Limit Pricing and the (In)Effectiveness of the Carbon Tax[*]

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Abstract

The conventional analysis of policy-induced changes in resource extraction is inconsistent with the actual way OPEC exerts its market power; we extend to non-renewable resources the limit-pricing theory. Facing a very inelastic demand, an extractive cartel seeks to induce the highest price that does not destroy its demand, unlike the conventional Hotelling analysis: the cartel tolerates some ordinary substitutes to its oil but deters high-potential ones. With limit pricing, policy-induced extraction changes do not obey the usual logic. For example, oil taxes have no effect on current oil production. The carbon tax not only applies to oil; it also penalizes its ordinary (carbon) substitutes, whose market shares are taken over by the cartel. Thus the carbon tax ambiguously affects current and long-term oil production.

I. Introduction

The analysis of policy-induced changes in the production of oil and other carbon-containing resources is still largely inspired by the schedule initiated by Harold Hotelling (1931). It points at an apparently essential aspect of carbon resources: their supply cannot solely be governed by extraction cost conditions, but should also respond to the scarcity value, i.e. the opportunity cost of exhausting limited reserves. Influential examples include, among many others, Chakravorty, Moreaux and Tidball (2008), Sinn (2008), Gaudet and Lasserre (2013), Golosov, Hassler, Krusell and Tsyvinski (2014), van der Ploeg and Withagen (2014)...

The analysis often neglects the role played by OPEC; a role that goes beyond the scene of the oil market as it affects the profitability of all other energy sources. Most of exploitable oil reserves, and the quasi-entirety of currently spare production capacities are still controlled by the cartel.[1] Whether in the short or long run, the balance of oil supply and demand fundamentally relies on the "call on OPEC", – the demand for the OPEC's oil, as coined by business analysts – to an extent that gives the cartel the notorious ability to raise prices by cutting production.[2] The often-made abstraction of OPEC may be justified on the ground that binding reserve limitations are to erode monopoly power in extractive resource markets (Stiglitz, 1976). But this view offers a controversial explanation to OPEC's behavior, that has already been challenged in various ways.[3] Furthermore, the constraint that OPEC's reserves are exhaustible has sometimes been considered "irrelevant" (Adelman, 1990, p. 1).

Whatever their treatment of reserves' exhaustibility, all conventional approaches to OPEC's monopoly power suffer the same major inconsistency. They necessarily come to

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[1] According to the US Energy Information Administration (EIA), 73 percent of proved oil reserves (recoverable at existing conditions) were controlled by OPEC members in 2013. Also according to the EIA, "OPEC member countries produce about 40 percent of the world's crude oil. [...] OPEC's oil exports represent about 60 percent of the total petroleum traded internationally." (Available at `http://www.eia.gov/finance/markets/supply-opec.cfm`).

[2] The EIA concludes that "Because of [its] market share, OPEC's actions can, and do, influence international oil prices". According to Adelman (2004, p. 20), they often do so "through trial and error".

[3] For example, see Lewis, Matthews and Burness (1979) on intertemporal distortions, Pindyck (1987) on reserve uncertainty, Gaudet and Lasserre (1988) on the endogeneity of reserves.

the usual, although never tested, condition[4] that the price elasticity of the demand for oil should be more than one. Indeed, a less-than-one elasticity would imply, as Stiglitz (1976) put it, that "one can obtain larger profits by reducing [the quantity]". Yet this prediction seems to be rejected by a fundamental, although often neglected, fact: the long-run price elasticity of the demand for oil is extremely low; Hamilton (2009a, 2009b) suggests that it should be expected to be lower than 0.25.[5] Moreover, recoverable oil alternatives to OPEC's oil empirically exhibit an extremely low supply elasticity (Golombek, Irarrazabal and Ma, 2013). Using these estimates, we establish that OPEC's demand has a less-than-one price elasticity.[6]

This apparent impasse resembles that addressed by Joe Bain (1949), in his time-honored analysis of limit-pricing industries. There are three possible methodological reactions in the face of the above theoretical inconsistency. One is to reject conventional economic theory by challenging the view that profit maximization is pursued or effectively reached; this paper does not follow that direction, already explored for instance by Cremer and Salehi-Isfahani (1980). Another one is to reject the cartel interpretation and to assume competition inside; in the extreme, monopoly power is exerted by Saudi Arabia alone, which leaves the theory unscathed. The third reaction – and that of this paper – is to examine further the structure of the energy market to account for why oil producers would not apparently benefit from higher prices.

Besides the resource exhaustibility constraint on the exercise of monopoly power, OPEC seems mainly concerned by the fact that too high oil prices trigger the entry of substitutes to its oil. This view has not been given much attention in the academic literature, but by business analysts (e.g. Stephen Schork),[7] and is receiving a renewed

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[4]In some studies (e.g. Stiglitz, 1976), less-than-one elasticity levels are assumed away, on the ground that a monopoly never operates in such regions of the demand curve. This simplifying restriction is often embedded in the form of the monopoly's gross revenue function; for instance, Lewis, Matthews and Burness (1979) assumed it to be decreasing with price everywhere.

[5]Krichene's (2005) estimate of the long-run price elasticity of the demand for crude oil is (absolute value) 0.26 for 1974-2004; according to Hamilton (2009a, pp. 217-218), since crude oil only represents about half the retail cost of final oil-based products like gasoline, the demand elasticity of the former is typically much lower than that of the latter (e.g. Hausman and Newey, 1995; Kilian and Murphy, 2014; references in Krichene, 2005, and in Hamilton, 2009a). See Hamilton (2009b, p. 192) on why the price elasticity of the crude oil demand should be expected to be even smaller now than over the last decades.

[6]See details in Section 5.

[7]The influential energy industry analyst reported to CNBC on August 16, 2010: "OPEC is more con-

interest with the recent market movements; the price drop initiated by OPEC in June 2014 is often seen as a reaction of the cartel to protect its market shares in front of technically-improved and very abundant shale oil deposits. At a time when oil prices were around US\$130, OPEC Secretary General Abdullah al-Badri already recognized that "[OPEC was] not happy with prices at this level because there will be destruction as far as demand is concerned".[8]

Estimates of the long-run marginal cost curve for liquid fuels (International Energy Agency, 2013, p. 228) exhibit a long and flat band that corresponds to the break-even prices of most very abundant deposits of unconventional oils (from shales and sands) and of virtually-unlimited capacities of synthetic fuels (from coal and gas). Besides, the long-run marginal cost of energy in general is bounded by energy sources like fusion power and future-generation high-potential biofuels. Thus OPEC's demand would be massively destroyed if the oil price was to remain around these break-even levels over some long enough period.

A.   Contribution

We first extend the limit-pricing theory (Bain, 1949; see the illuminating literature review of Tirole, 1988, p. 306, and the references therein) to a non-renewable-resource sector. A static monopoly facing a relatively-inelastic demand would always increase its profits by cutting production so as to charge higher prices. Yet when large-scale substitution possibilities destroy its demand at some break-even price, the monopoly secures its market share and profits by deterring those possibilities. Thus in optimum, it induces the maximum price that guarantees deterrence. Unlike the ordinary case where demand elasticity rises continuously with price, the entry of abundant substitution possibilities rules out the standard monopoly pricing and the rule of Abba Lerner (1934); hence the difference. When the intertemporal constraint that reserves are scarce is active, the limit-pricing monopoly's problem satisfies the Hotelling rule: the scarcity value of reserves (also the

cerned about long-term market share than they are about short-term price gains. (...). I speak with OPEC regularly, and [raising the entry barrier for alternative fuels] is consistently their main concern (...). The cheaper you make OPEC oil, the harder you make it to bring alternative fuels to bring on." (`http://www.theatlantic.com/business/archive/2010/08/why-opec-doesnt-mind-low-oil-prices/61557/`).

[8]E.g. `http://www.reuters.com/article/2012/05/03/us-opec-supply-idUSBRE8420UY20120503`).

opportunity cost of extraction) rises at the rate of interest. Yet, we find that this property plays no role in determining resource supply, as suggested by Adelman.

The possibility that limit pricing arises in non-renewable-resource markets with low demand elasticity has been anticipated by Salant (1977, p. 8) and Hoel (1978, p. 31), but has remained unexplored.[9] Besides its empirical foundations, the relevance of the limit-pricing theory for the oil market can be further substantiated on the ground of various accounts by OPEC-related personalities and commentators (e.g. Cairns and Calfucura, 2012).[10]. For example 40 years ago already, Jamshid Amuzegar recognized that "The first of [OPEC's] principles is that the price of oil should be equivalent to the cost of alternative sources of energy."[11]

Second, we revisit the effects of taxes – like the carbon tax – on a non-renewable resource – like oil – when limit pricing is exerted. Much research efforts currently revolve around the design of the optimal carbon tax; it is hoped that both the taxation of carbon resources like oil and the support to non-carbon substitutes are effective instruments to curb carbon emissions that are responsible for global warming. Moreover, relatively high tax rates are already applied to oil products in most countries. From existing governmental commitments and in light of current national and international policy discussions on climate change mitigation, it is to be anticipated that tax rates on carbon energies may further increase and that a more favorable fiscal treatment will be given to their non-carbon substitutes.

Yet, there exists no study of taxation-induced changes in non-renewable-resource quantities that consider the possibility that supply obeys a limit-pricing logic, whether in the literature on non-renewable-resource taxation (e.g. Gaudet and Lasserre, 2013) or in

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[9]See also Dasgupta and Heal (1979, p. 343) and Newbery (1981, p. 625). Those works especially dealt with the curious limit-pricing phase that may follow Stiglitz' (1976) non-renewable-resource monopoly pricing stage. In that context, see also the investigation on the green paradox by van der Ploeg and Withagen (2012, p. 353). A recent, related line of research was initiated by Gerlagh and Liski (2011, 2014b), where limit pricing arises from strategic interactions between resource producers and coordinated consumers; such interactions are absent here, as no-coordinated demand side is needed.

[10]They concluded from their analysis of OPEC behavior, that Saudi Arabia's (and OPEC's) dominant strategy is to "restrain the price to conserve its market in the long-run."

[11]In this interview (Time Magazine, October 14, 1974, p. 36), made famous by Dasgupta and Heal (1979), the Iran's Minister of the Interior and the Shah's right-hand oil expert was explaining that OPEC's strategy is to have the oil price following the industrialized countries' inflation.

the literature about market power on resource markets. Studies on the effect of taxes on resource monopolies are typically based on Stiglitz's (1976) Hotelling-type analysis; e.g. Bergstrom, Cross and Porter (1981) or Karp and Livernois (1992). We show that OPEC's limit pricing completely modifies the standard effects of large-scale environmental policies, to such an extent that exclusively relying on the conventional treatment leads to wrong conclusions. Thus our analysis is not only interesting for the methodology of economic applications to the oil market, but it is also critical for the design of public policies against a climate-change problem labeled "the ultimate commons problem of the twenty-first century" (Stavins, 2011).

B. Analysis' Structure and Principal Findings

We start with a very basic limit-pricing setup: a finite stock of homogenous resource (oil, say) is depleted by a monopoly that faces a relatively price-inelastic demand; substitution opportunities are summarized by a "backstop" technology, as coined by Nordhaus (1973, p. 532) – i.e. a perfect substitute producible without limit. Other aspects are progressively incorporated to the analysis.

In that first setting, we introduce a tax on the extracted flow of resource and we examine its effect in the spirit of Gaudet and Lasserre (2013). Unlike Hotelling models where only constant-present-value taxes are neutral (Dasgupta, Heal and Stiglitz, 1981), we show that resource taxes have in general no effect on current extraction, other than distributional. The goal of reducing the resource flow cannot be achieved in the short term by directly penalizing extraction.

An intuitively attractive way of reducing resource consumption may be to support the backstop substitute if it is environmentally better. On the contrary, we show that subsidizing the substitute at any date induces more extraction; unlike the "green paradox" (Sinn, 2008), the effect is contemporary rather than the result of an intertemporal substitution. This is the object of Section 2.

A backstop technology represents the possibility that the oil resource be largely or completely replaced in the long run, by a resource base capable of meeting virtually all demand requirements. Following Nordhaus' example, technologies like nuclear fusion

would provide such energy abundance that oil would no longer be economically scarce in the long run. Other similar examples are very abundant unconventional oil resources and the enormous capacity of liquefied coal and gas products. For any such oil substitute, there is a break-even price at which the resource massively flood the market.

In contrast, currently-exploited substitutes to OPEC's oil ordinarily offer limited substitution possibilities: the production of existing energy goods usually exhibits decreasing returns to scale because it relies on some scarce primary factors.[12] On these grounds, Section 3 introduces ordinary substitutes to the monopoly's resource that have imperfectly-elastic supplies, unlike the backstop. Each substitute is characterized by its entry price and has a rising marginal cost function.

Substitution possibilities that are used at current oil prices leave a (residual) demand for the cartel's resource; the curve of this demand progressively reflects the multiplicity of substitutes, with kinks and increasing demand elasticity at those kinks. On the one hand, the backstop has the potential to virtually destroy the cartel's demand. Profit maximization thus requires that it be deterred as in Section 2. On the other hand, ordinary substitutes are not sufficient threats to the resource market share to warrant deterrence. Extraction profits may increase with higher prices despite the fact that ordinary substitutes become economic, unlike the backstop. Limit pricing is compatible with ordinary substitutes being produced. In that context, resource taxes remain neutral and backstop subsidies retain their non-standard positive effect on extraction. In contrast, subsidies to ordinary substitutes increase their production and so induce a reduction in the extraction flow by the same amount, as is usually expected.

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[12]For non renewables (ordinary carbon products like deepwater oil, coal and gas, or uranium), scarcity arises from the finiteness of total exploration prospects and/or from the fact that low-cost available reserves specifically are limited. Similarly for standard biofuels, as well as for solar and wind energy production, scarcity arises from land limitations. For instance, at the microeconomic level of a wind turbine, returns to scale should be increasing because the turbine involves a fixed set-up cost and almost-constant marginal costs of maintenance; at the macroeconomic level however, the unit cost of wind energy output must be increasing both because of land supply limitations and because the marginal land is of worse quality as far as wind exploitation is concerned. See for instance Chakravorty, Magné and Moreaux (2008) and Heal (2009) on land requirements and large-scale substitution of fuel products. Land availability is considered an issue as soon as further use of land causes rents to rise. The same is true for hydropower exploitation: for example in Switzerland, the 25 projects of new hydroelectric power plants will exhibit an expected average unit cost that is twice as large as that of the existing plants (Swiss Federal Office of the Energy, 2013, p. 7).

The above results are also obtained in Appendix A, where a stationary and much simplified version of the model is presented.

The energy-market model of Section 3 allows to examine the carbon tax. Not only oil, but some of its energy substitutes contain carbon. The carbon tax is formally equivalent to several taxes, each being applied to one carbon-containing good, to the extent of its carbon content. According to the above results, the fact that the carbon tax is applied to the cartel's oil has no direct effect on the equilibrium resource quantity. The carbon tax may only be effective as it penalizes oil's carbon substitutes. Yet we highlight a mechanism by which the carbon tax tends to increase oil supply: the reduction in ordinary carbon substitutes (not-deterred, e.g. deepwater oil, coal, gas...) abandons market shares that the cartel is led to take over. When ordinary substitutes are less carbon intensive than oil is, the carbon tax may even increase the current flow of carbon.

In Section 4, we consider a Ricardian resource that is incompletely depleted: the cartel extraction may become uneconomic before exploitable reserves are exhausted. Throughout the limit-pricing exploitation period, taxation policies retain their effects on current extraction, but may further affect ultimately extracted quantities. When the carbon tax increases resource extraction, it shortens the resource exploitation period; its effect on the ultimately extracted quantity turns out to be ambiguous in that case.

Finally in Section 5, with further details in the Appendix, we discuss limit-pricing equilibria in less parsimonious models integrating various aspects of the oil market. First and foremost, the models of Sections 3 and 4 are isomorphic to one with a competitive fringe supplying the same resource as the cartel (e.g. Salant, 1976), once an ordinary substitute is interpreted as the fringe's production; it is in this section that we use recent elasticity estimates to assure the empirical relevance of the limit-pricing analysis for OPEC. We also elaborate on exploration and reserve development (e.g. Gaudet and Lasserre, 1988), as well as on the multiplicity of demand segments (e.g. Hoel, 1984).

II. A Simple Limit-Pricing Model and the Effects of Taxation Policies

This section presents a limit-pricing model of a homogenous non-renewable resource market, where substitution possibilities are solely represented by a backstop substitute. We study the effects of taxes on the resource and of subsidies to the substitute.

A. Static Limit Pricing

Consider first a single date t. At this date, a monopoly produces some energy resource flow q at a constant marginal cost $c_t > 0$.

The total energy demand is given by the function $\overline{D}_t(p)$ of its price p; it is continuously differentiable and strictly decreasing. We assume that the price elasticity of the energy demand is lower than unity all along the demand curve: $\xi_{\overline{D}t}(p) \equiv -\overline{D}'_t(p)p/\overline{D}_t(p) < 1$.

There is a backstop technology by which a competitive sector can produce a perfect substitute to the resource at a constant positive marginal cost $p_t^b > c_t$. The demand notion that is relevant to the monopoly is the residual demand it faces.[13] Let us denote it with $D_t(p) \leq \overline{D}_t(p)$. When $p < p_t^b$, the production of the substitute is not profitable and thus the residual demand for the resource amounts to the entire energy demand $D_t(p) = \overline{D}_t(p)$. When $p \geq p_t^b$, the substitute becomes profitable, so that the market establishes the resource price to $p = p_t^b$. Were the resource price strictly higher than p_t^b, the resource demand would be destroyed: $D_t(p) = 0$. For notational simplicity and without any consequence on our message, we assume as is standard that if $p = p_t^b$ consumers give priority to the resource: at this price, the monopoly may serve the entire demand $D_t(p_t^b) = \overline{D}_t(p_t^b)$, assumed to be strictly positive.

To sum up, we make the following assumption.

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[13]The presence of a competitive fringe producing an identical resource amounts to interpreting the residual demand for each price as being net of the fringe's supply for that price (e.g. Salant, 1976). More on that further below, in Section 5.

Assumption 1 (Low price elasticity of the resource demand)

For all prices $p < p_t^b$, the residual demand $D_t(p) = \overline{D}_t(p)$ for the monopoly's resource is strictly positive and exhibits a low elasticity

$$\xi_{Dt}(p) \equiv -D'_t(p)p/D_t(p) < 1; \tag{1}$$

at price $p = p_t^b$, the monopoly may serve any demand portion $q \in [0, \overline{D}_t(p_t^b)]$; for prices $p > p_t^b$, its demand vanishes.

Figure 1 illustrates the residual demand schedule and its kink at price $p = p_t^b$.
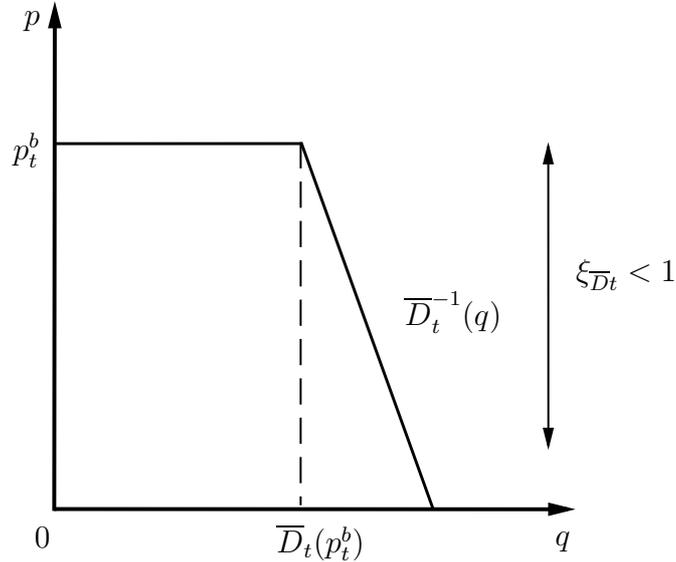

Figure 1: Residual demand for the resource with a backstop technology

Which production level maximizes the monopoly's profits in that context? If the monopoly supplies an amount q that is lower than the threshold quantity $\overline{D}_t(p_t^b) > 0$, it meets the demand at the resource market price is $p = p_t^b$; the monopoly's spot profit is $(p_t^b - c_t)q$, strictly increasing in q. With a higher supply $q > \overline{D}_t(p_t^b)$, the monopoly depresses the price below p_t^b; its spot profit as function of the resource quantity becomes $\left(\overline{D}_t^{-1}(q) - c_t\right)q$, which is strictly decreasing in q because demand is sufficiently inelastic.[14]

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[14]The less-than-one demand elasticity in Assumption 1 is sufficient, not necessary. It can easily be shown that the extraction profit $\left(\overline{D}_t^{-1}(q) - c_t\right)q$ is strictly decreasing for all elasticity levels $\xi_{\overline{D}t} < p_t^b/(p_t^b - c_t)$; a threshold greater than one in general, but equal to one with zero extraction costs.

Indeed, marginal profit may be written $p\left(1 - 1/\xi_{\overline{D}t}(p)\right) - c_t$, where $\xi_{\overline{D}_t}(p) < 1$ implies the term into parentheses to be negative. To sum up, the instantaneous profit is

$$\pi_t(q) = \begin{cases} (p_t^b - c_t)q, \text{ increasing, for } q \leq \overline{D}_t(p_t^b) \\ \left(\overline{D}_t^{-1}(q) - c_t\right)q, \text{ decreasing, for } q > \overline{D}_t(p_t^b) \end{cases}, \tag{2}$$

which is maximized by the supply level $q_t = \overline{D}_t(p_t^b)$ as depicted by Figure 2; this supply induces the limit price $p_t = p_t^b$, the maximum price that deters the entry of the backstop.
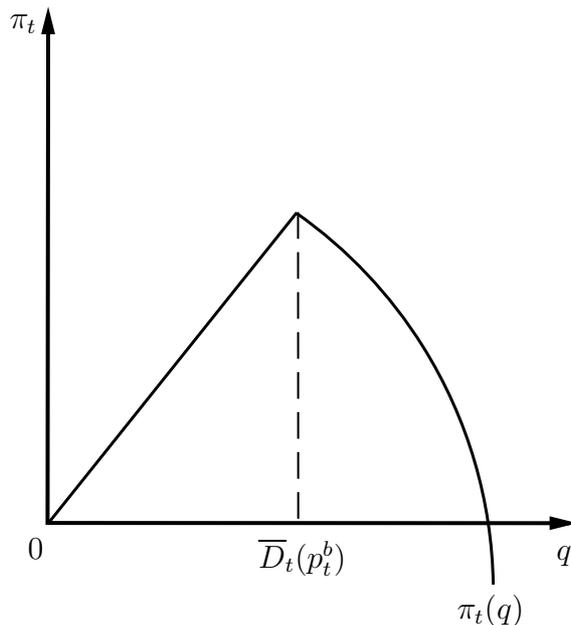
Figure 2: Instantaneous profit

The limit-pricing optimum differs from the usual optimum of a static monopoly characterized by the Lerner equality. Convention has it that the demand function is differentiable everywhere, so that its elasticity to price evolves continuously along its curve. As is well known in that context, a conventional monopoly always deviates from less-than-unity elasticity demand sections because it enjoys higher prices (e.g. Tirole, 1988, p. 66). In contrast, when the entry of a substitute at its break-even price causes a kink to the demand as per Assumption 1, the elasticity at this kink jumps from a low level to a very high level which reflects that the demand is destroyed. This entry threat maintains the profit-maximizing monopoly supply on the less-than-unity elasticity section of its demand; at the limit-pricing monopoly solution, higher elasticity levels are not observed.

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For our purpose to reexamine the effect of taxation policies when limit pricing is exerted, there is one fundamental difference between the standard-monopoly-pricing and the limit-pricing solutions. While the former varies with production costs in the way defined by the Lerner equality, the latter only depends on the entry price of the backstop as shown earlier in this section. An increase in the marginal production cost may not affect the limit-pricing monopoly, which also suggests that such monopoly may not react to tax penalties in the standard fashion.

B. Intertemporal Limit Pricing of Extraction

Consider now that the resource is non-renewable; it is available in a finite quantity $Q_0 > 0$, that is to be extracted over the continuum of dates $t \in [0, +\infty)$.

In that case, the monopoly's problem becomes intertemporal. Assuming a discount rate $r > 0$, the stream of discounted profits amounts to

$$\int_0^T \pi_t(q_t) e^{-rt} \, dt, \tag{3}$$

where the function $\pi_t(q_t)$ is given by the function (2) and where the terminal date $T \geq 0$ is endogenous. The monopoly chooses the extraction path $(q_t)_{t \geq 0}$ in such a way as to maximize (3) under the exhaustibility constraint

$$\dot{Q}_t = -q_t, \text{ with } Q_T \geq 0, \tag{4}$$

where Q_t denotes the remaining stock at date t, and $Q_0 > 0$ is given.

In such dynamic problems, the relevant instantaneous objective is the Hamiltonian function. The Hamiltonian at some date $t \geq 0$ does not only consist of the present-value static profit objective $\pi_t(q_t) e^{-rt}$; it is corrected by a linear term that reflects the opportunity cost of extracting the scarce resource. For the problem of maximizing (3) under (4), the Hamiltonian writes

$$\mathcal{H}(q_t, Q_t, \lambda_t, t) \equiv \pi_t(q_t) e^{-rt} - \lambda_t q_t, \tag{5}$$

where $\lambda_t \geq 0$ is the multiplier associated with constraint (4). λ_t must be interpreted as the discounted scarcity value of the resource. By the Maximum Principle, it is constant

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over time at the producer's optimum: $\lambda_t = \lambda$.[15] In current-value terms, it is equivalently rising at the rate of interest, like in the realm of Hotelling.

A necessary condition for the optimal choice of extraction q_t is that the Hamiltonian (5) is maximized at all dates of the extraction period. Since λq is linear in q, as well as $\pi_t(q)$ in (2) at the left of its maximum, it follows that the Hamiltonian is maximum for the same supply level $q_t^m = \overline{D}_t(p_t^b)$ as the instantaneous revenue $\pi_t(q)$ in (2), as long as the discounted marginal revenue $(p_t^b - c_t)e^{-rt}$ remains greater than the scarcity value λ (See Figure 3).
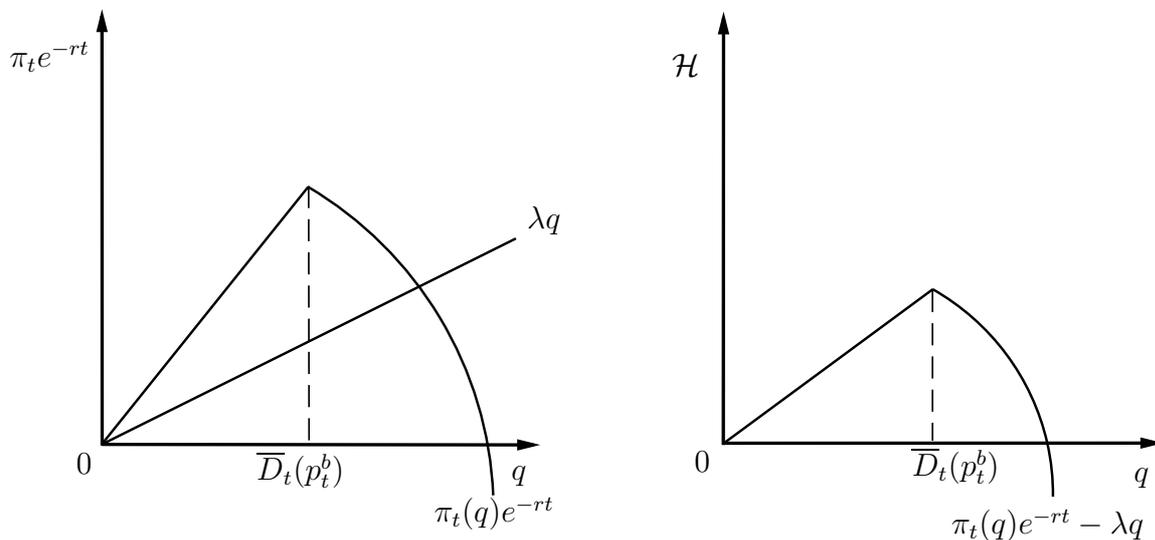

Figure 3: Instantaneous profit and Hamiltonian value

In the stationary version of the model, p^b and c are constant with $p^b > c$, so that the discounted marginal revenue $(p^b - c)e^{-rt}$ is strictly decreasing because of discounting. In the non-stationary model used here, it need not be so. For simplicity, we make the following assumption that excludes supply interruptions during the resource exploitation phase;[16] an assumption that will be maintained until Section 4.

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[15]The time independence of λ along the optimal producer path is standard in models of Hotellian resources. It arises from the fact that the Hamiltonian does not depend on Q_t because the resource is homogenous. In Section 4, we will examine the case of heterogenous resources.

[16]The analysis can easily accommodate supply interruptions, as when the limit price p_t^b falls short of c_t for some dates of the exploitation period. This would not modify the analysis in any insightful manner. Section 4 considers the possibility that limit-pricing extraction becomes uneconomic after some date.

Assumption 2 (Complete and uninterrupted extraction)

For all dates $t \geq 0$, the limit-pricing marginal revenue is strictly positive and strictly decreasing in present-value terms.

Absent any policy, the limit-pricing marginal revenue is $p_t^b - c_t$. By Assumption 2, for all $t \geq 0$, $p_t^b > c_t$ and $(p_t^b - c_t)e^{-rt}$ is strictly decreasing, as in the standard stationary treatment.

Assume, as a statement to be contradicted, that λ is nil. Since the present-value marginal revenue $(p_t^b - c_t)e^{-rt}$ is always strictly positive by Assumption 2, extraction must be $q_t^m = \overline{D}_t(p_t^b) > 0$ at all dates $t \geq 0$. Clearly, this would violate the exhaustibility constraint (4) in finite time.

Therefore we must conclude that the opportunity cost of extraction λ is strictly positive, so that the dynamic constraint (4) is active and the resource is economically scarce. Despite that, the rest of the resolution formally shows that the monopoly's optimum consists in the limit-pricing quantity $q_t^m = \overline{D}_t(p_t^b)$ from date 0, until the resource is exhausted; the reader may also directly refer to the heuristic verification that follows Proposition 1.

Now let us contradict that $p_0^b - c_0 < \lambda$. Were this true, by Assumption 2, $(p_t^b - c_t)e^{-rt}$ would fall short of λ for all $t \geq 0$; thus no extraction at all would be optimal. Since $p_t^b > c_t$ for all $t \geq 0$, this would be trivially dominated by some strictly positive extraction.

It follows that the marginal extraction profit $(p_t^b - c_t)e^{-rt}$ is greater than or equal to the opportunity cost λ, from date 0, until the terminal date T^m. At date T^m,

$$(p_{T^m}^b - c_{T^m})e^{-rT^m} = \lambda > 0, \tag{6}$$

i.e. extraction stops when the marginal extraction benefit meets the extraction opportunity cost. Since $\lambda > 0$, T^m must also be the exhaustion date: $Q_{T^m} = 0$. Since extraction is $q_t^m = \overline{D}_t(p_t^b) > 0$ all along the exploitation period $[0, T^m]$, the exhaustion date T^m is characterized by

$$\int_0^{T^m} \overline{D}_t(p_t^b) \, dt = Q_0. \tag{7}$$

Combining conditions (6) and (7) determines λ.

In the stationary model, the limit-pricing quantity $q^m = \overline{D}(p^b)$ is constant, that induces the limit price $p^m = p^b$. The terminal date T^m is given by $T^m = Q_0/\overline{D}(p^b)$, which

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implies $\lambda = (p^b - c)e^{-rT^m}$.

When Assumptions 1 and 2 are verified, the general properties of the limit-pricing equilibrium in absence of taxation policies are summarized by the following proposition.

Proposition 1 (Limit-pricing equilibrium)

1. *The monopoly supplies $q_t^m = \overline{D}_t(p_t^b) > 0$, and so induces the limit price $p_t^m = p_t^b$ that deters the backstop-substitute production, at each date t of the extraction period $[0, T^m]$;*

2. *The limit-pricing equilibrium leads to the complete exhaustion of the resource at the date T^m such that $\int_0^{T^m} \overline{D}_t(p_t^b) \, dt = Q_0$.*

It can easily be verified that deviations from this extraction path would decrease the sum of the monopoly's discounted profits. Two types of deviations are possible. First, consider reallocations of an infinitesimal quantity $\Delta > 0$ of resource from any date t to any date $t' \neq t$ such that $t, t' < T^m$. Reducing extraction by Δ at date t decreases present-value profits by $(p_t^b - c_t)\Delta e^{-rt}$ while increasing extraction at date t' decreases profits as well, since profits are decreasing for quantities exceeding the limit-pricing extraction q_t^m. Second, consider reallocations of an infinitesimal quantity $\Delta > 0$ of resource from any date $t \leq T^m$ to any date $t' > T^m$. Again, reducing extraction by Δ at date t decreases present-value profits by $(p_t^b - c_t)\Delta e^{-rt}$. On the other hand, increasing extraction at date t', from zero, by Δ, increases present-value profits by $(p_{t'}^b - c_{t'})\Delta e^{-rt'}$. However by Assumption 2, $(p_{t'}^b - c_{t'})e^{-rt'} < (p_t^b - c_t)e^{-rt}$, so that the overall effect on the discounted stream of profits remains negative.

This section shows a very peculiar characteristic of limit-pricing equilibria of non-renewable-resource markets. Unlike most dynamic problems, Proposition 1 implies that the maximization of the intertemporal profit objective (3) is compatible with pursuing the maximization of instantaneous profits (2) at each date of the exploitation period. This is so despite the fact that the dynamic exhaustibility constraint (4) is active. Thus if we observe that a resource monopoly maximizes instantaneous profits, we should not conclude that such rule is not (privately) optimal.

In other words, the limit-pricing theory reconciles two apparently incompatible analyses of oil supply. On the one hand, the conventional Hotellian treatment adopted by most resource economists requires that the dynamic dimension added by oil's exhaustibility be taken into account. On the other hand, Adelman (1990) and many energy analysts offer an interpretation of OPEC's behavior where the exhaustibility constraint plays no relevant role.

Besides, there are two noticeable differences between the limit-pricing equilibrium arising here and conventional Hotelling equilibria. First, throughout the extraction period, the equilibrium present-value marginal revenue $(p_t^b - c_t)e^{-rt}$ of the monopoly may be time varying – it is decreasing in the stationary model –, unlike Hotelling analysis where it is always constant, equal to the scarcity value λ; this is so despite the assumption that the resource is homogenous in both cases. Second, the stylized fact that the oil demand has a less-than-unity price elasticity at equilibrium is observed. This is incompatible with conventional treatments of monopoly power on resource markets, either because low-elasticity levels are assumed away (e.g. Stiglitz), or because the discontinuity of demand elasticity resulting from large-scale drastic substitution possibilities is not taken into account.

C.   Taxes on the Non-Renewable Resource

Let θ_t be a specific resource tax (or subsidy if negative) applied to the producer resource price p_t at each date $t \geq 0$ to determine the consumer resource price $p_t + \theta_t$.[17]

The consumer price at which the substitute becomes profitable is p_t^b, regardless of the tax. Therefore the resource supply that induces this limit consumer price remains the one given by the demand relation: $q_t = \overline{D}_t(p_t^b)$. With a lower supply, the substitute is profitable: the market establishes the (tax-inclusive) consumer resource price at level p_t^b and thus the resource producer price at level $p_t^b - \theta_t$. With a greater supply $q_t \geq \overline{D}_t(p_t^b)$, only the resource may be produced so that the (tax-inclusive) consumer price is given by the inverse demand $\overline{D}_t^{-1}(q_t)$: the price accruing to the producer becomes $\overline{D}_t^{-1}(q_t) - \theta_t$.

It turns out that the problem of the previous section is only modified to the extent

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[17]This is a consumer tax. As for instance in Bergstrom et al. (1981), its effect is formally equivalent to that of a tax falling on the producer.

that the instantaneous profit becomes

$$\pi_t(q) = \begin{cases} (p_t^b - \theta_t - c_t)q, \text{ increasing, for } q \leq \overline{D}_t(p_t^b) \\ \left(\overline{D}_t^{-1}(q) - \theta_t - c_t\right)q, \text{ decreasing, for } q > \overline{D}_t(p_t^b) \end{cases}. \tag{8}$$

The modification amounts to integrating the tax θ_t to the marginal cost c_t.

Let Assumption 2 apply in this context, where the cost c_t in the absence of policies is replaced here by $c_t + \theta_t$. The assumption implies focusing on taxes that leave extraction attractive along the exploitation period. First, the property that the limit-pricing marginal revenue $p_t^b - c_t - \theta_t$ remains positive for all $t \geq 0$ excludes so high taxes that would leave no extraction profits at all. Second, the property that $p_t^b - c_t - \theta_t$ is decreasing in present value excludes taxes (subsidies) that are falling (rising) too rapidly. The two conditions rule out the possibility that depletion be interrupted during the exploitation phase.[18]

Once Assumption 2 is adjusted that way, the analysis of the previous subsection follows through, unchanged, and the same limit-pricing equilibrium described in Proposition 1 is realized. Indeed the quantity that the monopoly needs to supply so as to deter the backstop production remains, at each date $0 \leq t \leq T^m$ of the exploitation period, $q_t^m = \overline{D}_t(p_t^b)$, regardless of whether the resource is taxed or not; in the limit-pricing equilibrium, the path of resource taxes has no effect on the monopolist's extraction. Meanwhile, its revenues are reduced by the tax burden $\theta_t \overline{D}_t(p_t^b)$ at each extraction date.

The following proposition summarizes the effect of resource taxes that are compatible with Assumption 2.

Proposition 2 (Effect of resource taxes)

Resource taxes leave resource extraction unchanged.

Extreme resource taxes eliminated by Assumption 2 might cause resource supply interruptions during the exploitation phase. First, too high taxes $\theta_t \geq p_t^b - c_t$ for some

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[18]In the stationary model, the assumption holds in particular for all constant taxes (and subsidies) $\theta < p^b - c$. It also holds for all rising taxes (falling subsidies), as well as for those taxes (subsidies) that are not too decreasing (increasing) over time. For example let a tax θ_t have an initial level $\theta > 0$ and be rising at a negative rate $\alpha < 0$: $\theta_t = \theta e^{\alpha t}$. It can easily be shown that Assumption 2 applies as long as $\alpha > 1 - (p^b - c)/\theta$, with $p^b - c > \theta$. In the time-dependent model where $p_t^b - c_t$ is decreasing, the set of admissible taxes is broader.

$t \leq T^m$ would expropriate the entire profit at the monopolist's optimum; the monopolist in that case would be better-off with no extraction. Second, taxes that are falling so rapidly that discounted marginal revenue is greater at distant dates $t > T^m$ than during the exploitation period would lead the monopolist to completely shift extraction away from the exploitation phase.

There also exist some neutral resource taxes in standard Hotelling models. Dasgupta et al. (1981) showed that specific resource taxes rising at the rate at which profits are discounted leave the extraction of a competitive sector unchanged; they do not modify the intertemporal no-arbitrage condition that prevails in any Hotelling competitive equilibrium. As noticed by Karp and Livernois (1992), this also applies under monopoly.[19] Also, under competition as well as in a monopoly, extreme taxes that eat the entire Hotelling rent do not warrant any extraction.

Although reminiscent of Dasgupta et al.'s (1981) and Karp and Livernois' (1992) neutrality result, the finding of Proposition 2 is much stronger. The novelty lies in the fact that resource taxation neutrality in limit-pricing equilibria does not require taxes to obey any particular dynamics.

D. Subsidies to the Backstop Substitute

An appealing alternative to taxing the resource may be to support the resource backstop substitute when it is environmentally better. Let γ_t^b be a specific subsidy to the backstop substitute, applied to the backstop's producer price, which is also its marginal cost p_t^b. Thus, the problem in absence of taxation is only modified to the extent that the price of the backstop substitute p_t^b should be replaced by the consumer net-of-subsidy price $p_t^b - \gamma_t^b$. Unlike a resource tax, a backstop subsidy γ_t^b always affects the limit-pricing equilibrium.

When the substitute consumer price is reduced to $p_t^b - \gamma_t^b$, the resource supply that deters its production rises to $\overline{D}_t(p_t^b - \gamma_t^b) > \overline{D}_t(p_t^b)$; indeed, more resource needs to be supplied to deter a cheaper backstop. Also, low resource quantities $q_t < \overline{D}_t(p_t^b - \gamma_t^b)$ that

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[19]In Hotelling equilibria, whether under competition or monopoly, there exists a family of optimal resource tax/subsidy paths. This family is indexed by a tax component Ke^{rt}, where K is some scalar. As Karp and Livernois (1992, p. 23) put it: "If the amount Ke^{rt} is added to [the optimal unit tax], the monopolist will still want to extract at the efficient rate, provided that the dynamics rationality constraint is satisfied (...)."

warrant the production of the substitute reduce the resource price to $p_t^b - \gamma_t^b$, so that the marginal extraction profit of the monopolist becomes $p_t^b - \gamma_t^b - c_t$.

We only consider subsidies that do not violate Assumption 2, so that an uninterrupted resource supply is warranted throughout the exploitation period. This avoids extreme subsidies that would cause resource supply interruptions. First, $p_t^b - \gamma_t^b - c_t > 0$, for all $t \geq 0$: the condition assumes away subsidies that would destroy extraction profits because the substitute would be available to consumers for a price $p_t^b - \gamma_t^b$ lower than the resource extraction cost c_t. Second, $p_t^b - \gamma_t^b - c_t$ remains decreasing in present value for all $t \geq 0$: the condition rules out backstop subsidies that are so decreasing over time that they would make extraction less attractive during the exploitation phase than at more distant dates.

With Assumption 2, the instantaneous extraction profit with backstop subsidies becomes

$$
\pi_t(q) = \left\{
\begin{array}{l}
(p_t^b - \gamma_t^b - c_t)q, \text{ increasing, for } q \leq \overline{D}_t(p_t^b - \gamma_t^b) \\
\left(\overline{D}_t^{-1}(q) - c_t\right)q, \text{ decreasing, for } q > \overline{D}_t(p_t^b - \gamma_t^b)
\end{array}
\right. ,
\tag{9}
$$

and the same dynamic analysis applies as before. It follows that at each date of the resource exploitation phase, the monopoly chooses the limit-pricing supply $q_t^m = \overline{D}_t(p_t^b - \gamma_t^b)$ that deters the backstop production.

Figure 4 illustrates how backstop subsidies shift the demand kink along the demand curve and modify the limit-pricing equilibrium. The following proposition summarizes the effect of subsidies to a backstop substitute in the context of this section.

Proposition 3 (Effect of subsidies to the backstop substitute)
Subsidies to the backstop substitute increase the resource current extraction.

If public policies aim at reducing current oil extraction, the model of this section yields a rather pessimistic message. Leave aside extreme policies that would cause supply disruptions: not only are resource taxes strongly neutral, but subsidizing the backstop substitute induces the monopoly to increase its supply.
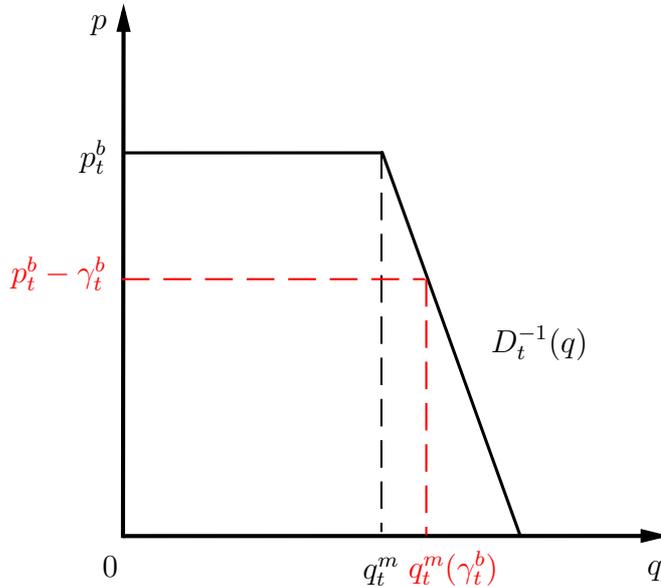
18

Figure 4: Limit-pricing equilibrium and the effect of a backstop subsidy

III. Ordinary Substitutes

A backstop technology is a standard and meaningful modeling device. It represents the possibility that the resource be completely replaced, as a result of a virtually-infinite resource base. Whether in conventional Hotelling-type equilibria or in the limit-pricing equilibrium of Section 2, such backstop technology is never used before the exhaustion date, after which it becomes the exclusive source of energy.

In contrast, empirical evidence shows that ordinary substitutes to oil are currently traded and consumed on energy markets, such some regular fuels and biofuels, and alternative energies. Yet, such substitutes meet a limited fraction of the energy demand.

In this section, we do away with the assumption that there is a single (backstop) substitute and allow for the possibility that some ordinary substitutes may be used along the resource extraction phase. Limit pricing to deter the backstop substitute is not incompatible with ordinary substitutes being produced during the resource exploitation phase.

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A. The Model

The elasticity of the residual demand is often interpreted as the extent of substitution opportunities (e.g. Lewis et al., 1979). Marshall (1920) argued that, ordinarily, demand curves should be expected to have the property that the price elasticity is increasing with price. In this section, there are several substitutes whose entries sequentially kink the resource demand and increase its elasticity.

The backstop substitute retains the same role as in Section 2; for prices greater than its entry price $p > p_t^b$, it offers an unlimited substitution opportunity that will induce the resource monopoly to deter its production.

We further consider ordinary substitutes. Like the backstop, ordinary substitutes are assumed to be perfect ones and are produced competitively.[20] Yet they only offer relatively limited substitution possibilities because their production exhibits decreasing returns to scale. In fact, ordinary substitutes offer so low substitution possibilities, that the resource monopoly does not find optimal to deter them. In brief, we define them in the following way, that will be given more precise grounds shortly below.

Definition 1 (Ordinary substitute)
With an ordinary substitute, Assumption 1 remains satisfied at all dates $t \geq 0$.

As already argued in the Introduction, the supply of energy goods that are not deterred by current prices is subject to limitations that typically arise because of the scarcity of some factors.[21] Whether this scarcity is static (e.g. land, as in the case of biofuels, and wind and solar energies) or dynamic (e.g. finite exploitable reserves, as in the case of non-renewable fuels), higher instantaneous prices always warrant a higher instantaneous supply, yet at some greater marginal costs.[22] Thus for simplicity, we assume that the production of substitutes is static and the only good that we explicitly treat as non

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[20]Similarly one may consider substitutability to be partial because some ordinary substitutes to oil only replace the resource for some uses (Hoel, 1984); the case of various uses with use-specific imperfect substitutes will be discussed in Section 5.

[21]See especially Footnote 12.

[22]In the case of a non-renewable competitively-produced substitute, supply is still characterized by the equalization of price with marginal costs, once marginal costs are adjusted to comprise the opportunity cost of extraction. See Sweeney (1993, pp. 775-776) and Daubanes and Lasserre (2014) for the conventional interpretation of the instantaneous supply of a non-renewable resource.

renewable is the resource supplied by the monopoly.

We consider for brevity a single ordinary substitute; as shall be clear, the analysis immediately accommodates more than one such substitutes. The ordinary substitute is produced for all prices strictly greater than $p_t^o > 0$. We further assume

$$c_t < p_t^o < p_t^b, \text{ for all } t \geq 0, \tag{10}$$

so as to exclude the uninteresting case where the ordinary substitute is deterred at the same time as the backstop.[23] Thus the ordinary substitute may be produced along the resource exploitation phase. We now examine the three sections of the residual resource demand curve, as is represented in Figure 5.
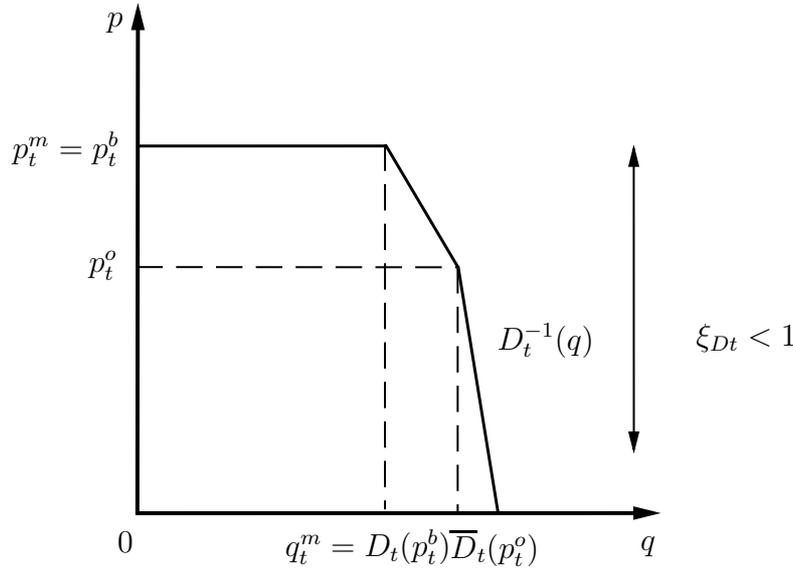
Figure 5: Residual demand and limit-pricing equilibrium with backstop and ordinary substitutes

ı) For all prices $p \leq p_t^o$, no substitute is competing with the resource at all. Hence, the residual demand that the monopoly is facing is the entire demand $D_t(p) = \overline{D}_t(p)$. Such range of prices is induced by sufficiently high monopoly extraction

$$q \geq \overline{D}_t(p_t^o) \tag{11}$$

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[23]In principle, there may be substitutes, backstop or ordinary, with entry prices exceeding the equilibrium limit price, that are not produced over the limit-pricing extraction phase.

over which

$$\pi_t(q) = \left(\overline{D}^{-1}(q) - c_t\right) q \text{ is decreasing} \tag{12}$$

by Assumption 1.

$\imath\imath$) For prices $p_t^o < p \leq p_t^b$, only the ordinary substitute is competing with the resource, as the resource price exceeds the entry price p_t^o, which is its marginal cost at the origin: $p_t^o \equiv C_t^{o\prime}(0) > 0$. Unlike the backstop, the ordinary substitute is unable to meet a large fraction of the resource demand without exhibiting substantial cost increase. Thus the marginal cost $C_t^{o\prime}(x)$ of producing a quantity x of ordinary substitute is differentiable, strictly rising and the ordinary-substitute supply function $S_t^o(p) \equiv C_t^{o\prime-1}(p)$ is continuous, with $S_t^o(p) > 0$ if and only if $p > p_t^o$.

Yet the price elasticity of the ordinary substitute's supply $\xi_{S^o t}(x) = C_t^{o\prime}(x) / \left(C_t^{o\prime\prime}(x)x\right)$ is low in the sense that the elasticity $\xi_{Dt}(q)$ of the residual demand $D_t(p) = \overline{D}_t(p) - S_t^o(p)$ satisfies the inequality

$$\xi_{Dt}(q) = \frac{e}{q}\xi_{\overline{D}t}(e) + \frac{x}{q}\xi_{S^o t}(x) < 1, \tag{13}$$

where $e = q + x$ is the total energy supply. This way, Assumption 1 is verified, as per Definition 1.

The range of prices $p_t^o < p \leq p_t^b$ over which only the ordinary substitute is produced is induced by the monopoly's intermediate supplies

$$D_t(p_t^b) \leq q < \overline{D}_t(p_t^o), \tag{14}$$

with

$$D_t(p_t^b) = \overline{D}(p_t^b) - S_t^o(p_t^b), \text{ assumed strictly positive.}^{24} \tag{15}$$

Over this range, it follows from (13) that

$$\pi_t(q) = \left(D_t^{-1}(q) - c_t\right) q \text{ is decreasing.} \tag{16}$$

$\imath\imath\imath$) For all prices $p > p_t^b$, the backstop has the capacity of meeting the entire demand while remaining more attractive than both the ordinary substitute and the resource.[25]

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[24]The assumption that $D_t(p_t^b) > 0$ despite the ordinary substitute is the counterpart of $\overline{D}_t(p_t^b) > 0$ in Section 2. This way, Assumption 1 is satisfied, which eliminates the uninteresting case where the backstop supply and the residual resource demand do not intersect at all.

[25]Instead of the backstop, limit pricing may seek to deter a substitute produced under decreasing – but

Since the backstop is supplied competitively, any monopoly's supply as low as

$$q < D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b) \tag{17}$$

induces the resource price $p = p_t^b$, under which

$$\pi_t(q) = \left(p_t^b - c_t \right) q, \text{ is increasing.} \tag{18}$$

To sum up, the instantaneous profit with an ordinary substitute writes

$$\pi_t(q) = \begin{cases} \left(p_t^b - c_t \right) q, \text{ increasing, for } q < D_t(p_t^b) \\ \left(D_t^{-1}(q) - c_t \right) q, \text{ decreasing, for } D_t(p_t^b) \leq q < \overline{D}_t(p_t^o) \\ \left(\overline{D}_t^{-1}(q) - c_t \right) q, \text{ decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases}, \tag{19}$$

and is thus maximized by the supply level

$$q_t^m = D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b). \tag{20}$$

Thus, once q_t^m of Section 2 is adjusted to become (20), the dynamic analysis of Section 2 applies as before under Assumption 2. The following proposition summarizes the properties of the limit-pricing equilibrium in the context of this section.

Proposition 4 (Limit-pricing equilibrium with an ordinary substitute)

In presence of an ordinary substitute,

1. *The monopoly supplies $q_t^m = D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b) > 0$ as per (20), and so induces the limit price $p_t^m = p_t^b$ that deters the backstop substitute's production, at all dates t of the extraction period $[0, T^m]$;*

2. *The limit-pricing equilibrium leads to the complete exhaustion of the resource at date T^m such that $\int_0^{T^m} D_t(p_t^b) \, dt = Q_0$;*

3. *All along the extraction period $[0, T^m]$, the ordinary substitute is produced in quantity $S_t^o(p_t^b) > 0$.*

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slowly – returns. Consider a substitute with a sufficiently high, although not infinite, supply elasticity; beyond its entry price, it may cause the residual demand to be sufficiently elastic for the monopoly's profit to be increasing. The analysis easily accommodates that case, for no additional interesting insight.

In the stationary model, the limit-pricing quantity $q^m = D(p^b) = \overline{D}(p^b) - S^o(p^b) > 0$ is constant, so that the exhaustion date is $T^m = Q_0 / \left(\overline{D}(p^b) - S^o(p^b) \right)$.

Absent taxation policies, the limit-pricing equilibrium at any date t of the exploitation phase is depicted in Figure 5. As far as taxation policies are concerned, the distinction between the deterred backstop and the on-use ordinary substitute, will turn out to be fundamental.

B.   Taxes on the Non-Renewable Resource

The same way as in Section 2, a unit consumer tax θ_t leaves unchanged the consumer price p_t^b at which the backstop substitute enters, and thus the limit extraction quantity $D_t(p_t^b)$, given by (20), that deters its entry. It also leaves the entry price p_t^o unchanged. Thus the tax only modifies the instantaneous revenue (19) to the extent that, for any extraction quantity q, the price accruing to the producer is the inverse demand $D_t^{-1}(q)$ reduced by the tax θ_t; as if the cost c_t was augmented by the levy θ_t.

When Assumption 2 is adjusted to the case of a resource tax, the instantaneous profit function becomes

$$\pi_t(q) = \begin{cases} \left(p_t^b - \theta_t - c_t \right) q, \text{ increasing, for } q < D_t(p_t^b) \\ \left(D_t^{-1}(q) - \theta_t - c_t \right) q, \text{ decreasing, for } D_t(p_t^b) \leq q < \overline{D}_t(p_t^o) \\ \left(\overline{D}_t^{-1}(q) - \theta_t - c_t \right) q, \text{ decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases} . \quad (21)$$

Thus to the extent that the tax does not violate Assumption 2 – it warrants no interruption of resource supply –, it will not affect the monopoly's limit-pricing path described in Proposition 4: the strong neutrality result of resource taxes (and subsidies) holds as per Proposition 2 in presence of an ordinary substitute.

C.   Subsidies to the Backstop Substitute

Subsidies to the backstop substitute also have the same effect as in Section 2, regardless of whether there is an ordinary substitute.

Consider a subsidy $\gamma_t^b \geq 0$ to the backstop substitute. Its price is reduced to $p_t^b - \gamma_t^b$, which is also the resource price whenever the backstop is profitable. The extraction quantity that deters the entry of the backstop substitute is thus increased to

$$D_t(p_t^b - \gamma_t^b) = \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b), \quad (22)$$

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instead of $D_t(p_t^b)$ as in (20).

As long as backstop subsidies leave a strictly positive limit-pricing revenue to the monopoly, as by Assumption 2, its revenue is only modified in this respect. It rewrites

$$\pi_t(q) = \begin{cases} \left(p_t^b - \gamma_t^b - c_t\right) q, \text{ increasing, for } q < D_t(p_t^b - \gamma_t^b) \\ \left(D_t^{-1}(q) - c_t\right) q, \text{ decreasing, for } D_t(p_t^b - \gamma_t^b) \leq q < \overline{D}_t(p_t^o) \\ \left(\overline{D}_t^{-1}(q) - c_t\right) q, \text{ decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases}, \quad (23)$$

with the exact same consequence as in Section 2 for the effect of γ_t^b: the equilibrium limit-pricing extraction q_t^m is increased as per (22).

Vice versa, if the deterred substitute was penalized by a tax, the monopoly would enjoy a higher limit price. For that, it would cut its extraction to $D_t(p_t^b - \gamma_t^b)$ as characterized by (22) with $\gamma_t^b \leq 0$.

D. Subsidies to (Taxes on) Ordinary Substitutes

In the limit-pricing equilibrium of Proposition 4, the production of the backstop substitute is deterred by the monopoly. Currently-used substitutes must all be ordinary substitutes that satisfy Definition 1. As this section shows, in a limit-pricing context, the effect of subsidies to currently-economic substitutes differs from the effect earlier identified of subsidies to the backstop.

With a subsidy $\gamma_t^o \geq 0$ to the consumption of the ordinary substitute, the resource price at which its production is profitable becomes $p_t^o - \gamma_t^o$. Thus the extraction level below which the substitute enters is reduced to $\overline{D}_t(p_t^o - \gamma_t^o)$ instead of $\overline{D}_t(p_t^o)$ in (11).

For all resource prices $p > p_t^o - \gamma_t^o$ – equivalently all extraction levels $q < \overline{D}_t(p_t^o - \gamma_t^o)$ – that warrant the production of the ordinary substitute, its supply expressed as a function of the resource price is augmented to $S_t^o(p + \gamma_t^o)$. Accordingly, the residual demand for the resource is reduced by the same amount: $D_t(p) = \overline{D}_t(p) - S_t^o(p + \gamma_t^o)$.

Hence at the entry price p_t^b of the backstop substitute, the subsidy γ_t^o increases the ordinary substitute's production to $S_t^o(p_t^b + \gamma_t^o)$ and reduces the residual demand faced by the monopoly by the same quantity; extraction to be supplied so as to deter the backstop's production is, instead of (20),

$$D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o), \quad (24)$$

25

lower than in absence of subsidy.

Definition 1 and Assumption 1 rule out the case where the ordinary substitute would satisfy the entire resource demand at some price below the backstop's price p_t^b. Thus by assumption, the residual resource demand at the limit price $D_t(p_t^b) = \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o)$ is strictly positive. This eliminates extreme subsidies γ_t^o that would make the ordinary substitute meet the entire energy demand, i.e. such that $S_t^o(p_t^b + \gamma_t^o) > \overline{D}_t(p_t^b)$, thus causing disruptions of resource supply.

Thus (19) rewrites

$$\pi_t(q) = \begin{cases} \left(p_t^b - c_t\right) q, \text{ increasing, for } q < \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o) \\ \left(D_t^{-1}(q) - c_t\right) q, \text{ decreasing, for } \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o) \le q < \overline{D}_t(p_t^o - \gamma_t^o) \\ \left(\overline{D}_t^{-1}(q) - c_t\right) q, \text{ decreasing, for } q \ge \overline{D}_t(p_t^o - \gamma_t^o) \end{cases} ,$$

$$(25)$$

where threshold quantities $\overline{D}_t(p_t^o - \gamma_t^o)$ and $\overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o)$ are reduced by the subsidy.

Thus the dynamic analysis of Section 2 follows through, and a limit-pricing equilibrium realizes, in which the monopoly supplies less, so as to induce the unchanged limit price p_t^b: $q_t^m = \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o)$, decreasing with γ_t^o. In Figure 6, the shift from the dark curve to the red curve depicts the reduction in the residual demand faced by the monopoly as a consequence of the subsidy to the ordinary substitute, and the resulting reduction in the limit-pricing resource quantity.

The message of the following proposition sharply contrasts with that of Proposition 3.

Proposition 5 (Effect of subsidies to the ordinary substitute)
Subsidies to an ordinary substitute,

   1. Increase the substitute current production;

   2. Reduce the resource current extraction by the same quantity.

Vice versa, taxes on an ordinary substitute reduce its current production and increase the resource demand by the same amount. Maximizing extraction profits requires serving the increased resource demand, as is illustrated by the shift from the red to the dark residual-resource-demand curve in Figure 6. Assume that in front of the reduction in the ordinary substitute's production, the monopoly was not adjusting its supply. Then,
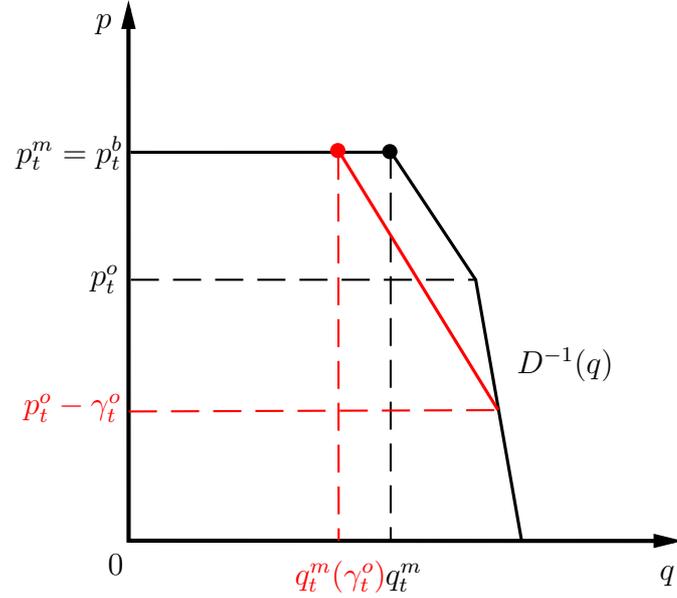
26

Figure 6: Limit-pricing equilibrium and the effect of a subsidy to the ordinary substitute

the backstop substitute would become profitable, and would meet the extra resource demand left by the ordinary substitute – the amount between the red and the dark kinks. Increasing its supply, so as to conquer the market share left by the ordinary substitute, at the backstop price p_t^b increases the monopoly's extraction profits until the backstop is completely excluded.

E. The Carbon Tax

The carbon tax is applied to the carbon content of energy goods. Thus the carbon tax is formally equivalent to several taxes, each applied to a carbon-energy good, to an extent that reflects its unit carbon content.

In particular, the carbon tax comprises a tax $\theta_t > 0$ on the oil resource of the monopoly, as earlier examined. The result of Proposition 2 is valid in the context of this section, which indicates that such tax has no direct effect on the monopoly's resource supply. The carbon tax has two potential indirect effects on resource extraction, of opposite directions.

The first effect may only manifest if the backstop substitute that is deterred by the monopoly contains carbon (e.g. unconventional oils, liquefied coal...). When the tax

penalizes the backstop (like a negative subsidy $\gamma_t^b \leq 0$), the result of Proposition 3 applies, that the monopoly is led to supply less resource so as to enjoy the augmented resource limit price at which the backstop enters. If the backstop does not contain carbon (e.g. fusion power), this effect vanishes as when $\gamma_t^b = 0$.

The second effect is of opposite direction. Some ordinary energy goods that compete with the oil resource at current prices do contain carbon (e.g. deepwater oil, some coal and gas). Thus the carbon tax acts as a tax on the ordinary substitute. Proposition 5 indicates that such tax (a negative subsidy $\gamma_t^o < 0$) leads the monopoly to increase its supply: the carbon tax penalizes ordinary carbon substitutes that abandon market shares; limit pricing leads the cartel to take over these market shares. In general, the latter mechanism limits the potential of the carbon tax to reduce the production of carbon goods.

These previously-established effects formally combine as follows. Merging expressions (22) and (24), the equilibrium resource extraction level with taxes on the backstop and ordinary substitutes appears to be, instead of $D_t(p_t^b)$ as per (20) in absence of policies,

$$q_t^m = \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o), \tag{26}$$

with $\gamma_t^b \leq 0$ and $\gamma_t^o < 0$. When the carbon tax penalizes the backstop ($\gamma_t^b < 0$), it increases the oil limit-price to $p_t^b - \gamma_t^b > p_t^b$ and thus reduces the total demand for oil and its ordinary substitute to $\overline{D}_t(p_t^b - \gamma_t^b)$. However, for any such limit price, it also reduces the supply of the ordinary substitute to $S_t^o(p_t^b - \gamma_t^b + \gamma_t^o)$, which tends to increase the residual demand for oil.

Therefore, when the backstop is relatively less affected by the carbon tax than the ordinary substitute is, the ordinary substitute production is reduced in equilibrium, and the carbon tax may increase oil extraction. This is the case depicted in Figure 7.
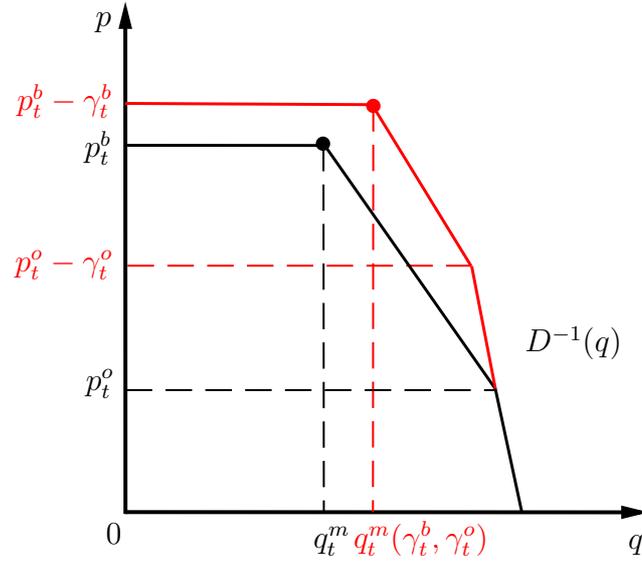
Figure 7: The effect of the carbon tax

Corollary 1 (Effect of the carbon tax)

The carbon tax on the carbon resource, on an ordinary carbon substitute, and on the backstop

1. *Always reduces the ordinary substitute current supply,*

2. *But only reduces the current demand for the resource and the ordinary substitute if the backstop is a carbon substitute.*

3. *When the carbon tax affects the backstop relatively less than the ordinary substitute, it increases the resource current extraction;*

4. *When the backstop is not a carbon substitute, the carbon tax*

   (a) *Reduces the ordinary substitute current production;*

   (b) *Increases the resource current extraction by the same amount.*

In the extreme case where the backstop is not at all penalized by the taxation of carbon (e.g. fusion power, future-generation biofuels...), the carbon tax does not affect

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the total consumption of oil and its ordinary substitute: it is then formally equivalent to a tax on the ordinary substitute as in the previous subsection, so that oil extraction exactly compensates the reduction in ordinary carbon substitute production.[26]

IV.   Resource Heterogeneity, Exploitation Duration and the Ultimately Extracted Quantity

In the simple limit-pricing model of Sections 2 and 3, resource reserves are completely depleted. In such context, our analysis showed that resource taxes like the carbon tax are very limited instruments to curb resource consumption and carbon emissions.

As a matter of fact, reserves of oil, of OPEC's oil in particular, are highly heterogenous (see for instance the discussion in Hamilton, 2009a, 225-226). One standard way, due to Hotelling (1931) and Gordon (1967), to take resource heterogeneity into account is to assume that marginal extraction costs increase as less reserves are to be extracted, as when the resource is Ricardian and its units are exploited in order of their respective costs. This approach has been recently used for instance by van der Ploeg and Withagen (2012, 2014) in works on the carbon tax.[27]

The "stock effects" just described notoriously introduce incentives to extract the resource less rapidly (Dasgupta and Heal, 1979). This section extends the limit-pricing model of Sections 2 and 3 to the Hotelling-Gordon cost representation. This extension makes the limit-pricing model comparable with the conventional non-renewable-resource monopoly model of Karp and Livernois (1992); it turns out that the limit-pricing outcome survives the introduction of stock effects.

Also with stock effects, the ultimately extracted quantity becomes endogenous because extraction can stop before the complete depletion of available reserves: too high extraction costs may not be met by the benefit derived from the last units to be extracted. Thus in principle, more reserves may become economic or uneconomic as a result of a policy.

---

[26]When units of the ordinary substitute contain more carbon than the resource (e.g. coal), the carbon tax still reduces carbon consumption in that case, despite the fact that the increase in resource supply compensates the decrease in the substitute's production. Vice versa, when the substitute is less carbon intensive than the resource (e.g. gas), the carbon tax increases carbon consumption.

[27]The view that exploited reserves contribute to increasing extraction costs has been initiated by Hotelling (1931, p. 152), consolidated by Gordon (1967), and perfected, among others, by Weitzman (1976) and Salant et al. (1983).

This possibility, assumed away by classical papers on the taxation of resource monopolies (Bergstrom et al., 1981; Karp and Livernois, 1992),[28] is often considered a fundamental aspect of climate policy.

A. The Model

Assume now that at each date $t \geq 0$ the marginal extraction cost c_t is given by the decreasing function

$$c_t \equiv C_t(Q_t) > 0 \tag{27}$$

of remaining reserves $Q_t \geq 0$; marginal cost increases as remaining reserves diminish. The function C_t is assumed differentiable everywhere.

To consider the possibility that extraction be incomplete, we do away in this section with Assumption 2 that the cost of extraction is always covered by extraction benefits. Absent any taxation policy, the discounted marginal limit-pricing revenue is $(p_t^b - c_t)e^{-rt}$ with $c_t = C_t(Q_t)$, which may now be negative.

In this context, at any date t when remaining reserves are Q_t, the instantaneous monopoly revenue (19) writes in a way similar to Section 3:

$$\pi_t(q, Q_t) = \begin{cases} \left(p_t^b - C_t(Q_t)\right) q, \text{ increasing or decreasing, for } q < D_t(p_t^b) \\ \left(D_t^{-1}(q) - C_t(Q_t)\right) q, \text{ decreasing, for } D_t(p_t^b) \leq q < \overline{D}_t(p_t^o) \\ \left(\overline{D}_t^{-1}(q) - C_t(Q_t)\right) q, \text{ decreasing, for } q \geq \overline{D}_t(p_t^o) \end{cases} . \tag{28}$$

For large supplies that deter the backstop, the monopoly revenue remains decreasing by Assumption 1 and Definition 1. However, $\pi_t(q, Q_t)$ may not always be increasing for low supplies $q < D_t(p_t^b)$ that warrant the backstop production. It retains the same form as before, and exhibits the same limit-pricing maximum $D_t(p_t^b) > 0$ only when the limit-pricing marginal revenue $p_t^b - C_t(Q_t)$ is positive. Otherwise, extraction is not economic for the monopoly; zero extraction is optimal.

As previously, the monopoly seeks to maximize its intertemporal stream (3) of discounted profits $\pi_t(q_t, Q_t)$ over the free extraction period $[0, T]$ under the exhaustibility constraint (4). At any date $t \geq 0$, its relevant instantaneous objective for the optimal

---

[28]Karp and Livernois (1992) specifically considered that reserves are fully exploited, despite stock effects and taxation.

choice of extraction q_t is given by the Hamiltonian

$$\mathcal{H}(q_t, Q_t, \lambda_t, t) \equiv \pi_t(q_t, Q_t) e^{-rt} - \lambda_t q_t, \tag{29}$$

where $\lambda_t \geq 0$ denotes the multiplier associated with (4).

As described in Section 2 (see Figure 3), the Hamiltonian admits the same maximum as the instantaneous revenue (28) whenever the discounted marginal profit $\left(p_t^b - C_t(Q_t)\right) e^{-rt}$ is greater than the extraction opportunity cost $\lambda_t \geq 0$. In that case, $\left(p_t^b - C_t(Q_t)\right) e^{-rt} > 0$, so that by (28) the optimal extraction is the limit-pricing supply $q_t^m = D_t(p_t^b)$.

In the spirit of Assumption 2, its following alternative assumes that limit-pricing marginal revenue decreases in present value; this is made for simplicity to eliminate supply disruptions along the exploitation period; phenomena of relatively minor economic interest. Unlike Assumption 2 however, the alternative Assumption 3 considers that extraction may become uneconomic.

Assumption 3 (Uninterrupted incomplete extraction)
The marginal limit-pricing revenue is strictly positive at date 0 for initial reserves $Q_0 > 0$; for all dates $t \geq 0$ and any given reserves $Q_0 \geq Q \geq 0$, it is continuously decreasing in present-value terms as long as it is positive.

Thus with no taxation policies, the marginal limit-pricing revenue $\left(p_t^b - C_t(Q_t)\right) e^{-rt}$ is positive at early dates, and decreases continuously with time for two reasons: for unchanged reserves by Assumption 3, and because diminishing reserves increase C_t by (27). Yet, unlike Sections 2 and 3, the value λ_t of the scarce resource underground is also decreasing in this context, to reflect that reserves exploited later are more costly: by the Maximum Principle, $\dot{\lambda}_t = C_t'(Q_t) q_t e^{-rt} < 0$ at each date t when an amount $q_t > 0$ is extracted. Appendix B shows that the marginal limit-pricing revenue $\left(p_t^b - C_t(Q_t)\right) e^{-rt}$ always decreases more rapidly than the opportunity cost λ_t.

Appendix B further shows that the marginal revenue $p_0^b - C_0(Q_0)$ initially exceeds λ_0. Thus the discounted marginal extraction revenue covers the scarcity value initially and until extraction stops at date T:

$$\left(p_t^b - C_t(Q_t)\right) e^{-rt} \geq \lambda_t, \ \forall t \in [0, T]. \tag{30}$$

All along the exploitation phase $[0, T]$, the limit-pricing extraction $q_t^m = D_t(p_t^b)$ is thus optimum, which induces the limit price $p_t^m = p_t^b$.

As far as the optimal terminal date T^m and abandoned reserves Q_{T^m} at that date are concerned, there are two possibilities, as detailed in Appendix B. Consider first that no reserves are abandoned, i.e. $Q_{T^m} = 0$. In that case, the limit-pricing extraction lasts until reserves are exhausted: T^m is such that

$$\int_0^{T^m} D_t(p_t^b) \, dt = Q_0, \tag{31}$$

as in Section 3. Full exhaustion may only be optimal if the marginal limit-pricing revenue is not becoming negative before the exhaustion date T^m given by (31).

Otherwise, the terminal date is such that marginal revenue becomes nil:

$$\left(p_{T^m}^b - C_{T^m}(Q_{T^m})\right) e^{-rT^m} = 0, \tag{32}$$

with

$$Q_{T^m} = Q_0 - \int_0^{T^m} D_t(p_t^b) \, dt. \tag{33}$$

Appendix B shows that the system jointly determines the date $T^m \geq 0$ when extraction stops, and abandoned reserves $Q_{T^m} \geq 0$ at that date – equivalently the ultimately extracted quantity $Q_0 - Q_{T^m} \leq Q_0$.

We have the following proposition that summarizes the properties of the limit-pricing equilibrium in the context of this section.

Proposition 6 (Limit-pricing equilibrium with incomplete extraction)

Under the assumptions of this section,

1. *The monopoly supplies $q_t^m = D_t(p_t^b) > 0$, and so induces the limit price p_t^b that deters the backstop substitute's production, at all dates of the exploitation period $[0, T^m]$;*

2. *Extraction is complete if there exists no date $T > 0$ such that the marginal revenue $p_T^b - C_T(Q_T)$ is nil with $Q_T = Q_0 - \int_0^T D_t(p_t^b) \, dt > 0$: in that case $Q_{T^m} = 0$ and T^m is given by (31);*

3. *Otherwise, extraction is incomplete: the terminal date T^m and abandoned reserves $Q_{T^m} > 0$ are determined by (32) and (33).*

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In the sequel, we examine how taxation policies affect this equilibrium.

B.   Taxation Policies

Assume, at each date $t \geq 0$, a tax $\theta_t \geq 0$ on the resource, a subsidy (tax) $\gamma_t^b \geq 0 (\leq 0)$ to the backstop and a subsidy (tax) to the ordinary substitute $\gamma_t^o \geq 0 (\leq 0)$. In light of the analysis of Section 3, the monopoly's profit at date $t \geq 0$, with reserves $Q_t \geq 0$, writes in that context

$$
\pi_t(q, Q_t) = \begin{cases} \left(p_t^b - \gamma_t^b - \theta_t - C_t(Q_t)\right) q, & \text{for } q < \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o) \\ \left(D_t^{-1}(q) - \theta_t - C_t(Q_t)\right) q, & \text{for } \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o) \leq q < \overline{D}_t(p_t^o - \gamma_t^o) \\ \left(\overline{D}_t^{-1}(q) - \theta_t - C_t(Q_t)\right) q, & \text{for } q \geq \overline{D}_t(p_t^o - \gamma_t^o) \end{cases},
$$
(34)

which has the same pattern as in (28). By Assumption 1 and Definition 1, extraction revenue (34) is decreasing for all quantities $q > \overline{D}_t(p_t^b) - S_t^o(p_t^b + \gamma_t^o)$ that deter the backstop substitute. By Assumption 3, extraction revenue is increasing for all $q < \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o)$, as long as the marginal revenue $p_t^b - \gamma_t^b - \theta_t - C_t(Q_t)$ is strictly positive.[29]

Thus for policies that satisfy Assumption 3 and Definition 1, the same analysis as in absence of policies applies, so that the limit-pricing equilibrium realizes as follows. At each date t of the exploitation period $[0, T^m]$, resource extraction becomes

$$
q_t^m = \overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o),
$$
(35)

that induces the limit-price $p_t^m = p_t^b - \gamma_t^b$. All along this period, it can easily be verified that the effects of θ_t, γ_t^b and γ_t^o on current extraction q_t^m remain those identified earlier in Propositions 2, 3 and 5.

When the resource is fully exhausted, the date at which exploitation ends is such that

$$
\int_0^{T^m} \left(\overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o)\right) dt = Q_0.
$$
(36)

In that case, backstop subsidies anticipate the terminal date because they increase current extraction during the exploitation period. In contrast, subsidies to ordinary substitutes reduce current extraction, and so induce a longer depletion.

---

[29] As for previous sections, Assumption 3 and Definition 1 amount to the following restrictions on the tax instruments under study. The resource tax and the backstop subsidy are not sufficiently high to make extraction uneconomic at early dates, and are not decreasing rapidly enough to make discounted marginal revenue increase. The subsidy to the ordinary substitute is not sufficiently high to destroy the (residual) resource demand.

When the marginal revenue $\left(p_T^b - \gamma_T^b - \theta_T - C_T(Q_T)\right) e^{-rT}$ becomes negative for positive remaining reserves $Q_T = Q_0 - \int_0^T q_t^m \, dt > 0$, extraction stops at the terminal date T^m characterized as follows:

$$\left(p_{T^m}^b - \gamma_{T^m}^b - \theta_{T^m} - C_{T^m}(Q_{T^m})\right) e^{-rT^m} = 0 \tag{37}$$

with

$$Q_{T^m} = Q_0 - \int_0^{T^m} \left(\overline{D}_t(p_t^b - \gamma_t^b) - S_t^o(p_t^b - \gamma_t^b + \gamma_t^o)\right) dt. \tag{38}$$

Marginal revenue in (37) is decreasing in the terminal date T^m and increasing in remaining reserves Q_{T^m} at that date. The remaining reserves in (38) are diminishing with the length of extraction T^m. Other things given, Appendix B shows that the two formulas systematically characterize the terminal date T^m and abandoned reserves Q_{T^m}, and can be used to examine the effects of any particular trajectory of tax instruments. It brings up the following general insights about the qualitative effects of policies with limit pricing.

There are two basic ways by which taxation policies may affect the marginal extraction revenue, thus the terminal date and abandoned reserves at that date. On the one hand, for unchanged remaining reserves Q_{T^m}, policies may deteriorate the marginal extraction revenue in (37) directly. On the other hand, policies that reduce (increase) current extraction q_t^m via (35), leave more (less) future reserves Q_{T^m} to be extracted as per (38), and so improve (deteriorate) the marginal revenue in (37) indirectly, because less reserves mean higher extraction costs.

For instance, since resource taxes do not affect current extraction (35) throughout the exploitation phase, they do not affect the reserves Q_t available for extraction at any date t. Thus they only anticipate the terminal date because they make extraction less profitable as per (37). It follows that resource taxes unambiguously reduce ultimately extracted reserves.

In contrast, for unchanged reserves, subsidies to ordinary substitutes do not affect directly the profitability of extraction in (37). Yet they reduce current extraction (35) all along the exploitation phase, so that, by (38), it takes longer to reach the cut-off level of remaining reserves that satisfy (37). Since extraction is less profitable over time, a later terminal date in (37) implies larger abandoned reserves.

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Backstop subsidies induce extraction (35) to increase along the exploitation phase, and thus contribute to greater extraction costs in (37). Simultaneously in (37), but for unchanged reserves, they directly deteriorate extraction profitability. For these two reasons, backstop subsidies anticipate the terminal date. Yet they imply a lower extraction over a shorter period and thus have an ambiguous effect on ultimately abandoned reserves.

Hence the following results.

Proposition 7 (Effect of policies with incomplete extraction)
When extraction is incomplete,

1. *Resource taxes shorten the extraction period and reduce the ultimately extracted quantity;*

2. *Subsidies to the backstop substitute shorten the extraction period but have an ambiguous effect on the ultimately extracted quantity;*

3. *Subsidies to the ordinary substitute extend the extraction period, but reduce the ultimately extracted quantity;*

Accordingly, the carbon tax ambiguously affects the duration of the resource exploitation and the ultimately abandoned resource reserves. For a brief illustration, the following corollary focuses on the most interesting case where the carbon tax increases current resource extraction over the exploitation period. More results can easily be derived using the graphical methodology described in Appendix B.

Corollary 2 (Effect of the carbon tax with incomplete extraction)

Consider that the carbon tax on the carbon resource, the ordinary and backstop carbon substitutes increases current resource extraction as per Corollary 1.

1. *If it penalizes more the resource than the backstop, the carbon tax*

    (a) *Shortens the extraction period;*

    (b) *Affects the ultimately extracted quantity ambiguously.*

2. *If it penalizes less the resource than the backstop, the carbon tax*

    (a) *Affects the extraction duration ambiguously;*

    (b) *Increases the ultimately extracted quantity.*

V. Discussion: Industrial Structure, Reserves' Production, Demand Segments...

This paper points at the empirical relevance of limit-pricing equilibria for the oil and energy market and shows that the effects of environmental taxation instruments in such context differ from conventional studies. In particular, taxes applied to flows of resources, when they warrant no supply disruption, are ineffective regardless of their time dynamics. As far as subsidies to oil substitutes are concerned, it is fundamental to make a distinction between two sorts of substitutes. On the one hand, limit pricing deters the entry of drastic substitution possibilities. Subsidies to a backstop substitute induce equilibrium extraction quantities to increase. On the other hand, substitutes to oil that are economic at current prices – we called them ordinary – offer less drastic substitution possibilities that are compatible with limit pricing. Unlike the backstop, subsidies to any currently in-use substitutes do reduce current extraction quantities by an amount that depends on their respective elasticity of supply.

While we have restricted attention to a single ordinary substitute for simplicity, extension to several such substitutes is immediate. Since the effect of subsidies depends on the supply elasticity of the substitute, the objective of reducing carbon-resource extraction

quantities in a cost-efficient manner may imply selecting non-carbon substitutes on the ground of their supply elasticity; an issue that is beyond the scope of the present work.

The simple model of Section 2 has focused on backstop substitution possibilities, and has assumed that the resource is entirely exhausted. Section 3 has completed the description of substitution possibilities, while Section 4 has considered incomplete resource exhaustion. Those extensions proved to neatly refine our results on the incidence of taxation policies. Yet, our results have been obtained in a relatively parsimonious model; one may question whether limit-pricing equilibria survive more complex setups. In the sequel, we discuss further aspects of the oil market.

A.   Competitive Fringe

The industrial structure of the oil market differs from the frequently-used monopoly model. The OPEC cartel controls the majority of exploitable oil reserves; yet non-OPEC reserves yield a substantial fraction of current oil production.[30] A more adequate representation of the monopoly power exerted in the oil-production sector must take into account that a competitive fringe limits the power of the dominant cartelized extractor as in the model initiated by Salant (1976).[31]

The analysis of Sections 3 and 4 easily extends to that case. Indeed, although the fringe's oil production is identical to the cartel's production, it is analogous to the ordinary substitute introduced in Section 3 (competitively-supplied, prefect substitute to the cartel's resource). Thus it can be represented in the same manner. The residual demand that the monopoly is facing is that fraction of the total oil demand that exceeds the fringe's production. Because of reserve limitations, the elasticity of non-OPEC oil supply is notoriously very limited, and is even more so as non-OPEC producers have virtually no spare production capacities.[32]

As will be argued shortly, it is sensible to consider that non-OPEC oil supply satisfies

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[30]See Footnote 1 for more details.

[31]Issues about coordination within the OPEC cartel are out of the scope of this discussion for simplicity. See for instance Griffin (1985). In the most extreme conceivable case, the cartel would be completely ineffective. Saudi Arabia would make the price alone, thanks to very large spare capacities; the fringe would consist of all other producers, OPEC members or not.

[32]According to Hamilton, "In the absence of significant excess production capacity, the short-run price elasticity of oil supply is very low." (Hamilton, 2009b).

Definition 1 of ordinary substitutes – equivalently, Assumption 1 holds in spite of the fringe –, by which OPEC's residual (net-of-fringe-supply) demand exhibits a lower-than-one price elasticity. Thus the limit-pricing analysis of Sections 3 and 4 carries over unchanged with the fringe and our results are relevant to the actual structure of the oil market.

Treating the fringe's oil supply as the ordinary substitute to the cartel's resource, x denotes the fringe's production and $e = q + x$ the total oil supplied. The question is to know whether the inequality in formula (13) is verified, by which the cartel's residual-demand elasticity ξ_{Dt} is lower than one:

$$\xi_{Dt}(q) = \frac{\xi_{\overline{D}t}(e)}{q/e} + \frac{x/e}{q/e}\xi_{S^\circ t}(x) < 1. \tag{13}$$

The formula gives this elasticity as a weighted sum of the elasticities of the total oil demand $\xi_{\overline{D}t}$ and of the fringe's supply $\xi_{S^\circ t}$; q/e and x/e are respectively the market shares of the cartel and the fringe.

It is possible to verify that recent (long-run) elasticity estimates satisfy relation (13). Market shares are currently about $q/e = 0.4$ and $x/e = 0.6$. For the price elasticity of the total oil demand, the value used in Hamilton (2009b) is 0.25, in line with Krichene's (2005) long-run estimate for the period 1974-2004. Hamilton (2009b, p. 192) argues that this elasticity should be expected to be even smaller. Taking this conservative value, basic algebra shows that (13) holds for any elasticity $\xi_{S^\circ t}$ of the fringe's supply such that

$$\xi_{S^\circ t} \leq 0.25. \tag{39}$$

For instance, Golombek et al.'s (2013) estimates of the (long-run) non-OPEC oil supply elasticity are between 0.11 and 0.25, depending on their model's specification, which is compatible with (39).

As the above numbers indicate, it is sensible to consider that Assumption 1 holds, by which the residual demand that OPEC is facing exhibits a less-than-one elasticity. Yet testing Assumption 1 requires further empirical research. For instance, the specification of existing empirical models assumes away the possibility that OPEC is limit pricing (e.g. Golombek et al., 2013, p. 8).

B. Reserves' Production

Section 4 assumes heterogenous reserves whose extraction cost rises as extraction goes. In that context, extraction may become uneconomic before reserves are completely depleted, so that in general taxation policies affect the exploitation duration, and the ultimately extracted quantity.

Another reason why policies may affect the ultimately exploited resource is that they discourage exploration and development efforts by which reserves become exploitable. In Appendix C, we borrow the approach of Gaudet and Lasserre (1988), also used for instance in Fischer and Laxminarayan (2005) or Daubanes and Lasserre (2014). In these models, the marginal cost of developing an amount of exploitable reserves is rising, as when resource units are developed in order of their respective development costs; reserves are established so as to equate the marginal development cost with the implicit value of marginal reserves.

This extension does not modify qualitatively the limit-pricing outcome, nor the earlier-identified effects of policies on ultimately developed and exploited quantities.

C. Multiple Demand Segments with Various Degrees of Substitutability

It is standard to rely on a unique decreasing function to describe the heterogeneity of the aggregate demand. Yet in reality, the oil demand is segmented. Segments mainly correspond to different uses of the resource (e.g. Hoel, 1984), and to different regions.

One particular resource use in one particular region can be represented by a particular demand function of a form similar to the demand of Section 3. Resource uses and regions may differ by their accessible possibilities of substitution, as well as by their regulation.

One can also consider substitutes to vary by their degree of substitutability with the resource. On the one hand, as imperfect substitutes only become profitable beyond a certain resource price, they introduce kinks to the oil demand as in Sections 3 and 4. On the other hand, imperfect substitutability amounts to a broader interpretation of the demand elasticity. On each segment, the sensitiveness of the resource demand at some resource price jointly reflects the elasticity of supply and the degree of substitutability of resource substitutes that are profitable at that price.

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Limit pricing in that context intuitively arises from the entry threat of sufficiently substitutable alternative sources, on large enough demand segments. For instance, in the interview mentioned in Footnote 7, the energy industry analyst Stephen Schork later clarified OPEC's "main concern" (CNBC on August 16, 2010): the "shift of the sentiment in the US *especially* towards alternative fuels." [our italics].

APPENDIX

A Appendix to Sections 2 and 3: The Simple Stationary Case

This appendix reproduces the analysis of Section 2 under stationary conditions. In the spirit of Section 3, it also introduces an ordinary substitute under the simplifying assumption that its supply is perfectly inelastic.

Assume that the resource marginal extraction cost c and the backstop marginal production cost p^b are constant with $p^b > c$. The total energy demand $\overline{D}(p)$ is stationary, and satisfies $\xi_{\overline{D}}(p) < 1$, for all $p > 0$ as per Assumption 1.

At each date t when there is some resource left to be exploited, the monopoly's instantaneous profit writes

$$
\pi(q) = \begin{cases} (p^b - c)q, \text{ increasing, for } q \leq \overline{D}(p^b) \\ \left(\overline{D}^{-1}(q) - c\right)q, \text{ decreasing, for } q > \overline{D}(p^b) \end{cases}, \tag{40}
$$

and is maximized by the supply $\overline{D}(p^b)$ which induces the limit price p^b that deters the backstop.

The intertemporal problem of maximizing the discounted stream of profits (3) under the exhaustibility constraint (4) implies the Hamiltonian function (5), where the scarcity value λ is constant. All along the extraction period $[0, T]$, the Hamiltonian is maximized by the same supply level

$$
q^m = \overline{D}(p^b)
$$

that maximizes the instantaneous profit.

Thus the maximized Hamiltonian

$$
\mathcal{H}(q^m, Q_t, \lambda, t) \equiv (p^b - c)q^m e^{-rt} - \lambda q^m \tag{41}
$$

is decreasing over time because profits are discounted at rate $r > 0$; in the stationary case, Assumption 2 is superfluous. It can easily be verified that the maximized Hamiltonian is initially positive because $p^b > c$ so that extraction is warranted. Also, one can verify that λ is strictly positive so that the exhaustibility constraint is not violated. Thus the resource is completely exhausted. At each date of the extraction period $[0, T^m]$, extraction is q^m, so that exhaustion occurs at the terminal date $T^m = Q_0/q^m$.

Since the duration of the exploitation period is free, the Hamiltonian must become nil at date T^m. This characterizes the scarcity value λ under limit pricing: $\lambda = (p^b - c)e^{-r(Q_0/q^m)}$, with $q^m = \overline{D}(p^b)$.

Effect of a Constant Resource Tax

Assume a constant tax on the resource $\theta > 0$ that leaves positive extraction profits: $\theta < p^b - c$. The producer price of the resource is reduced by θ, regardless of whether consumers are ready to pay $\overline{D}^{-1}(q)$ or p^b, as when the backstop is profitable.

Thus the instantaneous monopoly's profit becomes

$$\pi(q) = \begin{cases} (p^b - \theta - c)q, \text{ increasing, for } q \leq \overline{D}(p^b) \\ \left(\overline{D}^{-1}(q) - \theta - c\right)q, \text{ decreasing, for } q > \overline{D}(p^b) \end{cases}. \tag{42}$$

The same analysis as in absence of tax follows through, with $c + \theta$ instead of c. The limit-pricing equilibrium is not modified: it implies an unchanged extraction level $q^m = \overline{D}(p^b)$ at each date preceding $T^m = Q_0/q^m$.

Effect of a Constant Backstop Subsidy

Assume a constant subsidy to the backstop $\gamma^b > 0$. The price at which the backstop is profitable becomes $p^b - \gamma^b$ instead of p^b. Further assume that the backstop subsidy leaves positive extraction profits: $p^b - \gamma^b > c$. Then, the instantaneous profit of the monopoly writes

$$\pi(q) = \begin{cases} (p^b - \gamma^b - c)q, \text{ increasing, for } q \leq \overline{D}(p^b - \gamma^b) \\ \left(\overline{D}^{-1}(q) - c\right)q, \text{ decreasing, for } q > \overline{D}(p^b - \gamma^b) \end{cases}, \tag{43}$$

and the same analysis as in absence of policies applies with $p^b - \gamma^b$ instead of p^b. The limit-pricing equilibrium is thus modified. All along the extraction period, the monopoly's extraction is $q^m = \overline{D}(p^b - \gamma^b)$, which is greater than $\overline{D}(p^b)$ in absence of subsidies. The resource is exhausted earlier, at the terminal date $T^m = Q_0/\overline{D}(p^b - \gamma^b)$.

Inelastically-Supplied Ordinary Substitute

Assume that the demand the monopoly is facing is reduced by a constant amount S^o, exogenous, of a perfect substitute to the resource. Unlike the backstop, assume that this amount is limited so that it falls short of the monopoly's total demand: $S^o < \overline{D}(p^b)$. In that case, the limit-pricing extraction is modified as follows.

For any monopoly's supply q that deters the backstop, the resource price p is established in such a way that the market equilibrium $q = \overline{D}(p) - S^o$ realizes. Therefore, the supply that induces the limit price p^b is reduced to $\overline{D}(p^b) - S^o$ instead of $\overline{D}(p^b)$. Also, the inverse demand for the resource is reduced to $\overline{D}^{-1}(q - S^o)$.

Thus the monopoly's instantaneous profit becomes:

$$\pi(q) = \begin{cases} (p^b - c)q, \text{ increasing, for } q \leq \overline{D}(p^b) - S^o \\ \left(\overline{D}^{-1}(q - S^o) - c\right)q, \text{ decreasing, for } q > \overline{D}(p^b) - S^o \end{cases}, \tag{44}$$

which leads to the same dynamic analysis as before. The limit-pricing equilibrium realizes, with constant extraction $q^m = \overline{D}(p^b) - S^o$ until the exhaustion date $T^m = Q_0/\left(\overline{D}(p^b) - S^o\right)$.

B Appendix to Section 4: Elements of Proofs

The results of Section 4 are mostly shown in the main text. The main text also refers to the following elements.

Limit-Pricing Marginal Revenue and Scarcity Value

The limit-pricing marginal revenue, in present value terms, decreases more rapidly

than the multiplier λ_t; this can be shown as follows.

At any date t, when remaining reserves are Q_t and extraction is $q_t \geq 0$, the derivative of the discounted marginal revenue $\left(p_t^b - C_t(Q_t)\right) e^{-rt}$ with respect to time is

$$\frac{d\left(\left(p_t^b - C_t(Q_t)\right) e^{-rt}\right)}{dt} = \left[\frac{d\left(p_t^b - C_t(Q_t)\right)}{dt} - r\left(p_t^b - C_t(Q_t)\right)\right] e^{-rt} + C_t'(Q_t)q_t e^{-rt} \leq 0,$$

where the term between brackets is the increase in the discounted marginal revenue for given reserves. By Assumption 3, it is negative or zero. The second term $C_t'(Q_t)q_t e^{-rt}$ corresponds to the decrease in the marginal revenue that arises because reserves diminish. It is strictly negative when extraction is non zero, and zero otherwise.

By the Maximum Principle, the latter term is also the time derivative of λ_t:

$$\dot{\lambda}_t = -\frac{\partial \mathcal{H}(q_t, Q_t, \lambda_t, t)}{\partial Q_t} = C_t'(Q_t)q_t e^{-rt} \leq 0.$$

It follows that

$$\frac{d\left(\left(p_t^b - C_t(Q_t)\right) e^{-rt}\right)}{dt} \leq \dot{\lambda}_t \leq 0.$$

Extraction at Date 0

Consider, as a statement to be contradicted, that $p_0^b - C_0(Q_0) \leq \lambda_0$. Since the marginal revenue is decreasing more rapidly than $\lambda_t \geq 0$, then $\left(p_t^b - C_t(Q_t)\right) e^{-rt} \leq \lambda_t$, for all $t \geq 0$, where the equality may only hold as $\left(p_t^b - C_t(Q_t)\right) e^{-rt} = \lambda_t = 0$; some extraction may be optimal in that case, but for no profit at all. Clearly, this is dominated by some extraction at initial dates since by Assumption 3, $p_0^b - C_0(Q_0) > 0$. Thus we must conclude that $p_0^b - C_0(Q_0) > \lambda_0$.

Terminal Date and Ultimately Abandoned Reserves

Since the terminal date T when extraction stops is free, the Hamiltonian (29) – the relevant flow of extraction benefits – must be zero at that date. The standard transversality condition

$$\left(p_T^b - C_T(Q_T)\right) e^{-rT} = \lambda_T \tag{45}$$

must hold.

Also at the terminal date T, reserves left unexploited must be non negative by constraint (4):

$$Q_T \geq 0. \tag{46}$$

Therefore, another standard transversality condition must be satisfied, by which

$$\lambda_T Q_T = 0. \tag{47}$$

Hence two possibilities. Consider first that $Q_{T^m} = 0$. In that case, the limit-pricing extraction lasts until reserves are exhausted, so that T^m is characterized by (31).

Second, consider that $Q_{T^m} > 0$ because the extraction of the last units is uneconomic. By (47), this can only be compatible with reserves having no more value at the terminal

date T^m: $\lambda_{T^m} = 0$. In this case, the terminal date T^m must satisfy

$$\left(p_T^b - C_T(Q_T)\right) e^{-rT} = 0, \tag{48}$$

with

$$Q_T = Q_0 - \int_0^T D_t(p_t^b)\, dt; \tag{49}$$

a system that will turn out to uniquely characterize the terminal date T^m and abandoned reserves Q_{T^m}: hence (32) and (33).

We analyze this system now. By Assumption 3, the marginal revenue in (48) is initially positive for low T when Q_T in (49) is close to Q_0. If T does not exist such that, together with Q_T in (49), it implies the marginal revenue in (48) to take a zero value, then extraction continues until $Q_T = 0$. In that case, $Q_{T^m} = 0$ is solution as in the first possibility; T^m is given by (31), and the analysis is similar to that of Section 3 with complete exhaustion.

Thus the analysis of Section 4 is most interesting in the second possibility, when T exists such that $Q_T > 0$ in (49) and T jointly satisfy (48). In this case, the solution is obviously unique since the marginal revenue on the left-hand side of (48) strictly decreases as T increases and reserves Q_T diminish. Precisely, it is decreasing in T for a given Q_T, and strictly decreasing when it is taken into account that an increase in T goes hand in hand with a decrease in Q_T as per (49).

Focus now on that unique interior solution when it exists. For that, it will be useful to consider T and Q_T as two variables that separately affect (48); the effect of T on Q_T being encompassed in (49). In (48), the discounted marginal revenue on the left-hand side is decreasing in T and increasing with Q_T. Thus the equation defines a positive relationship between T and Q_T, that we denote with the following function:

$$T = T_1(Q_T), \text{ increasing.} \tag{50}$$

According to (49), a greater Q_T is associated with a shorter extraction period that lasts until a lower T. This negative relationship is represented by the function

$$T = T_2(Q_T), \text{ decreasing.} \tag{51}$$

The intersection of the T_1 and T_2 relations defines either the unique interior solution (Q_{T^m}, T^m) given by (32) and (33) when they cross at the right of the $Q_T = 0$ vertical axis $(Q_{T^m} > 0)$, or the complete-exhaustion solution $Q_{T^m} = 0$ earlier mentioned otherwise. The graphical representation of Figure 8 will be useful shortly to identify how this solution modifies with parametric policy changes.

Effects of Policies with Incomplete Extraction

The taxation policies under study in Section 4 are considered to satisfy Assumptions 1 and 3. In that context, the terminal date T^m and the ultimately abandoned reserves Q_{T^m} are characterized by (37) and (38), instead of (32) and (33). Under the same assumptions, the same analysis applies as in absence of policies: (37) and (38) can be represented with
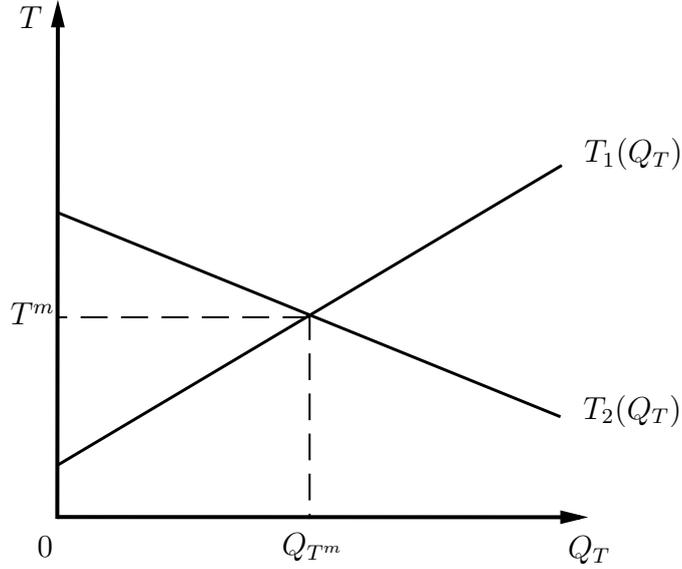
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Figure 8: Graphical characterization of T^m and Q_{T^m}

the T_1 and T_2 functions of (50) and (51), except that these functions now depend on policy parameters that enter (37) and (38).

We focus on the effects of taxation policies on the interior solution depicted in Figure 8. When the solution implies complete exhaustion, the analysis is the same as in Section 3 and is only concerned with the effects on current extraction levels over the exploitation period; effects on the length of this period are obvious. In what follows we derive the results presented in Section 4 by shifting the T_1 and T_2 curves of Figure 8 whose intersection characterizes Q_{T^m} and T^m in the limit-pricing equilibrium.

A tax $\theta_t > 0$, $\forall t \geq 0$, only affects (37). For given reserves Q_T, it brings backward the date T when the (tax-inclusive) marginal revenue becomes zero. Thus a rise in the tax amounts to shifting down the T_1 curve: it implies extraction until a lower T^m, and greater abandoned reserves Q_{T^m}.

A subsidy to the ordinary substitute $\gamma_t^o > 0$, $\forall t \geq 0$, only affects (38). For given terminal reserves Q_T, it brings forward the terminal date T at which those reserves will be reached. Thus a rise in the subsidy amounts to shifting up the T_2 curve: it implies extraction until a later T^m, and greater abandoned reserves Q_{T^m}. The opposite result is obviously obtained for a tax $\gamma_t^o < 0$, $\forall t \geq 0$.

A subsidy to the backstop substitute $\gamma_t^b > 0$, $\forall t \geq 0$, enters both (37) and (38). On the one hand, for given reserves Q_T, the subsidy brings backward the date T when the marginal revenue in (37) becomes zero; a subsidy rise amounts to shifting down the T_1 curve. On the other hand, for given abandoned reserves Q_T, the subsidy reduces the date T when those reserves will be reached in (38); a subsidy rise amounts to shifting down the T_2 curve. Those two changes to Figure 8 imply that subsidies to the backstop substitute imply a shorter extraction period, i.e. until a lower T^m. Yet they have an ambiguous effect

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on abandoned reserves Q_{T^m} and thus on the ultimately extracted quantity $Q_0 - Q_{T^m}$.

These results are summarized in Proposition 7. The results of Corollary 2 can easily be obtained in a similar way.

C   Appendix to Section 5: Costly Exploration and Development Efforts

In the context of Section 4, consider that reserves $Q_0 - Q_{T^m}$, before being exploited, need to be produced by costly exploration and development efforts. Following Gaudet and Lasserre (1988), assume that the production of those reserves takes place at date 0 and is subject to decreasing returns to scale because, as exploration prospects are finite, it must be more and more difficult to produce new reserves. When reserves' production is costly, it cannot be optimum to produce more than what is to be exploited. Formally, the cost of producing $Q_0 - Q_{T^m}$ is given by the increasing and strictly convex function $E(Q_0 - Q_{T^m})$. Let us further assume that $E'(0) = 0$ so as to avoid the uninteresting situation where the development cost induces the monopoly to produce no reserves at all.

The objective (3) of the monopoly now incorporates the reserve-development cost function E. Thus the monopoly's problem is

$$\max_{(Q_0 - Q_T),(q_t)_{t \geq 0}} \int_0^T \pi_t(q_t, Q_t)e^{-rt}\, dt - E(Q_0 - Q_T), \tag{52}$$

subject to (4), where T is a free variable.

Despite this modification of the objective, the Hamiltonian associated with the above problem is the same as in Section 4, given by (29). The integration of reserves' production into the monopoly's problem affects neither the analysis of the limit-pricing exploitation phase, nor the transversality condition (45), but the transversality condition associated with the non-negativity constraint (46).

Specifically, condition (47) is modified as follows. Q_0 may be entirely developed and completely exhausted as before and $Q_T = 0$ if development and extraction cost conditions make it profitable. Such is compatible with the marginal reserve-production cost being lower than the implicit value of marginal reserves: $E'(Q_0) \leq \lambda_T$. Yet when reserves are not completely developed and extracted, Q_T is strictly positive, and the implicit value of marginal reserves λ_T, instead of being equalized to zero as in absence of reserve production cost, is equalized to the marginal cost $E'(Q_0 - Q_T)$. The transversality condition associated with the non-negativity constraint (46) becomes

$$Q_T \left(\lambda_T - E'(Q_0 - Q_T) \right) = 0. \tag{53}$$

When $Q_T = 0$, things go as in absence of reserve-development efforts; no adjustment to Section 4 is needed. When $Q_T > 0$, the condition tells that instead of a zero value as in Section 4, λ_T equals the positive marginal cost of reserve production:

$$\lambda_T = E'(Q_0 - Q_T).$$

Thus condition (45) yields, instead of (32),

$$\left(p_{T^m}^b - C_{T^m}(Q_{T^m})\right) e^{-rT^m} - E'(Q_0 - Q_{T^m}) = 0, \tag{54}$$

where Q_{T^m} is still given by (33).

In that case, (54) and (33) form the system that uniquely characterizes the terminal date T^m and abandoned reserves Q_{T^m}. Since the left-hand side of (54) is increasing with Q_{T^m} in the same manner as (32), the new system retains the same properties as in the analysis of Section 4. Also, the system (54)-(33) only differs from (32)-(33) by the marginal development cost term $E'(Q_0 - Q_T)$. Since this term is not directly affected by the taxation policies considered in this paper, the interested reader can easily verify that the policies' effects established in Section 4 carry over to the case of this appendix.

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