Do Tournaments Solve the Adverse Selection Problem?

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September 23, 2013

Abstract. This paper provides a solution to a puzzle in the analysis of tournaments, that of why there is no agent discrimination in practice. The paper examines the problem of a principal contracting with multiple agents whose activities are subject to common shocks, when there is moral hazard and adverse selection. The presence of common shocks invites the use of relative performance evaluation to minimize the costs of moral hazard. But, in the additional presence of adverse selection, the analysis shows that at the optimum there may be no need for ex ante screening through menus of contract offers (i.e., for agent discrimination). This is so because the principal becomes better informed ex post about agent types, via the realization of common uncertainty, and can effectively penalize or reward the agents ex post. Thus, unlike the standard adverse selection problem without common uncertainty where the principal benefits from ex ante screening, it is shown that ex post sorting through relative performance evaluation reduces the scope for ex ante screening through menus, and eliminates it completely if agents are known to not be very heterogeneous. This is consistent with observed practice in industries where the primary compensation mechanism is a cardinal tournament which is uniform among agents.

Keywords: Tournaments; Contests.

JEL Codes: D82, D86.

Acknowledgments: I am grateful to the participants of the 2013 Conference of the Society for the Advancement of Economic Theory at MINES ParisTech in France for very useful comments.

1. Introduction

It is well established in contract theory that under adverse selection the uninformed party can do better by offering a menu of contract offers to the informed party than by offering a single contract.[1] Letting the agent self-select an offer from the menu enables the screening of the agent's type under appropriate incentive compatibility constraints. What is particularly puzzling is the fact that in certain industries where the primary compensation mechanism is relative performance evaluation via a cardinal tournament, uninformed parties offer a uniform tournament contract to all the agents. What explains this empirical deviation, that is, the lack of agent discrimination at the time of contracting, from the anticipated theoretical optimum?

Considerable research on tournaments has been motivated by broiler contracting, ever since Knoeber (1989) identified the importance of this application for tournament contracting. Cardinal tournaments have been in use by processor companies during the production of broilers (in houses owned and operated by growers) for decades now, and a lot of experience has been obtained with the optimal design and fine-tuning of such incentive schemes.[2] One empirical regularity in the production of broilers, which is quite puzzling, is the absence of agent discrimination at the time of initial contracting or at the time of subsequent recontracting, even after some information about grower abilities has been obtained. Ever since the work of Weitzman (1980), Freixas, Guesnerie and Tirole (1985) and Laffont and Tirole (1988), it has been suggested that recontracting under more onerous terms once some information about ability types is observed (i.e., the so called "ratchet effect") can distort agent incentives to perform initially, which can partially explain the absence of agent discrimination at the time of re-contracting.[3] However, the absence of screening at the initial stage of contracting is extremely puzzling. Lack of agent discrimination can also frequently be observed in the contracting of HMOs with physicians, in sales contracts and in determining annual raises for faculty.

A cardinal tournament rewards an agent based on his performance relative to that of his peers, specifically, agents receive a fixed base payment that induces participation, and a variable bonus which depends on the difference of an agent's performance from the average of his peer group.[4] A cardinal tournament is a type of *forcing contract* that resolves the

[1] For instance, see Page (1992, 1997).

[2] For the stylized facts related to the production of broilers see Knoeber (1989), Knoeber and Thurman (1994, 1995), Tsoulouhas and Vukina (1999) and Wu and Roe (2005, 2006).

[3] See Allen and Lueck (1998) for a discussion of the rachet effect in agriculture.

[4] Even though the founders of tournament theory, Lazear and Rosen (1981), focused on ordinal or rank-order tournaments, such tournaments are informationally wasteful when data on the agents' cardinal performance are available (as argued by Holmström (1982)), which is certainly the case with broiler production. Moreover, Tsoulouhas (2012) has shown that switching from ordinal to cardinal tournaments improves effi-

moral hazard problem through penalties for performance below the agent group average and rewards for performance above the group average. It also captures the essence behind a Crémer and McLean (1988) incentive scheme with correlated values. Starting with the seminal work of Lazear and Rosen (1981), Holmström (1982), Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Malcomson (1984), ordinal tournaments (i.e., tournaments based on rank) and cardinal tournaments have been shown to alleviate the moral hazard problem.[5] In addition to moral hazard on the agent side, cardinal tournaments, specifically, have been shown to alleviate the moral hazard problem on the principal side as well.[6] However, do cardinal tournaments alleviate the adverse selection problem? Lazear and Rosen (1981) argue that sorting between minor and major leagues would fail because agents would not self-select (all agents would want to be in the major leagues, that is, in the high ability group). Given that agents do not self-sort, agents in the resulting mixed leagues do not exert the efficient effort. In this spirit, adverse selection due to agent heterogeneity has not received enough attention in the tournament literature.[7]

This paper provides a solution to the puzzle of no agent discrimination at the time of contracting which is traced to the very nature of tournaments (or relative performance evaluation) as opposed to the absolute performance evaluation contracts that standard contract theory has considered. Specifically, tournaments screen and filter away common production shocks from the responsibility of agents. Therefore, ex post relative performance of an agent is a measure of an agent's ability subject to his idiosyncratic (luck) component. By contrast, ex post absolute performance of an agent is a measure of the agent's ability, realized common production shocks and idiosyncratic shock. In other words, to the extent that common shocks are relatively significant, ex post relative performance evaluation can improve the accuracy of inferences made about agent ability. Thus, agents can be penalized or rewarded ex post according to their inferred innate ability. The ex post ability to sort

ciency. Lastly, with rank-order tournaments and menus, each agent would have to self-select into different sets of prespecified prizes, which is arguably very difficult to determine and, hence, to the best of our knowledge, is non-existent in practice.

[5] An associated literature is the one on contests, often based on contest success functions. See Tullock (1980) for rent-seeking contests, Perez-Castrillo and Verdier (1992) for a more recent discussion of rent-seeking games, Skaperdas (1996) for an axiomatization of contest sucess functions, Wärneryd (2003) for uncertain prizes, and Konrad and Kovenock (2009) for contests with multiple rounds.

[6] See Carmichael (1983) and Tsoulouhas (1999).

[7] Early exceptions are O'Keefe, Viscusi and Zeckhauser (1984), Glazer and Hassin (1988), Bhattacharya and Guasch (1988) and Yun (1997). O'Keefe, Viscusi and Zeckhauser show that the Lazear and Rosen "climbing" problem can potentially be eliminated by increasing the prize spread, provided that the employer monitors the contestants less precisely. Yun examines a modification of the standard rank-order tournament, that penalizes a small number of contestants heavily, to show that such a scheme can achieve first best efficiency when there is moral hazard and adverse selection with risk-neutral agents. More recently, Eriksson, Teyssier and Villeval (2009) study the ex ante sorting effect of tournaments and its impact on the variability of effort.

agents, by obtaining informative signals about their abilities, and discriminate reduces the scope for ex ante agent discrimination (screening) via contract menus. By contrast, without tournaments, that is, with absolute performance evaluation, high agent performance would be rewarded even if it were the sole outcome of pure stochastic effects that are common among the agents. Thus, the contribution of the paper is to show that whereas ex ante agent discrimination through a screening contract makes perfect sense with absolute performance evaluation and in accordance with standard contract theory (for instance, see the analysis in Bolton and Dewatripont (2005)), no ex ante discrimination via a single pooling contract for each and every agent can be optimal under relative performance evaluation via cardinal tournaments.[8] The analysis characterizes an intuitive condition, which relies on a measure of agent heterogeneity, under which a pooling contract offer to all the agents dominates the offer of a screening contract menu. Specifically, the analysis shows that ex post sorting through relative performance evaluation completely eliminates the scope for ex ante screening through menus if agents are known to not be very heterogeneous. This is consistent with the stylized facts for the broiler industry, at least, where growers (farmers) share similar attributes. Thus, in all, tournaments do solve the adverse selection problem if agents are not very heterogeneous. This result is in sharp contrast with standard contract theory models without relative performance evaluation mechanisms where screening is always optimal regardless of agent heterogeneity. The degree of agent heterogeneity is very important under tournaments in that ex ante screening is inefficient if the agents are known to not be very heterogeneous, given that tournaments enable the ex post sorting of agents.

The intuition is that whereas screening contracts can be more efficient by tailoring agent incentives to their innate abilities, such fine-tuning comes at a monetary cost to the principal over the pooling contract offer, especially given the principal's ability to sort the agents ex post via relative performance evaluation. In other words, why bother to gain more information ex ante if you can get most of this information ex post at a lower cost? Only if agents are sufficiently heterogenous does it make sense to engage in ex ante screening in order to provide higher-power incentives for high ability types. If agents are not known to be sufficiently heterogeneous, the pooling contract increases expected profits because it does not require the satisfaction of self-selection incentive compatibility constraints. Moreover, even though the screening contract implements higher-power incentives for better ability types,

[8] Regarding the second, associated puzzle, of why principals do not discriminate agents even after agent types are learned through repeated observation of performance, in addition to the implications of a ratchet effect that were mentioned above, grower discrimination would add to their complaints about low pay and subjection to group composition risk (see Tsoulouhas and Vukina (2001)). Besides, the strict requirements of relationship-specific capital investments, in broiler production, and adherence to strict production rules effectively minimize the expression of higher ability into significantly and systematically higher performance.

these types need to be paid a lot more in order to perform. In other words, higher-power incentives for insignificantly better types is too costly.

The literature of adverse selection in tournaments is limited, however, our analysis is related to several recent papers that examine agent heterogeneity. Riis (2010), examines the efficiency of contests among ex ante heterogeneous but risk-neutral agents, when the prize structure can be modified to adjust for the agents' strategic behavior, for instance, by stipulating that the winner's prize is determined by the identity of the runner up. Konrad and Kovenock (2010) examine discriminating contests with stochastic abilities. Tsoulouhas *et al* (2007) consider CEO contests that are open to heterogeneous outsiders.[9]

A word of caution is in order. The paper does not characterize overall optimal contracts or tournaments. Instead, it focuses on the specific puzzle of why there is no agent discrimination under cardinal tournaments the way they are used in practice (at least in the case of contracts for salesmen, agricultural contracts and contracts for physicians contracting with HMOs).[10]

Section 2 presents the model. Section 3 examines the perfect information tournament as a benchmark for the analysis. Section 4 presents the case when the ability type of each agent is privately known to the agent, but the principal pools the agents by offering the same tournament contract to each of them without screening. Section 5 examines the case in which the principal screens the agents by offering them a menu and letting them self-select. Section 6 contrasts the pooling to the screening contract to obtain the core result in the paper. Section 7 concludes.

2. The model

A principal signs a contract with a finite number of agents, n. The number of agents is assumed to be determined exogenously.[11] Each agent i produces observable output x_i according to the production function $x_i = a_i + e_i + \eta + \varepsilon_i$, where a_i is the agent's finite ability, e_i is the agent's finite effort, η is a common shock inflicted on all agents and ε_i is an idiosyncratic shock. Agents know their own ability types at the time of contracting, but the principal does not. In addition to adverse selection there is moral hazard, in that each agent's effort and the subsequent realizations of the production shocks are private information to him, but the output obtained is publicly observed.[12] The price of output is normalized to 1. Each agent's type, a_i, independently follows a normal distribution with mean μ and finite variance σ_a^2. Independently of a_i, the common shock η follows a normal distribution with

[9] Tsoulouhas and Marinakis (2007), instead, analyze ex post agent heterogeneity.

[10] Also see Footnote 14 below.

[11] See Myerson and Wärneryd (2006) for a model where the set of players is a random variable.

[12] Note that tournaments alleviate the principal's moral hazard problem in providing lower quality inputs to the agent (see Carmichael (1983) and Tsoulouhas (1999)).

mean zero and variance σ_η^2 and, independently of a_i and η, the idiosyncratic shock ε_i follows a normal distribution with zero mean and finite variance σ_ε^2. Because cardinal data on the performance of each agent are assumed to be available, the principal compensates agents for their efforts based on their outputs by using a cardinal tournament.[13] The cardinal (or two-part piece rate) tournament is the payment scheme in which the compensation w_i to each agent i is determined by relative performance. Specifically,

$$w_i = b + \beta(x_i - \overline{x}) = b + \beta\left(\frac{n-1}{n}x_i - \frac{1}{n}\sum_{j\neq i} x_j\right), \tag{1}$$

where $\overline{x}$ is the average output obtained by all agents, b is the base payment and β is the bonus factor to be determined by the principal.[14] Note that under the tournament the total wage bill is proportional to the base payment b, that is, $\Sigma w_i = nb$. Thus, the principal's total payment to the agents and, hence, the expected payment per agent are independent of output ($Ew_i = b$), however, each agent's relative performance determines his share of the fixed total payments.

Agent preferences are represented by a CARA utility function of the form $u(w_i, e_i) = -\exp\left(-rw_i + \frac{r}{2}e_i^2\right)$, where $r \neq 0$ is the agent's coefficient of absolute risk aversion. Given the normality assumptions above, both x_i and w_i are normally distributed and, therefore, $u(\cdot)$ follows a lognormal distribution. An important property of the lognormal distribution is that its first moment has a closed form, which allows us to obtain analytical solutions for the contractual parameters.[15] Another advantage of this model is that the baseline results that can be obtained with homogeneous agents conform with those obtained by Lazear and Rosen (1981) who, even though they utilized more general utility functions, relied on first-order Taylor approximations of these functions. For ease of exposition, we normalize each agent's reservation utility to -1.[16] Thus, we assume that whereas each agent knows his true ability level, the "market" does not, therefore, all agents have the same opportunity cost and, hence, the same "reservation" utility.[17]

[13] Rank-order (ordinal) tournaments that ignore the agents' cardinal performance are informationaly wasteful in this case (Holmström (1982)).

[14] Tsoulouhas (2010) has shown that "hybrid" cardinal tournaments of the form $w_i = b + \beta x_i - \gamma\overline{x}$ are dominant over standard tournaments of the form $w_i = b+\beta(x_i - \overline{x})$, however, the former are more complicated to calculate and, to the best of our knowledge, they are not used in practice. Further, in the limit, that is, for a sufficinetly large number of workers, $\beta = \gamma$.

[15] This is so because $E[\exp(-rw_i + \frac{r}{2}e_i^2)] = \exp[m + \frac{\sigma^2}{2}]$, when $-rw_i + \frac{r}{2}e_i^2 \sim N(m, \sigma^2)$, which allows us to obtain a closed form solution for the expected utility.

[16] Note that the analysis is directly applicable to any normalization other than -1.

[17] Note that a similar assumption is made in Bolton and Dewatripont (2005, p. 53), in that the reservation utilities are normalized to zero for all agents. When the reservation utilities are type independent, there is no need to worry about countervailing incentives, which are present when the principal has to offer such a

3. The perfect information tournament

This section analyzes a benchmark for the analysis, which is the tournament if the type of each agent were known to the principal and all agents. If the agent's type a_i were known, the contractual parameters offered by the principal to each agent would depend on his type, and the principal would determine these parameters $(b(a_i), \beta(a_i))$ by backward induction. Under a tournament $w(a_i) = b(a_i) + \beta(a_i)(x_i - \overline{x})$, the agent's expected utility after he signs the contract but before exerting effort is

$$EU(e_i(a_i)|a_i, b(a_i), \beta(a_i)) = \tag{2}$$

$$= -\exp\left\{-r\left[\begin{array}{c} b(a_i) + \beta(a_i)\frac{n-1}{n}\left(a_i + e_i(a_i) - \overline{a_j}\right) - \beta(a_i)\frac{1}{n}\sum_{j\neq i} e_j(a_j) - \\ -\frac{(e_i(a_i))^2}{2} - \frac{r(\beta(a_i))^2}{2}\frac{n-1}{n}\sigma_\varepsilon^2 \end{array}\right]\right\},$$

where the expression in the square brackets is the certainty equivalent compensation for the agent of type a_i who has signed contract $(b(a_i), \beta(a_i))$, and $\overline{a_j} = \sum_{j\neq i} a_j/(n-1)$ is the average type of all agents other than i who are also participating in the tournament.[18] Note that when agent types are known, expectations are only over production uncertainties. Also observe that expected utility rises with increases in the expected payment from the principal, reductions in the effort level implemented by the principal and reductions in the variance of the payments. By backward induction, to ensure the compatibility of the contract with agent incentives to perform, the principal calculates the effort level that maximizes (2). First order conditions yield

$$e_i(a_i) = \frac{n-1}{n}\beta(a_i), \tag{3}$$

which implies that incentives to perform are fully determined by the incentives provided by the principal via the bonus factor $\beta(a_i)$. Note that ability affects the effort choice of the agent through the choice of the optimal bonus factor $\beta(a_i)$.[19] Given (3), to ensure the compatibility of the contract with agent incentives to participate, the principal selects the value of the base payment, $b(a_i)$, that satisfies the agent's individual rationality constraint with equality so that the agent receives no rents but still accepts the contract. Hence, the individual rationality constraint of a type a_i agent is:

$$EU(b(a_i), \beta(a_i)|a_i, e_i(a_i)) =$$

great deal to highly able agents in order to attract them that makes low ability agents mimic the high types.

[18] Given the modeling assumptions the certainty equivalent admits a mean-variance form.

[19] Ability would also affect the effort choice directly if the cost of effort was dependent on ability, but in this case the analysis would be intractable. This is because the agent's compensation inclusive of monetized effort costs would not follow a normal distribution, hence, utility would not follow a lognormal distribution.

$$= -\exp\left\{-r\left[b(a_i) + \beta(a_i)\frac{n-1}{n}\left(a_i - \overline{a_j}\right) - \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\sigma_\varepsilon^2\right)\left(\beta(a_i)\right)^2\right]\right\} = -1 \iff$$

$$\iff b(\beta(a_i)|a_i) = \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\sigma_\varepsilon^2\right)\left(\beta(a_i)\right)^2 - \frac{n-1}{n}\left(a_i - \overline{a_j}\right)\beta(a_i). \tag{4}$$

By setting b in accordance with (4) the principal can ensure agent participation at least cost. That is, the individual rationality constraint is binding and yields no rents to all agent types.

Given conditions (3) and (4), the principal maximizes expected total profit

$$E\Pi^{PI}(\beta(a_i)) = \sum_{i=1}^{n}\left[Ex_i - Ew_i\right] = n\left[\overline{a} + e_i(a_i) - b(\beta(a_i)|a_i)\right] = \tag{5}$$

$$= n\left[\overline{a} + \frac{n-1}{n}\beta(a_i)(1 + a_i - \overline{a_j}) - \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\sigma_\varepsilon^2\right)\left(\beta(a_i)\right)^2\right],$$

where $\overline{a} = \sum_i a_i/n$ is the average type of all competing agents. The solution to the principal's maximization problem satisfies

$$\beta(a_i) = \frac{1 + (a_i - \overline{a_j})}{\frac{n-1}{n} + r\sigma_\varepsilon^2},\ \forall a_i, \tag{6}$$

that is, the bonus factor β depends positively on the agent's ability a_i relative to the average ability $\overline{a_j}$ of other agents, because the principal implements higher-power incentives for relatively stronger agents. However, the larger the variance of the idiosyncratic shock, σ_ε^2, the lower the bonus factor, $\beta(a_i)$, because the weaker the link between the power of incentives and output. In order to provide correct incentives to the agents, note that the principal will hire an agent of type a_i only if $a_i > \overline{a_j} - 1$. This condition is equivalent to $a_i > \overline{a} - \frac{n-1}{n}, \forall i$, in other words, the principal picks a group of agents from the population so that the ability a_i of each agent in the group is larger than the average ability $\overline{a}$ of all the agents in the group minus $\frac{n-1}{n}$. The condition is clearly satisfied if it holds for the lowest ability agent in the group. Given condition (6), condition (4) implies

$$b(a_i) = \frac{1}{2}\frac{n-1}{n}\frac{1 - (a_i - \overline{a_j})^2}{\frac{n-1}{n} + r\sigma_\varepsilon^2} = \frac{1}{2}\frac{1 - (a_i - \overline{a_j})^2}{1 + \frac{n}{n-1}r\sigma_\varepsilon^2},\ \forall a_i. \tag{7}$$

In contrast to the bonus factor $\beta(a_i)$, the base payment $b(a_i)$ depends negatively on ability a_i relative to the average ability $\overline{a_j}$ of other agents, because relatively more able agents need weaker incentives to participate since the likelihood of them receiving a bonus payment is higher. Moreover, the less ability a_i differs from average ability $\overline{a_j}$ of other agents regardless of direction, that is, the more uniform the group is, the larger the base payment $b(a_i)$ needed to induce agent participation because the less likely it is for each agent to receive a bonus.

Moreover, assumption $a_i > \overline{a_j} - 1$ implies that $b(a_i) < 0$, $\forall i$ (if a_i is not smaller than $\overline{a_j} + 1$).[20] Note that neither the bonus factor nor the base payment depend on σ_η^2 because the principal filters away common shocks from the responsibility of the agents. Obviously, neither payment should depend on the variance of ability either, because abilities are assumed to be known in this benchmark. Given conditions (6) and (7), the principal's expected profit under tournament is

$$E\Pi^{PI} = \sum_{i=1}^{n} \left[\overline{a} + \frac{n-1}{n}\beta(a_i) - b(a_i) \right] = \sum_{i=1}^{n} \left[\overline{a} + \frac{1}{2}\frac{n-1}{n}\frac{(1+a_i-\overline{a_j})^2}{\frac{n-1}{n} + r\sigma_\varepsilon^2} \right]. \tag{8}$$

Condition (8) indicates that the principal's expected profit increases with the average ability of all agents and the relative ability of each agent, the latter having a positive impact on incentives and a negative impact on the expected payment per agent as measured by the base payment.

4. The tournament without screening

Next we turn to the case when the ability type a_i of each agent is privately known to the agent, but the principal pools the agents by offering the same tournament contract (b, β) to each of them without screening, so that $w_i = b + \beta(x_i - \overline{x})$. Whereas each agent knows his type, he does not know the type of any other agent which, given independent ability draws, he expects to be equal to $E(a_j|a_i) = E(a_j)$. The agent's expected utility after he signs the contract, but before exerting effort, is

$$EU(e_i|a_i, b, \beta) = \tag{9}$$

$$= -\exp\left\{ -r\left[b + \beta\frac{n-1}{n}\left(a_i + e_i - E(a_j)\right) - \beta\frac{1}{n}\sum_{j\neq i} e_j - \frac{e_i^2}{2} - \frac{r\beta^2}{2}\frac{n-1}{n}\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right) \right] \right\}.$$

The principal calculates the effort level that maximizes (9). First order conditions yield

$$e_i = \frac{n-1}{n}\beta. \tag{10}$$

The individual rationality constraint of a type a_i agent is:

$$EU(b, \beta|a_i, e_i) =$$

[20] Note that the agents will still participate because (7) is obtained by (4). This is feasible because the agents are not liquidity constrained. The optimality of cardinal tournaments under liquidity constraints has recently been examined by Marinakis and Tsoulouhas (2012). The optimality of tournaments under limited liability for the principal has been examined by Marinakis and Tsoulouhas (2013).

$$= -\exp\left\{-r\left[b+\beta\frac{n-1}{n}\left(a_i-E(a_j)\right)-\frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta^2\right]\right\} = -1 \iff$$

$$\iff b(\beta|a_i) = \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta^2 - \frac{n-1}{n}\left(a_i-E(a_j)\right)\beta. \tag{11}$$

But which ability type should be considered? Given the assumption that the number of agents n is exogenously determined, the principal will set $a_i = \widetilde{a}$ so that the required n agents of ability $a_i \geq \widetilde{a}$ are expected to accept the contract offer, that is, $\Pr(a_i \geq \widetilde{a})N = n$, where N is the total number of available agents. Note that by setting $a_i = \widetilde{a}$, the individual rationality constraint is non-binding for all types $a_i > \widetilde{a}$, because expected utility is increasing in ability a_i (see (9) above). Thus, this result is different from the First Best case where all agent types would get no rents, because the principal in the current case makes a single offer which must be expected to be accepted by n agents.

Given conditions (10) and (11)), the principal maximizes expected total profit

$$E\Pi^P(\beta) = \tag{12}$$

$$= \sum_{i=1}^{\Pr(a_i\geq\widetilde{a})N}\left[Ex_i - Ew_i\right] = \Pr(a_i \geq \widetilde{a})N\left[E(a_i)+e_i-b(\beta|\widetilde{a})\right] = n\left[E(a_i)+e_i-b(\beta|\widetilde{a})\right] =$$

$$= n\left[E(a_i)+\frac{n-1}{n}\beta(1+\widetilde{a}-E(a_j))-\frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta^2\right],$$

where $E(a_i)$ is the expected agent ability that is formed by the principal for agents who accept the contract offer. However, the principal and rational agents who receive independent ability draws should form the same expectation so that $E(a_i) = E(a_j)$. The solution to the principal's maximization problem, then, satisfies

$$\beta = \frac{1+(\widetilde{a}-E(a_i))}{\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)}, \tag{13}$$

and

$$b = \frac{1}{2}\frac{n-1}{n}\frac{1-(\widetilde{a}-E(a_i))^2}{\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)} = \frac{1}{2}\frac{1-(\widetilde{a}-E(a_i))^2}{1+\frac{n}{n-1}r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)}. \tag{14}$$

The principal will offer (b, β) to all agents, as determined by (14) and (13). Note that the bonus factor β depends positively on the "cut-off" ability $\widetilde{a}$ relative to the expected ability $E(a_i)$, because the principal implements higher-power incentives for relatively stronger agents, but the base payment b depends negatively on the "cut-off" ability $\widetilde{a}$ relative to the expected ability $E(a_i)$, because relatively more able agents need weaker incentives to participate. Similar to the benchmark perfect information case, we assume that the number of agents and the distribution of ability are such that $\widetilde{a} > E(a_i) - 1$, in order for the agents to be provided with correct incentives. Unlike the benchmark case though, parameters b and β depend negatively on the variance of ability, σ_a^2, because the higher the variance of ability the weaker the link between the power of incentives and output. Moreover, the less the "cut-off" ability $\widetilde{a}$ differs from expected ability $E(a_i)$, regardless of direction, that is, the more uniform the group is, the larger the base payment b needed to induce agent participation, because the less likely it is for each agent to receive a bonus. Given (13) and (14), the principal's expected profit under tournament is

$$E\Pi^P = n\left[E(a_i) + \frac{n-1}{n}\beta - b\right] = n\left[E(a_i) + \frac{1}{2}\frac{n-1}{n}\frac{(1+\widetilde{a} - E(a_i))^2}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}\right]. \tag{15}$$

Condition (15) indicates that the higher the expected agent ability $E(a_i)$ and the higher the "cut-off" ability $\widetilde{a}$ relative to the expected agent ability the higher the principal's expected profit. Note that, as Appendix A shows,

$$E(a_i) = \mu - \widetilde{a}F(\widetilde{a}) + \lim_{\delta \to -\infty} \int_{\delta}^{\widetilde{a}} F(a_i)da_i. \tag{16}$$

5. The tournament with screening

We now turn to the case when the ability type of each agent is privately known to the agent but the principal screens the agents by offering them a menu $\{b(a_i), \beta(a_i)\}$ instead of a single offer. The menu provides each agent with incentives to self-select the offer in the menu that is designed for his true type, so that the wage for agent type a_i is $w(a_i) = b(a_i) + \beta(a_i)(x_i - \overline{x})$. Thus, the principal is completely uninformed and expects any agent's ability to be equal to μ before the agent signs the contract, but knows the true ability a_i of each agent after the agent signs the contract. Whereas each agent knows his own type, he does not know the type of any other agent which, given independent ability draws, he expects to be equal to μ before and after signing the contract. The principal still determines the optimal contractual parameters by backward induction. The agent's expected utility after he self-selects and signs an offer $(b(a_i), \beta(a_i))$ in the menu, but before exerting effort, is

$$EU(e_i(a_i)|a_i, b(a_i), \beta(a_i)) = \tag{17}$$

$$= -\exp\left\{-r\left[\begin{array}{c} b(a_i) + \beta(a_i)\frac{n-1}{n}(a_i + e_i(a_i) - \mu) - \beta(a_i)\frac{1}{n}\sum_{j\neq i} e_j - \\ -\frac{(e_i(a_i))^2}{2} - \frac{r(\beta(a_i))^2}{2}\frac{n-1}{n}\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right) \end{array}\right]\right\}.$$

By backward induction, to ensure the compatibility of the contract with agent incentives to perform, the principal calculates the effort level that maximizes (17). First order conditions yield

$$e_i(a_i) = \frac{n-1}{n}\beta(a_i), \forall a_i. \tag{18}$$

Given (18), to ensure the compatibility of the contract with agent incentives to participate, the principal selects the value of the base payment, $b(a_i)$, that satisfies the individual rationality constraint of type a_i agent:

$$EU(b(a_i), \beta(a_i)|a_i, e_i(a_i)) =$$

$$= -\exp\left\{-r\left[\begin{array}{c} b(a_i) + \beta(a_i)\frac{n-1}{n}(a_i - \mu) - \\ -\frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i)^2 \end{array}\right]\right\} \geq -1 \iff$$

$$\iff b(\beta(a_i)|a_i) \geq \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)(\beta(a_i))^2 - \frac{n-1}{n}(a_i - \mu)\beta(a_i). \tag{19}$$

Thus, a base payment $b(\beta(a_i)|a_i)$ that satisfies condition (19) is individually rational.

By appealing to the Extended Revelation Principle, in order to ensure the compatibility of the contract with each agent type's incentives to select the menu offer that is designed for his true type, the principal needs to make sure that the following additional incentive compatibility constraint is satisfied for every type a_i :

$$a_i = \arg\max_{\widehat{a}_i} EU(b(\widehat{a}_i), \beta(\widehat{a}_i)|a_i, e_i(\widehat{a}_i)) = \tag{20}$$

$$= -\exp\left\{-r\left[b(\widehat{a}_i) + \beta(\widehat{a}_i)\frac{n-1}{n}(a_i - \mu) - \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)(\beta(\widehat{a}_i))^2\right]\right\}.$$

The following Lemma is applicable to this constraint.

Lemma 1 *The incentive compatibility constraint (20) is equivalent to:*

$$\frac{db(a_i)}{da_i} + \left[(a_i - \mu) - \left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i)\right]\frac{n-1}{n}\frac{d\beta(a_i)}{da_i} = 0, \ \forall a_i, \tag{21}$$

$$\frac{d\beta(a_i)}{da_i} \geq 0, \ \forall a_i, \tag{22}$$

where constraint (21) is the local incentive compatibility constraint and constraint (22) is the monotonicity constraint.

Proof. The F.O.C. of the maximization problem in (20) imply

$$\frac{db(\widehat{a}_i)}{da_i}+\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(\widehat{a}_i)\right]\frac{n-1}{n}\frac{d\beta(\widehat{a}_i)}{da_i}=0 \tag{23}$$

is satisfied $\forall a_i=\widehat{a}_i$.

The S.O.C. of the maximization problem imply

$$\begin{aligned}\frac{d^2b(\widehat{a}_i)}{da_i^2}+\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(\widehat{a}_i)\right]\frac{n-1}{n}\frac{d^2\beta(\widehat{a}_i)}{da_i^2}- \\ -\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\frac{n-1}{n}\left(\frac{d\beta(\widehat{a}_i)}{da_i}\right)^2+\frac{n-1}{n}\frac{d\beta(\widehat{a}_i)}{da_i} \quad \leq \quad 0\end{aligned} \tag{24}$$

is satisfied $\forall a_i=\widehat{a}_i$.

First note that the F.O.C. above implies condition (21). Second, observe that differentiating (21) with respect to a_i implies

$$\begin{aligned}\frac{d^2b(a_i)}{da_i^2}+\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(a_i)\right]\frac{n-1}{n}\frac{d^2\beta(a_i)}{da_i^2}- \\ -\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\frac{n-1}{n}\left(\frac{d\beta(a_i)}{da_i}\right)^2+2\frac{n-1}{n}\frac{d\beta(a_i)}{da_i} \quad = \quad 0\end{aligned} \tag{25}$$

The latter condition and the S.O.C. imply

$$\frac{n-1}{n}\frac{d\beta(a_i)}{da_i}\geq 0, \tag{26}$$

which in turn implies (22). Thus, so far we have proved that the incentive compatibility constraint (20) implies the local incentive compatibility constraint (21) and the monotonicity constraint (22).

Next, suppose that both the local incentive compatibility and the monotonicity constraints (21) and (22) hold. We need to show that the incentive compatibility constraint (20) is satisfied. Suppose that there exist a_i such that the local incentive compatibility constraint (21) and the monotonicity constraint (22) hold, but the incentive compatibility constraint (20) is violated for at least one $\widehat{a}_i\neq a_i$. Thus,

$$\begin{aligned}-\exp\left\{-r\left[b(a_i)+\beta(a_i)\frac{n-1}{n}(a_i-\mu)-\frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)(\beta(a_i))^2\right]\right\}< \\ <-\exp\left\{-r\left[b(\widehat{a}_i)+\beta(\widehat{a}_i)\frac{n-1}{n}(a_i-\mu)-\frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)(\beta(\widehat{a}_i))^2\right]\right\}.\end{aligned} \tag{27}$$

By integrating we obtain

$$\int_{a_i}^{\widehat{a_i}} -\exp\left\{-r\left[\frac{db(\theta_i)}{d\theta_i}+\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(\theta_i)\right]\frac{n-1}{n}\frac{d\beta(\theta_i)}{d\theta_i}\right]\right\}d\theta_i>0. \tag{28}$$

Constraint (22) implies $\frac{d\beta(\theta_i)}{d\theta_i}\geq 0$. Assuming $\widehat{a_i}>a_i$, it follows that

$$\begin{aligned}&\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(\theta_i)\right]\frac{n-1}{n}\frac{d\beta(\theta_i)}{d\theta_i}< \\ <\ &\left[(\theta_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(\theta_i)\right]\frac{n-1}{n}\frac{d\beta(\theta_i)}{d\theta_i}.\end{aligned} \tag{29}$$

Hence, (21) implies

$$\int_{a_i}^{\widehat{a_i}} -\exp\left\{-r\left[\frac{db(\theta_i)}{d\theta_i}+\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(\theta_i)\right]\frac{n-1}{n}\frac{d\beta(\theta_i)}{d\theta_i}\right]\right\}d\theta_i<0, \tag{30}$$

which contradicts (28). A similar result can be obtained if $\widehat{a_i}<a_i$. ■

Therefore, in order to characterize the optimal menu $\{b(a_i),\beta(a_i)\}$, with $w(a_i)=b(a_i)+\beta(a_i)(x_i-\overline{x})$, and given condition (18), the principal maximizes expected total profit

$$\begin{aligned}E\Pi^S(b(a_i),\beta(a_i))&=\sum_{i=1}^n[Ex_i-Ew_i]=\sum_{i=1}^n[\mu+e_i(a_i)-b(a_i)]= \\ &=\sum_{i=1}^n\left[\mu+\frac{n-1}{n}\beta(a_i)-b(a_i)\right],\end{aligned} \tag{31}$$

subject to the individual rationality constraint (19), the local incentive compatibility constraint (21) and the monotonicity constraint (22).

But, in the presence of the incentive compatibility constraint (21), is the individual rationality constraint (19) binding or non-binding? The individual rationality constraint would be binding for every a_i if $b(\beta(a_i)|a_i)$ could be reduced for every a_i so that (19) held as an equality without hurting the local incentive compatibility constraint (21) or the monotonicity constraint (22). But reducing $b(\beta(a_i)|a_i)$ so that (19) holds as an equality for every a_i may require a non-parallel shift in the $b(a_i)$ schedule which could, in principle, affect $\frac{db(a_i)}{da_i}$. The following Lemma shows that $b(\beta(a_i)|a_i)$ can be reduced so that (19) holds as an equality without hurting the local incentive compatibility constraint (21), assuming that the monotonicity constraint is non-binding. Thus, the individual rationality constraint is binding.

Lemma 2 *Assuming that the monotonicity constraint (22) is non-binding for every a_i, the individual rationality constraint (19) is binding for every a_i.*

Proof. First note that without the individual rationality constraint (19), the local incentive compatibility constraint (21) is binding. This is so because the objective function without the individual rationality constraint (19) or the local incentive compatibility constraint (21) is strictly increasing (and linear) in $\beta(a_i)$ and strictly decreasing (and linear) in $b(a_i)$. Therefore, the optimal $\beta(a_i)$ is the highest possible, for every a_i, with $\beta(a_i) > 0$ in order to provide correct incentives to the agent and $\beta(a_i) < +\infty$ because of the assumption of finite effort, and the optimal $b(a_i) \to -\infty$, for every a_i. This solution violates the local incentive compatibility constraint (21).

Second, without the individual rationality constraint (19), by substituting the binding local incentive compatibility constraint (21) into the objective function, the maximization problem reduces to choosing $b(a_i)$ and $\beta(a_i)$ to maximize

$$\sum_{i=1}^{n} \left[\mu + \frac{n-1}{n} \frac{\left[\frac{n}{n-1} \frac{b'(a_i)}{\beta'(a_i)} + (a_i - \mu) \right]}{\left(\frac{n-1}{n} + r \left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2 \right) \right)} - b(a_i) \right], \tag{32}$$

which is strictly decreasing (and linear) in $b(a_i)$ and is independent of $\beta(a_i)$. Thus, $b(a_i) \to -\infty$, $\forall a_i$, which violates the individual rationality constraint (19) for every a_i, given a finite number of agents. ■

The result in Lemma 2 is in sharp contrast with standard adverse selection theory models without tournaments (for instance, see Bolton and Dewatripont (2005), pp. 47-56, 77-81 and 82-88) where the individual rationality constraint of all types other than the lowest is non-binding. Here, agents need not be given information rents in order to participate and select the offer in the menu $\{b(a_i), \beta(a_i)\}$ that was designed for their true ability type. The rationale is that the principal becomes better informed ex post about agent types through the realization of common uncertainty which he filters through the average output $\overline{x}$ obtained by the agents. By the *strong law of large numbers* $\overline{x}$ does provide an informative signal about the value of common shocks. Thus, the principal can effectively penalize or reward the agents ex post without the need for providing them with excessive incentives ex ante. Further, as the next lemma indicates, the local incentive compatibility constraint is also binding for almost all types but one.

Lemma 3 *Assuming that the monotonicity constraint (22) is non-binding for every a_i, the local incentive compatibility constraint (21) is binding for every $a_i \neq \mu - 1$, and is non-binding for $a_i = \mu - 1$.*

Proof. First note that, without the local incentive compatibility constraint (21), the individual rationality constraint is binding because the principal will find it optimal to reduce $b(a_i)$, $\forall a_i$, so that the agent receives no rents. The solution then satisfies conditions similar to (7) and (6) under perfect information, but with μ instead of $\overline{a_j}$, because the average type of agents is not known to the principal and the agents in this case, which also introduces uncertainty about the types of other agents captured by σ_a^2. Hence,

$$\beta(a_i) = \frac{1 + (a_i - \mu)}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}, \ \forall a_i, \tag{33}$$

and

$$b(a_i) = \frac{1}{2}\frac{1 - (a_i - \mu)^2}{1 + \frac{n}{n-1} r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}, \ \forall a_i. \tag{34}$$

By differentiating (19) with equality with respect to a_i we obtain:

$$\frac{db(a_i)}{da_i} = \tag{35}$$

$$= \frac{n-1}{n}\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i)\frac{d\beta(a_i)}{da_i} - \frac{n-1}{n}(a_i - \mu)\frac{d\beta(a_i)}{da_i} - \frac{n-1}{n}\beta(a_i) =$$

$$= \left[\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i) - (a_i - \mu)\right]\frac{n-1}{n}\frac{d\beta(a_i)}{da_i} - \frac{n-1}{n}\beta(a_i), \ \forall a_i.$$

Therefore,

$$\frac{db(a_i)}{da_i} + \left[(a_i - \mu) - \left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i)\right]\frac{n-1}{n}\frac{d\beta(a_i)}{da_i} = \tag{36}$$

$$= \left[\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i) - (a_i - \mu)\right]\frac{n-1}{n}\frac{d\beta(a_i)}{da_i} - \frac{n-1}{n}\beta(a_i) +$$
$$+ \left[(a_i - \mu) - \left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i)\right]\frac{n-1}{n}\frac{d\beta(a_i)}{da_i} =$$

$$= -\frac{n-1}{n}\beta(a_i), \ \forall a_i.$$

Given (33), it follows that

$$\frac{db(a_i)}{da_i}+\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(a_i)\right]\frac{n-1}{n}\frac{d\beta(a_i)}{da_i}= \tag{37}$$

$$=-\frac{n-1}{n}\frac{1+(a_i-\mu)}{\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)},\ \forall a_i.$$

However,

$$-\frac{n-1}{n}\frac{1+(a_i-\mu)}{\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)}=0 \tag{38}$$

only if $a_i=\mu-1$. Thus, the the local incentive compatibility constraint (21) is automatically satisfied and, hence, non-binding if $a_i=\mu-1$, and is binding otherwise. ■

Lemma 3 indicates that without the local incentive compatibility constraint (21), if agents were offered a menu $\{b(a_i),\beta(a_i)\}$ with $b(a_i)$ satisfying (34) and $\beta(a_i)$ satisfying (33), only type $a_i=\mu-1$ would trivially self-select the offer meant for his type.

Given condition (18), and given Lemmas 2 and 3, for $a_i\neq\mu-1$, the principal maximizes expected total profit subject to a binding individual rationality constraint (19) with equality, and a binding local incentive compatibility constraint (21), assuming that the monotonicity constraint (22) is non-binding. Note that (21) implies

$$\frac{db(a_i)}{da_i}+\left[(a_i-\mu)-\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)\beta(a_i)\right]\frac{n-1}{n}\frac{d\beta(a_i)}{da_i}=0,\ \forall a_i,\ \iff$$

$$\iff\ \beta(a_i)=\frac{\left[\frac{n}{n-1}\frac{b'(a_i)}{\beta'(a_i)}+(a_i-\mu)\right]}{\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)},\ \forall a_i, \tag{39}$$

where $b'(a_i)\equiv\frac{db(a_i)}{da_i}$ and $\beta'(a_i)\equiv\frac{d\beta(a_i)}{da_i}$. Hence, for $a_i\neq\mu-1$, the principal chooses $b(a_i)$ and $\beta(a_i)$ to maximize

$$E\Pi^S(b(a_i),\beta(a_i))=\sum_{i=1}^{n}\left[Ex_i-Ew_i\right]=n\left[\mu+e_i(a_i)-b(a_i)\right]= \tag{40}$$

$$=\sum_{i=1}^{n}\left[\mu+\frac{n-1}{n}\beta(a_i)-b(a_i)\right]=$$

$$=\sum_{i=1}^{n}\left[\begin{array}{c}\mu+\frac{n-1}{n}\frac{\left[\frac{n}{n-1}\frac{b'(a_i)}{\beta'(a_i)}+(a_i-\mu)\right]}{\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)}-\\-\frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)\right)(\beta(a_i))^2+\frac{n-1}{n}(a_i-\mu)\beta(a_i)\end{array}\right],$$

which leads to the following Lemma.

Lemma 4 *Assuming that the monotonicity constraint (22) is non-binding for every a_i, the contract menu $\{b(a_i), \beta(a_i)\}$, with $w_i(a_i) = b(a_i) + \beta(a_i)(x_i - \overline{x})$, satisfies*

$$\beta(a_i) = \frac{(a_i - \mu)}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}, \ \forall a_i > \mu, \tag{41}$$

$$b(a_i) = -\frac{1}{2}\frac{n-1}{n}\frac{(a_i - \mu)^2}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}, \ \forall a_i > \mu, \tag{42}$$

and

$$\beta(a_i) = 0, \forall a_i \leq \mu, \tag{43}$$

$$b(a_i) = 0, \forall a_i \leq \mu. \tag{44}$$

Proof. The F.O.C. of the maximization problem above satisfy

$$-\frac{n-1}{n}\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\beta(a_i) + \frac{n-1}{n}(a_i - \mu) = 0, \tag{45}$$

hence

$$\beta(a_i) = \frac{(a_i - \mu)}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}.$$

Therefore, (19) implies,

$$\begin{aligned} b(a_i) &= \frac{1}{2}\frac{n-1}{n}\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)\frac{(a_i - \mu)^2}{\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)^2} - \\ &\quad -\frac{n-1}{n}(a_i - \mu)\frac{(a_i - \mu)}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)} = \\ &= -\frac{1}{2}\frac{n-1}{n}\frac{(a_i - \mu)^2}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}. \end{aligned}$$

Conditions (33) and (34) prove (43) and (44) for $a_i = \mu - 1$. Nevertheless, if $a_i \leq \mu$ the profit maximizing principal would always offer $\beta(a_i) = 0$ and $b(a_i) = 0$ to provide correct incentives. ■

In the remaining analysis, and in accord with section 4 above, we assume that $\Pr(a_i > \mu)N \geq n$, that is there are enough agents whose type is above the population average. Lemma 5 below completes the analysis by noting that the monotonicity constraint, which was assumed to be non-binding earlier, is indeed non-binding.

Lemma 5 *The monotonicity constraint (22) is non-binding for every $a_i > \mu$.*

Proof. Given condition (41), it follows that

$$\frac{d\beta(a_i)}{da_i} = \frac{1}{\left(\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)\right)} > 0,$$

which completes the proof. ■

Given the analysis above, the principal's expected profit under tournament with screening before agents accept the contract is

$$E\Pi^S = \sum_{i=1}^{n}\left[\mu + \frac{n-1}{n}\beta(a_i) - b(a_i)\right] = \sum_{i=1}^{n}\left[\mu + \frac{n-1}{n}\frac{(a_i-\mu) + \frac{1}{2}(a_i-\mu)^2}{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}\right]. \qquad (46)$$

6. The dominant tournament

In this section we determine the dominant tournament when the ability type of each agent is privately known to the agent, given equations (15), with $\widetilde{a} > E(a_i) - 1$ and $\Pr(a_i \geq \widetilde{a})N = n$, where N is the total number of available agents, and (46), with $a_i > \mu$ and $\Pr(a_i > \mu)N \geq n$. The proposition that follows compares the screening to the pooling tournament.

Proposition 6 *Assuming $\widetilde{a} > E(a_i) - 1$ and $a_i > \mu$, the screening menu of contract offers $\{b(a_i), \beta(a_i)\}$, with $w(a_i) = b(a_i) + \beta(a_i)(x_i - \overline{x})$, dominates the pooling contract offer (b, β), with $w_i = b + \beta(x_i - \overline{x})$, from the principal's perspective iff*

$$\frac{\sum_{i=1}^{n}(a_i - \widetilde{a} + A)\left[1 + (\widetilde{a} - E(a_i) + 1) + (a_i - \mu)\right]}{n} - 2A\frac{(n-1) + r\left(\sigma_a^2 + n\sigma_\varepsilon^2\right)}{(n-1)} \geq 1, \quad (47)$$

where $E(a_i)$ satisfies condition (16) and $A \equiv E(a_i) - \mu$.

Proof. Equations (15) and (46) yield:

$$E\Pi^S \geq E\Pi^P \iff \qquad (48)$$

$$\iff \sum_{i=1}^{n}\left[(a_i-\mu) + \frac{1}{2}(a_i-\mu)^2\right] \geq nA\frac{\frac{n-1}{n} + r\left(\frac{\sigma_a^2}{n} + \sigma_\varepsilon^2\right)}{\frac{n-1}{n}} + n\frac{1}{2}(1 + \widetilde{a} - \mu - A)^2 \iff$$

$$\iff \sum_{i=1}^{n}\left[1+2(\widetilde{a}-a_i-A)+(\widetilde{a}-\mu-A)^2-(a_i-\mu)^2\right] \leq -2nA\frac{\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)}{\frac{n-1}{n}} \iff$$

$$\iff \sum_{i=1}^{n}(a_i-\widetilde{a}+A)\left[1+(\widetilde{a}-E(a_i)+1)+(a_i-\mu)\right] \geq n+2nA\frac{\frac{n-1}{n}+r\left(\frac{\sigma_a^2}{n}+\sigma_\varepsilon^2\right)}{\frac{n-1}{n}},$$

which completes the proof. ■

Equation (47) provides the main result of the paper. Observing that the LHS of the equation is a crude measure of variability in agent types, the equation is stating that when agent type variability is small, the principal does not benefit by offering a menu of contracts to screen the agents. That is, expected profit for the principal is greater under a uniform or pooling contract offer to all agents, provided that agent ability is known to not vary much. Only if agent ability varies significantly it is profitable for the principal to screen the agents in order to provide higher-power incentives to more able agents.

7. Conclusions

The result obtained in section 6, stating that when agent type variability is small the principal does not benefit by offering a menu of contracts to screen the agents, is in sharp contrast with standard adverse selection theory models without tournaments (for instance, see Bolton and Dewatripont (2005), pp. 47-56, 77-81 and 82-88). The rationale is that, under tournaments, and regardless of whether the principal makes a pooling or a screening offer, the principal becomes better informed ex post about an agent's ability type through the realization of common uncertainty which he filters through the average output $\overline{x}$ obtained by the agents. By the *strong law of large numbers* $\overline{x}$ provides an informative signal about the value of common shocks. Thus, the principal can effectively penalize (reward) the agents ex post through a reduced (increased) ex post monetary reward. The feasibility of penalizing an agent ex post through a tournament reduces the scope for screening agent types ex ante through the offer of a menu they have to chose from, even though it facilitates the implementation of screening contracts without the need for paying excessive information rents. Provided that agent ability is known to not vary much among agents, if the principal observes a better performance, he interprets it as coming from idiosyncratic luck and there is no need to reward luck in order to provide correct incentives. Only if agents are known to be sufficiently heterogeneous, then it is optimal for the principal to screen the agents with individualized offers through a menu. In that case, when the principal observes a high performance, it is more likely that it is coming from a more able type who must be rewarded accordingly through an individualized offer. Further, if agents are not known to be sufficiently heterogeneous, the pooling contract increases expected profits because it does not require the satisfaction of self-selection incentive compatibility constraints. Moreover, even though the screening contract implements higher-power incentives for better ability types, these types need to be paid a lot more in order to perform, which reduces the principal's profit. In all, the intuition is that whereas screening contracts can be more efficient by tailoring agent incentives to their innate abilities, such fine-tuning comes at a monetary cost to the principal over the pooling contract offer, especially given the principal's ability to sort the agents ex post via relative performance evaluation. In other words, why bother to gain more information ex ante if you can get most of this information ex post at a lower cost?

The analysis of this paper provides a solution to a puzzle in tournament theory, that of why there is no ex ante agent discrimination in practice. Unlike standard contract theory where, regardless of agent heterogeneity, the uninformed party can do better through agent discrimination rather than by offering a single contract, that is, by offering a contract menu to the informed party and letting him self-select, the present analysis shows that, unless agent ability types are known to vary sufficiently, there is no point in agent discrimination. This is so because agent ability types can be inferred ex post through the tournament and be penalized or rewarded accordingly. By contrast, without tournaments, that is, with absolute performance evaluation, high agent performance would be rewarded even if it were the sole outcome of pure common luck!

A second, associated puzzle, is why principals do not discriminate agents even after agent types are learned through repeated observation of performance (see, for instance, Leegomonchai and Vukina (2005) who found no evidence of ex post agent discrimination through a strategic allocation of varying quality inputs to the agents). But the analysis can help provide a solution to that puzzle as well. The best application of our analysis is the empirically observed lack of agent discrimination in broiler contracting through tournaments, even though tournaments have been used by processing companies (integrators) for decades, yielding plenty of experience to them in order to fine-tune the implementation of tournaments. The solution to the seeming puzzle is that processors know that growers (i.e., chicken farmers) are sufficiently homogeneous to not warrant discrimination. Such discrimination would add to grower complaints about low pay and grower group composition risk (see Tsoulouhas and Vukina 2001). It could also have detrimental effects on agent effort for they would fear a *rachet effect* if individualized contract offers with more stringent terms were made in the future following success. Besides, the strict requirements of relationship-specific capital investments and adherence to strict production rules effectively minimize the expression of higher ability into significantly and systematically higher performance.

Appendix A: Proof of condition (16)

Given that only ability types $a_i \geq \widetilde{a}$ would accept the contract offer, the expected type of an agent who accepts the contract is $\int_{\widetilde{a}}^{\infty} a_i f(a_i) da^i$. Given that the population mean is μ, it follows that

$$\int_{\widetilde{a}}^{\infty} a_i f(a_i) da_i = \mu - \int_{-\infty}^{\widetilde{a}} a_i f(a_i) da_i.$$

However,

$$\int_{-\infty}^{\widetilde{a}} a_i f(a_i) da_i = \lim_{\delta \to -\infty} \int_{\delta}^{\widetilde{a}} a_i f(a_i) da_i.$$

Integrating by parts, it follows that

$$\int_{\delta}^{\widetilde{a}} a_i f(a_i) da_i = \int_{\delta}^{\widetilde{a}} a_i F'(a_i) da_i = \widetilde{a} F(\widetilde{a}) - \delta F(\delta) - \int_{\delta}^{\widetilde{a}} F(a_i) da_i.$$

Hence,

$$\begin{aligned} \int_{\widetilde{a}}^{\infty} a_i f(a_i) da_i &= \mu - \widetilde{a} F(\widetilde{a}) + \lim_{\delta \to -\infty} \delta F(\delta) + \lim_{\delta \to -\infty} \int_{\delta}^{\widetilde{a}} F(a_i) da_i = \\ &= \mu - \widetilde{a} F(\widetilde{a}) + \lim_{\delta \to -\infty} \delta 0 + \lim_{\delta \to -\infty} \int_{\delta}^{\widetilde{a}} F(a_i) da_i = \\ &= \mu - \widetilde{a} F(\widetilde{a}) + \lim_{\delta \to -\infty} \int_{\delta}^{\widetilde{a}} F(a_i) da_i. \end{aligned}$$

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