Polarization and Income Inequality: A Dynamic Model of Unequal Democracy

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Abstract

Recent empirical work has demonstrated a strong correlation between growing income inequality and polarization of political elites in the United States. We provide a dynamic model of party competition in which two policy motivated parties compete both for votes and donations in order to win elections. Parties know the distribution of voters and donors but are unsure of the relative importance of each group. As a consequence, parties take divergent (polarized) positions and election outcomes are stochastic. Moving policy outcomes to the right increases income inequality and shifts the distribution of donor ideal points to the right. Along the equilibrium path, polarization and income inequality are positively correlated. Increased polarization results more from the policy changes of the party favoring policies to the right rather than the party favoring the left. Finally, a victory by the right favoring party in any election implies more polarization in the future.

1   Introduction

We provide a dynamic model of party competition to analyze three recent empirical findings. The first of these findings is due to McCarty, Poole and Rosenthal (2003, 2006) who demonstrate a tight correlation between growing income inequality as measured by the GINI coefficient and the polarization of elected officials in the U.S. Congress. McCarty et. al., (2006) also find that the increase in polarization is largely due to a rightward shift by Republicans. Finally, Bartels (2008) finds a consistent pattern in which Republican presidential victories produce greater redistribution towards the wealthy than Democratic victories.

In our model, agents have Euclidean policy preferences over a unidimensional policy space. An agent supports a party if she prefers that party's policy position to the position of the other party. We assume that some agents are voters and some are donors and that parties require support from both groups to win election. Two parties (Democrat and Republican) take positions in the policy space that attract *support* within each set of agents. We assume that the distribution of voters and donors are different with the median donor located to the right of the median voter.

Rather than explicitly model the process by which parties translate support from both groups into actual votes, we assume a *vote share function* that maps levels of support from both groups and the realized value of uncertain parameter–importance of money–to vote shares. We assume that parties are policy motivated with Democrats preferring policy outcomes to the left and Republicans outcomes to the left. Since the importance of money is uncertain, the parties choose distinct policies (i.e., election outcomes are *polarized*) election outcomes are stochastic.

In our model, election outcomes effect the election environment in later periods. Specifically, we assume that higher (i.e., more conservative) policies yield greater in-

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equality. We call this assumption, the *effectiveness of policy.* We also assume that an increase in inequality moves the donor distribution to the right. We call this assumption the policy *relevance of inequality.* The two assumptions together constitute what we call the *feedback effect.* In our model, all agents are myopic and hence the feedback effect is the only intertemporal linkage.

Our model yields three results all of which are generated by the following mechanism: greater inequality shifts the donor distribution to the right. The Republican party responds to this shift by choosing a more conservative policy. The response of the Democratic party depends on the dispersion of donor ideal points. If the donor distribution is sufficiently concentrated, the Democratic party also moves to the right when inequality increases. If the donor distribution is dispersed, then the Democratic party moves to the left. In either case, *polarization increases as inequality increases.* Moreover, even when the Democratic party shifts left, it moves less in absolute terms than the Republican party. That is, *the bulk of the polarization is due to a rightward shift by the Republican party.* Since the Republican party chooses more conservative policies than the Democratic policy and since more conservative policies yield greater inequality, it follows that a Republican election victory results in greater inequality and polarization next period than a Democratic election victory. When the donor distribution is concentrated a Republican victory increases inequality and polarization in the long-run as well.

However, when the donors are dispersed, a Republican election victory causes the Democratic party to move leftward. Therefore, a Republican victory in period t followed by a Democratic victory in $t+1$ may result in less inequality in period $t+2$ than two consecutive Democratic victories in periods t, $t+1$. Nevertheless, we show that *a Republican victory in any period increase expected inequality and polarization in*

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all subsequent periods when feedback is linear. Thus, our model is consistent with all three of the empirical observations discussed in the first paragraph above.

The paper is organized as follows: In section 1.1 below, we review the literature. Section 2 contains the description of the stage game and Proposition 1 which establishes the existence, uniqueness of a pure strategy equilibrium. Section 3 describes the dynamic game with feedback effects and contains our main results and an example. Section 4 is the conclusion. Proofs are in an appendix.

1.1   Literature Review

Our single period model is a version of the spatial model of party competition due to Hotelling (1929) and Downs (1957). In Downs' formulation, two candidates take positions in a unidimensional policy space and in equilibrium converge on the median voter's ideal point. Wittman (1983) considers an important variant of the model in which candidates care about policy outcomes rather than winning office (see also Calvert (1985); and Roemer (1994,1997, 2001)). In these models, the median voter result holds when there is no uncertainty about the median voter's ideal point. However, with sufficient uncertainty, parties diverge and take policies symmetric around the median voter's expected ideal point.

Models of policy motivated parties are subject to equilibrium existence problems. Roemer (2001 and 2007) considers a variant in which candidate positions must be approved by multiple factions. A *party-unanimity Nash equilibrium* (PUNE) requires that there exists no alternative policy that is strictly preferred by all the factions in the party. Roemer also looks extensively at the relationship between party competition and income inequality but his models do not produce a correlation between polarization and income inequality nor are they explicitly dynamic. Krasa and Polborn (2010) develop

a new model and survey the current literature. Bernhardt, Duggan and Squintani (2008) look at the normative implications of policy motivated parties. They show that when the location of the median voter is stochastic, the polarized equilibrium with policy motivated parties may yield higher welfare than the equilibrium office motivated parties.

Acemoglu and Robinson (2001), Bartels (2008), Battaglini and Coate (2008) and Battaglini (2012) provide alternative models in which policy outcomes in previous periods can influence political competition in later periods. Acemoglu and Robinson (2001) explore the dynamic interactions between different kinds of political institutions. They typically consider models with different constituencies (e.g., elites and voters) but do not explicitly consider the long run dynamics of party competition. Battaglini (2012) considers a dynamic model of elections with redistribution but does not focus on income inequality or polarization.

Campante (2011) offers a static model in which individuals, as in our model, vote *and* make political contribution. Campante obtains a result analogous to our baseline pivotal agent result. Furthermore, since he identifies policies with tax rates, his model establishes a connection between inequality and redistribution: at low levels of inequality, increasing inequality increases redistribution. However, eventually, further increasing inequality yields less redistribution. Bai and Lagunoff (2013) consider a model of weighted voting that may favor elites and provide results that are useful for empirically determining the presence of unequal voting systems. Their model produces non-median voter outcomes but they do not consider policy motivated parties so there is no polarization.

Prato (2013) provides a model that connects feedback processes, polarization and income inequality. In Prato's model two policy motivated parties have different pref-

erences over levels of redistribution. In each period, the election determines the level of redistribution, which in turn, determines the preference distribution for the next period. Polarization occurs because parties are forward looking and care about the future preference distribution. In our model the connection between income inequality and polarization occurs even though all agents are myopic.

There are a variety of models in the literature that examine the relationship between campaign contributions and policy (e.g., Baron (1994); Grossman and Helpman (1996)). In Baron's model, parties require money to persuade uninformed voters. In our model, it is as if parties use money to mobilize those who prefer the party's position. Baron finds that when policy outcomes are non-excludable, both parties converge on the median voter's ideal point. The Baron and Grossman-Helpman papers explicitly consider donors' incentives to make campaign contributions.

The central findings that stimulated the recent burst of interest in income inequality are found in the seminal work of Piketty and Saez (2001). The empirical literature in political science has focused extensively on issues related to redistribution and income inequality (see Bartels (2008) and McCarty et. al., (2006) for literature reviews.) While there is general agreement that party elites have polarized (see McCarty et. al., (2006)) there is disagreement about whether voters have become more polarized as well (see Fiorina (2005) and Krasa and Polborn (2012)).

2   Elections with Voters and Donors

In each period, two parties $\{D, R\}$–the Democratic party, D, and Republican party R–choose policies in a unidimensional policy space. Party D chooses policy y while R chooses policy z. Let $(y, z) \in \mathcal{R}^2$ denote a generic policy profile. For any such policy

profile we call x the *average policy*

$$x = (z + y)/2$$

and δ is the *polarization* between the party's positions.

$$\delta = z - y.$$

The policy choices generate a set of potential supporters among two sets of agents: voters and donors with ideal points distributed along the closed interval $\mathcal{R}$. We assume agents support the party whose position they prefer. An agent with ideal point ω prefers D if

$$|y - \omega| < |z - \omega| \text{ or}$$
$$\omega < y = z$$

If the inequalities above are reversed, then ω prefers R. Thus, we assume that if the parties choose the same policy, agents with ideal points to the left of that policy prefer D while those with ideal points to the right of that policy prefer R. When the two parties choose distinct policies, we ignore any agent whose ideal point is equidistant from both policies. We also ignore any type $\omega = y = z$.

Voter ideal points are distributed according to the logistic distribution F with mean 0 and standard deviation $\sqrt{\pi/12}$. That is,

$$F(x) = \frac{1}{1 + e^{-x}}$$

In the standard Downsian model, voter preferences translate directly into electoral support. In our model, parties require money to translate voter support into actual

votes. Donor ideal points are distributed according to the logistic distribution G with mean $\mu > 0$ and standard deviation $r\sqrt{\pi/12} > 0$. That is,

$$G(x) = \frac{1}{1 + e^{-\frac{x-\mu}{r}}}$$

We assume that both voter ideal points and donor ideal points have a logistic distribution to ensure the existence of a pure strategy equilibrium. For a given distribution of voter and donor ideal points, support from each group for each party is determined by the midpoint x as follows:

$$V_D = \begin{cases} F(x) & \text{if } y \leq z \\ 1 - F(x) & \text{if } y > z \end{cases}$$

$$M_D = \begin{cases} G(x) & \text{if } y \leq z \\ 1 - G(x) & \text{if } y > z \end{cases}$$

and

$$V_R = 1 - V_D$$

$$M_R = 1 - M_D.$$

Thus, all agents with preferences below the average party position support D unless its position is strictly to the right of R's position (i.e., $y > z$). Henceforth, we will only describe the outcome for the case in which $z \geq y$. The remaining case is analogous and will never occur in equilibrium. Thus $V_D = F(x)$, $M_D = G(x)$, $V_R = 1 - F(x)$ and $M_R = 1 - G(x)$.

2.1 Vote Share Function

We assume that the vote share of a party with voter support V and donor support M depends upon the importance of money α. The vote share function is a standard

contest function as below

$$Q(V, M, \alpha) = \frac{V^{1-\alpha}M^{\alpha}}{V^{1-\alpha}M^{\alpha} + (1-V)^{1-\alpha}(1-M)^{\alpha}}$$

The economics literature has used such contest functions extensively (see Jia, Skaperdas and Samarth (2012) for a review of the literature). This functional form makes our game simple to analyze and has the following desirable properties: when money doesn't matter ($\alpha = 0$) a party's vote share is $Q(V, M, 0) = V$ i.e., exactly equal to its share of voter support. Conversely, when only money matters ($\alpha = 1$) vote share is determined entirely by the party's share of money i.e., $Q(V, M, 1) = M$. For a fixed level of donor support $M \in (0, 1)$ if a party's share of voter support gets sufficiently close to one (i.e., $V \approx 1$) the party vote share is $Q(V, M, \alpha) \approx 1$. Similarly, fixing the party's voter support, as the party share of donor support gets large ($M \approx 1$) it receives almost all the votes ($Q(V, M, \alpha) \approx 1$).

To motivate this functional form, suppose that a party's voters decide to turnout in the election as a function of their perceived civic duty to vote (see Riker and Ordeshook (1968) or Feddersen and Sandroni (2006) for examples of such models). To make things simple assume that the voter's perceived civic duty is increasing in the amount of money the party receives in donations. The fraction of a party's vote support V_i that turns out to vote given donations m_i and importance of money α is $m_i = V_i m_i^{\alpha}$ for $i \in \{D, R\}$. Let $V = V_D$, and hence $V_R = 1 - V$. Then, the actual votes for the parties are:

$$v_D = V m_D^{\alpha}$$

$$v_R = (1 - V) m_R^{\alpha}$$

That is, turnout is increasing in the amount of money donated to the party but, fixing the amount of money, an increase in the importance of money decreases turnout. This

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makes sense since when money doesn't matter all of the party's supporters can be expected to turnout.

Next, we assume that the amount of money m each party can raise from its donors is an increasing function of its level of donor support M and the degree to which the party trails in voter support $(1 - V)$. The first assumption is quite intuitive and the latter assumption is consistent with models of costly participation with asymmetric support (see, for example, Feddersen and Sandroni (2006) and Krasa and Polborn (2009)) in which the equilibrium probability of participation is always higher among those in the minority. The actual level of donor support may also depend on a variety of non-party specific factors e.g., the distance between the party positions δ; the importance of money, α, as long as these other parameters are the same for both parties and only change the marginal rate of donations. Hence, let ϵ be the vector of parameters that influence the marginal rate of contributions q to both parties and suppose let $q(\epsilon) > 0$ for all ϵ. Let m_i be the donations to party $i \in \{D, R\}$ with M and V the donor and voter support for party D:

$$m_D = M(1 - V)q(\epsilon)$$

$$m_R = (1 - M)Vq(\epsilon)$$

Then, substituting m_i into v_i establishes that the vote share of party D is

$$\frac{Vm_D^\alpha}{Vm_D^\alpha + (1 - V)m_R^\alpha} = \frac{V^{1-\alpha}M^\alpha}{V^{1-\alpha}M^\alpha + (1 - V)^{1-\alpha}(1 - M)^\alpha}$$

Hence, we have derived the vote share function Q above. Party R vote share is $1 - Q(V, M, \alpha) = Q(1 - V, 1 - M, \alpha)$.

2.2 The Pivotal Agent

A party wins the election if $Q > 1/2$; loses if $Q < 1/2$ and the election is tied otherwise. For a given average policy x, we let $\alpha(x)$ denote the α that produces a tie. That is, $\alpha(x)$ is the α that solves

$$Q(F(x), G(x), \alpha) = 1/2.$$

If there exists no α that satisfies the equation above, then either $Q(F(x), G(x), \alpha) < 1/2$ for all α or $Q(F(x), G(x), \alpha) > 1/2$ for all α. If the former holds, we set $\alpha(x) = 0$ and we let $\alpha(x) = 1$ in the latter case.

Our assumption that the voter and donor distributions are logistic combined with our specification of the vote share function imply that

$$\alpha(x) = \begin{cases} 0 & x < 0 \\ \frac{r \cdot x}{\mu - (1-r) \cdot x} & 0 \le x \le \mu \\ 1 & \mu \le x \end{cases}$$

This is easy to see since

$$\frac{V^{1-\alpha} M^\alpha}{V^{1-\alpha} M^\alpha + (1-V)^{1-\alpha}(1-M)^\alpha} = \frac{1}{2}$$

implies

$$(1 - \alpha) x = -\alpha (x - \mu) / r.$$

We call the agent whose ideal point x_p solves this equality

$$x_p = \frac{\alpha \mu}{\alpha + r(1 - \alpha)}$$

the *(ex post) pivotal agent.* This agent is the analog of the median voter in our model. If the midpoint of the party positions is equal to x_p, then each party receives $1/2$ of the vote share. The pivotal agent plays a similar role in our model to the role that the

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median voter plays in a Downs/Wittman type model. Note that both the distribution of voters and donors affect the location of the pivotal agent, not just the median. The pivotal agent's ideal point is strictly between the median voter and median donor's ideal point.

Note also that changing the distribution of F below its median or changing the distribution of G above its median would have no effect on the location of the pivotal agent. In our model, analogous to conventional spatial models, the location of the pivotal agent's ideal point is the only connection between voter and donor preferences and party behavior. Thus our model is consistent with Bartels' (2008) finding that the policy preferences of those to the left of the median voter have no impact on the location of the pivotal agent. However, it is also the case in our model that those to the right of the median donor are unimportant as well.

Parties care only about implemented policy. If policy w is implemented D receives a payoff of $-w$ and R receives a payoff of w in that period. We assume that α is uniformly distributed over the unit interval. Given that α is uniformly distributed, $\alpha(\cdot)$ is the probability that party D wins the election. Thus parties' expected payoffs are as follows:

$$D(y, z) = (z - y) \cdot \alpha((y + z)/2) - z$$
$$R(y, z) = -D(y, z)$$

Proposition 1 below establishes the existence and uniqueness of a pure strategy equilibrium for the stage game. In equilibrium, both parties choose policies that are polarized and equidistant from the expected pivotal agent's ideal point (the pivotal agent's ideal point when $\alpha = 1/2$).

Proposition 1 *The stage game Nash Equilibrium is unique. In equilibrium, each party*

wins the election with probability $1/2$, $y = \frac{(1-r)\mu}{(1+r)^2}$ *and* $z = \frac{(1+3r)\mu}{(1+r)^2}$.

To understand the stage game equilibrium, note that for large values of r, donor support is relatively insensitive to policy positions and hence plays a relatively minor role in party competition and party competition is reduced to a competition for voters. Therefore, both y and z approach the median voter's ideal point for arbitrary large values of r. Conversely, as r gets closer to 0, voter support is relatively insensitive to policy and hence y, z converge to the median donor's ideal point μ.

Note that increasing μ always increases z but increases y if and only if $r < 1$. Hence, R always "follows the money" while D's response is ambiguous: it follows the money whenever the marginal return from accommodating donors is high. When it is low, D compensates for the loss of donor support that arises from the increased μ and R's response to this increase by increasing its advantage among voters. Moreover, an increase in μ *always* induces a bigger (absolute) change in R's policy than in D's policy and hence always increases polarization.

In the next section, we assume that the election outcome in a given period affects the mean of the donor distribution in the next period. Our interpretation of this assumption is that election outcomes affect inequality, which in turn, affects the donor distribution. We call this relationship between policy and inequality the *feedback effect* and investigate the circumstances in which the feedback effect yields the empirical observations that McCarty, Poole and Rosenthal and Bartels identify.

3   Dynamic Equilibrium with Feedback Effects

Bartels (2008) finds an empirical connection between income inequality and policy outcomes. There is compelling evidence that support for redistributive policies and

income are inversely related (see Alesina and Guiliano (2009)). Bartels, Page and Seawright (2011) find that policy preferences of the wealthy are considerably more conservative on economic issues compared to the population as a whole. Thus, there is a foundation for a connection between wealth and policy preferences. It is intuitive then to assume that a change in policy can produce a change in inequality and that such a change can, in turn, produce a change in the distribution of donors.

To model the interaction between policy and inequality, we assume that the mean of the donor distribution μ depends on the state s which we identify with the level of inequality. Higher values of s denote greater inequality and we assume that μ is a nondecreasing function of s. That is, greater increases the mean of the donor distribution. We also assume that policy affects inequality. To simplify the analysis, we assume that the state s depends only on the policy outcome of the previous election and is a nondecreasing function of it; that is, we assume that more conservative policies yield greater income inequality.

We call the first assumption above, that μ is a nondecreasing function of s the policy *relevance of inequality* and the second assumption, that s is a nondecreasing function of w, the previous period's policy outcome, the *effectiveness of policy*. The two assumption together constitute the *feedback effect* which is the focus of our analysis. We assume that all voters, donors and both parties are *myopic* and do not consider the consequences of the current policy outcome on future behavior. As a result, an equilibrium for the dynamic game is simply a sequence of the equilibria of the stage game with parameters $r, \mu(s_t)$ where $s_t = s(w_{t-1})$ for $t > 1$ and w_{t-1} is the random policy outcome of the previous election. We let s_1 be the initial level of inequality.

Let $y(s)$ and $z(s)$ denote the equilibrium policy positions (described in Proposition 1) of party 1 and party 2 in state s; that is, $y(s) = \frac{(1-r)\cdot\mu(s)}{(1+r)^2}$ and $z(s) = \frac{(1+3r)\cdot\mu(s)}{(1+r)^2}$.

Let θ_t be a sequence of iid random variables that take on the values 0 or 1, each with equal probability. We identify $\theta_t = 1$ with an R victory and $\theta_t = 0$ with a D victory in period t and 0. Let s_t denote the state in period t and w_t denote the policy outcome in period t. Since the election outcome is random, the initial state together with the stage game equilibrium described in Proposition 1 induces a Markov process: $X_t = (s_t, \mu_t, y_t, z_t, \delta_t, \theta_t, w_t)$. The transitions of these variables are as follows:

$$s_t = s(w_{t-1})$$
$$\mu_t = \mu(s_t)$$
$$y_t = y(s_t)$$
$$z_t = z(s_t)$$
$$\delta_t = z_t - y_t$$
$$w_t = \theta_t \cdot z_t + (1 - \theta_t) \cdot y_t$$

Proposition 2 below establishes that the feedback effect causes inequality and polarization to move together. Hence, our model offers a mechanism that yields the main empirical observation of McCarty, et. al. (2006). The proposition relates the existing level of inequality to the current polarization.

Proposition 2 *With probability 1, $(\delta_\tau - \delta_t) \cdot (s_\tau - s_t) \geq 0$.*

To see how the proposition follows immediately from the comparative statics of the stage game equilibrium described in Proposition 1, note that since s is the only state variable of the stage game, $s_\tau > s_t$ and the relevance of inequality implies $\mu(s_\tau) \geq \mu(s_t)$. Since δ is increasing in μ, the desired result follows. Note that co-movement of inequality and polarization requires the effectiveness of policy and hence requires

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the full force of the feedback effect: if policy were not relevant; that is, if $s(\cdot)$ were constant, then it would not be possible to observe variation in inequality.

McCarty et. al. (2006) offer another, related observation; they note that growing polarization is disproportionately due to changes in Republican policy positions. To see what this means in the context of our model, consider two arbitrary dates t, τ such that $\delta_\tau > \delta_t$. Hence, there is greater polarization at date τ than at t. Since R always chooses a policy to the right of D's policy, this means that either $z_\tau - z_t > 0$ or $y_t - y_\tau > 0$ or both. In all cases, to say that polarization is disproportionately due to Republican policy changes means that when polarization increases, R moves more to right than D moves to the left; that is, $z_\tau - z_t > y_t - y_\tau$. Our next proposition establishes that, under the hypothesis of the previous proposition, this is indeed the case.

Proposition 3 *With probability 1, $(\delta_\tau - \delta_t)(z_\tau - z_t - y_t + y_\tau) \geq 0$.*

To see why Proposition 3 holds, note that by Proposition 1, in absolute terms, increasing μ always has a greater effect on z than on y. Since μ is an increasing function of s and z and δ are both increasing functions of μ, the desired conclusion follows.

Our final result establishes that polarization is greater if R wins an election rather than D. More precisely, let s_τ^{it} denote "inequality at date τ conditional on $\theta_t = i$ at date $t < \tau$." That is, s_τ^{1t} is any random variable that is distributed according to the conditional distribution of s_τ given an R election victory at a date $t < \tau$. We define y_τ^{it}, z_τ^{it} and δ_τ^{it} in a similar fashion.

The cdf H stochastically dominates the cdf $\hat{H}$ if $H(\beta) \leq \hat{H}(\beta)$ for all β. Equivalently, if H is the cumulative of Y and $\hat{H}$ is the cumulative of Z, we say that Y

stochastically dominates Z whenever H stochastically dominates $\hat{H}$. Proposition 4 below establishes that under the hypothesis of the propositions above, ξ_τ^{1t} stochastically dominates ξ_τ^{0t} and hence ξ_τ for $\xi = s, w, x, y, z$ and δ. Thus, an R victory in any period means greater inequality, more conservative policies and more polarization in all future periods.

Proposition 4 *If $r \leq 1$, then ξ_τ^{1t} stochastically dominates ξ_τ^{0t} for $\xi = s, y, z, \delta$ and $t < \tau$.*

Proposition 4 above provides a strong statement of Bartels' empirical observation that future inequality polarization increases after a Republican election victory provided that the donors not too dispersed. Its simple proof is in the appendix. Here we provide the intuition behind it. Recall that when $r \leq 1$, both R and D move rightward as inequality increases and more rightward policies yield more inequality in the next period. Hence, fixing any sequence of election outcomes, more inequality at time $t + 1$ means more inequality in all future periods. But, a Republican election victory at time t yields more inequality at time $t + 1$ than a Democratic election victory at time t. Hence, a Republican election victory means more inequality than a Democratic election victory in all future periods. By Proposition 2 above, this implies more polarization as well.

For any r, an R election victory in any period yields more inequality and hence more polarization for the next period than a D election victory. The relationship between election outcomes and inequality in the long run is more complicated when $r < 1$. We investigate this question in the next subsection.

3.1 Linear Feedback

In this section, we assume that both the relationship between the election outcome and inequality and the relationship between inequality and the donor mean is linear. Although these assumptions are difficult to assess empirically, they are adequate for our purposes since our goal is to show that a positive correlation between Republican election victories and polarization holds even in the setting not covered by Proposition 4 (i.e., $r > 1$) provided that "level effects" are not too strong. Below, we prove a weaker version of Proposition 4 for $r > 1$ by comparing *expected* correlation after an R victory to *expected* correlation after a D victory. Such a comparison is reasonable since the McCarty et. al. procedure for estimating polarization itself relies on averaging policy positions across different parts of the policy spectrum.

Since the case of $r \leq 1$ is covered by Proposition 4, we assume $r > 1$. Let $s(w) = w + 1 - b$ and $\mu(s) = s$ and hence both s, μ are increasing linear functions.[1] To simplify the subsequent notation, let

$$a = \frac{1 - r}{(1 + r)^2} \quad \text{and} \quad b = \frac{1 + 3r}{(1 + r)^2}$$

Then, let $S = [1 - b + a, 1]$ and $W = [a, b]$. Since $1 - b + a > 0$, $\mu(s) > 0$ for $s \in S$. It is easy to verify that $y(s), z(s) \in W$, whenever $s \in S$ and $s(w) \in S$ whenever $w \in W$. It follows that s_{t+1} is in the interval S whenever s_t is in that interval. Hence, we assume that $s_1 \in S$ so that $s_t \in S$ for all t. When $r > 1$, $\mu(\cdot)$, $s(\cdot)$ have the functional forms above and $s \in S$, we say that the feedback is linear.

Let $E[\xi_\tau | \theta_t = i]$ be the expected level of ξ in period τ conditional on $\theta_t = i$. Hence, $E[\xi_\tau^{it}] = E[\xi_\tau | \theta_t = 1]$. In particular, $E[s_\tau^{1t}]$ is the expected level of inequality in period

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[1]For the analysis of this section to go through, the parameters of the linear functions $s(\cdot), \mu(\cdot)$ do not matter. We choose this particular specification to simplify the notation.

τ conditional on an R victory in period t. Proposition 5 below establishes that in the linear dynamic game, *expected* inequality and polarization in every future period are higher after an R win than the corresponding levels after a D win.

Proposition 5 *If feedback is linear, then $E[\xi_\tau^{1t}] > E[\xi_\tau^{0t}]$ for $\xi = s, \delta$ and $t < \tau$.*

An immediate consequence of Proposition 5 is that total expected inequality and total expected polarization increase when R wins an election. That is, $\sum_{j=t+1}^{\tau} E[\xi_j^{1t}] > \sum_{j=t+1}^{\tau} E[\xi_j^{0t}]$ for $\xi = s, \delta$ and $t < \tau$. The proof of Proposition 5 is in the appendix. To see the intuition that underlies the proof, note that since $b + a > 0$: (1) Expected inequality next period, s_{t+1}, is an increasing function of the inequality s_t. (2) Moreover, since the feedback is linear, it is only the expected inequality that matters. (3) By Proposition 2, polarization is an increasing function of the inequality and hence higher inequality means higher polarization. (4) Finally, since R chooses more conservative policies than D, an R win means greater inequality and polarization next period and by (1)-(3) greater expected inequality and polarization in all subsequent periods.

3.2   An Example

In this section, we provide a numerical non-linear example and compute the stationary distributions of equilibrium policies. The example serves both as an illustration of Propositions 2, 3 and also enables us to establish the weaker statement in Proposition 5 even when feedback is not linear.

Let $s_1 = 16$, $\mu(s) = s$ for all s. Then, define,

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s(w) = \begin{cases}
8 & \text{if } w < -2 \\
24 + w & \text{if } -2 \leq w < -1 \\
16 & \text{if } -1 \leq w < 5 \\
8 + \frac{8w}{5} & \text{if } 5 \leq w < 15 \\
32 & \text{otherwise}
\end{cases}
$$

Let $r = 3$, $S = \{8, 16, 32\}$ and $W = \{-4, -2, -1, 5, 10, 20\}$. It is easy to verify that in this example, only donor means and inequality levels in the set S and hence policies in W will be observed in equilibrium. Party D chooses $y < 0$ while R chooses $z > 0$.

Let $\pi(s) = 1/3$ for all $s \in S$ and $\hat{\pi}(w) = 1/6$ for all $w \in W$. It also easy to verify that π and $\hat{\pi}$ are the stationary distributions for inequality and the policy outcome respectively; that is, the distribution of s_t and w_t will converge to π and $\hat{\pi}$ respectively as t goes to infinity.

Thus, the effect of an election outcome diminishes over time and in the long run, disappears entirely. Let $\Delta = \{6, 12, 24\}$. It is easy to verify that

$$
\delta(s) := z(s) - y(s) = 3s/4
$$

and hence only polarization levels in Δ will be observed along an equilibrium trajectory. Thus, polarization is a nondecreasing function of inequality as established in Proposition 2. Note that even though the feedback is not linear, polarization is a linear function of inequality.

When polarization increases by 6 units, from 6 to 12, only 1 unit of this increase is due to the change in D's policy position while 5 units of the change is due to R. Similarly, when polarization increase by 12 units from 12 to 24, D's policy change

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accounts for 2 units of this change and R's policy change accounts for the remaining 10 units. Hence, as established in Proposition 3, R causes more of the polarization than D.

As we noted in our discussion of the linear feedback model, when $r > 1$, even though polarization increases with inequality, a categorical result such a Proposition 4 does not hold. In fact, because feedback is not linear, not even the weaker conclusion of Proposition 5 holds. To see this note that straightforward calculations together with the fact that $s_1 = 16$ reveal that conditional on an R victory in period 1, s_4, the inequality in period 4, is 8 with probability 1/4; 16 with probability 1/2; and 32 with probability 1/4. While if D wins in period 1, μ_4 is 8 with probability 1/2 and 32 with probability 1/2. Hence,

$$E[s_4^{11}] = 18 < 20 = E[s_4^{01}]$$

That is, expected inequality and therefore expected polarization in period 4 conditional R winning the election in period 1 is less than the corresponding expectation conditional on an D victory. Nevertheless, a weaker result holds: total expected inequality and total expected polarization from period $t + 1$ to τ conditional on an R victory in any period $t < \tau$ is always greater than total expected inequality and polarization from period $t + 1$ to τ conditional on a D victory in period t. Hence, while expected inequality and polarization may decrease in some periods, the overall effect of an R election victory is to increase both.

Fact $\displaystyle\sum_{j=t+1}^{\tau} E[\xi_j^{1t}] > \sum_{j=t+1}^{\tau} E[\xi_j^{0t}]$ for $\xi = s, \delta$ and $t < \tau$.

We relegate the details of the calculations necessary for verifying the fact above to

the appendix. These calculations also reveal that

$$\lim_{\tau \to \infty} \left(\sum_{j=2}^{\tau} E[s_j^{1t}] - \sum_{j=2}^{\tau} E[s_j^{0t}] \right) = 80/3$$

$$\lim_{\tau \to \infty} \left(\sum_{j=2}^{\tau} E[\delta_j^{1t}] - \sum_{j=2}^{\tau} E[\delta_j^{0t}] \right) = 20$$

Hence, over time, the effect of an R win dies out but the total effect adds up to 20. A period 1 R victory yields 18 units more polarization in period 2 than a D victory in period 1. While the bulk of this effect of an R election victory occurs in period 2, on average, an R victory increases polarization in the immediate future and total polarization afterwards.

4 Conclusion

In this paper we develop a simple model of party competition to investigate the link between polarization and income inequality. Our dynamic stochastic model yields an equilibrium connection between polarization and income inequality only if the following four conditions are met: (1) money matters, donations must influence party competition; (2) feedback effects link policy outcomes and the distribution of income and donors; (3) parties must behave as if they care about policy outcomes; and (4) parties have imperfect information about the distribution of donors or the importance of money.

Without the feedback effect, policy may be polarized but it will always be centered on a fixed pivotal agent's ideal point and polarization will be constant unless some exogenous factors are impacting the distribution of income and donors. For polarization to exist, parties must be uncertain about the location of the pivotal agent. If money

doesn't matter in elections, then the feedback effect does not emerge and polarization would not vary with inequality even if the location of the median voter location were uncertain.

Our model also provides a foundation for cross national comparisons. Countries in which money is less important in elections (e.g., due to public funding or limitations on contributions) should be expected to have lower polarization and inequality. More generally, most of the key parameters in our model as well as the predictions are directly observable from available data or data that can be easily collected. The key inputs of the model are the policy preferences of voters which is available and the policy preferences of donors (which is collectible).

Our model is simple enough to allow a variety of extensions. For example, we assume throughout the paper that all agents are myopic. Clearly, in the dynamic setting with feedback effects farsighted policy motivated parties would have incentives to change their policies. Similarly, farsighted voters and donors would have preferences over the party positions that take into account the impact of a party's victory on future policy outcomes. We hope to explore the implications of non-myopic preferences in future work.

We only allow for a single dimension for political conflict. But the model could be extended to allow parties to take positions on multiple issue dimensions. Such a model would permit the analysis of how polarization on the economic issue would impact voter and donor support and party behavior on other issues.

Another useful extension would endogenize the importance of money. The preferences over the importance of money for policy motivated parties seem pretty clear: party D would like to decrease the importance of money while R would like to increase it. The preferences of voters and donors are less clear. If the importance of money is

decided by a political process in which money matters, it is an open question whether a winning coalition of agents would exist that would be willing to change its importance.

5   Appendix

5.1   Proof of Proposition 1

We first show that in any pure strategy equilibrium $y < z$ and $x \in (0, \mu)$. That is, the parties must take different positions with the D to the left of R and the midpoint of the party positions must be between the median voter and median donor's ideal point. To see this, suppose that the midpoint $x \leq 0$. Now, if $y \leq z$ then party R wins the election for sure and can still win for sure by choosing $z + \varepsilon$ for small enough ε and increase his payoff. If $x \leq 0$ and $y > z$, D wins for sure and can increase his payoff by choosing z instead of y. A symmetric argument rules out $x \geq \mu$. Suppose $x \in (0, \mu)$ and $y > z$, then D can increase his payoff by choosing z. If $x \in (0, \mu)$ and $y = z$, then R can increase his payoff by choosing $z + \varepsilon$ for small enough ε.

So, in any pure strategy equilibrium, we must have $x \in (0, \mu)$ and both parties have a positive probability of winning i.e., $\alpha(x) \in (0, 1)$. A similar argument as above shows that $y < z$. We now show that there is a unique pair of positions that satisfy the first order conditions for an equilibrium.

For $y < z$, the probability that D wins the election is

$$\alpha(x) = \begin{cases} 0 & x \leq 0 \\ \frac{rx}{\mu - (1-r)x} & 0 < x < \mu \\ 1 & x \geq \mu \end{cases}$$

For any function ξ, let ξ_i (ξ_{ii}) denote the partial derivative of ξ (ξ_i) with respect to

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its i'th argument. The first order conditions require $D_1 = R_2 = 0$. Now, at any equilibrium (y, z) where $x = (y + z)/2$ and $\delta = z - y$, it must be the case that

$$D_1 = -\alpha + \frac{\delta \alpha'}{2} = 0$$
$$R_2 = 1 - \alpha - \frac{\delta \alpha'}{2} = 0$$

Since $\alpha(x) = \frac{1}{2}$, we obtain

$$x = \frac{\mu}{1 + r}.$$

The first order conditions also imply

$$\delta = \frac{1}{\alpha'} = \frac{4\mu r}{(1 + r)^2}$$

So the pair (y, z) where

$$y = x - \frac{\delta}{2} = \frac{(1 - r)\mu}{(1 + r)^2}$$
$$z = x + \frac{\delta}{2} = \frac{(1 + 3r)\mu}{(1 + r)^2}$$

is the only possible pure strategy equilibrium. It remains to show that the pair is an equilibrium. Clearly, it is never a best response for D to choose a policy $\hat{y} > z$ and R to choose a policy $\hat{z} < y$. Hence, to establish that (y, z) is an equilibrium, it is enough to show that $D(\cdot, z)$ is strictly concave on $(-\infty, z]$ and $R(y, \cdot)$ is concave on $[y, \infty)$. Hence, we need

$$D_{11} = -\alpha' + \frac{\delta}{4} \cdot \alpha'' < 0$$
$$R_{22} = -\alpha' - \frac{\delta}{4} \cdot \alpha'' < 0$$

whenever $\hat{y} \leq z$ and $\hat{z} \geq y$. Note that $\mu - (1-r)(z+\hat{y})/2 > 0$ and $\mu - (1-r)(\hat{z}+y)/2 > 0$ at such $\hat{y}, \hat{z}$. It is easy to verify that $\alpha' = \frac{\mu r}{(\mu - (1-r)x)^2}$ and $\alpha'' = \frac{2\mu r(1-r)}{(\mu - (1-r)x)^3}$. Then, since

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$\mu > 0$, straightforward manipulations of the two display equations above reveal that $D_{11}(\cdot, z) < 0$ is equivalent to $(1 - r)(1 + 3r) < (1 + r)^2$ and $R_{22}(y, \cdot) < 0$ equivalent to $(1 - r)^2 < (1 + r)^2$ both of which follow from $r > 0$. QED

5.2   Proof of Proposition 4

Fix the level of inequality in period t at s_1. Without loss of generality, we set $t = 1$. Also fix a sequence of realizations $(\hat{\theta}_2, \ldots, \hat{\theta}_{\tau-1})$ of election outcomes. Then, let s_j^1 be the level of inequality in period j given the realizations $(1, \hat{\theta}_2, \ldots, \hat{\theta}_{j-1})$ of election outcomes and let s_j^0 be the level of inequality in period j given the realizations $(0, \hat{\theta}_2, \ldots, \hat{\theta}_{j-1})$. Note that $s_2^1 = z(s_1) > y(\hat{s}_1) = s_2^0$. Moreover,

$$s_{j+1}^i = s\left(\hat{\theta}_j \cdot z(s_j^i) + (1 - \hat{\theta}_j) \cdot y(s_j^i)\right)$$

for $i = 0, 1$ and $t + 1 \leq j \leq \tau$.

Since $s_{j+1}^1 > s_{j+1}^0$ and $s(\cdot)$, $y(\cdot)$ and $z(\cdot)$ are all increasing functions, $s_\tau^1 > s_\tau^0$. Since each sequence of realizations of θ_j's for $j = t + 1, \ldots, \tau - 1$ yields a higher level of inequality given an R win in period t than a D win in period t, it follows that s_τ^{1t} stochastically dominates s_τ^{0t}. Finally, since s_τ^{1t} stochastically dominates s_τ^{0t}, $\delta_\tau^{it} = z(s_\tau^{it}) - y(s_\tau^{it})$ and by Proposition 1, $z(\hat{s}) - y(\hat{s})$ is increasing in $\hat{s}$, it follows that δ_τ^{1t} stochastically dominates δ_τ^{0t}. QED

5.3   Proof of Proposition 5

Let $s \in S$ be the state in period t. Without loss of generality, we assume $t = 1$. Let $\gamma = (\hat{\theta}_2, \ldots, \hat{\theta}_\tau)$ be a sequence of realization of election outcomes from period 2 to τ and let Γ be the set of all such realizations. Then, let $n = \tau - 2$ and $s^i(\gamma)$ be inequality

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in period τ given initial state s, $\theta_1 = i$ and election outcome sequence γ. It is easy to verify that

$$s^1(\gamma) - s^0(\gamma) = (b - a)s \cdot a^{k(\gamma)} \cdot b^{n-k(\gamma)}$$

where $k(\gamma)$ is the number of zeros in the vector γ. Hence,

$$E[s_\tau^{11}] - E[s_\tau^{01}] = 2^{-n}(b - a)s \cdot \sum_{\gamma \in \Gamma} a^{k(\gamma)} \cdot b^{n-k(\gamma)}$$

Then,

$$\sum_{\gamma \in \Gamma} a^{k(\gamma)} \cdot b^{n-k(\gamma)} = \sum_{i=0}^{n} \binom{n}{i} a^i \cdot b^{n-i} = (b + a)^n$$

Since s, $b-a$ and $b+a$ are all strictly positive, we have $E[s_\tau^{11}] - E[s_\tau^{01}] > 0$. Polarization is a linear function of the current state:

$$\delta(\hat{s}) = z(\hat{s}) - y(\hat{s}) = \frac{4r\hat{s}}{(1 + r)^2}$$

Hence, $E[s_\tau^{11}] - E[s_\tau^{01}] > 0$ implies $E[\delta_\tau^{11}] - E[\delta_\tau^{01}] > 0$. QED

5.4   Proof of the Fact and Related Calculations

Let π_t be the distribution of s_t. If inequality in period t is 8, then an R victory or a D victory both send the state to 16. Hence, the election outcome has no effect on future inequality or polarization. Note also that $\pi_j(8) = \pi_j(32)$ for all j and hence $\pi_j(8) < 1$. Moreover, an R victory if $s_t = 32$ or 16 both yield $s_{t+1} = 32$. Similarly, a D victory if $s_t = 32$ or 16 both yield $s_{t+1} = 8$.

Hence, to prove the Fact, we need only show that it holds if $s_t = 16$; that is, if the Fact holds for $t = 1$; it holds for all t. Let π_j^{11}, π_j^{01} be the distributions of inequality conditional on an R win in period 1 and on a D win period j respectively. Hence, $\pi_{j+2}^{0t} = \pi_{j+1}^{1t}$ for all $j > 1$. Therefore, the total difference inequality that results from

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an R winning rather that D in period 1 is the difference in inequality that results in period 1; i.e., 4 plus the difference in inequality associated with π_τ^{11} and π_3^{01}; that is,

$$\sum_{j=2}^{\tau} E[s_j^{11}] - \sum_{j=2}^{\tau} E[s_j^{01}] = 4 + \sum_{\hat{s} \in S} \hat{s} \cdot \pi_\tau^{11}(\hat{s}) - \sum_{\hat{s} \in S} \hat{s} \cdot \pi_3^{01}(\hat{s})$$

$$= 8 + 8\pi_\tau^{11}(8) + 16\pi_\tau^{11}(16) + 32\pi_\tau^{11}(32) > 0$$

The proves the fact for inequality and since polarization is a linear function of inequality for it as well. QED

Since π_t converges to π, the uniform distribution on S, the expression above converges to $80/3$; that is, the total difference in inequality is $80/3$ which implies that the total difference in polarization is $\frac{3}{4}\frac{80}{3} = 20$ as indicated in Section 3.2.

References

[1] Acemoglu, D., and James Robinson (2001), "A Theory of Political Transitions," *American Economic Review*, 91(4), pp. 938-63.

[2] Alesina, A. and Paula Giuliano. (2009) "Preferences for Redistribution", NBER Working Paper 14285.

[3] Atkinson, Anthony B., and Thomas Piketty ed. (2010), *Top Incomes: A Global Perspective,* Oxford and New York: Oxford University Press.

[4] Bernhardt, D., J. Duggan and F. Squintani. (2008), "The Case for Responsible Parties," *American Political Science Review*, 103 (1), 570-587.

[5] Baron, David. (1994) "Electoral Competition with Informed and Uninformed Voters," *American Political Science Review*, 88 (1), 33–47.

[6] Bartels, Larry M., (2008), *Unequal Democracy: The Political Economy of the New Gilded Age*, Princeton: Princeton University Press.

[7] Battaglini, M. (2012) "A Dynamic Theory of Electoral Competition", working paper, Princeton University.

[8] Battaglini, M and S. Coate. (2008), "A Dynamic Theory of Public Spending, Taxation and Debt", *American Economic Review*, 98(1), 201-236.

[9] Calvert, R.. (1985) "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence," *American Journal of Political Science*, 29 (1), 69–95.

[10] Campante, F.. (2011) "Redistribution in a Model of Voting and Campaign Contributions," *Journal of Public Economics*, 95 (7-8), 646–656.

[11] Cooper, Michael J., Huseyin Gulen, Alexei V. Ovtchinnikov. (2010). "Corporate Political Contributions and Stock Returns," *The Journal of Finance,* 65 (2), 687-724.

[12] Downs, A.. (1957) *An Economic Theory of Democracy.* New York: HarperCollins.

[13] Feddersen, Timothy and Alvaro Sandroni. (2006). "A Theory of Participation in Elections." *American Economic Review*, 96 (4): 1271-1282.

[14] Fiorina, Morris P., Samuel J. Abrams and Jeremy C. Pope. (2005) *Culture Wars? The Myth of a Polarized Electorate*, New York: Pearson Longman.

[15] Grossman, G. and E. Helpman. (1996) "Electoral Competition and Special Interest Politics," *Review of Economic Studies*, 63 (2), 265–286.

[16] Hotelling, H. (1929) "Stability in Competition," *Economic Journal*, 39, 41-57.

[17] Jia, Hao., Stergios Skaperdas and Samarth Vaidya. (2012). "Contest Functions: Theoretical Foundations and Issues in Estimation." *International Journal of Industrial Organization.*

[18] Karabarbounis, L. (2011) "One Dollar One Vote", *Economic Journal,* Volume 121, Issue 553, pages 621–651.

[19] Krasa, Stefan and Mattias Polborn (2009). "Is mandatory voting better than voluntary voting?" *Games and Economic Behavior* 66(1): 275-291.

[20] Krasa, Stefan and Mattias Polborn (2010). "Competition between specialized candidates." *American Political Science Review* 104(4): 745-765.

[21] Krasa, Stefan and Mattias Polborn (2012) "Elites of Masses? A structural model of policy divergence, voter sorting and apparent polarization in U.S. presidential elections, 1972-2008." Working paper. University of Illinois.

[22] McCarty, Nolan, Keith T. Poole and Howard Rosenthal. 2006. *Polarized America: The Dance of Ideology and Unequal Riches,* Cambridge: The MIT Press.

[23] Page, Benjamin, Larry Bartels, and Jason Seawright. (2011). "Democracy and the Policy Preferences of Wealthy Americans." APSA 2011 Annual Meeting Paper.

[24] Piketty, Thomas and Emmanuel Saez. (2003), "Income Inequality in the United States, 1913-1998", *Quarterly Journal of Economics*, 118(1):1-39.

[25] Roemer, John. (1994) "A Theory of Policy Differentiation in Single-Issue Electoral Politics," *Social Choice and Welfare*, 11 (4), 355–380.

[26] Roemer, John. (1995) "Political Cycles," *Economics & Politics*, 7 (1), 1–20.

[27] Roemer, John. (1997) "Political-Economic Equilibrium When Parties Represent Constituents: The Unidimensional Case," *Social Choice and Welfare*, 14 (4), 479–502.

[28] Roemer, John. (1998) "Why the Poor Do Not Expropriate the Rich in Democracies: An Old Argument in New Garb," *Journal of Public Economics*, 70 (3), 399–442.

[29] Roemer, John.. (1999) "The Democratic Political Economy of Progressive Income Taxation," *Econometrica*, 67 (1), 1–19.

[30] Roemer, John.. (2001) *Political Competition: Theory and Applications.* Cambridge, MA: Harvard University Press.

[31] Roemer, J.. (2006) "Party Competition Under Private and Public Financing: A Comparison of Institutions," *Advances in Theoretical Economics*, 6 (1), Article 2.

[32] Wittman, D.. (1983) "Candidate Motivation: A Synthesis of Alternative Theories," *American Political Science Review*, 77 (1), 142–157.