Public Goods, Taxation and Laffer Curves

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Introduction

(a) We set out a simple three good model, with one a public good, and N identical households. We solved numerically for a first best case and observed differences when the elasticity of substitution in the utility function changed. Each agent had a CES utility function.
(b) We turned to a second best case (a "market" model) in which an agent takes the level of the public good as a parameter and optimizes with respect to her level of the private good and her amount of leisure. The second best solution was very similar to the first best counterpart when the elasticity of substitution was less than unity. For the elastic case however ($\sigma = 2$), the second best results were very different from those for the corresponding first best solution. Welfare was halved and there was much substitution of leisure for work. The Laffer curve (government revenue as a function of the tax rate) peaked at a value of the tax rate close to that which yielded the maximum utility when the elasticity of substitution was low ($\sigma = 2/3$).

analysis

(c) The case of $\sigma = 2$ yielded a Laffer curve peaking at the tax rate at unity, a corner. We took up the implications of distinct distributions of income between two groups of agents. We observed exact substitution of more of the private good and more leisure for agents endowed with more capital. The two groups kept supplying the same amount of labor in total but one gained when members of that group were allotted more capital initially. "National income" and aggregate tax revenue remained unchanging as endowments were re-allotted.

(d) We emphasized that standard income incidence analysis takes the government good as being private and thus this approach is yielding wrong results IF IN FACT THE GOVERNMENT GOOD IS A SAMUELSON PUBLIC Good. We presented two worked examples of this public good funding bias.

analysis

Each of N agents has utility function $u(q_c, H_l, q_g)$. H_l is defined by $24 - H_c - H_g$ where H_c is hours worked by the agent in the (private) consumer goods sector and H_g is hours worked by the agent in the government goods sector. q_g is a pure Samuelson public good. Every agent consumes quantity q_g and pays Samuelson price p_g/N per unit of q_g. An agent is optimizing her utility given the prevailing prices for the private and public goods. $p_g q_g$ is the total cost of producing the public good in the economy. Total production of the private good is Nq_c.

$$Nq_c = f(K_c, NH_c)$$

for K_c capital (say buildings and machines) used in the consumption goods sector. We have

$$q_g = g(K - K_c, NH_g)$$

for K the current endowment of capital to the economy, currently. q_g is a flow of services that government produces rather than something like infrastructure that is durable.

Equilibrium is defined in part by the first order conditions for a household in

$$\frac{u_{q_c}}{u_{q_g}} = \frac{p_c}{(p_g/N)} \text{ and } \frac{u_{q_c}}{u_{H_l}} = \frac{p_c}{w}$$

where p_c and p_g are output prices of a unit of q_c and q_g respectively. w is marginal value to the agent of an hour of her endowment of time. u_{q_c}, u_{q_g} and u_{H_l} are first derivatives of the utility function. Each agent has the budget constraint: $p_c q_c + (p_g/N) q_g = rK/N + w[H_c + H_g]$ for r the rental rate on capital.

analysis

For the producers of Nq_c and q_g, equilibrium requires

$$\frac{f_{K_c}}{f_{[NH_c]}} = \frac{r}{w} \text{ and } \frac{g_{[K-K_c]}}{g_{[NH_g]}} = \frac{r}{w}.$$

f_{K_c} and $f_{[NH_c]}$ are first derivatives of $f(.)$ and $g_{[K-K_c]}$ and $g_{[NH_g]}$ are first derivatives of $g(.)$; and zero profit conditions

$$p_c Nq_c = rK_c + wNH_c \text{ and } p_g q_g = r[K - K_c] + wNH_g.$$

The above six relations, plus the two production functions, plus the time constraint $24 = H_l + H_c + H_g$ and $p_c = 1$ form a system of ten equations in q_c, q_g, H_l, H_c, H_g, K_c, p_c, p_g, w and r.

For the specification u as $[\alpha q_c^{-\beta} + zH_l^{-\beta} + (1-\alpha-z)q_g^{-\beta}]^{-1/\beta}$, $Nq_c = K_c^{alc}[NH_c]^{1-alc}$, and $q_g = [K - K_c]^{a\lg}[NH_g]^{1-a\lg}$, we solve for a Samuelson equilibrium with parameters N=50; α=0.4; z=0.2; alc=0.2; alg=0.35; K=40; β=0.5 ($\sigma = 2/3$, inelastic case): Table 1

	u	q_c	q_g	H_c	H_g	H_l
Sam	13.4085	6.8293	69.9137	13.0188	3.3060	7.6753

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p_g	r	w	K_c
1.5265	2.6412	.4197	25.8574

elastic case: We repeated the numerical investigation above with β=-.5 ($\sigma = 2$). ($\tau = 0.966312$), we obtained the results in the first line of Table 3

$\tau = 0.966312$	u	q_c	q_g	H_c	H_g	H_l
Exog τ	61.89	0.42	344.5	0.95	22.2	0.85
Sam	61.892	0.419	344.5	0.95	22.2	0.847

...

p_g	r	w	K_c
1.74	5.36	0.35	0.78
1.74	5.359	0.352	0.78

More on Exog τ below.

MARKET SOLUTION (an agent is a q_G taker)
$u(q_c, H_l, q_g)$ by choice of q_c and H_l subject to
$q_c = (1-\tau)[w * [24 - H_l] + \frac{rK}{N}]/p_c$ and $H_l = 24 - H_c - H_g$. This implies that each agent is choosing q_c and H_l to satisfy

$$\frac{u_{q_c}}{u_{H_l}} = \frac{p_c}{(1-\tau)w}. \quad (1)$$

Formally, the Samuelson "tax model" above is altered with the substitution of ABOVE EQUATION for the first best equation .... $\frac{u_{q_c}}{u_{H_l}} = \frac{p_c}{w}$. WELFARE ANALYSIS: Take tax rate from first best solution and solve the second best model... then compare.

ELASTIC CASE: Utility is 1/2 the first best. q_g is about 1/3 r is 1/3 the earlier value; wage is higher by 1/3. We might infer that this reflects the considerably smaller number of hours being worked in this second best case. Labor has become scarcer in a certain sense. Table 4

τ	u	q_c	q_g	H_c	H_g	H_l
0.63	31.0334	1.867	111.22	3.259	4.509	16.23
0.65	31.0023	1.716	111.89	2.982	4.499	16.52

...

p_g	r	w	K_c
1.429	1.857	0.458	10.051
1.424	1.823	0.460	9.412

INESLASTIC CASE: Labor is 1 hour LESS relative to the Samuelson optimum. Utility a small amount less. The Laffer Curve continues to be peaking at the tax "limit". For this case, q_c and u are tending to zero with q_g becoming large as tax rate rises. Table 5.

τ	u	q_c	q_g	H_c	H_g	H_l
0.239	13.3476	6.486	67.349	12.21	3.12	8.673
0.999	0.0689	0.012	349.20	0.028	22.44	1.537

...

p_g	r	w	K_c
1.512	2.513	0.425	25.813
1.741	5.322	0.352	0.023

INCOME TAX INCIDENCE

Feldstein, Martin (1999) "Tax Avoidance and the Deadweight Loss of the Income Tax", *Review of Economics and Statistics*, 81:4, pp. 674-80.
Saez, Emmanuel, Joel Slemrod and Seth H. Giertz (2012) "The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review", *Journal of Economic Literature*, 50:1, pp. 3-50.

INCOME TAX INCIDENCE: market model above and perburb tax rate upward

$$u_{q_c}\frac{dq_c}{d\tau}+u_{H_l}\frac{dH_l}{d\tau}+u_{q_g}\frac{dq_g}{d\tau}\ \left(=\frac{du}{d\tau}\right)$$
$$\text{or } \frac{dq_c}{d\tau}+\left[\frac{(1-\tau)w}{p_c}\right]\frac{dH_l}{d\tau}+\frac{u_{q_g}}{u_{q_c}}\frac{dq_g}{d\tau}\ \left(=\frac{1}{u_{q_c}}\frac{du}{d\tau}\right),\text{ in numeraire units.}$$

The standard incidence result is marginal deadweight loss per agent (eg. Saez, Slemrod and Giertz (2012)) is $\frac{dq_c}{d\tau}+\left[\frac{(1-\tau)w}{p_c}\right]\frac{dH_l}{d\tau}+\frac{u_{q_g}}{u_{q_c}}\left[\frac{dq_g}{d\tau}-\frac{\Delta T^i}{p_g}\right]$ with $\left[\frac{dq_g}{d\tau}-\frac{\Delta T^i}{p_g}\right]=0$ because the tax rate increase yields just enough new revenue ΔT^i from agent i that the cost of the increase in government product consumed by agent i is exactly covered. Our observation here is that when q_g is a public good, $\left[\frac{dq_g}{d\tau}-\frac{\Delta T^i}{p_g}\right]$ is positive and instead it is $\left[\frac{dq_g}{d\tau}-\left\{\frac{\Delta T^i}{p_g}+\sum_j^{N-1}\frac{\Delta T^j}{p_g}\right\}\right]$ that equals zero. *** Should use $[N*\Delta T]/p_g$ and not $\Delta T/p_g$. ***

We proceed to illustrate with our second best model with $\sigma = 2/3$. We solve the model twice, with $\tau = 0.2$ and $\tau = 0.23$. The results are reported in Table 8.

Table 8.

	u	q_c	q_g	H_c	H_g	H_l	p_g	r
$\tau = 0.2$	13.2772	6.7884	56.4220	12.69	2.58	8.7366	1.5039	2
$\tau = 0.23$	13.3435	6.5559	64.8268	12.32	2.99	8.6879	1.5104	2

Our central observation is that $\Delta q_g = [64.8268 - 56.4220] = 8.44$ in Table 8 is a good approximation to $[N * \Delta T]/p_g$ for ΔT an agent's increase in income taxes being paid. ΔT works out to be 0.2616, making $[N * \Delta T]/p_g$ approximately 8.8 numeraire units. Deadweight loss is underestimated when the government good has the characteristic of Samuelson publicness. (DO NOT USE $\Delta T/p_g$ IN ESTIMATING DWL WITH PUBLIC GOODS... USE $[N * \Delta T]/p_g$)

PERSONALIZED "SAMUELSONIAN" INCOME TAX RATES: We start with $p_g q_g = \tau[p_g q_g + Np_c q_c]$ and proceed to get

$$\frac{p_g / N}{p_c} = \left[\frac{\tau}{1-\tau}\right] \frac{q_c}{q_g}.$$

We need an expression for τ such that $\left[\frac{\tau}{1-\tau}\right] \frac{q_c}{q_g} = \frac{u_g}{u_c}$. Clearly

$$\tau = \frac{u_g q_g}{u_g q_g + u_c q_c}$$

is the desired expression. For our case of N identical households, this expression for τ is a personalized rate of income tax completely analogous to the personalized public goods prices or charges in the original Samuelson model. Hence we are able to delete the equation expressing the demand for q_g from the 10 equations defining the Samuelson equilibrium and replace that equation with $p_g q_g = \left[\frac{u_g q_g}{u_g q_g + u_c q_c}\right] [p_g q_g + Np_c q_c]$. The new 10 equation system yields the same solution as the first 10 equation system.

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analysis

end ......

End