and understatement more often than other European continental languages (Wierzbicka, 2006).

[3] Shardt, Arlie, My Memo Said What? New York Times, February 16, 2000. http://www.nytimes.com/2000/02/16/opinion/my-memo-said-what.html

[4] Al Gore, March 19, 1999.

of protesters assembled in downtown Minsk and regional centers Wednesday evening, as they have for five weeks running, police and plainclothes goons were waiting for them. Squads of men in tracksuits formed human chains to break up the gatherings, seizing everyone in their path. Many were punched or kicked on the ground before being dragged away into unmarked buses."[5]

In our model, these patterns, exaggeration, understatement and irony, arise in equilibrium as a trade-off between two forces: on the one hand the conflict of interest, which is the incentive for the Sender to mislead the Receiver, and on the other hand, some sort of preference of the Sender for certain messages, that depends on his information. In our main formulation, following Kartik (2009) and Kartik, Ottaviani and Squintani (2007), we assume that the Sender does not like to lie. More precisely, words have a litteral meaning and absent the influence motive, the Sender would prefer to say the truth and to use this plain litteral meaning. Therefore distortions from honesty and litteral meaning occur only because of the strategic motive to manipulate the Receiver.

In an extension, following Chen (2011) and Kartik, Ottaviani and Squintani (2007), we study a situation where the preference of the Sender for certain words arises endogenously because the Receiver is naive with some exogenous probability, meaning that he interprets the Sender's words litterally, without taking into account the Sender's strategic incentive. This in turn causes the Sender to prefer certain messages.

Besides preferences over words, another important feature of our model is that we assume that the language is vague. People often use words that do not have a well-defined meaning, such as the words "critical" and "seriously" in

"The situation is critical. We take it very seriously."

Following Shannon and Weaver (1949) and Blume and Board (2013), we model this noise as an additive component that is the same accross all messages. While this is obviously a strong assumption, it serves as an approximation for the fact that

[5] Jason Motlagh, *Time*, July 7, 2011.
http://content.time.com/time/world/article/0,8599,2081858,00.html#ixzz2g19ne2ZU

all words are vague to some extent. This assumption has the following important implication: when exaggeration is used in equilibrium, it is more informative than understament. The intuition is simple: when the Sender exaggerates, he covers the noise and his message provides more information on his private information. It should be noted that unlike models where the Sender chooses the precision of his signal (as in Kamenica and Gentzkow, 2010), in our model, the Sender does not consciously choose the precision of his signal. In particular, any given Sender type is unable to change the precision of the signal. All he can do is to mislead the Receiver. Informativeness arises as an endogenous equilibrium phenomenon, as in Crawford and Sobel (1982).

We find that equilibria always exist and that sometimes, multiple equilibria coexist, even in a restricted startegy space, the space of linear strategies. There can be up to five linear equilibria, all of them separating, yet only partially informative because of the vagueness of the language. There is always at least one, but at most three *straight talking* equilibria, in which both players' strategies are increasing. Exactly one of the following statements holds: either (i) there is exactly one *truthful* or *exaggerating* equilibrium or (ii) there are between one and three *understated* equilibria. This resulty is important, as it indicates that there is never an indeterminacy on whether the Sender understates or exaggerates. We characterize the frontier in the parameter set, between the undertstatement and exaggeration regions. In addition to the straight-talking equiibria, there can also be up to two *ironic* equilibria, where both players' strategies are decreasing.

We obtain various comparative statics result. We show that increasing the Sender's sensitivity to the state, i.e. how much he wants the receiver to react to the state, always increases the amount of information that is transmitted. This is the case even if the Sender is already more sensitive to the state than the Receiver. This result contrasts where Crawford and Sobel (1982), where increasing the conflict of interest always decreases the amount of information transmitted in equilibrium. We also obtain the surprising result that decreasing the lying cost may have the same effect. In both cases, the intuition is simple: a more sensitive Sender, or one who is less reluctant to lie exaggerates more in equilibrium, and this results in more information being transmitted.

In another result, we show that when the Sender is less sensitive than the Receiver, from the point of view of the Receiver, the optimal level of vagueness is not zero. Again, the intuition is simple. Increasing vagueness commits the Receiver to react less to the Sender's message, since it is mechanically less informative. This in turn results in the Sender exaggerating more and revealing more information in equilibrium. This increase in information revealed may compensate for the increased vagueness of the language, a finding that echoes results by Myerson (1991), Blume, Board and Kawamura (2007) and Goltsman, Hörner, Pavlov and Squintani (2009) in models of noisy cheap talk. Last, we consider an extension where the Sender is not perfectly informed: he only observes a noisy signal of the state. We show that when the Sender is less sensitive to the state than the Receiver, it may be better for the receiver to listen to a less well informed speaker, a finding that echoes a result by Ivanov (2010) in the context of cheap talk communication. The intuition is similar as in the case of the noise increase. Hearing a Sender whom he knows is less informed commits the Receiver to react less to the Sender's message. In equilibrium, the Sender may reveal more of his information. This increase in the information the Sender reveals may compensate the decrease in the information he has in the first place.

# 2 Related literature

In addition to the papers already cited, our model is related to the litterature on endogenous signalling, starting with Mirman and Matthews (1983) and Kyle (1985).[6] In those models where firms or insiders signal private information through quantities or prices in the presence of noise, the message (price or quantity) is also costly to the Sender in the sense that it enters his payoff through its profit function, but this type of cost is quite different than the one we consider here. The applications and interpretations of theses models are also very different from ours.

Our paper also contributes to the literature on pragmatics, which studies the question of how context contributes to meaning (Grice, 1975). In particular, in recent years, a growing literature has emerged that uses game theoretical models to adress this question (Pinker, Nowak and Lee, 2008; Mialon and Mialon, 2013; Board and Blume, 2013).

[6] A recent paper by Gendron-Saulnier and Santigini (2013) uses a noisy signalling model to analyze informational properties of price-discrimination strategies.

# 3 The model

First, nature draws the Sender's type $\theta \in \Theta = \mathbb{R}$ and a noise level $\epsilon \in \mathbb{R}$ from a joint distribution. The Sender observes $\theta$ and sends a message $m \in M = \mathbb{R}$. The receiver then observes $y = m + \epsilon \in Y = \mathbb{R}$ and chooses an action $a \in A = \mathbb{R}$. The payoffs $U^i(a, \theta)$, for $i \in \{R, S\}$ are then realized. A pure strategy for the Sender is a function $\mu : \Theta \to M$. A pure strategy for the Receiver is a function $\alpha : Y \to A$. A Bayesian Nash-Equilibrium is a strategy profile $(\mu, \alpha)$ such that

$$E_\epsilon \left[ U^S \left( \alpha \left( \mu \left( \theta \right) + \epsilon \right), \theta \right) \mid \theta \right] \geq E_\epsilon \left[ U^S \left( \alpha \left( m + \epsilon \right), \theta \right) \mid \theta \right].$$

for all $m \in M$, and all $\theta \in \Theta$ and

$$E_{\theta,\epsilon} \left[ U^R \left( \alpha \left( y \right), \theta \right) \mid \mu \left( \theta \right) + \epsilon = y \right] \geq E_{\theta,\epsilon} \left[ U^R \left( a, \theta \right) \mid \mu \left( \theta \right) + \epsilon = y \right].$$

for all $a \in A$, and all $y \in Y$.

We assume that the Receiver's payoff is

$$U^R(a, \theta) = -\left(a - r\theta\right)^2,$$

and that the Sender's payoff is

$$U^S(a, \theta) = -\left(a - s\theta\right)^2 - k\left(\theta - m\right)^2.$$

Here $\theta$ and $\epsilon$ are independent real random variables that we are assume to be normally distributed with zero zero expectation and variances $\sigma_\theta^2 > 0$ and $\sigma_\epsilon^2 > 0$. Let

$$v = \frac{\sigma_\epsilon^2}{\sigma_\theta^2} > 0.$$

The parameters $r$ and $s$ are real numbers. They represent how responsive respectively the receiver and the sender are to the sender's type. Without loss of generality we assume $s \geq 0$.[7] Following Kartik (2009), the real number $k > 0$ parametrizes the sender's cost of lying. If $k$ is large, the cost of lying is high. In the limit $k \to 0$ the sender's message is pure cheap-talk.

[7] Treating $-\theta$ rather than $\theta$ as the type of the sender transforms a model with $s < 0$ into one with $s > 0$.

# 4 Linear equilibria

We look for equilibria in which the sender uses a linear strategy $\mu(\theta) = \beta\theta$ and the receiver uses a linear strategy $\alpha(y) = \lambda y$. The unique best reply of the receiver to a linear strategy $\beta$ of the sender is a linear strategy $\lambda$ satisfying

$$\lambda = \frac{r\beta}{\beta^2 + v}. \tag{1}$$

This equation follows upon observing that the receiver's best response is given by $rE[\theta \mid y]$ and that from the linear conditional expectation property of normally distributed random variables we have

$$E[\theta \mid y] = \frac{\beta y}{\beta^2 + v}.$$

The unique best reply of the sender to a linear strategy $\lambda$ of the receiver is linear with parameters $\beta$ satisfying

$$\beta = \frac{k + s\lambda}{k + \lambda^2}. \tag{2}$$

This equation follows from substituting $a = \lambda(m + \epsilon)$ into the expression for the sender's payoff, taking expectations with respect to $\epsilon$ and then maximizing with respect to $m$.

Therefore $(\beta, \lambda)$ is an equilibrium if it solves the system

$$\begin{cases} \lambda = \frac{r\beta}{\beta^2+v} \\ \beta = \frac{k+s\lambda}{k+\lambda^2}. \end{cases} \tag{3}$$

Furthermore, we can write the player's expected payoffs as functions of $(\beta, \lambda)$ and the exogenous parameters:

$$u_S(\beta, \lambda) = -\left[(\lambda\beta - s)^2 + \lambda^2 v + k(1-\beta)^2\right]\sigma_\theta^2. \tag{4}$$

$$u_R(\beta, \lambda) = -\left[(\lambda\beta - r)^2 + \lambda^2 v\right]\sigma_\theta^2. \tag{5}$$

## 4.1 Straight-talking, irony and babbling

We refer to an equilibrium as *straight talking* if $\beta > 0$ holds. An equilibrium with $\beta = 0$ is *babbling* and an equilibrium with $\beta < 0$ is *ironic.*

Substituting the first equation from (3) into the second yields that $\beta$ is part of and equilibrium if and only if

$$k\beta\left(\beta^2+v\right)^2+r^2\beta^3-k\left(\beta^2+v\right)^2-sr\beta\left(\beta^2+v\right)=0 \tag{6}$$

holds. Because we have assumed $k>0$ the left side of (6) is strictly smaller than zero at $\beta=0$ and converges to $\infty$ for $\beta\rightarrow\infty$. If follows that there is no babbling equilibrium and that at least one straight talking equilibrium exists.

The intuition for the non-existence of a babbling equilibrium is the following: if $y$ had no informational content, the receiver would find it optimal to chose the response $a=c$ no matter which signal he receives. Given that lying is costly and his message does not affect the receiver's response the sender will however find it optimal to choose $m=\theta$, implying that $y$ carries informational content.

Expanding the polynomial equation (6) can be rewritten as

$$k\beta^5-k\beta^4+\left[2kv+r^2-sr\right]\beta^3-2kv\beta^2+\left[kv^2-srv\right]\beta-kv^2=0. \tag{7}$$

Form Descartes' rule of signs it is then immediate that there is a unique straight talking equilibrium if

$$2kv+r^2-sr\leq 0 \tag{8}$$

holds. This is because when this inequality holds, then $kv-sr\leq 0$ also holds an an implication. Similarly, there is no ironic equilibrium if

$$kv-sr\geq 0 \tag{9}$$

holds. This is because when this inequality holds, then $2kv+r^2-sr\geq 0$ also holds as an implication.

As we will see below (9) does a good job at capturing the conditions which preclude the existence of ironic equilibria, namely high values of $k$ and $v$ and low values of $r$ and $s$ (observe, in particular, that no ironic equilibrium can exist for $s=0$ or $r\leq 0$), but the sufficient conditions for uniqueness of a straight talking equilibrium in (8), which requires not only $k$ and $v$ to be small, but also $s>r>0$, can be much improved.

We obtain the following result.

**Proposition 1.** *For any parameters, there can be at most three straight talking equilibria and at most two ironic talking equilibria. There are parameters for which the game has two ironic and three straight talking equilibria.*

*Proof.* TBA. □

## 4.2 Changing parameters

Let $\gamma = \lambda\beta$. This parameter measures how strongly the receiver's response varies with the underlying type of the sender. Multiplying both equations in the system (3) by $\beta$ (which introduces an artificial root at $\beta = 0$ which does not correspond to an equilibrium) we can rewrite these equations as

$$\gamma = f(\beta) = \frac{r\beta^2}{v + \beta^2} \tag{10}$$

and

$$g(\beta, \gamma) = k\beta^2 + \gamma^2 - k\beta - s\gamma = 0. \tag{11}$$

and identify the linear equilibria of the model with the solutions $(\beta, \gamma)$ of these equations satisfying $\beta \neq 0$.

Equation (11) is equivalent to

$$\frac{\left(\beta - \frac{1}{2}\right)^2}{\left(\sqrt{\frac{k+s^2}{4k}}\right)^2} + \frac{\left(\gamma - \frac{s}{2}\right)^2}{\left(\sqrt{\frac{k+s^2}{4}}\right)^2} = 1 \tag{12}$$

and thus describes an ellipse. Let

$$\underline{\gamma} = \frac{s}{2} - \sqrt{\frac{k+s^2}{4}},\ \overline{\gamma} = \frac{s}{2} + \sqrt{\frac{k+s^2}{4}}, \underline{\beta} = \frac{1}{2} - \sqrt{\frac{k+s^2}{4k}} \text{ and } \overline{\beta} = \frac{1}{2} + \sqrt{\frac{k+s^2}{4k}}.$$

For all $\gamma \in [\underline{\gamma}, \overline{\gamma}]$ let $\beta_1(\gamma) \leq \beta_2(\gamma)$ be the two reals such that $(\beta_1(\gamma), \gamma)$ is the left part of the graph of the ellipse and $(\beta_2(\gamma), \gamma)$ is the right part of the graph of the ellipse. More precisely:

$$\beta_1(\gamma) = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{s\gamma - \gamma^2}{k}} \tag{13}$$

$$\beta_2(\gamma) = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{s\gamma - \gamma^2}{k}}. \tag{14}$$

Similarly, for all $\beta \in [\underline{\beta}, \overline{\beta}]$, let $\gamma_1(\beta) \leq \gamma_2(\beta)$ be the two reals such that $(\beta, \gamma_1(\beta))$ is the lower part of the graph of the ellipse and $(\beta, \gamma_2(\beta))$ is the upper part of the graph of the ellipse. More precisely

$$\gamma_1(\beta) = \frac{s}{2} - \sqrt{\frac{s^2}{4} + k(\beta - \beta^2)} \tag{15}$$

$$\gamma_2(\beta) = \frac{s}{2} + \sqrt{\frac{s^2}{4} + k(\beta - \beta^2)}\,. \tag{16}$$

Observe that every equilibrium $(\beta, \gamma)$ must satisfy $\underline{\beta} \leq \beta \leq \overline{\beta}$ and $\underline{\gamma} \leq \gamma \leq \overline{\gamma}$.

### 4.3 Straight-talking equilibria

Here we consider the case $r > 0$.[8] In addition we assume $s > 0$.[9]

We have already established that there is at least one straight talking equilibrium. As we have seen in examples, there can be up to three straight-talking equilibria. We say that a straight-talking equilibrium is truthful if $\beta = 1$, exaggerated if $\beta > 1$ and understated if $\beta < 1$. In principle, one could expect that different straight-talking equilbria could belong to different categories. We show next that this is not the case: all straight-talking equilibria belong to the same category. Morever, when the straight-talking equilibria are either truthful or exaggerated, there is in fact a unique straight-talking equilibrium.

**Theorem 1.** *There exists an equilibrium with*

$$\beta \begin{array}{c} > \\ = \\ < \end{array} 1$$

*if and only if the relation*

$$r \begin{array}{c} < \\ = \\ > \end{array} s(1+v)$$

*holds. In the first two cases, there is a unique straight-talking equilibrium.*

[8] For $r = 0$ it is trivial that the unique linear equilibrium has $(\beta, \lambda) = (1, 0)$. The case $r < 0$ will be considered separately.

[9] The case $s = 0$ is covered in most of the following, so that it can be used for illustrative purposes. However, the result establishing uniqueness for low $k$ uses the assumption $s > 0$.

*Proof.* First, it is clear that there is an equilibrium satisfying $\beta = 1$ if and only if $f(1) = s$ holds, [10] which is in turn is equivalent to $r = s(1+v)$. Furthermore, if $f(1) = s$ holds, there can be no other straight talking equilibrium but $(\beta, \gamma) = (1, s)$ because $\gamma_2(\beta) > s > f(\beta) > 0 > \gamma_1(\beta)$ holds for all $\beta \in (0,1)$ and $f(\beta) > s > \gamma_2(\beta) \geq \gamma_1(\beta)$ holds for all $\beta \in (1, \overline{\beta})$. Second, consider an equilibrium sastifying $\beta > 1$ holds. Because $\gamma_2(\beta) \geq s$ and $\gamma_1(\beta) \leq 0$ holds for all $\beta \in [0,1]$ and $f(\beta)$ is strictly increasing such an equilibrium exists if and only if $f(1) < s$ holds. Consequently, a necessary and sufficient condition for the existence of an exaggerating equilibrium is

$$r < s(1+v) \tag{17}$$

and if this condition holds there can be no straight talking equilibrium with $\beta \leq 1$. We show next that, in addition, that there cannot be more than one exaggerating equilibrium. The difficult case in proving this result is the one in which $f(\overline{\beta}) < s/2$ holds, meaning that the receiver is much less reactive than the sender would like him to be. Third and last, because straight talking equilibria exist and (as we have seen above) for

$$r > s(1+v)$$

there can be no truthful or exaggerating equilibria, this condition is necessary and sufficient for the existence of an understated equilibrium. □

*Proof.* Let $(\beta^*, \gamma^*)$ be the straight talking equilibrium with the smallest $\beta > 0$. If $\beta^* = \overline{\beta}$ it is immediate that there is no other straight talking equilibrium. Suppose $1 < \beta^* < \overline{\beta}$ and $\gamma^* = \gamma_2(\beta^*)$ holds. Because $f$ is strictly increasing and $\gamma_2$ is strictly decreasing on the interval $[\beta^*, \overline{\beta}]$ it is immediate that $f(\beta) > \gamma_2(\beta)$ holds for all $\beta \in (\beta^*, \overline{\beta}]$. Using the inequality $\gamma_2(\beta) \geq \gamma_1(\beta)$ this implies there is no straight talking equilibrium with $\beta > \beta^*$. It remains to consider the case $1 < \beta^* < \overline{\beta}$ and

[10] Assuming $b = c = 0$ the interpretation of the case $f(1) = s$ is the following: if the sender reports his type truthfully (that is, choose $\beta = 1$) the best response of the receiver is to choose $\lambda = s$ so that conditional on $\theta$ the receiver's expected action is the one the sender wants him to choose. Hence, there is no incentive for the sender to distort his message from the truth and $(\beta, \lambda) = (1, s)$ is an equilibrium. Similarly, $f(1) < s$ means that if the sender reports his type truthfully then from the sender's perspective the receiver's best response isn't sufficiently reactive to the underlying state of the world. This implies that the sender has an incentive to exaggerate by choosing $\beta > 1$.

$\gamma^* = \gamma_1(\beta^*)$. The slope of $f$ at $\beta \in (0, \overline{\beta})$ is

$$f'(\beta) = \frac{2rv\beta}{(\beta^2+v)^2} = 2\frac{vf(\beta)}{r\beta^2}\frac{f(\beta)}{\beta}.$$

From (10) we have $vf(\beta) < r\beta^2$, implying

$$f'(\beta) < 2f(\beta)/\beta. \tag{18}$$

The slope of $\gamma_1$ at $\beta \in (0, \overline{\beta})$ is

$$\gamma_1'(\beta) = \frac{k\left(\beta - \frac{1}{2}\right)}{\sqrt{\frac{s^2}{4} + k\left(\beta - \beta^2\right)}} = \frac{k\left(\beta - \frac{1}{2}\right)}{\frac{s}{2} - \gamma_1(\beta)}.$$

From (11) we have $k(\beta - 1) = \gamma_1(\beta)(s - \gamma_1(\beta))/\beta$. Using $\gamma_1(\beta) < s/2$ this implies

$$\gamma_1'(\beta) > \frac{\gamma_1(\beta)}{\beta}\frac{(s-\gamma_1(\beta))}{(\frac{s}{2} - \gamma_1(\beta))} > 2\frac{\gamma_1(\beta)}{\beta}. \tag{19}$$

From (18) and (19) it follows that $f(\beta) < \gamma_1(\beta)$ holds for all $\beta \in (\beta^*, \overline{\beta})$ (because at any point of intersection of $f$ and $\gamma_1$ in the interval $[0, \overline{\beta})$ the slope of $f$ is strictly smaller than the slope of $\gamma_1$, implying that there can be at most one such intersection). Using the inequality $\gamma_2(\beta) \geq \gamma_1(\beta)$ this implies there is no straight talking equilibrium with $\beta > \beta^*$.[11] □

As an implication of Theorem 1, the only circumstance in which multiple straight-talking equilibria may exist is when the inequality $r > s(1+v)$, i.e. when straight-talking equilibria are undertstated. In the appendix, we study this case in further detail and provide conditions for uniqueness and multiplicity. We say that an understated equilibrium is *slightly understated* if $\beta \in [1/2, 1)$ holds and that it is *strongly understated* if $\beta \in (0, 1/2)$ holds.

## 4.4 Ironic equilibria

Ironic equilibria can only exists if $r > 0$ and $s > 0$ holds, so we impose these parameter restrictions throughout this section.

[11] This is a bit sloppy because as it stands the argument only precludes the existence of an equilibrium in $(\beta^*, \overline{\beta})$, so one should also argue that there can be no additional equilibrium at $\overline{\beta}$. But as the "slope" of $\gamma_1$ is infinite there, it seems obvious enough (but painful to write down properly) that there can be no such equilibrium.

A simple necessary condition for the existence of ironic equilibria is that the inequality

$$-\frac{r}{2\sqrt{v}}\beta \geq \gamma_1(\beta) \tag{20}$$

holds for some $\beta \in (\underline{\beta}, 0)$. The expression on the left side of this inequality is obtained by observing that - by the same argument as in the one used in the proof of Proposition 14 for the case $\beta > 0$ - the inequality $f(\beta) \leq \frac{r}{2\sqrt{v}}\beta$ holds for all $\beta < 0$.

Because $\gamma_1(0) = 0$, $\gamma_1'(0) = -\frac{k}{s}$ and $\gamma_1$ is strictly convex inequality (20) holds for some $\beta < 0$ if and only if $rs \geq 2k\sqrt{v}$, so that the condition $rs < 2k\sqrt{v}$ precludes the existence of an ironic equilibrium. Suppose on the other hand that the inequality $rs \geq 2k\sqrt{v}$ holds and that, in addition, $-\sqrt{v} \geq \underline{\beta}$ holds. Then we have

$$f(-\sqrt{v}) = \frac{r}{2} \geq \frac{k}{s}\sqrt{v} > \gamma_1(-\sqrt{v}),$$

implying the existence of (at least) two ironic equilibria. We have thus shown

**Proposition 2.** *If $rs < 2k\sqrt{v}$ then no ironic equilibrium exists. If $rs \geq 2k\sqrt{v}$ and*

$$\sqrt{v} \leq \frac{1}{2}\sqrt{1+\frac{s^2}{k}} - \frac{1}{2} \tag{21}$$

*then at least two ironic equilibria exist.*

In particular, fixing all other parameter values ironic equilibria exist if

- $v$ is small
- $k$ is small
- $s$ is large

Provided that $v$ and $k$ are sufficiently small relative to $s$ to ensure the inequality in (21), ironic equilibria will also exist whenever $r$ is large enough.

# 5 Stable equilibria

We say that an equilibrium $(\beta^*, \lambda^*)$ is stable if there exists a neigborhood of $\beta^*$ such that for any initial condition $\beta_0$ in the neighborhood, the composed best response dynamic

$$\begin{cases} \lambda_n = \frac{r\beta_n}{\beta_n^2+v} \\ \beta_{n+1} = \frac{k+s\lambda_n}{k+\lambda_n^2} \end{cases}$$

converges to $(\lambda^*, \beta^*)$.

In the generic situation in which we have an odd number of straight talking equilibria, the smallest and the loudest straight talking equilibrium may or may not be stable. If there are ironic equilibria the larger of these (in absolute value) may or may not be stable. The smaller one is necessarily unstable. If there is more than one straight talking equilibrium the first and (in case this is possible) the third of these will be stable. Moreover, when there are there are three straight talking equilibria, we know that all of them are understated. The loudest equilibrium is necessarily stable and the middle one unstable. We summarize this result in the following proposition.

**Proposition 3.** *If there are three (necessarily understated) straight talking equilibria, the slightest, that is the one with the highest absolute value of $\beta$, is necessarily stable. Moreover, in that case, the iteration of the composed best response dynamic from $\beta^* = 1$ converges to the slightest understated equilibrium. The medium one is necessarily unstable. If there are two ironic equilibria, the one with the lowest absolute value of $\beta$ is necessarily unstable.*

*Proof.* The presence of three straight talking equilibria implies that all three are understated. In particular, the loudest one $(\beta^*, \lambda^*)$ is such that the best response of the sender is negative at $\lambda^*$. This and the presence of three straight talking equilibria further implies that the best response of the receiver is also negative ta $\beta^*$. Finally, the best-response of the sender cross the receiver's from above, which implies that $(\beta^*, \lambda^*)$ is stable. □

If one considers "truth-telling" as the natural starting point of a dynamic, then one would tend to focus on the largest straight talking equilibrium. If ones considers babbling as the natural starting point of a dynamics, then one would tend to focus on the smallest straight talking equilibrium. It is not immediately apparent how one would want to tell a story about the emergence of ironic equilibria.

# 6 Informational content and welfare

A standard question in models such as ours is "How much information is transmitted in equilibrium?" In a model with normally distributed random variables it is natural to measure "how much information" by considering the ratio of the precisions of the receiver's posterior (after having observed the message) and prior (before having observed the message) forecast of $\theta$. The prior precision is $1/\sigma_\theta^2$. The posterior precision after having observed the signal $\beta\theta + \epsilon$ is $1/\sigma_\theta^2 + \beta^2/\sigma_\epsilon^2$ (this uses standard formulas for the precision of normally distributed random variables). Hence, the ratio of the precisions is simply

$$1 + \beta^2/v$$

(to be evaluated at the equilibrium value of $\beta$).[12]

The following result follows directly from the definitions.

**Proposition 4.** *The informational content of an equilibrium is proportional to $\beta^2$. The loudest straight talking equilibrium is also the most informative one. Any straight talking equilibrium is more informative than any ironic equilibrium.*

Multiplying both sides of the equation for the receiver's best response by $\lambda$ and using the definition $\gamma = \lambda\beta$ we obtain that for any profile $(\beta, \lambda)$ on th receiver's best response, the relation

$$\lambda^2 v = (r - \gamma)\gamma \tag{22}$$

holds. Substituting this into the formulas for player's expected utilities given in (4) and (5) we obtain.

**Lemma 1.** *Let $(\beta, \lambda)$ be an equilibrium with $\beta\lambda = \gamma$. Then the corresponding equilibrium utilities are given by*

$$u_S(\beta, \lambda) = -\left[(\gamma - s)^2 + (r - \gamma)\gamma) + k(1 - \beta)^2\right]\sigma_\theta^2 - \frac{kv + (r - \gamma)r}{kv}(c - b)^2.$$
$$u_R(\beta, \lambda) = -r\,(r - \gamma)\,\sigma_\theta^2.$$

Using these eexpressions, one can immediately rank the equilibria, from the point of view of the receiver's welfare. This is because in any equilibrium, $\gamma \leq r$ holds and $u_R$ is increasing in $\gamma$. Therefore the receiver prefers the equilibrium which has the

[12] As we have seen before, the value $\beta = \sqrt{v}$ plays a special role in our analysis as this is the value of $\beta$ at which the receiver's best response $\lambda$ takes it maximal value. We may observe that at this value of $\beta$ the measure of informational efficiency is equal to 2 – which happens to be its equilibrium value in the Kyle model. Observe too, that as far as the cost resulting from the presence of noise is concerned the value $\beta = \sqrt{v}$ is the worst possible one as it leads to the maximal possible value of $\lambda^2 v$.

highest value of $\gamma$, which is also the most informative one. This gives the following result.

**Proposition 5.** *If these equilibria exist, the receiver ranks equilibria as follows. The gentle ironic is the worst (if there is any), followed by the loud ironic (if there is any), followed by the gentle straight talking (if there is any), followed by the middle straight talking (if there is any), followed by the loudest equilbrium, which is the receiver's preferred equilibrium.*

In the simple case where $b = c$, the sender's utility over pairs $(\beta, \gamma)$ on the receiver's best response curve is given by

$$(2s - r)\,\gamma - k(1 - \beta)^2.$$

If $2s > r$, this implies that the sender's ideal point $(\beta^\circ, \gamma^\circ)$ on the receiver's best response curve is such that $\beta^\circ > 1$. In this case, the sender's utility is single-peaked in $\beta$ for positive values of $\beta$. Moreover, between two points $(\beta, \gamma)$ and $(-\beta', \gamma')$, with $0 < \beta' \leq \beta \leq 1$, on the receiver's best response curve, the sender prefers the first. In other words, if an ironic pair is preferred to a straight talking one, the former must be louder. But when $2s > r$, we know that there is a unique straight talking equilibrium and either no or two ironic equilibria. In this case, it is clear that the unique straight talking equilibrium is preferred to the two ironic equilibria, if they exist. How the sender ranks the two ironic equilibria is not immediately clear.

If $r > 2s$, this imples that the sender's ideal point is $(1, 0)$, while if $r < s$, his ideal point is $(1, +\infty)$. In both cases, it is not immediately clear how the sender ranks equilibria. In the second case, if the loudest equilibrium is understated (or if it does not exaggerate too much), this will be the sender's preferred equilibrium. If it exagerates a lot, then the middle straight talking equilibrium is preferred. In any case, one of these two equilibria is preferred to all the others. The most gentle straight talking equilibrium comes next. The ranking between the two ironic equilibria is again unclear.

# 7 Comparative statics

In this section, we establish comparative statics results on equilibria and welfare. We study changes in the sender's sensitivity $s$, in the lying cost $k$, and in the vagueness of the language $v$.

## 7.1 Changing the sender's sensitivity

The effect of increasing $s$ is easy to understand in terms of the $(\beta, \gamma)$ diagram. We keep the points $(0,0)$ and $(1,0)$ fixed and "stretch" the ellipse by pulling at the points $(0,s)$ and $(1,s)$ in the vertical direction. Consequently, the effect will be the following:

- $r > 0$, straight talking either exaggerated or slightly understated equilibrim, or any strongly understated equilibrium that is not the middle one: both $\beta$ and $\gamma$ increase.

- $r > 0$, straight talking, strongly understated equilibrium in the middle: both $\beta$ and $\gamma$ decrease.

- $r > 0$, ironic equilibria: Starting from a situation in which no ironic equilibria exist (for low $s$), at some point an ironic equilibrium appears and splits into two ironic equilibria. The one with the greatest absolute value of $\beta$ moves towards an even higher $\beta$ (in absolute value) and a higher value of $\gamma$. The other one moves towards the origin, i.e. a lower absolute value of $\beta$ and a lower $\gamma$.

- $r < 0$ : if there is a unique one, it moves towards the origin, i.e. a lower $\beta$ and also a lower $\gamma$ in absolute value.

## 7.2 Changing the cost of lying

The effect of increasing $k$ is easy to understand by thinking in terms of the $(\beta.\gamma)$ diagram. We keep the points $(0,0), (s,0)$, $(1,0), (1,s)$ fixed and "stretch" the ellipse by pulling at the points $(1/2, \overline{\gamma})$ and $(1/2, \underline{\gamma})$ in the vertical direction. Consequently, the effect will be the following:

- $r > 0$, straight talking, exaggerated equilibria: $\beta$ and $\gamma$ fall as $k$ increases.

- $r > 0$, straight talking, understated equilibria: If there is a unique straight talking equilibrium $\beta$ and $\gamma$ increase as $k$ increases. If $v \geq 1/4$ the equilibrium will thus simply trace along the $f$-curve. If $v < 1/4$ more interesting situations may arise: For sufficiently low $k$ we have a unique strongly understated equilibrium which then may either morph continuously into a slightly understated equilibrium or at some critical value of $k$ "suddenly" a second larger strongly understated equilibrium appears which then "splits" into two equilibria with the larger of these equilibria then moving smoothly into the slightly understated domain and the smaller one of this pair (for which $\beta$ and $\gamma$ are decreasing with $k$) merging with the smallest strongly understated equilibrium, leaving the slightly understated equilibrium as the unique one for sufficiently high $k$.

- $r > 0$, ironic equilibria: Starting from a situation in which two of these exists, the one with the greatest absolute value of $\beta$ moves closer to the origin as $k$ increases and the one with the smallest absolute value of $\beta$ will move in the opposite direction until they both merge and disappear.
- $r < 0$: if there is a unique one $\beta$ and the absolute value of $\gamma$ increase as $k$ increases. Multiplicity story is akin to the one for the $r > 0$ case.

An interesting implication of these results and the ones obtained in the previous section is the fact that when $0 < r < s(1 + v)$ the receiver's equilibrium utility is actually decreasing in $k$. One might have thought that it is always to the receiver's advantage if the sender's incentive to mislead him is reduced.

**Proposition 6.** *Suppose that* $0 < r < s(1+v)$ *holds. Then the informational content and the receiver's expected utility are decreasing in* $k$ *at the (unique) exaggerating equilibrium.*

The intuition of this result is simple. As moral norms against lying becomes weaker, the equilibrium involves more exaggeration. In equilibrium, information is encoded in a more exaggerated language, which implies a better transmission of information, as the relative importance of the noise decreases, due to how "loud" the sender speaks.

#### 7.2.1 Limit as $k \to \infty$

For large enough $k$ we have a unique equilibrium $(\beta_k, \gamma_k)$ satisfying $\beta_k \to 1$ and $\gamma_k \to r/(1 + v)$.

In terms of the welfare analysis it is not immediately obvious whether the term $k(1-\beta)^2$ converges to zero. However, using (11) we know that

$$k(1-\beta)^2 = (\gamma - s)\gamma \left[\frac{1}{\beta} - 1\right] \tag{23}$$

holds in every equilibrium. As $\beta \to 1$ and $\gamma$ has a finite limit, it follows that $k(1-\beta)^2$ converges to zero as $k$ converges to infinity.

$$u_S = -\frac{(r-s)^2 + s^2 v}{1+v}\sigma_\theta^2 + (c-b)^2 \text{ and } u_R = -\frac{r^2 v}{1+v}\sigma_\theta^2$$

(One should check whether the above is what one gets "in the limit", that is, by simply presuming that the sender is an automaton who has to tell the truth. I think it should. That would help in clarifying that the remaining costs result from the fact that (a) the seller gets his way in expectation with (b) the noisiness of the communication channel imposing an additional cost hurting both players.)

#### 7.2.2 Limit as $k \to 0$

For $r < 0$ there will be a unique equilibrium for $k$ small enough and this converges to babbling, that is the limit is $(0, 0)$.

For $r > 0$ and sufficiently small $k$ there are exactly three equilibria, two ironic ones and a straight talking equilibrium. The gentler (unstable) ironic equilibrium converges to $(\beta, \lambda) = (0, 0)$, which is the babbling equilibrium. (As none of the other equilibria converges to babbling this establishes a sense in which the babbling equilibrium – that always exists when $k = 0$ – is not stable.)

For the loud ironic and the straight talking equilibria, there are two cases to consider: $s \geq r$ and $s < r$.

- $r \leq s$. In this case, the sequence of straight talking equilibria $(\beta_k, r)$ converges to $(+\infty, r)$ and the sequence of load ironic equilibria converges to $(-\infty, r)$.In either case, the informational content of equilibrium goes to infinity and the equilibrium utility of the receiver converges to 0 – which is the receiver's ideal outcome. Understanding what happens to the sender's payoff is a bit more challenging. Suppose, first, that $b = c$ holds, so we can ignore the last term in the sender's payoff. Using (23) and $\beta \to \infty$, $\gamma \to r$, we find that the term

$k(1-\beta)^2$ converges to $(s-r)r$ – hence, unless $r = s$ the expected lying costs to be borne by the sender do not converge to zero. The sender's utility converges to $(s-r)r$. [13] Now, let's assume $b \neq c$. The question then is what happens to the term $(r-\gamma)r/kv$ as $k$ converges to zero. This is not obvious as $r-\gamma$ and $k$ both go to zero, so we might want to take a closer look at $(r-\gamma)/k$. In fact, rather than doing that let us return to the expression $\lambda^2/k$ as the one describing the expected cost of "lying about the intercept." This term can be rewritten [14] as

$$\frac{\lambda^2}{k} = \frac{\beta - 1}{\beta}\frac{\gamma}{s-\gamma}. \tag{24}$$

As $\beta$ goes to infinity, the first fraction goes to 1, demonstrating that in the case $r < a$ we get a strictly positive limit given by $r/(s-r)$. (I find it somewhat puzzling that this expression is strictly increasing in $r$.) In the special case $s = r$ we get that the cost converges to infinity. (So if there is only conflict about the intercept the costs go off to infinity. If, however, there is an additional conflict about slope that this effect gets tempered - provided $r < s$ holds.)

- $r > s$. In this case, the two equilibrium values of $\beta$ converges to the positive and negative solution of

$$\frac{s}{r} = \frac{\beta^{*2}}{\beta^{*2} + v},$$

i.e.

$$\beta^* = \pm\sqrt{\frac{v}{1 - \frac{r}{s}}}$$

and $\gamma^* = s$.

Observe the limit of the straight talking equilibrium can be gentle, loud, truthful, or exaggerated depending on how the value of the ratio $r/s$, e.g. $r = s(1+v)$ implies $\beta^* = 1$ etc. (the case distinction is exactly in line with what we have seen before).

Observe: In the case $b = c$ there are no lying costs in the limit and in expectation the sender gets his most preferred action. Nevertheless, the sender does not

[13] Observe that keeping $s$ fixed this term is maximized for $r = s/2$. This is consistent with the highest possible amount of exaggeration occurs when $f(\overline{\beta}) = s/2$.

[14] Here is a somewhat roundabout way of doing this. Multiply the sender's best response condition by $\lambda$ rather than $\beta$ to obtain $\lambda^2(s-\gamma) = k(\gamma - \lambda)$ and then eliminate the $\lambda$ on the right side by using $\lambda = \gamma/\beta$.

obtain his bliss utility as $\lambda$ converges to a finite limit, implying that the noise cost-term $\lambda^2 v$ does not vanish in the limit, but converges to $(r - s)s > 0$. If $b \neq c$ it is clear from the calculations that we did above and the fact that $\beta$ cannot converge to 1 that the expected cost of lying about the intercept go to infinity.

## 7.3 Changing the vagueness of the language

Increasing $v$ flattens the function $f$.

Consider straight talking equilibria for $r > 0$ and suppose equilibrium is unique. The equilibrium value of $\beta$ will then be increasing in $v$ until we hit the point at which $f(\overline{\beta}) = s/2$. Thereafter the equilibrium value of $\beta$ is decreasing and converges to 1 as $v \to \infty$. If $v$ is sufficiently small (and $r$ sufficiently large) that $\beta < 1/2$ holds for small enough $v$, then the equilibrium value of $\gamma$ will first be increasing in $v$ and then – once $\beta = 1/2$ has been hit – decreasing in $v$.

Considering the ironic equilibria for $r > 0$,it is clear that these will cease to exist for $v$ sufficiently large. As long as they exist, the gentle (unstable) one will move away from babbling when $v$ increases, whereas the comparative statics of the louder ironic equilibrium (I am again taking it for granted that there are at most two ironic equilibria) are determined by whether equilibrium sits on $\gamma_2$ or $\gamma_1$. If it sits on $\gamma_2$, then the absolute value of $\beta$ is increasing in $v$ until it reaches $\beta = \underline{\beta}$ is reached; thereafter we move on $\gamma_1$ with $\beta$ decreasing until we bump into the unstable equilibrium and both disappear. Throughout $\gamma$ is decreasing in $v$ for the stable equilibrium.

Consider the case $r < 0$ under the additional assumption that we have uniqueness. Then $\beta$ is increasing in $v$ and $\gamma$ is decreasing in $v$ for $\beta < 1/2$ and increasing thereafter.

### 7.3.1 Limiting behavior of all equilibria when the language vagueness is low

Let $r > 0$.

Keeping all other parameters fixed, in the limit where $v$ goes to 0 :

- If

$$\frac{r}{s} \leq 1,$$

there is a unique positive equilibrium, that converges to $(\beta_2(r), r)$, i.e. $\beta = \beta_2(r)$ and $\lambda = \frac{r}{\beta_2(r)}$. There are also two negative equilibria. One that converges to $(0,0)$, i.e. $\beta = 0$ and $\lambda = -\frac{k}{s}$, and another one that converges to $(\beta_1(r), r)$, i.e. $\beta = \beta_1(r)$ and $\lambda = \frac{r}{\beta_1(r)}$.

- If
$$1 < \frac{r}{s} < \frac{1}{2} + \frac{\sqrt{\frac{k}{s^2} + 1}}{2},$$
there are three positive equilibria. Two of them converge respectively to $(\beta_1(r), r)$ and to $(\beta_2(r), r)$. The third one converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = +\infty$. There are also two negative equilibria. One that converges to $(0,0)$, i.e. $\beta = 0$ and $\lambda = -\frac{k}{s}$, and another one that converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = -\infty$.

- If
$$\frac{1}{2} + \frac{\sqrt{\frac{k}{s^2} + 1}}{2} < \frac{r}{s},$$
there is again a unique positive equilibrium that converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = +\infty$. There are also two negative equilibria. One that converges to $(0,0)$, i.e. $\beta = 0$ and $\lambda = -\frac{k}{s}$, and another one that converges to $(0, s)$, i.e. $\beta = 0$ and $\lambda = -\infty$.

### 7.3.2 Optimal vagueness

Noise has a direct negative on the welfare of both players. It has also an indirect strategic effect. For positive (negative) equilibria that lie on some increasing (decreasing) portion of the ellipse, a noise increase has a positive effect on the receiver. The opposite is true for equilibria that lie on some decreasing (increasing) portion of the ellipse.

The welfare effect on the sender is more complex, because the lying cost also plays a role.

Assume $r > 0$ and focus on straight talking equilibria.

If $r \leq s$ (so that the unique straight talking equilibrium is always exaggerating) it is clear that the receiver prefers noise to be as small as possible and in the limit for $v \to 0$ obtains his bliss point with $\gamma = r$.

If $r > s$ the question of the optimal noise level is more interesting (and we have to grapple with the problem that there might be multiple equilibria).

In particular, when $s < r < s/2 + \sqrt{s^2 + k}/2$ we have already seen that there are two equilibria such that the receiver obtains his bliss point in the limit as $v \to 0$. There is, however, also a third equilibrium in which $\gamma$ converges to $s$. So there is a risk that the receiver may end up in the "wrong" equilibrium.

If $r$ is greater than the upper bound just given, pushing $v$ to zero is not what the receiver wants to do. Rather he wants to choose $v$ such that equilibrium occurs at $\beta = 1/2$ – which is the best the receiver can hope for. It is not immediately obvious to me, though, that this equilibrium must then be unique. If it is not, the same question as in the previous case arise. This is summarized in the following result.

**Proposition 7.** *Suppose that* $r > \overline{\gamma}$. *The informational content and the expected utility of the receiver are both increasing in* $v$ *in* $\left(0, \frac{r-\overline{\gamma}}{4\overline{\gamma}}\right]$ *and decreasing in* $v$ *on* $\left[\frac{r-\overline{\gamma}}{4\overline{\gamma}}, +\infty\right)$.

# 8 Extensions

In this section we consider three extensions of the model. In the first one, we consider different preferences over actions that include a constant term in the sender's bias. In the second one, we drop the assumption that the receiver perfectly observes $\theta$. In the third, we study a model where the sender does not have a lying cost, but where the receiver can be naive with some probability. We show that this alternative model can be mapped to our model.

## 8.1 Constant sender's bias

In this section, we assume that the Receiver's payoff is

$$U^R(a, \theta) = -(a - [r\theta + b])^2,$$

and that the Sender's payoff is

$$U^S(a, \theta) = -(a - [s\theta + c])^2 - k(\theta - m)^2.$$

The parameters $b$, $c$, $r$ and $s$ are real numbers. As before, without loss of generality we assume $s \geq 0$. We look for equilibria in which players use "linear" strategies.

$$\mu(\theta) = \beta\theta + \mu_0$$
$$\alpha(y) = \lambda y + \alpha_0.$$

The unique best reply of the receiver to a linear strategy $(\beta, \mu_0)$ of the sender is linear with parameters $(\lambda, \alpha_0)$ satisfying

$$\lambda = r\frac{\beta}{\beta^2 + v} \text{ and } \alpha_0 = b - \lambda\mu_0. \tag{25}$$

These equations follow upon observing that the receiver's best response is given by $rE[\theta \mid y] + b$ and that from the linear conditional expectation property of normally distributed random variables we have $E[\theta \mid y] = \frac{\beta}{\beta^2+v}(y - \mu_0)$. The second equality has the following interpretation. The expected message sent induces the expected preferred action of the receiver. The unique best reply of the sender to a linear strategy $(\lambda, \alpha_0)$ of the receiver is linear with parameters $(\beta, \mu_0)$ satisfying

$$\beta = \frac{k + s\lambda}{k + \lambda^2} \text{ and } \mu_0 = \lambda\frac{c - \alpha_0}{k + \lambda^2}. \tag{26}$$

These equations follow from substituting $a = \lambda(m + \epsilon) + \alpha_0$ into the expression for the sender's payoff, taking expectations with respect to $\epsilon$ and then maximizing with respect to $m$.

If $(\beta, \lambda)$ solves the system 3, which is equivalent to $(\beta, \lambda)$ being an equilibirum of the model anayzed in sections 2 and 3, then the system

$$\begin{cases} \alpha_0 = b - \lambda\mu_0 \\ \mu_0 = \lambda\frac{c-\alpha_0}{\lambda^2+k}. \end{cases}$$

has a unique solution given by

$$\mu_0 = \frac{\lambda}{k}(c - b) \text{ and } \alpha_0 = b - \frac{\lambda^2}{k}(c - b). \tag{27}$$

Therefore the problem of finding equilibria can be reduced to the problem of finding the solutions $(\beta, \lambda)$ to the system (3). Furthermore, taking the solution to (27) into account, we can we can write the player's expected payoffs as functions of $(\beta, \lambda)$ and the exogenous parameters:

$$u_S(\beta,\lambda) = -\left[(\lambda\beta - s)^2 + \lambda^2 v + k(1-\beta)^2\right]\sigma_\theta^2 - \frac{k+\lambda^2}{k}(c-b)^2. \tag{28}$$

$$u_R(\beta,\lambda) = -\left[(\lambda\beta - r)^2 + \lambda^2 v\right]\sigma_\theta^2. \tag{29}$$

Setting $b = 0$, and $c > 0$, we obtain the following interesting comparative statics results on changes in $c$.

**Proposition 8.** *Let $b = 0$ and $c > 0$. An increase in c has no effect on the equilibrium values of $\lambda$ and $\beta$. The equilibrium values of $\alpha_0$ decrease. The equilibrium values of $\mu_0$ increase if the equilibrium is straightalking ($\lambda > 0$) and decrease if the equilibrium is ironic ($\lambda < 0$). Such a change has no impact on the welfare of the receiver in each of the (possibly many) equilibria. It decreases the welfare of the sender.*

## 8.2 The sender's competence

We consider the effect of an additional source of noise in the system. We no longer assume as in the main model that the sender perfectly observes $\theta$. The sender may be more or less compentent, where this competence is defined as the precision of the sender's measurment (or the inverse of the variance of the measurement error). Ws show that under certain conditions, decreasing the sender's competence can improve communication and the quality of the information obtained by the receiver. A similar result is established by Ivanov (2010) in a model without language vaguness nor lying cost. The coarseness of the sender's information is modelled in a very different way: the sender only knows that his type lies within a certain interval.

Suppose then that the sender does not perfectly observe $\theta$, but only $\theta + \delta$, where $\delta$ is a normally distributed random variable with mean 0 and variance $\sigma_\delta^2$. We assume that the variables $\theta$, $\delta$ and $\varepsilon$ are independent. It is convenient to define

$$w = \frac{\sigma_\delta^2}{\sigma_\theta^2}.$$

Note that the case $w = 0$ coincides with the main model: it is the case where the sender is perfectly informed on $\theta$. As in the main model, we look for equilibria where the sender sends a message equal to $\beta(\theta + \delta)$ and the receiver takes an action equal to $\lambda y$. One can show that the best response of the receiver is

$$\lambda = \frac{r\beta}{v + \beta^2(1+w)}$$

while the best response of the sender is

$$\beta = \frac{k + \frac{s}{1+w}\lambda}{k + \lambda^2}.$$

This model can be mapped in our main model as follows. Consider the change of variable

$$s' = \frac{s}{1+w};\ r' = \frac{r}{1+w};\ v' = \frac{v}{1+w} \text{ and } k' = k.$$

We have the following result.

**Proposition 9.** *The profile $(\beta, \lambda)$ is an equilibrium of the game with an imperfectly competent sender with parameters $s$, $r$, $v$, $k$ and $w$ if and only if the same profile $(\beta, \lambda)$ is an equilibrium in the main model, with parameters $s'$, $r'$, $v'$ and $k'$.*

We are interested in the effect of a decrease in the sender's competence (i.e. an increase in $w$) on the equilibrium strategies and on the receiver's welfare. An increase in $w$ affects both the best-responses of the receiver and of the sender: it decreases both of them. In the case of a straight-talking equilibrium that is either exaggerated, truthtelling or slightly understated, these effects work in the same direction: they decrease the equilibrium value of $\gamma = \beta\lambda$ and therefore decrease the receiver's welfare. This result is expected: the sender is less informed and this decrease in information is passed on to the receiver. In the case of a strongly understated equilibrium, the effects work in opposite directions. The decrease of the best response of the receiver tends to increase $\gamma$, while the decrease of the best response of the sender tends to decrease $\gamma$. The magnitude of the later effect is turned-off when $s = 0$ and is dominated by the former when $s$ is small. Thus we obtain the following result.

**Proposition 10.** *When $s$ is small enough and the equilibrium is straight-talking and strongly understated, a decrease of the sender's competence increases the welfare of the receiver.*

This intuition for this result is that in a strongly understated equilibrium, the receiver reacts too much from the point of view of the sender. Decreasing the sender's comptence commits the receiver to be less reactive. In equilibrium, the sender understates less, which means that he passes more of the information is possesses. This increase can be large enough to compensate for the coarsening of his own information. This result echoes Ivanov's (2010). It also provides an insight on when such a phenomenon can occur: in situations wher $s$ is small and where the equilibrium is understated to begin with.

## 8.3 Naive receivers

We now consider an alternative source of preferences over specific words. Even if the sender does not have psychological costs of lying and maximizes only the expectation of the utility he derives from the receiver's action, that is $-(a-s\theta)^2$, the receiver could be naive with positive probability $\kappa \in (0,1)$ and strategic with probability $1-\kappa$. A naive receiver interprets messages litterally, without taking into account strategic incentives of the sender. This is because he believes that the sender is reporting his type in a truthful manner. This behavior in turn creates an endogenous message cost for the sender. Similar behavioral types were studied by Chen (2011), in a game of communication without exogenous noise. We will show that in our context, this model is in a certain sense equivalent to ours. This means that in our model, the lying cost $k$ need not be taken literally, as it can arise endogenously from the possibility in the sender's mind that the receiver may be naive.

We distinguish two cases, depending on whether or not the naive receive is aware that the message sent by the sender is subject to noise. In either case a naive receiver has the same preferences as a rationnal one, he maximizes the expected value of his utility $-(a-r\theta)^2$. A rationnal and a naive receiver only differ in their belief over $\theta$. As in the main model, given that the sender sends message $\beta\theta$ when his type is $\theta$, the best response of a rationnal receiver is to choose action $\lambda y$, with

$$\lambda = \frac{r\beta}{v+\beta^2}. \tag{30}$$

### 8.3.1 The naive receiver is also unaware of the noise

We consider here the case where the naive receiver is also unaware of the noise. Upon receiving the garbled message $y$, the naive receiver believes that the sender's type is $y$. He thus chooses action $ry$. Given that the sender expects the receiver to take action $ry$ with probability $\kappa$ and $\lambda y$ with probability $1-\kappa$, he maximizes

$$\mathbb{E}_{\varepsilon}\left[-(1-\kappa)\left(\lambda(m+\varepsilon)-s\theta\right)^2-\kappa\left(r(m+\varepsilon)-s\theta\right)^2 \mid \theta\right]$$

which is maximized by sending the message $m = \beta y$ such that

$$\beta = \frac{\left(\frac{\kappa}{1-\kappa}\right) r^2 + r\lambda}{\left(\frac{\kappa}{1-\kappa}\right) r^2 + \lambda^2} \frac{s}{r}. \tag{31}$$

Consider the following change of variable:

$$\widetilde{\beta} = \beta \frac{r}{s};\ \widetilde{\lambda} = \lambda;\ \widetilde{r} = \frac{r^2}{s};\ \widetilde{v} = \frac{vr^2}{s^2};\ \widetilde{s} = r \text{ and } \widetilde{k} = \frac{\kappa}{1-\kappa} r^2.$$

Then $(\beta, \lambda)$ solves the system (30)(31) if and only if $\left(\widetilde{\beta}, \widetilde{\lambda}\right)$ is a solution of the system (3) with parameters $\widetilde{r}$, $\widetilde{v}$, $\widetilde{s}$ and $\widetilde{k}$, i.e. if and only if it is an equilibrium in our main model with those parameters. It is now clear that this model maps to our main model in the sense made clear by the following proposition.

**Proposition 11.** *Let $(\beta, \lambda)$ be an equilibrium of of the game without lying cost, where the receiver is possibly naive and unaware of the noise with parameters $r$, $s$, $v$ and $\kappa$. Then $\left(\widetilde{\beta}, \widetilde{\lambda}\right)$ is an equilibrium in the main model with parameters $\widetilde{r}$, $\widetilde{v}$, $\widetilde{s}$ and $\widetilde{k}$.*

In particular, it is clear that increasing the probability that the receiver is naive is equivalent to an increase in the lying cost, at least from the point of view of the effect on the equilibrium. One can also see that the effect of an increase of $\kappa$ on the welfare of the strategic receiver has the same direction as an increase of $k$ on the welfare of the receiver in the main model.

#### 8.3.2 The naive receiver is aware of the noise

We now turn to the more complicated case where the naive receiver is aware of the noise. Upon receiving the garbled message $y$, the naive receiver only differs from a rationnal one in that he believes that the sender is truthful, i.e. plays $\beta = 1$. This implies that his optimal action is $r^* y$, with $r^* = \frac{r}{v+1}$, instead of $ry$ in the version of the model where the naive receiver is also unaware of the noise. The rest of the analysis is identical in the two versions of the models. We obtain the best response functions:

$$\begin{cases} \lambda = \frac{r\beta}{v+\beta^2} \\ \beta = \frac{\left(\frac{\kappa}{1-\kappa}\right)\left(\frac{r}{1+v}\right)^2 + \left(\frac{r}{1+v}\right)\lambda}{\left(\frac{\kappa}{1-\kappa}\right)\left(\frac{r}{1+v}\right)^2 + \lambda^2} \frac{s(1+v)}{r}. \end{cases} \tag{32}$$

Consider the following change of variable:

$$\widehat{\beta} = \beta \frac{r}{s(1+v)};\ \widehat{\lambda} = \lambda;\ \widehat{r} = \frac{r^2}{s(1+v)};\ \widehat{v} = \frac{vr^2}{s^2(1+v)^2};\ \widehat{s} = \frac{r}{1+v} \text{ and } \widehat{k} = \frac{\kappa}{1-\kappa}\left(\frac{r}{1+v}\right)^2.$$

Then $(\beta, \lambda)$ solves the system (32) if and only if $\left(\widehat{\beta}, \widehat{\lambda}\right)$ is a solution of the system (3) with parameters $\widehat{r}$, $\widehat{v}$, $\widehat{s}$ and $\widehat{k}$, i.e. if and only if it is an equilibrium in our main model with those parameters.. It is now clear that this model maps to our main model in the sense made clear by the following proposition.

**Proposition 12.** *Let $(\beta, \lambda)$ be an equilibrium of of the game without lying cost, where the receiver is possibly naive but aware of the noise with parameters $r$, $s$, $v$ and $\kappa$. Then $\left(\widehat{\beta}, \widehat{\lambda}\right)$ is an equilibrium in the main model with parameters $\widehat{r}$, $\widehat{v}$, $\widehat{s}$ and $\widehat{k}$.*

In particular, it is clear that increasing the probability that the receiver is naive is equivalent to an increase in the lying cost, at least from the point of view of the effect on the equilibrium. One can also see that the effect of an increase of $\kappa$ on the welfare of the strategic receiver has the same direction as an increase of $k$ on the welfare of the receiver in the main model.

# 9 Conclusion

TBA.

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# 10 Appendix A: uniqueness and multiplicity of understated equilibria

In this appendix, we study under what conditions on the parameters there exists a unique understated equilibrium. From Proposition ?, we already know that an understated equilibrium exists if and inly if the inequality

$$r > s(1+v)$$

holds. We assume in this appendix that this condition holds. Because $\beta \in (0,1)$ implies $\gamma_1(\beta) < 0$ every understated equilibrium satisfies $\gamma = \gamma_2(\beta)$.

Because $\gamma_2$ is strictly decreasing on the interval $[1/2, 1]$ and $f$ is strictly increasing there can be at most one slightly equilibrium and the conditions $f(0.5) \leq \gamma_2(0.5)$ and $f(1) > \gamma_2(1)$ are necessary and sufficient for existence of a slightly equilibrium. As we will see below the condition $f(0.5) \leq \gamma_2(0.5)$ is not sufficient to exclude the possibility that besides the slightly equilibrium there are also strongly understated ones. If, however, the stronger condition $f(0.5) \leq s$ holds, there can be no strongly understated equilibrium as $\gamma_2(\beta) > s > f(\beta)$ then holds for all $\beta \in (0, 1/2)$. We thus have the following result.

**Proposition 13.** *An equilibrium satisfying* $1/2 \leq \beta < 1$ *exists if and only if*

$$(1+v)s < r \leq [1+4v]\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right]. \tag{33}$$

*Furthermore, there is at most one such equilibrium and if*

$$(1+v)s < r \leq (1+4v)s \tag{34}$$

*holds then the unique equilibrium satisfying* $1/2 < \beta < 1$ *is also the unique equilibrium satisfying* $\beta > 0$.

For large enough values of $k$ it follows from (33) that a slightly understated equilibrium exists provided the condition $(1+v)s < r$ is satisfied. Intuition suggests that for large value of $k$ there can be no strongly understated equilibrium as it is too expensive for the sender to stray far from $\beta = 1$, implying that under these circumstance a slightly equilibrium not only exists but is also the unique straight talking equilibrium. The following proposition formalizes this intuition.

**Proposition 14.** *Suppose*

$$(1+v)s < r \leq 4\sqrt{v}\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right] \tag{35}$$

*holds. Then there exists a unique straight talking equilibrium which satisfied* $1/2 \leq \beta < 1$.

*Proof.* Define $\lambda_f : (-\infty, +\infty) \to (0, \infty)$ by $\lambda_f(\beta) = f(\beta)/\beta = r\beta/(\beta^2 + v)$. The derivative of this function is

$$\lambda_f'(\beta) = \frac{v - \beta^2}{\beta^2 + v)^2},$$

implying that $\lambda_f(\beta)$ is unimodal on $[0, \infty)$ with the unique maximum occurring at $\beta = \sqrt{v}$. Hence, we have $f(\beta) \leq \lambda_f(\sqrt{v})\beta$ for all $\beta \in [0, \overline{\beta}]$. Because $\gamma_2$ is concave and $\gamma_2(0) \geq 0$ holds, it follows that the condition $\gamma_2(1/2) \geq \lambda_f(\sqrt{v})/2$ is sufficient to imply $\gamma_2(\beta) > f(\beta)$ for all $\beta \in (0, 1/2)$. Calculating $\lambda_f(\sqrt{v})$ yields the result. □

Observe that (as must be the case) the rightmost side in (35) is smaller than the rightmost side in (33), but that for $v = 1/4$ these two expressions are identical, implying that in this special case whenever a slightly understated equilibrium exists it is the unique straight-talking equilibrium. More generally, as asserted in the following proposition, there is a unique straight talking equilibrium whenever $v \geq 1/4$ holds.

**Proposition 15.** *If* $v \geq 1/4$ *there exists a unique straight talking equilibrium.*

*Proof.* Define $\lambda_2 : (0, \overline{\beta}) \to (0, \infty)$ by $\lambda_2(\beta) = \gamma_2(\beta)/\beta$. Because $\gamma_2$ is strictly concave and $\gamma_2(0) \geq 0$ holds, the function $\lambda_2$ is strictly decreasing on $(0, 1/2]$ whereas for $v \geq 1/4$ the function $\lambda_f$ (defined in the proof of the previous proposition) is strictly increasing on $(0, 1/2]$. It follows that the functions $\lambda_2$ and $\lambda_f$ have either zero or one intersections on $(0, 1/2)$. In the first case every straight talking equilibrium satisfies $\beta \geq 1/2$ and it follows from previous results (Propositions **??**, **??**, and 13) that there is a unique straight talking equilibrium. In the second case there is a unique gentle equilibrium and we must have $\lambda_f(1/2) > \lambda_2(1/2)$, implying $r > [1 + 4v]\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right]$. From previous results (Propositions **??**, **??**, and 13) the later condition precludes the existence of a straight talking equilibrium with $\beta \geq 1/2$, implying that the unique gentle equilibrium is also the unique straight talking equilibrium. □

We know that the condition

$$r > [1 + 4v] \left[ \frac{s}{2} + \frac{\sqrt{s^2 + k}}{2} \right] \tag{36}$$

is sufficient for the existence of a gentle equilibrium and ensures that all straightforward equilibria are gentle. If $v \geq 1/4$ holds, the previous result implies that (36) is necessary and sufficient for the existence of a gentle equilibrium and ensures it uniqueness. Without the additional condition $v \geq 1/4$ we are neither assured that (36) is necessary for existence of a gentle equilibrium (rather we have the weaker necessary conditions $r > \max\{(1+4v)s, 4\sqrt{v}\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right]\}$) nor do we have a uniqueness result.

At the cost of replacing (36) by a stronger condition, the following proposition extends our previous uniqueness result for gentle equilibria to the case $v < 1/4$.

**Proposition 16.** *If*

$$r \geq 2 \left[ \frac{s}{2} + \frac{\sqrt{s^2 + k}}{2} \right] \tag{37}$$

*then there there exists a unique straight talking equilibrium. If $v < 1/4$ this equilibrium satisfies $\beta < 1/2$.*

*Proof.* Condition (37) is equivalent to $f(\sqrt{v}) \geq \gamma_2(1/2)$. Suppose $v < 1/4$. Then (37) implies (36), so that all straight talking equilibria satisfy $\beta < 1/2$. Uniqueness follows by observing that both $f$ and $\gamma_2$ are increasing on $[0, 1/2]$ so that $f(\sqrt{v}) \geq \gamma_2(1/2)$ implies $f(\beta) > \gamma_2(\beta)$ for all $\beta \in [\sqrt{v}, 1/2]$. Because $\lambda_f$ is strictly increasing and $\lambda_2$ is strictly decreasing on $(0, \sqrt{v})$ it follows that $f$ and $\gamma$ have at most one intersection on $[0, 1/2]$. If $v \geq 1/4$ uniqueness follows from the preceding result. □

Our results so far establish that multiple straight talking equilibria can only exist if there is at least one gentle equilibrium. We now provide an upper bound on $k$ which ensures that if there is a slightly understated equilibrium, it must be the unique straight talking equilibrium, thus establishing uniqueness of straight talking equilibrium for sufficiently small lying costs.

**Proposition 17.** *If*

$$k \leq rs\frac{v}{(v + 1/4)^2} \tag{38}$$

*then there is a unique equilibrium with $\beta > 0$.*

*Proof.* We first observe that the first and second derivatives of $f$ are

$$f'(\beta) = 2rv\frac{\beta}{(v+\beta^2)^2}, \quad f''(\beta) = 2rv\frac{(v-3\beta^2)}{(v+\beta^2)^3}. \tag{39}$$

Hence on $(0,\infty)$ the function $f$ has a unique inflection point $\beta^\circ = \sqrt{v/3}$. Suppose there exist equilibria satisfying $0 < \beta < 1/2$. (Otherwise uniqueness of straight talking equilibria is immediate from the previous results.) Let $\beta^*$ be the smallest such equilibrium. It must then be the case that $f'(\beta^*) \geq \gamma'(\beta^*)$ holds. If $f'(\beta^*) > \gamma_2'(\beta^*)$ and there is no other equilibrium satisfying $\beta < 1/2$, it follows that $f(1/2) > \gamma_2(1/2)$ holds, precluding the existence of any equilibrium with $\beta \geq 1/2$, so that there is a unique straight talking equilibrium. Similarly, unless there is another equilibrium satisfying $\beta < 1/2$, $f'(\beta^*) = \gamma_2'(\beta^*)$ and $f''(\beta^*) > \gamma_2''(\beta^*)$ implies $f(1/2) > \gamma_2(1/2)$ and thus uniqueness of straight talking equilibria. Hence, if there are multiple straight talking equilibria, we must either have (i) $f'(\beta^*) = \gamma_2'(\beta^*)$ and $f''(\beta^*) \leq \gamma''(\beta^*)$ or (ii) there exists a second equilibrium with $1/2 > \beta^{**} > \beta^*$ satisfying $f'(\beta^{**}) \leq \gamma_2'(\beta^{**})$. In either case we have the existence of an equilibrium satisfying $0 < \beta < 1/2$, $f'(\beta) \leq \gamma_2'(\beta)$ and $\beta > \beta^\circ$: In case (i) the conclusion $\beta > \beta^\circ$ follows because $\gamma_2$ is strictly concave, so that $f''(\beta^*) \leq g''(\beta^*)$ implies $f''(\beta^*) < 0$; in case (ii) it follows because we have $\gamma_2'(\beta^*) \leq f'(\beta^*)$ and $f'(\beta^{**}) \leq \gamma_2'(\beta^{**})$ and therefore, from the concavity of $\gamma_2$, $f'(\beta^{**}) < f'(\beta^*)$. This implies the inflection point of the $f$ lies on $[0, \beta^{**})$. To finish the proof it thus suffices to show that condition (38) implies that there is no $\beta \in (\beta^\circ, 1/2)$ satisfying $f'(\beta) \leq \gamma_2(\beta)$. Towards this end we first observe that $\beta \in (\beta^\circ, 1/2)$ implies

$$f'(\beta) \geq f'\left(\frac{1}{2}\right) = \frac{rv}{\left(v+\frac{1}{4}\right)^2}.$$

Second, since $\gamma_2'$ is strictly decreasing, we have that $f'(\beta) \leq \gamma_2'(\beta)$ implies

$$f'(\beta) < \gamma_2'(0) = \frac{k}{s}.$$

Therefore, if both of these conditions holds we have be that

$$\frac{v}{\left(v+\frac{1}{4}\right)^2} < \frac{k}{rs},$$

contradicting (38). $\square$

## 10.1 Summary of sufficient conditions for a unique straight-talking equilibrium

Uniqueness of straight talking equilibrium is assured if the communication schedule is sufficiently noisy, that is $v \geq 1/4$. Depending on the other parameters, this case is consistent with the equilibrium value of $\beta$ taking any value in $(0, \overline{\beta}]$.

For small noise, that is $v < 1/4$, the following are sufficient for uniqueness:

- $r$ is sufficiently small relative to $s$, that is $r \leq (1 + 4v)s$. In this case the equilibrium value of $\beta$ satisfies $\beta > 1/2$. Observe, in particular, that if $r \leq s$ uniqueness holds independently of the other parameter values.
- $r$ is sufficiently large relative to $s$, that is $r > s+\sqrt{s^2 + k}$. Under the assumption $v < 1/4$ in this case the equilibrium value of $\beta$ satisfies $\beta < 1/2$.
- Large lying costs $k$, that is $r \leq 4\sqrt{v}\left[\frac{s}{2} + \frac{\sqrt{s^2+k}}{2}\right]$. In this case the equilibrium value of $\beta$ satisfies $\beta \geq 1/2$.
- Small lying cost $k$, that is $k \leq rs\frac{v}{(v+1/4)^2}$.

## 10.2 Sufficient conditions for multiple straight talking equilibria

From the above sufficient conditions for uniqueness, multiple straight talking equilibria can exist only if $v$ is small and the other parameters are in some intermediate range. So far we we have noted that multiplicity of straight talking equilibria requires that all straight-talking equilibria be understated, that at least one of them be strongly understated, and that at most one of them be slightly understated. We have not, however, provided explicit conditions on the underlying parameters which are sufficient for the existence of multiple straight talking equilibria. Here we fill this gap.

The basic idea is very simple: Assuming $r < \frac{s}{2} + \frac{\sqrt{s^2+k}}{2}$ ensures that there exists a straight-talking equilibrium satisfying $\beta > 1/2$. On the other hand, as the proof of the following proposition demonstrates, for sufficiently low $v$ a strongly understated equilibrium exists whenever $s < r$ holds.

**Proposition 18.** *Let*

$$s < r < \frac{s}{2} + \frac{\sqrt{s^2 + k}}{2} \tag{40}$$

*and suppose*

$$\sqrt{v} \leq \min\{\frac{s}{k}\hat{\rho}, \frac{1}{2\hat{x}}\} \tag{41}$$

*where* $\hat{\rho} = \max_{x \geq 0} \left[ r\frac{x}{1+x^2} - s\frac{1}{x} \right] > 0$ *and* $\hat{x} > 0$ *is a solution to this maximization problem. Then there exist multiple straight-talking equilibria.*

*Proof.* We first demonstrate that $\hat{\rho}$ and $\hat{x}$ are well defined and satisfy $\hat{\rho} > 0$ and $\hat{x} > 0$. Towards this end observe that

$$\rho(x) = r\frac{x}{1+x^2} - s\frac{1}{x} = \frac{1}{x(1+x^2)} \left[ (r-s)x^2 - s \right],$$

so that the assumption $r > s$ in (40) implies that $\rho(x)$ is strictly positive for sufficiently large $x$. As $\rho(0) < 0$ and $\lim_{x\to\infty} \rho(x) = 0$ it follows that $\hat{\rho}$ and $\hat{x}$ are well-defined and satisfy $\hat{\rho} > 0$ and $\hat{x} > 0$. Now set $\hat{\beta} = \hat{x}\sqrt{v}$. From (41) we have $\hat{\beta} \leq 1/2$. Because $\gamma_2$ is strictly concave, $\gamma_2(0) = s$, and $\gamma_2'(0) = \frac{k}{s}$ we have $\gamma_2(\hat{\beta}) < s + \frac{k}{s}\hat{x}\sqrt{v}$. We also have $f(\hat{\beta}) = r\hat{x}^2/(1+\hat{x}^2)$, so that (41) implies $f(\hat{\beta}) > g(\hat{\beta})$. It follows that there exists a strongly understated equilibrium. As the second inequality in (40) implies that there exists an equilibrium with $\beta > 1/2$, it follows that there are multiple straight talking equilibria. □

# 11 Appendix B: the case $s = 0$

The special case $s = 0$ in which the sender does not care about the state $\theta$ is of interest because here the effect of lying costs on the equilibrium analysis are particularly easy to understand. We first observe that in this case there are no ironic equilibria. All equilibria have to be straight talking (because $\underline{\beta} = 0$) and understated (because $\overline{\beta} = 1$ and $\gamma_1(\overline{\beta}) = \gamma_2(\overline{\beta}) = 0$.

The uniqueness results from the previous section apply to the case $s = 0$ (with the "small $k$"-bound stated in Proposition 17 and the bound $r \leq [1 + v]$ never being satisfied for any values of the remaining parameters). In particular, equilibrium is unique if $v \geq 1/4$ holds, with the equilibrium being loud if $r \leq [1 + 4v]\sqrt{k}$ and gentle otherwise. If $v < 1/4$ there is a unique equilibrium if $r \leq 2\sqrt{v}\sqrt{k}$ (in which case the equilibrium is loud) or $r \geq \sqrt{k}$ (in which case the equilibrium is gentle). Observe that the latter of these conditions implies that as in the case $s > 0$ equilibrium is unique for sufficiently small lying costs.

Even in the case $s = 0$ multiple equilibria may occur. To see this, suppose the inequality $r \leq \frac{1}{2}\sqrt{k}$ holds, ensuring that for all $v > 0$ a loud equilibrium exists. Provided that $v < 1/4$ holds, it follows that there must be multiple equilibria if the inequality

$$f(\sqrt{v}) \geq \gamma_2(\sqrt{v}) \Leftrightarrow r \geq 2\sqrt{\sqrt{v} - v}\sqrt{k} \tag{42}$$

is satisfied. For values of $v$ satisfying the inequalities $\sqrt{v} - v < 1/16$ and $v < 1/4$ the interval $(2\sqrt{\sqrt{v} - v}\sqrt{k}, \frac{1}{2}\sqrt{k})$ is non-empty and for all $r$ in this interval multiple equilibria exist. Hence, provided that $v < \underline{v} \approx 0,0044$ holds, we have multiple equilibria.

## 12 Appendix C: the case $r < 0$

We first observe that in this case there are no ironic equilibria. From our general existence result a straight talking equilibrium exists. Because $f(\beta) < 0$ holds for $\beta > 0$ all straight talking equilibria must feature $\gamma < 0$ and must thus be understated and satisfy $\gamma = \gamma_1(\beta)$.

Following the same logic as in the case $r > 0$ some results are clear:

- Loud equilibria exist if and only if $f(1/2) > \gamma_1(1/2)$ which translates to

  $$r \geq [4v + 1]\left[\frac{s}{2} - \frac{\sqrt{s^2 + k}}{2}\right].$$

  Furthermore, there is at most one loud equilibrium.

- The condition

  $$r \geq 4\sqrt{v}\left[\frac{s}{2} - \frac{\sqrt{s^2 + k}}{2}\right]$$

  ensures that there is no gentle equilibrium. Hence, if this condition holds there is a unique equilibrium which is loud.

- If $v \geq 1/4$ there is a unique equilibrium.

# Figures

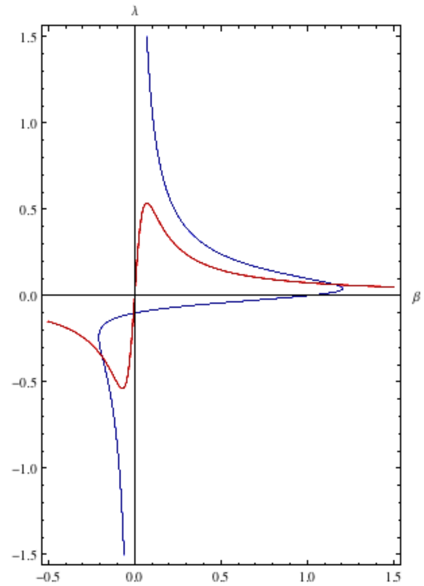



Figure 1: Best response diagram. Receiver in red and sender in blue. Parameter values: $r = 0.076$; $s = 0.1$; $v = 0.005$; $k = 0.01$.

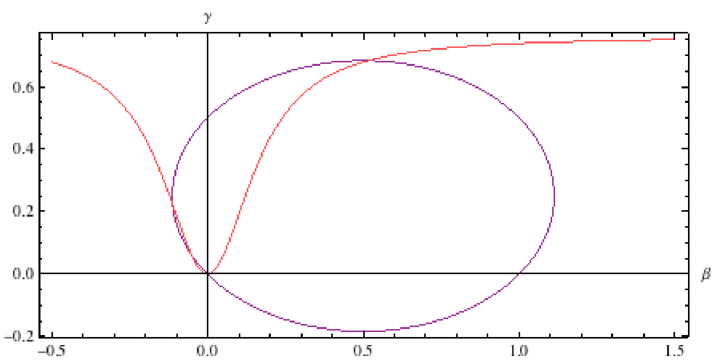



Figure 2: Best response diagram in $(\beta, \gamma)$ coordinates. Receiver in red and sender in blue. Parameter values: $s = 0.5$; $r = 0.76$; $v = 0.03$; $k = 0.5$.

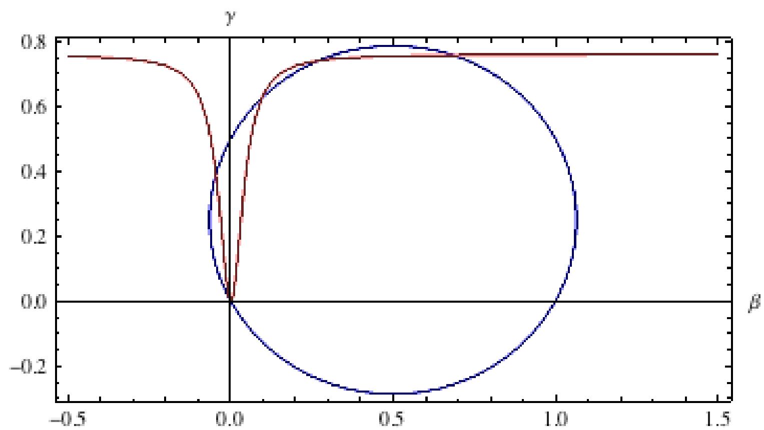



Figure 3: Best response diagram in $(\beta, \gamma)$ coordinates. Receiver in red and sender in blue. Parameter values: $s = 0.5$; $r = 0.76$; $v = 0.002$; $k = 0.9$.

Parameters: $s = 0.5$; $r = 0.76$; $\sigma^2 = 0.002$; $k = 0.9$.

Exaggerating equilibria

Parameters: $s = 0.5$; $r = 0.3$; $\sigma^2 = 0.002$; $k = 0.9$.

Exaggerating equilibria

Parameters: $s = 0.5$; $r = 0.76$; $\sigma^2 = 2$; $k = 0.9$.

Three understated equilibria

Parameters: $s = 0.5$; $r = 0.76$; $\sigma^2 = 0.002$; $k = 0.9$.

Increasing the conflict of interest

Changing the sender's lying cost (exaggerated region)

Decreasing the sender's lying cost (understated region)

Increasing vagueness

Decreasing the sender's competence

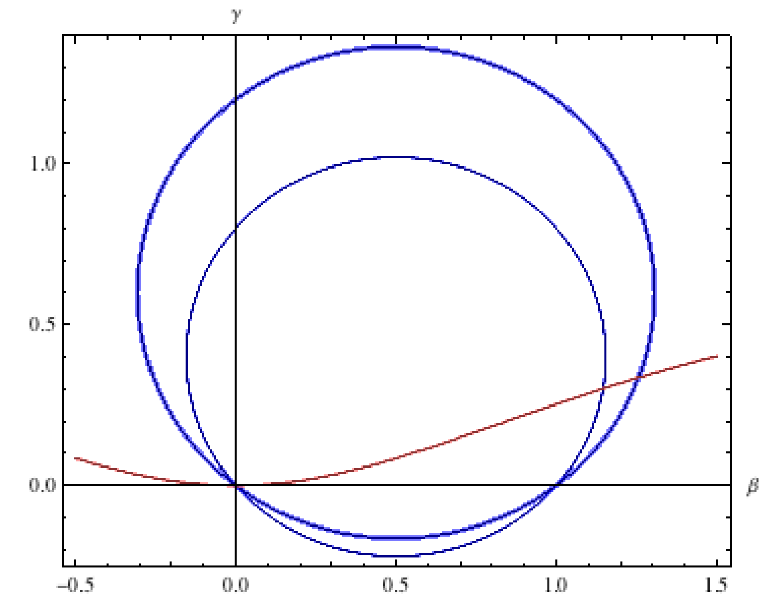



Figure 4: Increasing the conflict of interest. Parameter values $s_1 = 0.8$ (light blue) and $s_2 = 1.2$ (thick blue); $r = 0.76$; $v = 2$; $k = 0.9$.

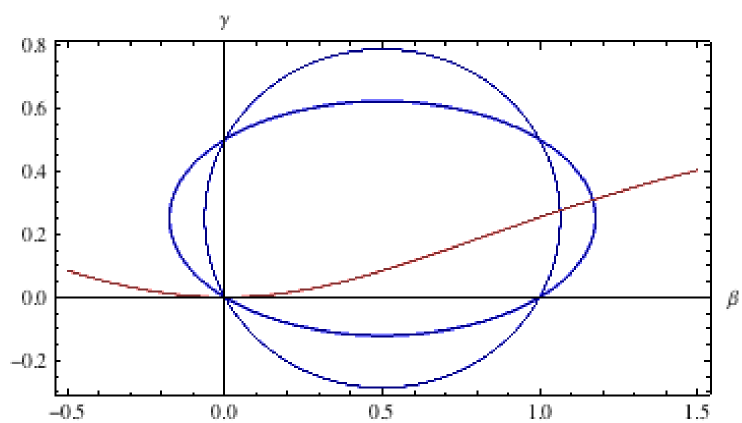



Figure 5: Decreasing the lying cost. Parameter values ; $s = 0.5$; $r = 0.76$; $v = 2$; $k_1 = 0.9$ (light blue) and $k_2 = 0.3$ (thick blue).

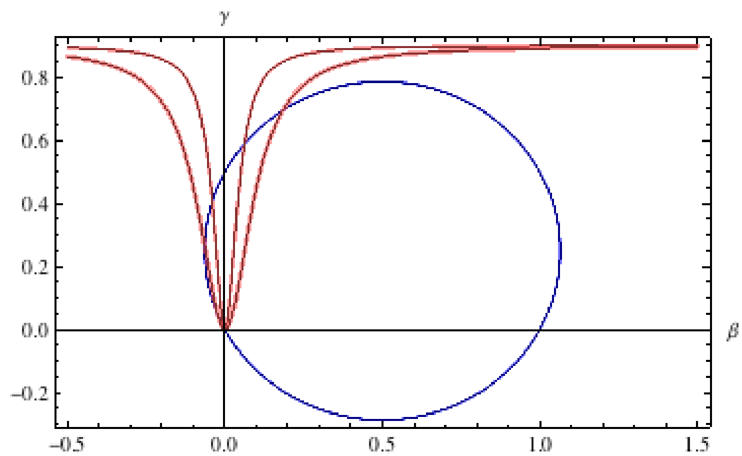



Figure 6: Increasing the noise level. Parameter values: $s = 0.5$; $r = 0.9$; $v_1 = 0.002$ (light red) and $v_2 = 0.01$ (thick red); $k = 0.9$.

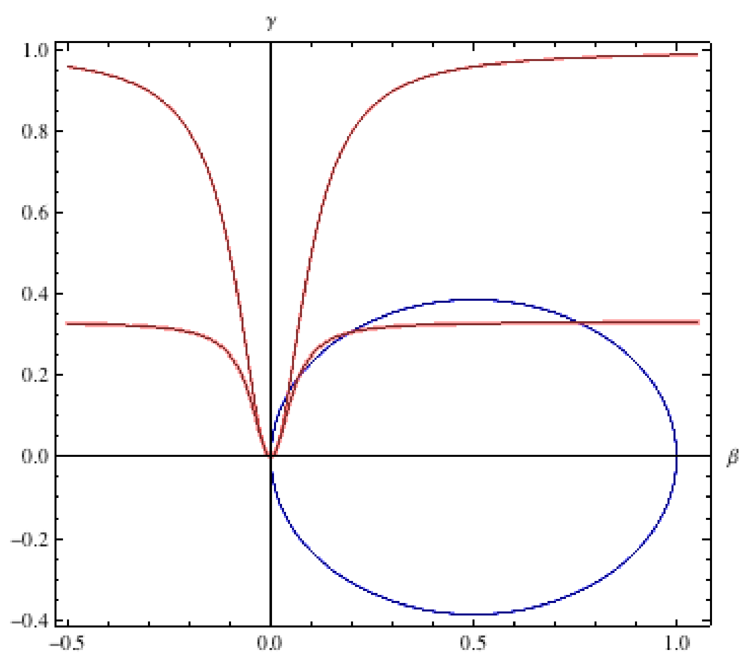
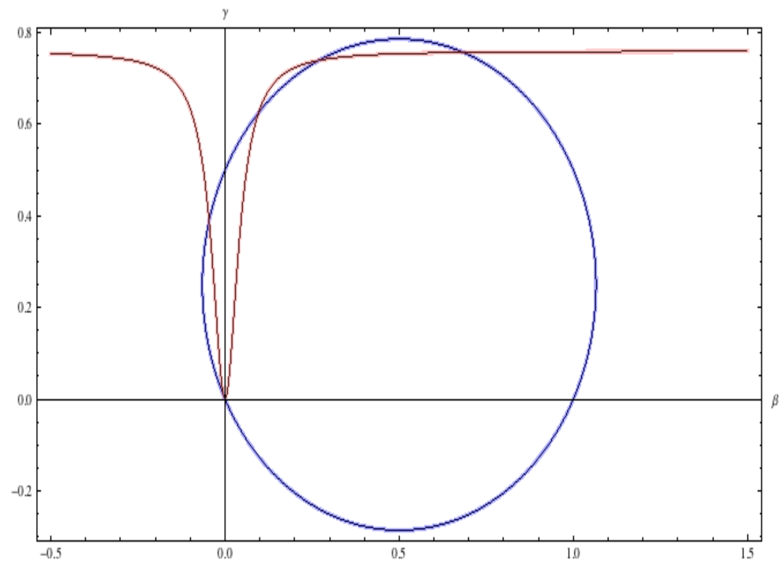



Figure 7: Decreasing the sender's competence. Parameter values: $s = 0$; $r = 1$; $v = 0.01$; $u_1 = 1$ (light red) and $u_2 = 3$ (thick red).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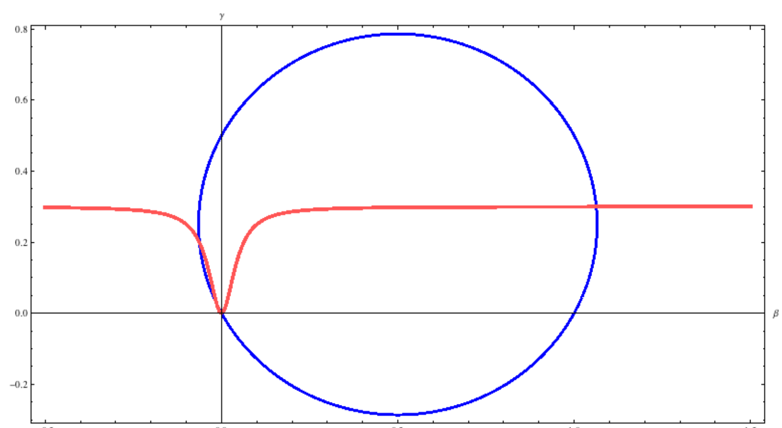
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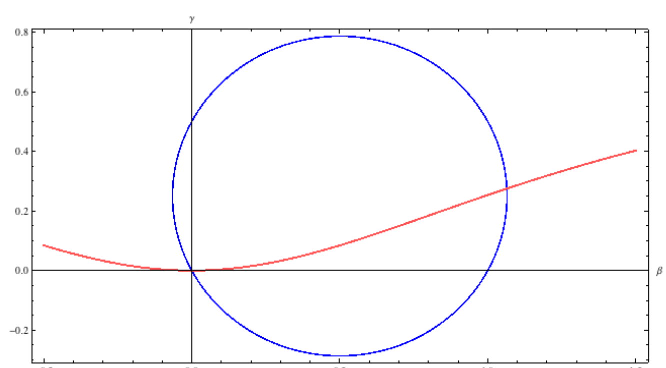
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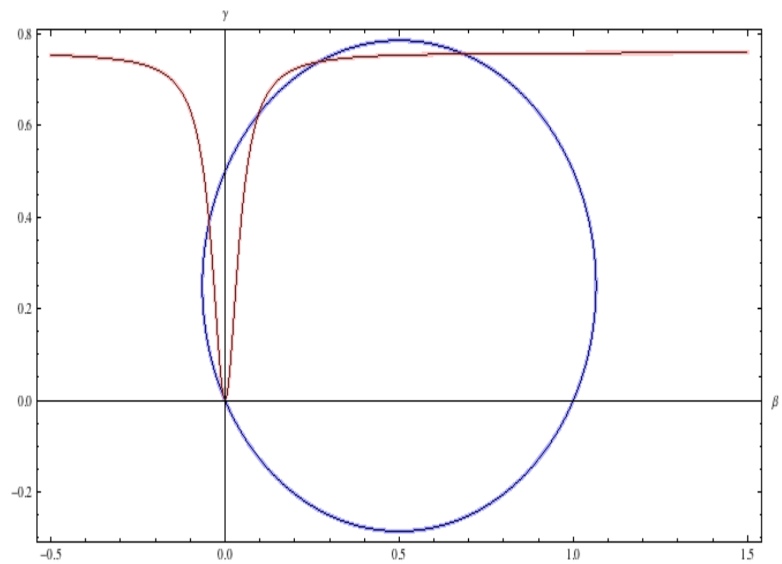
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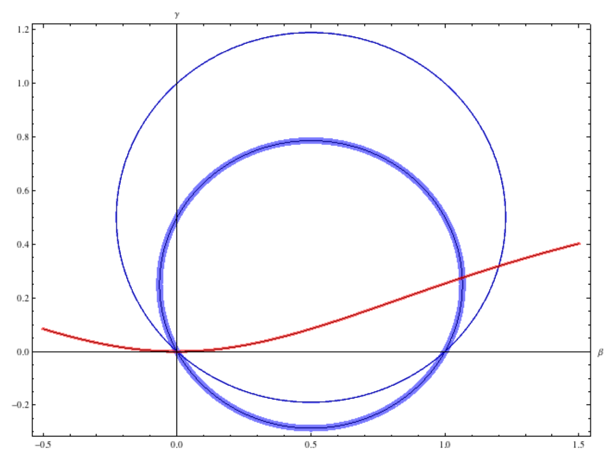
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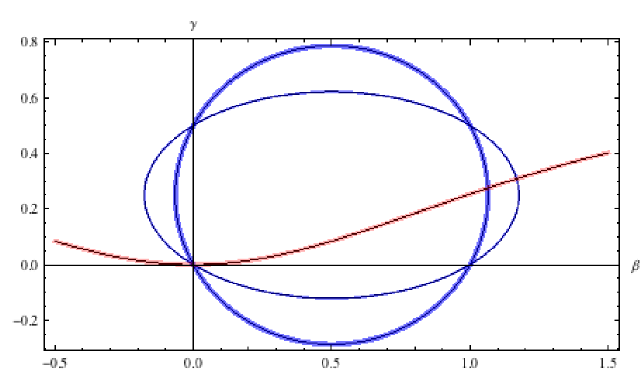
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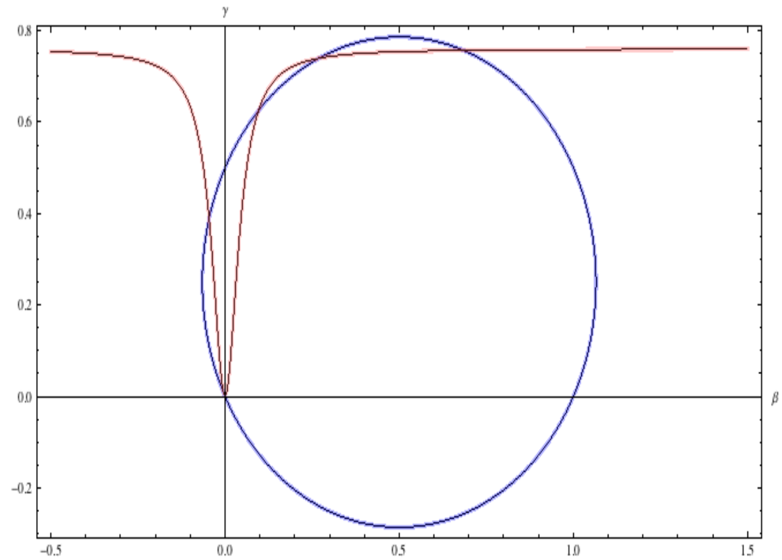
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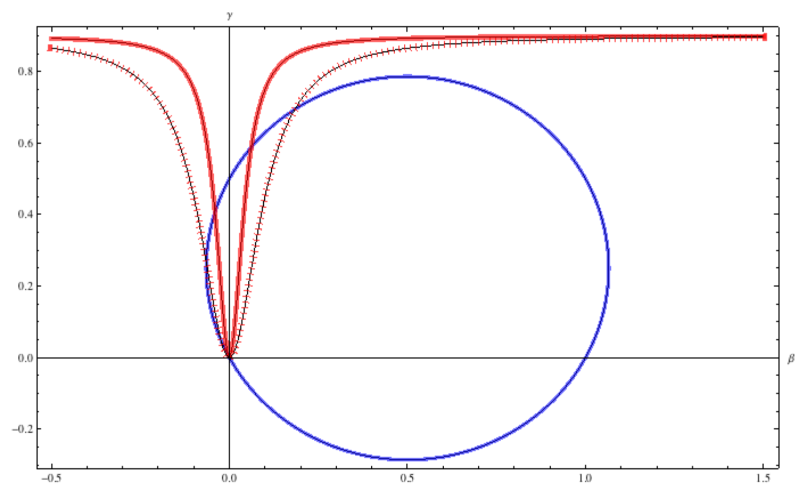
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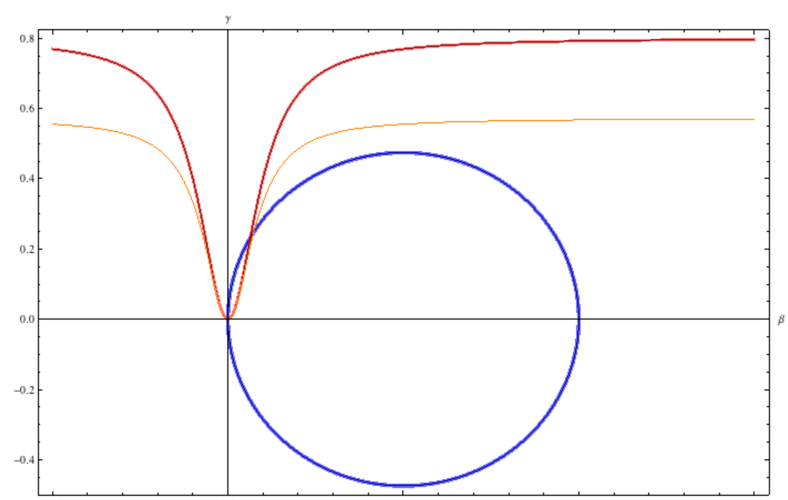
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