

# Overlapping Generations, Natural Resources and The Optimal Quality of Property Rights

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## Abstract

This paper investigates the merits for a renewable resource economy to have partial property rights. Can partial property rights be socially optimal in an otherwise perfect economy? If so, under which circumstances? In a decentralized perfectly competitive economy involving a renewable natural resource and overlapping generations, we show that optimal institutions should make it possible to infringe on a resource stock. The quality of property rights on the resource is defined as the proportion of the resource that can be appropriated rather than left under open access. With quasi-linear preferences and a strictly concave renewable resource growth function, we show that there always exists a quality of property rights leading to optimal steady-state extraction and resource stock levels. Full private appropriation of the resource stock can lead to overaccumulation of the resource asset. When property rights are complete and a perfectly competitive economy is dynamically inefficient, households appropriate themselves a share of the resource stock that should optimally be used in production. The optimal quality of property rights then involves some limitation to open access to counter the tragedy of the commons, but not full private appropriation.

Key words: Institutions; property rights; natural resource; overlapping generations.

JEL Classification: K11, Q20.

# 1 Introduction

This paper investigates the merits for a renewable resource economy to have partial property rights. We show that, in a perfectly competitive economy where agents live finite lives, optimal institutions should make it possible to infringe on a resource stock. The quality of property rights on the resource is defined as the proportion of the resource that can be appropriated rather than left under open access. The answers are important for policy: when natural resources are overextracted as a result of too weak institutions, the distance to optimal institutions may be shorter than commonly believed.

For infinitely lived agents, in a deterministic economy with complete property rights and no market failure, the competitive equilibrium is Pareto optimal provided that the number of goods and agents are finite. Crucial to the definition of the competitive equilibrium is the condition that property rights be complete and perfectly defined. When the economy involves the extraction of a renewable resource, the dynamic path of that economy and its steady-state equilibrium are also optimal under perfect competition, given that perfect competition implies complete markets. The optimal steady-state is stable. However, property rights on the resource are often missing; open access leads to overexploitation and the tragedy of the commons.

With overlapping generations (OLG) models, the situation is different. Whether or not a renewable natural resource is exploited, the steady-state equilibrium of a perfectly competitive OLG economy need not be Pareto efficient. The first theorem of welfare may fail to apply because there is an infinite number of finitely lived agents. However, not every equilibrium is inefficient. Efficiency is linked to the marginal productivity of capital; the Cass criterion (Cass, 1972) gives necessary and sufficient conditions for efficiency. The possibility of inefficiency arises from the fact that the competitive growth equilibrium of an OLG economy may involve excessive savings. In an OLG economy using a renewable resource, excessive savings would take the form of insufficient harvesting and has been shown to be possible (Kemp and Long, 1979; Koskela *et al.*, 2002).

This paper formally investigates these Pareto inefficiencies in terms of the quality of property rights. In an overlapping generations model with quasi-linear preferences and a strictly concave renewable-resource growth function, we show that there always exists a quality of property rights that leads to optimal steady-state extraction and resource stock level. Under standard assumptions on preferences, technology, and resource dynamics, we establish the optimal steady-state quality of property rights and show that the steady-state is saddle stable. Our analytical results are illustrated

by numerical calculations.

The paper is organized as follows. Section 2 examines the literature. Section 3 presents the basic structure of the model. Section 4 characterizes the competitive equilibrium. In section 5, the conditions of existence, the number of decentralized steady-states and the local stability properties of those equilibria are studied. Section 6 provides a characterization of the efficient steady-state. Section 7 studies the existence of an optimal quality of property rights and determines its expression as a function of technology, preferences and stock dynamics. Numerical calculations with parametric specifications and a graphic analysis are presented in section 8. We conclude in the last section.

## 2 Relation to the literature

Our analysis builds on two major strands of the economics literature. One addresses the question of whether complete property rights are necessary to optimally exploit a natural resource (Engel and Fisher, 2008; Costello and Kaffine, 2008; Croutzet and Lasserre, 2013). The other strand considers the question of efficiency and/or equity in the exploitation of a natural resource when agents have finite lives and different generations coexist. In this latter strand, extensively reviewed by Farmer and Bednar-Friedl (2010), property rights are considered either complete (Kemp and Long, 1979; Mourmouras, 1991; Olson and Knapp, 1997; Koskela *et al.*, 2002, Brechet and Lambrecht, 2011), absent (Mirman and To, 2005; Karp and Rezai, 2014) or partial (Balestra *et al.*, 2010). Finally, in an OLG model with endogenous fertility and without a natural resource, Schoonbrodt and Tertilt (2014) investigate whether children should have property rights on their entire labor income.

Engel and Fisher (2008) consider how a government should contract with private firms to exploit a natural resource where an incentive to expropriate those firms exists in the good state of the world where profits are high. Engel and Fisher consider three sources of potential inefficiencies: uncertainty, market power and an irreversible fixed cost. This paper considers a perfectly competitive economy with no market failure. Costello and Kaffine (2008) study the dynamic harvest incentives faced by a renewable resource harvester with insecure property rights. A resource concession is granted for a fixed duration after which it is renewed with a known probability only if a target stock is achieved. They show that complete property rights are sufficient for economically efficient harvest but are not necessary. The idea is that if the target stock is set sufficiently high, then when the appropriator weighs the

extra benefit of harvesting now against the expected cost of losing renewal, the appropriator may choose a similar path to infinite tenure and complete rights. This paper differs from Engel and Fisher (2008) and Costello and Kaffine (2008) in that complete rights are no longer a sufficient condition for efficiency: complete rights can be inefficient. Croutzet and Lasserre (2013) show that partial property rights can be optimal in the presence of market power: the optimal quality of property rights depend on the number of firms, on technology through the elasticity of input productivity and on preferences through the price elasticity of demand. That paper is in partial equilibrium and is essentially static. The present paper characterizes the steady-state equilibria of an OLG economy, studies its dynamic stability properties and compares competitive and efficient steady-state equilibria.

Using an OLG model with complete property rights, Kemp and Long (1979) demonstrate that a competitive economy with constant population may under-harvest a renewable resource as a consequence of the resource being inessential for production. They assume constant resource growth. Mourmouras (1991) considers interactions between capital accumulation and natural exploitation in Diamond's (1965) overlapping generations model. He shows that both a low rate of resource regeneration relative to population growth and a low level of savings may lead to the unsustainable use of a renewable resource, despite the existence of complete property rights. In this paper, complete property rights are not assumed; property rights can be complete, absent or partial. The quality of property rights is an institutional parameter taken as given by individual agents. The natural resource is assumed to be essential for production and a strictly concave renewable resource-growth function is assumed. Olson and Knapp (1997) analyze competitive allocations of an exhaustible resource in an OLG economy and characterize the behavior of resource extractions and prices when they are endogenously determined by preferences and technology.

Our model and methodology are similar to the renewable resource model and the approach of Koskela *et al.* (2002). However, in the paper by Koskela *et al.*, property rights are not the focus of the analysis and are assumed complete. Our model explicitly considers the role of the quality of the property rights in the dynamics of the economy: resource extraction and price paths evolve endogenously considering the quality of property rights at each date. Our model admits the model of Koskela *et al.* (2002) as a special case when property rights are assumed complete in each period. Brechet and Lambrecht (2011) consider an overlapping generations economy in which firms' technology is CES and combines labor, physical capital and a natural resource. They consider an economy in which households have a warm

glow resource bequest motive. They shed light on the interplay between the resource bequest motive and the substitutability/complementarity relationship between capital and the natural resource in the determination of the use of the resource at the equilibrium. In this paper, consistent with the traditional walrasian representation of a perfectly competitive market, we do not assume intergenerational altruism nor a bequest motive: agents care only about their own lifetime welfare.

In contrast with the previous literature, Mirman and To (2005) consider an OLG model where property rights on the renewable resource are absent. Young agents use the extracted resource as a vehicle for savings and have market power on the resource market. Our model is also an OLG model; however, agents use the non-extracted resource as savings vehicle and, save for the possibility of partial property rights on the resource, the economy is perfectly competitive for all generations. Karp and Rezai (2014) use a two-sectors OLG model, with log linear additive intertemporal utility, to study the intergenerational effects of a tax that protects a renewable resource in open access. The old agents benefit from the environmental improvement (i.e., increase in the steady state level of stock and extraction) resulting from the tax. Absent a transfer, the tax harms the young agents by decreasing their real wages. They show that a Pareto improving tax can be implemented under various political economy settings. In this paper, there is only one sector, and property rights exist on the renewable resource but their quality is to be determined. The absence of property rights is only an extreme case of our model. Although our results with partial property rights bear some similarity with those of Karp and Rezai (2014), partial property rights differ from Pigovian taxes in the sense that the quality of property rights, as an institution, is not a handily available policy instrument; it is a durable, secular characteristic of an economy. Although, as underlined by Copeland and Taylor (2009), they are not an immutable characteristic of an economy, their dynamics may still be thought of as slow-motioned, short-term stationary; the quality of property rights evolves as a result of long-term decisions such as public investments in the judiciary system and/or law enforcement, negotiations, compromises and/or cultural changes. Moreover, unlike the Pigovian tax, partial property rights do not involve the collection, management, or redistribution by the government of the share of goods that failed to be appropriated.

Balestra *et al.* (2010) investigate the optimal number of plots (or property rights) to maximize the stock of a natural resource whose evolution depends on both spatial spillovers amongst private owners (the higher the number of plots the less likely spatial spillovers occur) and maintenance

cost of each plot (the higher the number of plots, the smaller the plots, the lower the maintenance cost of each plot). They consider an overlapping generations model with a renewable resource where a government decides the division of the resource in plots at each date. Each plot is assigned to a community that must manage it. Within each community, a representative young harvests and a representative old owns the capital (in the form of extracted resource). There are two sources of market power: as in Mirman and To (2005), within each community, the young has a form of market power as she decides how much to harvest taking into account the equilibrium of the production inputs market where she meets her contemporaneous old; the second source of market power is the different communities playing a Cournot-Nash game. The authors compare the non-cooperative and cooperative outcomes and show that the gain from cooperation is remarkable. They study how a fiscal policy could decentralize the cooperative outcome. In this paper, the economy is perfectly competitive and the natural resource growth function meets standard assumptions (i.e., no biological spillovers are assumed): for instance, a logistic growth function meets our assumptions on the resource growth.

Schoonbroodt and Tertilt (2014) question the economic rationale of pronatalist policies. They consider an OLG model where capital and labor are inputs in production, with fertility choice and parental altruism. When the cost of bearing children is positive, they show that parents' appropriation of children's income is rendered necessary to have a non-zero equilibrium fertility.

### 3 The model

We use a standard OLG model similar to the one used by Koskela *et al.* (2002). Our assumptions allow us to use Koskela *et al.*'s model as a benchmark when property rights are complete at all dates. We consider an overlapping generations economy without population growth where agents live for two periods. We assume that agents maximize the intertemporally additive, quasi-linear lifetime utility function:

$$V = u(c_1^t) + \beta u_2(c_2^t) \tag{1}$$

with  $u_2(c_2) = c_2$  where  $c_i^t$  denotes the period  $i = 1, 2$  consumption of a consumer-worker born at time  $t$  and  $\beta = \frac{1}{1+\delta}$  with  $\delta$  being the exogenous rate of time preference. For the first-period utility function,  $u' > 0$ ,  $u'' < 0$ , and  $\lim_{c \rightarrow \infty} u'(c) = 0$  and  $\lim_{c \rightarrow 0} u'(c) = \infty$ . The youngs are endowed with

one unit of labor, which they supply inelastically to firms in the consumption goods sector. Labor earns a competitive wage. The representative young consumer-worker uses the wage to buy the consumption good and to buy the stock of renewable resource that remains after production as savings to be used during her retirement. In addition to trading in the resource market, the young can also participate in the financial market by borrowing or lending<sup>1</sup>. The representative old rentier sells the stock of renewable resource and the financial assets bought when she was young to buy the consumption good during her retirement.

The representative firm produces the consumption good under a constant returns to scale technology that transforms the harvested resource  $H_t$  and labor  $L_t$  into output:  $F(H_t, L_t)$ . The technology can be expressed in factor-intensive form as  $f(h_t) = \frac{F(H_t, L_t)}{L_t}$  with the standard properties  $f' > 0$  and  $f'' < 0$ . Furthermore, we assume that the Inada conditions are verified:  $\lim_{h \rightarrow 0} f'(h_t) = \infty$  and  $\lim_{h \rightarrow \infty} f'(h_t) = 0$ , where  $h_t$  is the *per capita* harvest. The assumption of a representative firm is not restrictive because with constant returns to scale, the number of firms does not matter and production is independent on the number of firms which use the same technology. Moreover, since the firm has constant returns to scale technology, profits are zero in equilibrium. Also, as noted by De la Croix and Michel (2002), we may assume that firms live forever. This would not change the results as the firm's problem in any case is a static one.

The growth of the renewable resource is  $g(x_t)$ , where  $x_t$  denotes the beginning of period  $t$  *per capita* stock of the resource;  $g(x_t)$  is strictly concave and there are two values  $x = 0$  and  $x = \tilde{x}$  for which  $g(0) = g(\tilde{x}) = 0$ . Consequently, there is a unique value  $\hat{x}$  at which  $g'(\hat{x}) = 0$ , where  $\hat{x}$  denotes the stock providing the maximum sustainable yield (MSY). A logistic growth function  $g(x) = ax - \frac{1}{2}bx^2$  meets these assumptions.

The renewable resource in this model has two roles. It is both a savings vehicle between generations and an input in the production of a consumption good. The market for the resource operates in the following manner. At the beginning of period  $t$  the old agent owns the stock  $x_t$ ; the stock increases by the current growth to  $x_t + g(x_t)$ . If property rights are complete, she sells the stock (growth included) to the firm, which then chooses the harvest  $h_t$  to be used as input in the production of the consumption good. The firm then sells the remaining resource stock  $x_{t+1}$  to the young, who becomes the

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<sup>1</sup>The second vehicle for savings is not necessary for our demonstration. It is introduced to streamline the presentation and render explicit the underlying arbitrage condition between the resource and the financial assets.

old agent in the next period. The firm only plays the role of an intermediary between the generations; it does not extract any surplus from its activities. Surpluses are allocated between the generations by the price system.

With complete property rights, the natural growth of the resource yields a profit for its owner. The transition equation for the resource is:

$$x_{t+1} = x_t + g(x_t) - h_t \quad (2)$$

where  $h_t$  denotes the resource stock harvested by the firm for use as an input in production. The initial stock  $x_t$  and its growth,  $g(x_t)$ , can be put aside to feed into next period's stock or used to contribute to the current period's harvest.

Let  $\theta_t \in [0, 1]$  be an indicator for the quality of property rights on the resource owned by the old agent at date  $t$  with  $\theta_t = 1$  corresponding to complete rights and  $\theta_t = 0$  corresponding to the absence of property rights. All other property rights in the economy are assumed complete. When property rights on the stock of resource are partial, the firm can harvest a proportion of the resource owned by the old agent without paying for it. At the beginning of period  $t$ , the firm appropriates for free a proportion  $(1 - \theta_t)$  of the quantity of resource  $h_t$  it harvests for production and buys the rest of the quantity it needs,  $\theta h_t$ , from its owner at the going resource price  $p_t$ . Then, the remaining resource stock augmented by its natural growth, a quantity of  $x_t - h_t + g(x_t)$ , is transmitted to the next generation at price  $p_t$ . Altogether the old thus obtains the amount  $p_t(x_t - (1 - \theta)h_t + g(x_t))$  from the resource; the firm harvests the quantity  $h_t$  at cost  $p_t\theta h_t$ ; the young receives a quantity  $x_{t+1} = x_t + g(x_t) - h_t$  which she pays to the old at the market price  $p_t$  out of her wage income  $w_t$ .  $\theta_t$  is exogenous to individuals and firms.

The periodic budget constraints are thus:

$$c_1^t + p_t x_{t+1} + s_t = w_t \quad (3)$$

$$c_2^t = p_{t+1} [x_{t+1} + g(x_{t+1}) - (1 - \theta_{t+1})h_{t+1}] + R_{t+1}s_t \quad (4)$$

where  $R_{t+1} = 1 + r_{t+1}$  is the return factor on the financial asset and  $s_t$  represents savings by the young on the financial market. At the equilibrium,  $s_t$  will be zero so that the resource is the only savings vehicle. According to equation (4), the old agent consumes her savings, including the interest and the income she gets from selling the resource. From equations (3) and (4), the intertemporal budget constraint is:

$$c_1^t + \frac{c_2^t}{R_{t+1}} = w_t + \frac{p_{t+1} [x_{t+1} + g(x_{t+1}) - (1 - \theta_{t+1})h_{t+1}] - R_{t+1}p_t x_{t+1}}{R_{t+1}} \quad (5)$$



## 4 Competitive equilibrium

To study the competitive equilibrium, we follow De la Croix and Michel (2002)'s approach and distinguish the temporary equilibrium and the intertemporal equilibrium. The temporary equilibrium is, according to Hicks (1939), "such that all agents are reaching their best position subject to the constraint by which they are bound, and with the expectations they have at the moment". The intertemporal equilibrium, or equilibrium over time, must be "such that it is maintainable over a sequence, the expectations on which it is based, in each single period, being consistent with one another" (Hicks, 1965).

### 4.1 Temporary equilibrium

The temporary equilibrium of period  $t$  is a competitive equilibrium given price expectations. It is such that: (i) the representative agent optimizes her lifetime utility subject to both her budget constraint in each period and her price expectations, and, (ii) all markets clear at period  $t$ . The temporary equilibrium gives the equilibrium value of the current variables, including current prices as a function of the past and of the expectations about the future.

Consumptions at each period  $c_1^t$  and  $c_2^t$  by an individual of generation  $t$  and the demand for the resource stock as savings  $x_{t+1}$ , are determined as a solution to the following utility's maximization problem:

$$\max_{c_1^t, c_2^t, x_{t+1}} u(c_1^t) + \beta c_2^t$$

subject to the appropriate nonnegativity constraints and the intertemporal budget constraint (5). It gives the following first-order conditions for  $c_1^t$ ,  $c_2^t$  and  $x_{t+1}$  at the interior solution with  $\lambda$  a non-negative multiplier:

$$\begin{aligned} u'(c_1^t) &= \lambda \\ \beta &= \frac{\lambda}{R_{t+1}} \\ \lambda p_{t+1} \frac{[(1 + g'(x_{t+1})) - (1 - \theta_{t+1})(1 + g'(x_{t+1}))]}{R_{t+1}} &= \lambda p_t \end{aligned}$$

Rearranging the system of first-order conditions leads to

$$u'(c_1^t) = \beta R_{t+1} \tag{6}$$

$$p_t u'(c_1^t) = \beta p_{t+1} \theta_{t+1} (1 + g'(x_{t+1})) \tag{7}$$

Recalling that  $u'_2 = 1$ , equation (6) is the first Euler equation which provides that, in an optimal plan, the marginal utility cost of saving equal the marginal utility benefit obtained by saving. More specifically, the opportunity cost (in terms of current utility) of saving one more unit in the current period in the form of financial assets must be equal to the benefit of having  $R_{t+1}$  more units in the next period. This benefit is the discounted additional utility that can be obtained next period through the increase in consumption by  $R_{t+1}$  units. Rearranging equation (6), an alternative interpretation follows from:

$$\frac{u'(c_1^t)}{\beta} = R_{t+1}$$

the utility marginal rate of intertemporal substitution  $\frac{u'(c_1^t)}{\beta}$  should be equal to the marginal rate of transformation  $R_{t+1}$  which is the rate at which savings in the form of financial assets allow an agent to shift consumption from period  $t$  to  $t + 1$ . Equation (7) is the second Euler equation which indicates that the opportunity cost (in terms of current utility) of saving the value of one more unit in the current period in the form of resource stock must be equal to the benefit of having  $\theta_{t+1}(1 + g'(x_{t+1}))$  more units valued  $p_{t+1}$  each in the next period. This benefit is the discounted additional utility that can be obtained next period through the increase in consumption by  $\theta_{t+1}(1 + g'(x_{t+1}))\frac{p_{t+1}}{p_t}$  units. Rearranging equation (7), an alternative interpretation follows from:

$$\frac{u'(c_1^t)}{\beta} = \theta_{t+1}(1 + g'(x_{t+1}))\frac{p_{t+1}}{p_t}$$

the utility marginal rate of intertemporal substitution should be equal to the marginal rate of transformation  $\theta_{t+1}(1 + g'(x_{t+1}))\frac{p_{t+1}}{p_t}$  which is the rate at which savings in the form of resource stock allow an agent to shift consumption from period  $t$  to  $t + 1$ . Equations (6) and (7) together imply the arbitrage condition for the two assets at the equilibrium:

$$R_{t+1} = \theta_{t+1}(1 + g'(x_{t+1}))\frac{p_{t+1}}{p_t} \quad (8)$$

which provides that the interest factor should be equal to the resource price adjusted growth factor considering the quality of property rights. When savings behavior is optimized, we see from equation (8) that the price paths of the resource stock adjusts itself to the quality of the property rights. In other words, the firm and the young pay for the stock exactly what it is worth considering the quality of the property rights.

We now consider the market clearing conditions:

$$c_1^t + c_2^{t-1} = f(h_t) \quad (9)$$

is the consumption good market clearing condition.

$$x_{t+1} + \theta_t h_t = x_t + g(x_t) - (1 - \theta_t)h_t \quad (10)$$

is the renewable resource stock market clearing condition.

$$s_t = 0 \quad (11)$$

The fact that the arbitrage condition (equation (8)) is verified, that there is only one type of consumer per generation (i.e., no intragenerational heterogeneity) and no government debt, forces the asset market clearing condition to be such that savings is 0 for all  $t$ .

The first-order conditions for the firm's profits maximization are

$$f'(h_t) = \theta_t p_t \quad (12)$$

$$f(h_t) - h_t f'(h_t) = w_t. \quad (13)$$

They determine the demand for the factors of production  $H_t$  and  $L_t^2$  from their marginal costs  $\theta_t p_t$  and  $w_t$ . From these equations, it is clear that the firm has zero profits at the optimum. The resource price is endogenous in this economy. However, in a partial equilibrium analysis, we note that, for a given resource price, the marginal cost of the resource is lowered as  $(1 - \theta_t)$  is appropriated from the old at no cost by the firm. Equation (12) defines the quantity of resource harvested as an implicit function of the quality of property rights; for a given price, the derivative of that implicit function<sup>3</sup> is negative as  $f'' < 0$ : the more partial the property rights on the resource stock, the higher the quantity harvested. When  $\theta_t \rightarrow 0$ , we have:  $h_t \rightarrow x_t + g(x_t)$ : the resource is exhausted in period  $t$ . This is an illustration of the traditional tragedy of the commons when preferences are quasi linear and harvest costs are zero.<sup>4</sup>

Equation (13), on the other hand, defines the wage as an implicit function of the quality of the property rights; for a given resource price, the derivative of that implicit function is negative<sup>5</sup>: the more partial the property rights on the resource stock, the higher the wage. We define the intertemporal equilibrium in the next paragraph.

<sup>2</sup>The labor market also clears and we have:  $L_t = L \forall t$  as there is no population growth.

<sup>3</sup> $\psi(h_t, \theta_t) = f'(h_t) - \theta_t p_t$  leading to  $\frac{\partial h_t}{\partial \theta_t} = \frac{p_t}{f''(h_t)} < 0$

<sup>4</sup>In some fishery models, harvest costs increase as the resource stock diminishes, preventing extinction.

<sup>5</sup>We have  $\Lambda(\theta_t, w_t) = f(h_t) - h_t f'(h_t) - w_t = 0$  leading to  $\frac{\partial w_t}{\partial \theta_t} = -[\frac{\partial h_t}{\partial \theta_t} f'(h_t) - \frac{\partial h_t}{\partial \theta_t} f'(h_t) - h_t f''(h_t)] = h_t f''(h_t) < 0$

## 4.2 Intertemporal equilibrium

In this economy, the link between two periods  $t$  and  $t + 1$  is given by the resource dynamics and by the rational expectations on resource prices and property rights quality<sup>6</sup>. Using the transition equation for the renewable resource stock (10) and the first-order conditions for profit maximization (12) and (13) to eliminate input prices from the first-order condition for the resource stock (7), the intertemporal equilibrium is, for a given initial resource stock  $x_1$ , a sequence of temporary equilibria that satisfies for all  $t \geq 0$  the following conditions:

$$x_{t+1} = x_t + g(x_t) - h_t \quad (14)$$

$$f'(h_{t+1})\beta\theta_t[1 + g'(x_{t+1})] = u'[f(h_t) - f'(h_t)h_t - \frac{1}{\theta_t}f'(h_t)x_{t+1}]f'(h_t) \quad (15)$$

where we have also used the periodic budget constraints (3) and (4). In this paper, we consider a special sequence of equilibria: steady-state equilibria having the property that  $x$  and  $h$  do not change over time.

## 5 Competitive steady-states equilibria: existence, number and stability properties

Consistent with the durable nature of the quality of property rights, the study focuses on the steady-states of the dynamic system defined by equations (14) and (15). In addition, the quality of property rights is assumed constant over time,  $\theta_t = \theta \forall t$ , in what follows. Prior to addressing whether the steady states are optimal in sections 6 and 7, the conditions of existence and the number of steady-states are defined in this section using the approach of Koskela *et al.* (2002) adapted to a context involving partial property rights. The different phases of the dynamical system and the local stability properties of the steady-states are also determined.

### 5.1 Existence of steady-states

If steady-states exist, they are solution to the following system obtained from (14) and (15) with  $\Delta x_t = 0$  and  $\Delta h_t = 0$ :

$$h = g(x) \quad (16)$$

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<sup>6</sup>There is no uncertainty in this economy so that rational expectations are equivalent to perfect foresight.

$$u'[f(h) - f'(h)(h + \frac{x}{\theta})] = \beta\theta[1 + g'(x)] \quad (17)$$

In order to ensure that this system has at least one solution, we need to modify Koskela *et al.* (2002)'s conditions of existence to take into account the possibility of partial property rights as follows:

$$(1 + g'[x_c(\theta)])\theta\beta \geq u'[c_{1m}(\theta)] \quad (18)$$

with

$$x_c(\theta) = \arg \max[f(g(x)) - f'(g(x))(g(x) + \frac{x}{\theta})] \quad (19)$$

and

$$c_{1m}(\theta) = f[g(x_c(\theta))] - f'[g(x_c(\theta))][g(x_c(\theta)) + \frac{x_c(\theta)}{\theta}] \quad (20)$$

is the maximized first-period steady-state consumption. In other words, for a steady-state to exist, the marginal utility of the highest possible consumption in the first-period should be lower than the discounted benefits (taking into account the quality of the property rights) from the growth of the resource stock which maximizes the first-period consumption. If it is higher, it is not worth waiting to consume. With a very low discount factor or very weak property rights, consumers may not want to consume anything in any future period and, therefore, no decentralized steady-state equilibrium exists. In what follows, we call  $\bar{\theta}$  the minimum quality of property rights for which steady-states exist.

A question arises: how restrictive is this condition of existence? An answer can be given through a numerical illustration. If consistent with Arrow (1995), we assume an annual pure rate of time preference of 2%/year and assume that a period lasts 35 years in our model, we have  $\beta = 0.55$ . With logarithmic preferences for the first period, with the Cobb-Douglas production function and the logistic growth function used in our numerical illustration (section 8), we find  $\bar{\theta} \simeq 0.8$ . Moreover, as we will be mainly interested in situations where there may be overaccumulation of resource at the steady-state, it is even more likely that this condition of existence will be verified as those require a high discount factor (i.e., a low rate of time preference): in our numerical illustration, a discount factor higher than  $\beta = 0.7158$  (corresponding to  $\bar{\theta} \simeq 0.65$ ) leads to overaccumulation. In what follows, we consider situations where equation (18) is verified.

## 5.2 Number of steady-states

In order to determine the number of steady states, we need to first define the two isoclines corresponding to the system of equations (14) and (15) and

then compare their slopes to see when, and how, they intersect. The first isocline is obtained from (14) when  $\Delta x_t = 0$  but  $h_t$  can vary over time:

$$h_t = g(x) \quad (21)$$

For the second isocline, it is helpful to see that equation (15) defines  $h_t$  as an implicit function of  $x_{t+1}$  and then, using (14) consider that implicit function when  $\Delta h_t = 0$  and  $x_t$  can vary over time<sup>7</sup>:

$$\Psi(h, x_t) = 0$$

with:

$$\Psi(h, x_t) = u'[f(h) - f'(h)h - \frac{1}{\theta}f'(h)(x_t + g(x_t) - h)] - \beta\theta[1 + g'(x_t + g(x_t) - h)] \quad (22)$$

The slope of (21) is:

$$\left. \frac{dh_t}{dx_t} \right|_{\Delta x_t=0} = g'(x) \quad (23)$$

The slope of (22) is  $\left. \frac{dh_t}{dx_t} \right|_{\Delta h_t=0} = -\frac{\Psi_x(h, x_t)}{\Psi_h(h, x_t)}$  leading to:

$$\left. \frac{dh_t}{dx_t} \right|_{\Delta h_t=0} = \frac{(u'' \frac{f'}{\theta} + \beta\theta g'')(1 + g')}{u''[f' - f''(\frac{x}{\theta} + h)] + \beta\theta g''} > 0 \quad (24)$$

While the slope in (23) can be positive, null, or negative, the slope in (24) is always positive in the neighbourhood of an equilibrium given the assumptions on the utility function and because in the steady-state equilibrium  $(1 + g') = \frac{R}{\theta} > 0$ . It can also be shown that  $(h = 0, x = 0)$  is a point of (21) where  $(h > 0, x = 0)$  is a point of (22)<sup>8</sup>. Therefore, by a similar rationale to Koskela *et al.* (2002)'s first proposition, we find that when there are steady-state equilibria (i.e., equation (18) is verified), there are at least two of them, except for the rare case, where the Euler equation and the growth curve are tangent to each other. Besides, when two steady-states exist, the isocline associated with the Euler equation necessarily cuts the growth curve first from above and then from below. On the portion of the growth curve

<sup>7</sup>Recalling that the quality of property rights is now constant.

<sup>8</sup>When the quality of property rights is explicitly considered, Koskela *et al.* (2002)'s proof must be amended as follows: when  $x \rightarrow 0$ , the second term on the Right Hand Side (RHS) of equation (22) approaches some finite number when  $\theta \in [\bar{\theta}, 1]$ . For  $\Psi = 0$  to hold, the first term of the RHS of equation (22) must also approach some finite number. Koskela *et al.* (2002) show that it happens for some strictly positive finite value of  $h$ .

where  $g'(x) \leq 0$ , there can only be one steady-state equilibrium because the slope of the Euler (equation (24)) is always positive. In what follows, we concentrate on the case of two steady-states; i.e., the isocline associated with the Euler equation cuts the growth curve from above in the case of the equilibrium with the smaller level of resource stock:

$$\left. \frac{dh_t}{dx_t} \right|_{\Delta h_t=0} < \left. \frac{dh_t}{dx_t} \right|_{\Delta x_t=0}.$$

The isocline associated with the Euler equation cuts the growth curve from below in the equilibrium with the larger level of resource stock:

$$\left. \frac{dh_t}{dx_t} \right|_{\Delta h_t=0} > \left. \frac{dh_t}{dx_t} \right|_{\Delta x_t=0}$$

We call  $x_1^D$  and  $x_2^D$  those decentralized steady-state equilibria with  $x_1^D < x_2^D$ .

### 5.3 Stability properties of the steady-states

To study the stability properties of the steady-states, the different phases of the dynamical system are defined (phase-diagrams for specific sets of parameters are drawn in section 8 - Figure 1), then the local stability properties are determined.

#### 5.3.1 Phases of the dynamical system

The paths, for which  $x_{t+1} \geq x_t$  and  $h_{t+1} \geq h_t$ , are now considered. It follows from (14) that:

$$x_{t+1} \geq x_t \iff g(x_t) \geq h_t$$

Therefore, in the  $\{x, h\}$  space,  $x$  increases inside the area delimited by  $g(x_t)$  and  $x$  decreases outside that area. It follows from (15) and our assumptions on the production function that:

$$h_{t+1} \geq h_t \iff f'(h_{t+1}) \leq f'(h_t) \iff \frac{u'[f(h_t) - f'(h_t)h_t - \frac{1}{\theta}f'(h_t)x_{t+1}]}{\beta\theta[1 + g'(x_{t+1})]} \leq 1$$

This defines the area above the  $\Delta h_t = 0$  isocline, which is made clear in the next paragraph. Therefore,  $h$  increases above the  $\Delta h_t = 0$  isocline and decreases below.

### 5.3.2 Local stability properties

Equations (14) and (15) can be rewritten as:

$$x_{t+1} = x_t - h_t + g(x_t) = G(x_t, h_t) \quad (25)$$

$$f'(h_{t+1}) = \left[ \frac{u'[f(h_t) - f'(h_t)h_t - \frac{1}{\theta}f'(h_t)x_{t+1}]}{\beta\theta[1 + g'(x_{t+1})]} \right] f'(h_t) \quad (26)$$

Substituting (25) into (26) leads to:

$$\Xi(x_t, h_t) = f'(h_{t+1}) - \left[ \frac{u'[f(h_t) - f'(h_t)h_t - \frac{1}{\theta}f'(h_t)[x_t - h_t + g(x_t)]]}{\beta\theta[1 + g'(x_t - h_t + g(x_t))]} \right] f'(h_t) = 0 \quad (27)$$

which defines a two arguments implicit function for  $h_{t+1}$ :

$$h_{t+1} = F(x_t, h_t) \quad (28)$$

The planar system describing the dynamics of the resource stock and harvesting now consists of (25) and (28). The stability of the steady-states depends on the eigenvalues of the Jacobian matrix of the partial derivatives of the system:

$$J = \begin{bmatrix} G_x & G_y \\ F_x & F_y \end{bmatrix}$$

The eigenvalues of the Jacobian are studied in Appendix A and a proof of the following proposition, which is an extension from Koskela *et al.* (2002)'s proposition 2 to an economy where property rights can be partial, is provided.

**Proposition 1** *When the quality of property rights is explicitly considered, in the case of concave resource growth with two steady-states, the steady-state equilibrium associated with a larger natural stock is saddle stable while the steady-state equilibrium associated with a smaller stock is unstable. To the extent that the steady-states exist, the stability properties of the steady-states do not depend on the quality of property rights.*

## 6 Efficient steady-state equilibria

De la Croix and Michel (2002) point out that the conditions for long run intergenerational efficiency depend on whether only the younger generation is considered in the steady-state or both the initial older generation and



the younger generation are considered. We follow Diamond (1965)'s seminal article which defines "golden age" paths by excluding the initial older generation. The social planner's problem is therefore to maximize the lifetime welfare of a representative individual subject to the constraint that the aggregate consumption is equal to production:

$$\max_{(c_1, c_2, x)} W = u(c_1) + \beta c_2$$

subject to:

$$h = g(x) \tag{29}$$

$$c_1 + c_2 = f(h) \tag{30}$$

As pointed out by Diamond (1965) in an economy where capital and labor were used as production inputs, such a maximization problem decomposes naturally into two separate problems: that of optimizing the height of the consumption constraint; and that of dividing this amount of consumption between the different periods of life. Here, resource and labor are used as production inputs, optimizing the height of the consumption constraints (equation (30)) means selecting the optimal *per capita* level of harvest. Note that the optimality of *per capita* harvest is independent of the exact division of consumption. Equations (30) and (29) define the maximum *per capita* harvest as the solution of:

$$g'(x^*) = 0 \tag{31}$$

$$h^* = g(x^*) \tag{32}$$

with  $x^*$  the optimal stock and  $h^*$  the optimal harvest levels at the steady-state. We note that equation (31) defines the maximum sustainable yield which is the Golden Rule level of resource stock and that equation (32) defines the Golden Rule level of harvest.

The second problem is to define the optimal intertemporal lifetime allocation of the maximized amount of total consumption obtained with  $h^*$  and  $x^*$ . The solution of the social planner's problem subject to the constraints (30), (31) and (32) is:

$$u'(c_1) = \beta \tag{33}$$

In the next section, we compare the efficiency conditions with the conditions defining our decentralized steady-state equilibria.

## 7 Optimal property rights

In our economy, the quality of property rights is represented by a single parameter. It is generally not possible to hit two birds with one stone. The problem of maximizing total consumption and the one of optimizing the allocation of that maximized consumption between the two lifetime periods are separate. In what follows, we focus on the first problem: we investigate whether a quality of property rights can be found to maximize total consumption. We discuss the second problem at the end of the section.

Let  $x_i^D(\theta)$ ,  $i = 1, 2$ , represent the decentralized steady-state equilibria associated with property right quality  $\theta$ ,  $\theta \in [\bar{\theta}, 1]$ . At  $x_i^D(\theta)$ ,  $i = 1, 2$ ,  $h = g[x_i^D(\theta)] \geq 0$ , (29) and (30) are verified as they are constraints considered in the decentralized optimization problem. Efficient resource stock and harvest must verify equations (31) and (32). First, consider the situations where property rights are complete,  $\theta = 1$ , and focus on the steady-state with the larger stock. Koskela *et al.* (2002) have shown that  $x_2^D(1)$  may or may not be optimal depending on the value of the parameters on technology, preferences and resource dynamics. A Pareto optimal competitive equilibria with complete property rights is such that:

$$g'(x^{*D}(1)) = 0 \quad (34)$$

The set of parameters implying a non efficient steady-state equilibrium in presence of complete rights is defined by:

$$g'[x^D(1)] < 0 \quad (35)$$

That is

$$x^D(1) > x^* \quad (36)$$

We must prove that, in situations where equation (35) holds,  $\theta^* \in [\bar{\theta}, 1]$  exists such that, at the steady-state, resource stock and harvest are at their first-best levels. The first-best level of stock is defined by equation (31).  $\theta^* \in [\bar{\theta}, 1]$  must verify:

$$x^D(\theta^*) = x^* \quad (37)$$

For a given quality of property rights,  $x^D(\theta)$  is the solution of equation (17):

$$u'[f(h) - f'(h)(h + \frac{x}{\theta})] = \beta\theta[1 + g'(x)]$$

which defines  $x$  as an implicit function of  $\theta$ :

$$\Omega(x, \theta) = u'[f(h) - f'(h)(h + \frac{x}{\theta})] - \beta\theta(1 + g'(x)) = 0 \quad (38)$$

From the implicit function theorem, we know that:

$$\frac{dx}{d\theta} = -\frac{\frac{d\Omega}{d\theta}}{\frac{d\Omega}{dx}}$$

From equation (18), we know that a steady-state equilibrium exists only if  $g'(x) > -1 \forall \theta \in [\bar{\theta}, 1]$  as the marginal utility of consumption is positive. When  $-1 < g' < 0$ , we have

$$\frac{\partial \Omega(x, \theta)}{\partial \theta} = \frac{x}{(\theta)^2} f'(h) u''(c_1) - \beta(1 + g'(x)) < 0$$

and, recalling that  $h = g(x)$  at the steady state,

$$\frac{\partial \Omega(x, \theta)}{\partial x} = [-g'(x) f''(h) h - \frac{g'(x) f''(h)}{\theta} x - \frac{f'(h)}{\theta}] u''(c_1) - \beta \theta g''(x) > 0$$

Therefore,

$$\frac{dx}{d\theta} > 0 \quad (39)$$

The more partial the property rights, the lower the resource stock. From equations (36) and (39), we find  $\theta^* < 1$ . Let's prove that  $\theta^* \geq \bar{\theta}$ . When  $\theta = \bar{\theta}$ , we have

$$u'[f(h) - f'(h)(h + \frac{x}{\theta})] - \beta \bar{\theta}(1 + g'(x)) = 0 \quad (40)$$

From equation (18), we know that  $x_c(\bar{\theta})$  is solution of equation (40).  $x_c(\bar{\theta})$  is the level of stock that maximizes the consumption in the first period. We know that any harvest corresponding to a level of stock beyond the maximum sustainable yield can also be reached with a level of stock below the maximum sustainable yield. Moreover, for any given level of harvest, a higher consumption will be achieved with a lower stock<sup>9</sup>. Therefore, in order to maximize consumption in the first period,  $x_c(\bar{\theta})$  must be such that  $x_c(\bar{\theta}) \leq x^*$ . Hence,  $\theta^* \geq \bar{\theta}$ .

Using equations (17), (31) and (32),  $\theta^*$  is the solution of

$$u'[f(h^*) - f'(h^*)(h^* + \frac{x^*}{\theta^*})] = \beta \theta^* \quad (41)$$

**Proposition 2** *When a perfectly competitive OLG renewable resource economy is dynamically inefficient, partial property rights on the resource can always lead to first-best steady-state levels of resource stock and harvest.*

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<sup>9</sup>  $\frac{dc_1}{dx} \Big|_{h \text{ constant}} = -\frac{f'}{\theta} < 0$ .

From equation (41), it is also clear that the optimal quality of property rights depends on preferences, on technology and on resource stock. Finally, one can verify that  $\theta^*$  does not solve the problem of the optimal intertemporal allocation of the maximized consumption (equation (33)). However, property rights on both labor and the production output are assumed complete in this paper. It can be shown in a rationale similar to Croutzet (2013) that an efficient quality of property rights on labor income, not necessarily complete, can optimally reallocate the maximized consumption between the two lifetime periods.

## 8 Numerical illustrations

To shed further light on the properties of the model and contrast the results with those with complete rights, the same parametric example as in Koskela *et al.* (2002) is used. The first-period utility function, the production function, and the resource growth function are assumed to be:

$$u(c_1) = \ln c_1 \quad (42)$$

$$f(h) = h^\alpha \text{ with } 0 < \alpha < 1 \quad (43)$$

$$g(x) = ax - \frac{1}{2}bx^2 \quad (44)$$

The economically interesting parameters are the output elasticity of the resource  $\alpha$  which determines the price elasticity of resource demand, and the discount factor  $\beta$ . Equation (44) is the logistic growth function for renewable resources. With these specifications, equations (16) and (17) reduce to:

$$h = ax - \frac{1}{2}bx^2 \quad (45)$$

$$\frac{1}{(1-\alpha)h^\alpha - \alpha h^{\alpha-1} \frac{x}{\theta}} = \beta\theta(1+a-bx) \quad (46)$$

Koskela *et al.* (2002)'s situation where a steady state equilibrium can be inefficient under complete rights (when  $\theta = 1$ ) is replicated by choosing  $a = 1$ ,  $b = 0.001$  which imply the Golden Rule level of stock and harvest  $\hat{x} = 1000$  and  $\hat{h} = 500$  and by choosing  $\alpha = 0.15$ . Four scenari summarized in Table 1 are considered. Golden rule stock and harvest levels depend on the resource growth parameters and are the same for all scenari. A perfectly competitive economy is considered in all scenari, with the exception of partial property rights assumed in scenari 3 and 4. Scenario 1 considers a situation where the

OLG economy has a decentralized Pareto optimal steady-state at the larger stock; resource stock and harvest are at their Golden Rule levels; intertemporal utility is optimized. In scenario 2, the discount factor is  $\beta = 0.90$ , property rights remain complete: the OLG economy now exhibits dynamic inefficiency: at the decentralized steady-state, the stock level is above its Golden Rule level, harvest is below its Golden Rule level. As  $x_2^D > \hat{x}$ , the optimality condition  $g'(x_2^D) \geq 0$  is not verified:  $x_2^D$  is inefficient with complete rights. It is also saddle stable as it is the steady-state with the larger stock. Scenario 3 differs from scenario 2: property rights are no longer complete. The optimal quality of property rights is computed, using equation (46), we find  $\theta^* = 0.8675$ . The decentralized steady-state at the larger stock with  $\theta^* = 0.8675$  leads to first-best resource stock and harvest levels. In other words, in order to reach the first-best levels of resource stock and harvest, the firm must harvest 13.25% of the old agent stock without paying for it. If the entire stock was protected, the steady-state equilibrium with the larger stock would not lead to first-best resource stock and harvest as was shown in scenario 2. In scenario 4, we consider property rights weaker than the optimal quality,  $\theta = 0.8$ , the OLG economy exhibits inefficiency due to too weak property rights: at the decentralized steady-state, both resource stock and harvest are below their golden rule levels. The resource is overextracted.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Golden rule stock level	1000	1000	1000	1000
Golden rule harvest level	500	500	500	500
Discount factor	0.7158	0.9	0.9	0.9
Property rights quality	1	1	0.8675	0.8
Resource stock	1000	1131.09	1000	914.386
Harvest level	500	491.408	500	496.374
Production	2.54007	2.53347	2.54007	2.5373
Resource price	0.000762	0.000773	0.000878	0.000958
Wage	2.15906	2.15345	2.15906	2.1567
Consumption first-period	1.39704	1.27875	1.28072	1.27989
Consumption second-period	1.14303	1.25473	1.25935	1.25741
Utility first-period	0.334354	0.245879	0.247421	0.246774
Utility second-period	1.14303	1.25473	1.25935	1.25741
Intertemporal utility	1.15254	1.37513	1.38084	1.37844

Table 1: Numerical Illustrations

The graph on the left-hand side of Figure 1 shows that the steady-state with the larger stock of resource is saddle stable and inefficient as it is located to the right of the maximum sustainable yield ( $x^D(1) = 1131.09 > \hat{x} = 1000$  and  $h_2^D(1) = 491.408 < \hat{h} = 500$ ). The graph on the right-hand side of Figure 1 shows the steady-state with optimal incomplete property rights ( $x(\theta^*) = \hat{x}$ ,  $h(\theta^*) = \hat{h}$ ) and that it is saddle stable.

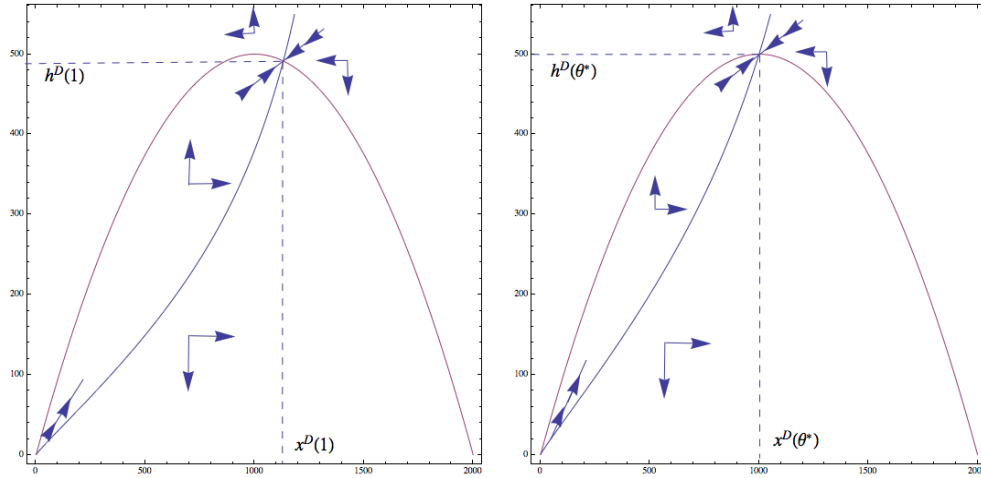


Figure 1: Phase diagrams of the dynamical system with complete and partial property rights respectively.

## 9 Conclusion

Complete property rights can lead to resource overaccumulation at the steady-state. When property rights are complete and a perfectly competitive economy is dynamically inefficient, households appropriate themselves a share of the resource stock that should optimally be used in production. In other words, the paradigm according to which inefficiencies are a consequence of weak institutions that allow such ill behavior as theft is partly reversed. Inefficiencies can also be a consequence of too strong property rights. Efficient institutions may involve partial property rights. In this paper, we have shown that there always exists a quality of property rights, though not necessarily complete, leading to steady-state optimal resource extraction and resource stock. Optimal partial property rights increase the lifetime welfare of all individuals. We have also shown that steady-states with optimal partial property rights are saddle stable.

While overfishing, deforestation, endangered species may result from institutions that are too weak, this paper delivers a message of hope: although property rights may need to be strengthened in those situations, they do not always need to be complete to achieve efficiency. Strong, efficient institutions often need to fall short of imposing complete property rights. Beyond a certain quality of property rights, strengthening them further is inefficient.

In an OLG economy where conventional capital rather than a renewable resource is used as the savings vehicle, one may conjecture that, in presence of dynamic inefficiencies, efficient capital accumulation can be reached when agents cannot fully appropriate the returns from savings. However, conventional capital differs from a renewable resource. First, the services from the whole capital stock, not an extracted share of the stock, constitute the relevant production input. Second, capital depreciation is always a negative contribution to capital growth while resource growth is usually positive at relevant stock levels. However, consistent with the rationale of this paper, our preliminary results indicate that efficiency requires property rights on some goods in the economy to be partial.

## Appendix A Proof of Proposition 1

We rewrite (14) and (15) as:

$$x_{t+1} = x_t - h_t + g(x_t) = G(x_t, h_t) \quad (47)$$

$$f'(h_{t+1}) = \left[ \frac{u'[f(h_t) - f'(h_t)h_t - \frac{1}{\theta}f'(h_t)x_{t+1}]}{\beta\theta[1 + g'(x_{t+1})]} \right] f'(h_t) \quad (48)$$

Substituting (47) into (48) leads to:

$$\Xi(x_t, h_t) = f'(h_{t+1}) - \left[ \frac{u'[f(h_t) - f'(h_t)h_t - \frac{1}{\theta}f'(h_t)[x_t - h_t + g(x_t)]]}{\beta\theta[1 + g'(x_t - h_t + g(x_t))]} \right] f'(h_t) = 0 \quad (49)$$

which defines a two arguments implicit function for  $h_{t+1}$ :

$$h_{t+1} = F(x_t, h_t) \quad (50)$$

The planar system describing the dynamics of the resource stock and harvesting now consists of (47) and (50) and the stability of the steady-states depends on the eigenvalues of the Jacobian matrix of the partial derivatives of the system:

$$J = \begin{bmatrix} G_x & G_y \\ F_x & F_y \end{bmatrix}$$

The determinant of the Jacobian is  $D = G_x F_h - G_h F_x$  and the trace is  $T = G_x + F_h$ . The characteristics polynomial is:

$$p(\zeta) = \zeta^2 - T\zeta + D = 0$$

From the stability theory of difference equations (Azariadis, 1993), we know that, for a saddle point, the roots of  $p(\lambda)$  need to be on both sides of unity. Thus, we need that  $D - T + 1 < 0$  and  $D + T + 1 > 0$  or  $D - T + 1 > 0$  and  $D + T + 1 < 0$ . We find:

$$\begin{aligned} G_x(x_t, h_t) &= 1 + g'(x_t) \\ G_h(x_t, h_t) &= -1 \end{aligned}$$

and the implicit function theorem gives:

$$F_{x_t} = -\frac{\Xi_{x_t}(x_t, h_t)}{\Xi_{h_{t+1}}(x_t, h_t)} \text{ and } F_{h_t} = -\frac{\Xi_{h_t}(x_t, h_t)}{\Xi_{h_{t+1}}(x_t, h_t)}$$

That is,



$$F_x(x_t, h_t) = \frac{1}{\beta\theta f''(h_{t+1})} \left[ \frac{-\frac{1}{\theta}(f'(h_t))^2 u''(c_t)[1 + g'(x_t)]}{(1 + g'(x_{t+1}))} \right] - \frac{1}{\beta\theta f''(h_{t+1})} \left[ \frac{f'(h_t)u'(c_t)g''(x_{t+1})(1 + g'(x_t))}{[(1 + g'(x_{t+1}))]^2} \right]$$

$$F_h(x_t, h_t) = \frac{1}{\beta\theta f''(h_{t+1})} \left[ \frac{f''(h_t)u'(c_t)}{(1 + g'(x_{t+1}))} \right] + \frac{1}{\beta\theta f''(h_{t+1})} \left[ \frac{f'(h_t)u''(c_t)\left[\frac{f'(h_t)}{\theta} - f''(h_t)\left(\frac{x_t + g(x_t) - h_t}{\theta} + h_t\right)\right]}{(1 + g'(x_{t+1}))} \right] + \frac{1}{\beta\theta f''(h_{t+1})} \left[ \frac{f'(h_t)u'(c_t)g''(x_{t+1})}{(1 + g'(x_{t+1}))^2} \right]$$

Evaluating the elements of the Jacobian at the steady state and utilizing the facts that  $u' = \beta\theta(1 + g')$  and  $h = g$ :

$$G_x(x_t, h_t) = 1 + g'$$

$$G_h(x_t, h_t) = -1$$

$$F_x(x_t, h_t) = \frac{-\frac{1}{\theta}(f')^2 u'' - f'\beta\theta g''}{\beta\theta f''} < 0$$

$$F_h(x_t, h_t) = 1 + \frac{\frac{(f')^2 u''}{\theta}}{\beta\theta f''(1 + g')} - \frac{f'u''[\frac{x}{\theta} + h]}{\beta\theta(1 + g')} + \frac{f'g''}{(1 + g')f''} > 0$$

Based on these partial derivatives, the trace T and the determinant D of the characteristic polynomial can be calculated to be:

$$D = G_x F_h - G_h F_x$$

$$= (1 + g') + \frac{\frac{(f')^2 u''}{\theta}}{\beta\theta f''} - \frac{f'u''[\frac{x}{\theta} + h]}{\beta\theta} + \frac{f'g''}{f''} + \frac{-\frac{1}{\theta}(f')^2 u'' - f'\beta\theta g''}{\beta\theta f''}$$

which simplifies to:

$$D = (1 + g') - \frac{f'u''[\frac{x}{\theta} + h]}{\beta\theta} > 0$$

and

$$\begin{aligned} T &= G_x + F_h \\ &= 2 + g' + \frac{\frac{(f')^2 u''}{\theta}}{\beta \theta f''(1+g')} - \frac{f' u''[\frac{x}{\theta} + h]}{\beta \theta (1+g')} + \frac{f' g''}{(1+g') f''} > 1 \end{aligned}$$

It is easy to see that  $D + T + 1 > 0$  holds. The nature of the stability of the steady-states then depends crucially on the sign of  $D - T + 1$ . Calculating  $D - T + 1$  gives:

$$\frac{1}{\beta \theta} \left[ -f' u''[\frac{x}{\theta} + h] \left[ \frac{g'}{1+g'} \right] - \frac{f'}{f''(1+g')} \left( \frac{f'}{\theta} u'' + \beta \theta g'' \right) \right] \quad (51)$$

To determine the sign of  $D - T + 1$ , we compare the slopes of the growth curve and the consumer optimization condition at the steady state. At the steady-state, the slope of the Euler equation is:

$$\left. \frac{dh_t}{dx_t} \right|_{\Delta h_t=0, \Delta x_t=0} = \frac{(u'' \frac{f'}{\theta} + \beta \theta g'')}{-f'' u''(\frac{x}{\theta} + h)} > 0$$

At the larger steady-state, the consumer first-order condition cuts the growth curve from below and we have:

$$\frac{(u'' \frac{f'}{\theta} + \beta \theta g'')}{-f'' u''(\frac{x}{\theta} + h)} > g'$$

which can be rewritten as, keeping in mind that  $-f'' u''[\frac{x}{\theta} + h] < 0$ ,

$$0 < g'(-f'' u''(\frac{x}{\theta} + h)) - (f' u'' + \beta \theta g'')$$

Finally, multiplying both sides by  $f'$  and dividing both sides by  $f'' < 0$  and  $1 + g'$ , we obtain:

$$0 > -f' u''(\frac{x}{\theta} + h) \left[ \frac{g'}{1+g'} \right] - \frac{f'}{f''(1+g')} \left( \frac{f'}{\theta} u'' + \beta \theta g'' \right) \quad (52)$$

Multiplying both sides of equation (52) by  $\frac{1}{\beta \theta}$  implies:

$$\frac{1}{\beta \theta} \left[ -f' u''[\frac{x}{\theta} + h] \left[ \frac{g'}{1+g'} \right] - \frac{f'}{f''(1+g')} \left( \frac{f'}{\theta} u'' + \beta \theta g'' \right) \right] < 0 \quad (53)$$

The left hand-side of equation (53) is  $D - T + 1$  (equation (51)) which proves that  $D - T + 1 < 0 \forall \theta \in [\bar{\theta}, 1]$ .

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