On oligopoly with positive network effects and incompatible networks

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Abstract

We consider symmetric oligopolies with positive network effects where each firm has its own proprietary network, which is incompatible with that of its rivals. We provide minimal conditions for the existence of (non-trivial) symmetric equilibrium in a general setting. We analyze the viability of industries with firm-specific networks, and show that the prospects for successful launch decrease with more firms in the market. This is a major reversal from the case of single-network industries. In terms of the comparative statics effects of entry, we find that this model has properties that are closer to regular Cournot oligopoly than to the case of single-network industries. Finally, we compare the viability and market performance of oligopolistic industries with compatible and incompatible networks, and show that viability, output and social welfare are higher for the former. However, the comparison of industry price, profits and consumer surplus is ambiguous.

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1 Introduction

Industries with network externalities are characterized by a complex demand structure wherein consumers' individual demands are positively interdependent: a consumer's willingness to pay for the product depends in a significant way on the number of consumers who will, or are expected to, purchase (a compatible version of) the same product. Examples of such industries are common in everyday life, ranging from telecommunications devices, such as telephone, fax, and instant messaging, to various fashion goods, compact discs and software products, among many others. Rohlfs (1974) pioneered the study of interdependent demand systems and proposed a simple and tractable model for dealing with such industries. Katz and Shapiro (1985) provided the first study of network industries under imperfect competition. To reduce the model to a static framework, they proposed a notion of Cournot equilibrium with an endogenously derived inverse demand function that reflects economy-wide rational expectations about the right market size.[1] Katz and Shapiro (1985) investigate two different Cournot-type oligopolies with homogeneous products and network effects. The first model features goods that are completely compatible across firms, and is thus referred to as the single-network industry. In the second model, two goods produced by two different firms are completely incompatible; such industries thus have firm-specific networks.[2] Amir and Lazzati (2011) provide a generalization and expansion of the results relating to Katz and Shapiro's (1985) first model. The present paper aims to perform the same task for their second model, and then to provide a comparative study of some of the key properties of the two models.

Among the many industries with firm-specific networks, one can name some well known cases: video games consoles;[3] personal computers (early on, IBM and Macintosh computers were not

[1]Veblen (1899) was first to bring up the key observation that interdependent demands in the form of fashion, fads and bandwagon effects can lead to substantial novel implications of a macroeonomic nature. In addition, Leibenstein (1950) was the first attempt to model in a very elementary manner demand systems in the presence of network effects.

[2]A third, hybrid case, often prevails in practice: an industry may have a few incompatible standards with each standard being adopted by a subset of the firms, in such a way that firms with the same standard have mutually compatible products. The Betamax-VHS standard war that we mention later on went through a phase of such coalitional rivalry.

[3]For consoles, network externalities arise from two separate sources. As more consumers purchase a particular console, a buyer's valuation of that same console increases, as he will have more people to play and share games with, and he also (rationally) expects to have a larger variety of games available for his own use due to software developers' higher incentives to create more games.

compatible); video cassette recorders; and digital music systems such as digital compact cassette and mini disc (see Church and Gandal, 1992; Cusumano, Mylonadis and Rosenbloom, 1992; and Katz and Shapiro, 1986).[4]

In addition to the division of network industries along the important dimension of inter-firm compatibility, these industries can also instructively be classified according to whether the product is a pure or a mixed network good. The former derive their value solely from network externalities (and have no intrinsic value when purchased in isolation), and include most telecommunications devices as typical examples. Mixed network goods instead possess both intrinsic and network value components, and include software and fashion goods as typical examples. An important simplifying assumption in Katz and Shapiro (1985) is that these two components appear additively in a consumer's utility function, and are thus independent. One implication is that the lowest possible demand function, the one corresponding to an expectation of a zero market size, is positive. It follows that their results apply only to mixed network goods. In contrast, Amir and Lazzati (2011) consider a more general demand structure allowing for interacting pure and network values, which is adopted by the present paper. The key advantage of this general demand structure is that it nests the important case of pure network goods as a possibility. For such goods, the problem of viability turns out to be of crucial importance: whether or not the launch of a new industry featuring a pure network good succeeds or totally fails depends critically on how expectations affect the set of possible inverse demand functions.[5]

A diverse set of factors might lie behind firms' decisions to make their product compatible or not. According to Shapiro and Varian (1988), this question is always of significant, and sometimes of

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[4]The highly publicized standards war between Betamax and Video Home System (VHS) in the 1980s is worth recalling. These were two incompatible formats of home video cassette recorders (VCRs), differing in the sizes of the cassettes for the VCRs, the tape-handling mechanisms and the technology to read the tapes. After a long battle, Sony and the firms that adopted its format stopped producing the Betamax (see Cusumano, Mylonadis and Rosenbloom, 1992 for more details). This case helps us to illustrate the emergence of asymmetric equilibria in the presence of firms that could be considered symmetric given the similarities between the two technologies. At the beginning of the 1980s, VHS had a higher market share, and by the end of the decade, Betamax was out of the market.

[5]Failure of new launches is of course also related to the cost structure, in addition to the expectations dimension. By contrast, for regular (non-network) industries, viability always amounts to a simple condition to the effect that demand is sufficiently high relative to production costs.

critical, importance for the market performance of network industries. A key factor is the perception by one or more firms that they can win a standards war in an unregulated setting and drive potential rivals out of the market. This may happen due to first-mover advantages (i.e., early entry), quality differences in the firms' products. Firm specific networks may also emerge due to refusals by firms to adopt a rival's standard and abandoning their own, which may be due to a perceived prohibitive cost of doing so, hubris, or brand protection, among other factors. There are also multiple reasons for a single network to emerge, often owing to some form of government intervention. In his detailed study of the history and evolution of many network industries, Rohlfs (2003) argues forcefully that a single network is always preferable from a welfare perspective, and often also from the firms' standpoint, in particular when the viability of the industry itself is at stake.

The main aim of this paper is to investigate the general properties of oligopolies with identical firms (for simplicity), and firm-specific network effects. As such, this paper may be seen as the counterpart to the study of single-network industries by Amir and Lazzati (2011) for industries with firm-specific networks. Imposing a general structure on the model compatible with the Cournot oligopoly counterpart exhibiting strategic substitutes, we begin with a general existence result for symmetric Cournot equilibrium with rational expectations (as defined by Katz and Shapiro, 1985). Although such models may possess both symmetric and asymmetric equilibria, we focus in this paper only on the former. This choice is due to a number of different reasons. The first is that symmetric equilibria allow for a more direct comparison with the single-network case. The second is that, while clearly very important for the case of firm-specific networks, asymmetric equilibria raise issues that are beyond the scope of our static analysis. The main such issue is that of firms endogenously exiting the market, a possibility that is best investigated in an intrinsically dynamic setting (see e.g., Chen, Doraszelski and Harrington, 2009).

Since our setting allows for pure network goods as a special case, we also provide an existence argument for non-trivial equilibria, i.e., ones with strictly positive output. This result is clearly needed since the trivial equilibrium is always present for a pure network good, as a sort of self-fulfilling expectation. As a result, the issue of viability then arises naturally.

Beyond these foundational results, the first issue of substantial economic and policy interest is that of industry viability, the study of which is based on the asymptotic properties of the usual expectations-augmented Cournot adjustment process.[6] The existence of a non-trivial equilibrium guarantees that the industry is either viable or conditionally viable.[7] As to the factors that have direct influence on viability, we focus on exogenous technological progress and exogenous entry. While the effect of the former is positive, which is quite intuitive, an increase in the number of firms in the industry actually lowers its viability. In other words, for industries with firm-specific networks, monopoly leads to the highest prospects for viability! This is in sharp contrast to the case of single-network industries wherein further entry raises the prospects for viability (Amir and Lazzati, 2011). These two opposite results taken together from a thorough vindication of the conclusions on viability reached by Rohlfs (2003) by introspection through his extensive case studies: as far as viability is concerned, the case of firm-specific networks is unambiguously inferior to the single-network case, except in the case of monopoly for which the two models fully coincide.

The next part of the paper provides a thorough investigation of the effects of exogenous entry on overall market performance. It is well known that such effects in network industries differ substantially from those found in regular industries (when constant returns to scale is assumed in both types of industries).[8] [9] One finding is that industry output may go either way in response to entry, and we derive sufficient conditions for each possibility. This is in contrast to both regular Cournot oligopoly and single-network industry. As to the effects of entry on profit, per-firm output and social welfare, we find that, surprisingly, the firm-specific network case is closer to regular Cournot oligopoly than to the single-network case. Thus, under very mild conditions, social welfare increases with entry while per-firm output and profit decrease. The latter result constitutes a sharp reversal from the case of single-network industries.

The differences in the comparative statics of market performance with respect to entry bring out another key difference between the two different types of network industries. As suggested by Amir and Lazzati (2011), for single-network industries, the Katz-Shapiro equilibrium concept is one

[6]Recall that in their seminal work, Katz and Shapiro (1985) do not consider the viability problem, since their separable demand function directly rules out pure network goods.

[7]As in Amir and Lazzati (2011), these notions are defined via the convergence of the said adjustment process to a non-zero equilibrium from any or from a sufficiently high, initial belief about the network size.

[8]For such differences, see e.g., Economides (1996), Economides and Flyer (1997), Katz and Shapiro (1985), and Amir and Lazzati (2011), among many others.

[9]This makes for an instructive comparison since strongly increasing returns to scale creates "perverse" comparative static in regular Cournot industries as well (Amir and Lambson, 2000).

of co-opetition, a hybrid of competition and cooperation. Firms are indeed partners in working together to build a joint network, and then Cournot-type rivals in competing for customers once the network is built. The present results indicate that the analogous equilibrium concept does not display much in the cooperation dimension as far as industries with firm-specific networks are concerned. This again is a major reversal from the single-network case.

The last part of the paper addresses a comparison, in terms of overall market performance, between the two different types of network industries. The main finding here may be summarized as follows: with the same number of firms operating in both cases, the single-network case offers superior performance than the firm-specific network case in terms of equilibrium output, profit, and social welfare. Here again, the main thrust of Rohlfs' (2003) conclusions are confirmed, although some qualifying assumptions are needed and provided.

As to the organization of the paper, it proceeds in the chronology described in the summary of our results given above. The proofs are all gathered in the last section. As a final remark, several examples with closed-form solutions are provided throughout the paper to illustrate key conclusions of interest in a manner accessible to all.

2 Oligopoly with firm-specific networks

2.1 The model and the assumptions

In this section we describe the model, which is a static game of oligopolistic competition in an industry with a homogeneous good with positive network effects and complete inter-firm incompatibility. This is a market situation where the firms produce perfect substitutes and the consumers' willingness to pay for any good is increasing in the number of agents that purchase the good from the same firm. In other words, the goods are not compatible across firms; rather every firm possesses its own network. This model is a generalization of the second model of oligopoly with network effects, the one with complete incompatibility, introduced by Katz and Shapiro (1985). In equilibrium, every firm maximizes its profit given the output of the rest of the firms, with each firm's output matching its own expected network size. This corresponds to the so-called Cournot equilbrium with fulfilled expectations. This notion of equilibrium is due to Katz and Shapiro (1985) and will be formally defined below.

Every firm in the market faces the inverse demand function $P(z, s)$, where z denotes the total output in the market and s the expected size of the firm's network. Each firm has its individual network with expected size s, which is not necessarily the same across firms (but will be at the symmetric equilibria that we consider). If every consumer purchases at most one unit of the good, s accounts for the expected number of agents that will purchase the good from that same firm.

The firms face the same linear cost of production.[10] Hence, for given s, firm i chooses the output that maximizes its profit given by

$$\pi(x, y, s) = xP(x + y, s) - cx,$$

where $c \geq 0$ is the unit cost, x is the firm's output level and y is the joint output of the other $(n-1)$ firms.[11] The firm does not get to choose s; rather, this is an exogenous expectations variable for the firm, in that the firm does not have the power to influence consumers' expectations about its own network size (just as in Katz and Shapiro, 1985).

Then, the firm's best reaction correspondence is given by

$$x(y, s) = \arg\max\{\pi(x, y, s) : x \geq 0\}.$$

Alternatively, one can think of firm i as choosing total output $z = x + y$ that maximizes

$$\tilde{\pi}(z, y, s) = (z - y)P(z, s) - c(z - y),$$

with best-reaction correspondence

$$z(y, s) = \arg\max\{\tilde{\pi}(z, y, s) : z \geq y\},$$

given a particular y and s. Then, it should be the case that $z(y, s) = x(y, s) + y$.

The equilibrium of this game is given by what Katz and Shapiro (1985) called a rational expectations Cournot equilibrium (henceforth, RECE), defined as follows.

[10] Although our approach can easily handle a more general cost function, we abstract away from cost curvature effects, since we wish to stress that the departures from standard oligopoly results that we are about to establish are all due to demand-side network effects. This is in contrast to the results of Amir and Lambson (2000) where such departures for regular Cournot oligopoly are due to increasing returns to scale in production.

[11] We chose to restrict atttention to constant returns to scale in production to emphasize that our departures from traditional conclusions are due to demand-side, as opposed to supply-side, increasing returns. For the role of the latter in Cournot competition, see Amir and Lambson (2000).

Definition 1 *A RECE consists of a vector of individual outputs* $(x_1, x_2, \ldots, x_n)$ *and a vector of expected networks sizes* $(s_1, s_2, \ldots, s_n)$ *such that:*

(1) $x_i \in \arg\max\{xP(x + \sum_{j \neq i} x_j, s_i) - cx : x \geq 0\}$, *and*

(2) $x_i = s_i$, *for all* $i \in \{1, 2, \ldots, n\}$.

Although widely accepted as an appropriate solution concept for industries with network effects, the RECE concept is nevertheless still somewhat controversial. A full discussion of its actual scope, and its pros and cons, is provided in Amir and Lazzati (2011), and the reader is referred to that discussion for the case of firm-specific networks as well.

Throughout the paper, the subindex n is added to any variable to denote that the variable in question is in equilibrium. The subindex i is usually dropped for simplicity since we focus on symmetric equilibria in this paper. We will also often refer to RECE simply as "equilibrium".

The following basic assumptions will be in effect throughout the paper:

(A1) $P : [0, \infty) \times [0, \infty) \to [0, \infty)$ is twice continuously differentiable, $P_1(z, s) < 0$ and $P_2(z, s) > 0$.

(A2) $x_i \leq K$, for each firm i.

(A3) $P(z, s)$ is a log-concave function of z for each s, i.e.,

$$P(z, s)P_{11}(z, s) - P_1^2(z, s) \leq 0, \text{ for all } (z, s).$$

The first assumption is standard in the literature. The usual law of demand is captured by $P_1(z, s) < 0$. The part $P_2(z, s) > 0$ reflects the positive network effects or demand-side economies of scale, i.e., consumers' willingness to pay when more people are expected to buy the good increases.

Recall that (A1) and (A2) imply that $\partial\tilde{\pi}(z, y, s)/\partial z\partial y \triangleq -P_1(z, s) > 0$, hence, every selection of the total output best-response correspondence, $z(y, s)$, is increasing in y for each s. This property alone guarantees the existence of a symmetric Cournot equilibrium for each s, as well as the abscence of asymmetric Cournot equilibria. For a detailed discussion, see Amir and Lambson (2000).

The capacity constraint assumption (A2) is only for the convenience of avoiding technical issues with unbounded outputs, with the magnitude of K being immaterial to the results.

Assumption (A3) is well known to ensure that for fixed network size s, the Cournot oligopoly is a game of strategic substitutes, i.e., will have reaction curves that are decreasing in rivals' total output, in addition to implying a unique Cournot equilibrium for given s (Amir, 1996). Nonetheless, Assumption (A3) is not crucial for the existence of RECE here; in fact, we could replace it with the log-supermodularity of demand and then use supermodularity arguments and Tarski's fixed point theorem (similar to the approach taken in Amir and Lazzati, 2011).

We impose no special restriction on the value of $P(z, 0)$. This characteristic of the inverse demand function allows the model to account for pure and mixed network goods. Pure network goods are those that do not have stand-alone value, i.e., $P(z, 0) = 0$, meaning that if the expected size of the firm's network is zero, no consumer will value this good at all. On the other hand, $P(z, 0) > 0$ reflects a mixed network good, one that the consumers value to some extent even if the expected size of the network is zero. As in Amir and Lazzati (2011), both possibilities are allowed.

Throughout the paper, we shall find it instructive to compare the results pertaining to the present model to those of the related model with complete inter-firm compatibility (Katz and Shapiro, 1985, and Amir and Lazzati, 2011). In so doing, we shall refer to the former model as oligopoly with firm-specific networks (or with complete incompatibility, or just incompatibility), and to the latter as oligopoly with a single-network (or with complete compatibility, or just compatibility).

2.2 Existence of symmetric equilibrium and viability

This subsection considers the existence of RECE, or for short simply, equilibrium, as a result of some minimal structure. Under Assumptions (A1)-(A3), maintained throughout, we will show that at least one symmetric equilibrium exists. In this paper, we focus on the symmetric RECE since this part of the model itself is rich in interesting results that complement those of the single-network model and allow for an instructive comparison.[12]

Recall that for each fixed n and s, under Assumptions (A1)-(A3), the standard Cournot oligopoly with inverse demand $P(Z, s)$ possesses a unique and symmetric Cournot equilibrium (Amir and Lambson, 2000). Denote the per-firm equilibrium output by the single-valued function $q_n(s)$. All the proofs are shown in the Appendix.

Theorem 1 *Under Assumptions (A1)-(A3), for each $n \in N$,*

(a) the function $q_n(s)$ is continuously differentiable, and

(b) the Cournot oligopoly with firm-specific networks has (at least) one symmetric RECE.

[12] Asymmetric equilibria are quite different in nature, and appear more suitable for a dynamic analysis. Thus this is set aside for further research. Katz and Shapiro (1985) do consider asymmetric equilibria.

In industries with network effects, it is quite possible to face a situation where the only equilibrium is the trivial one, where all the firms choose to produce zero output. Such an industry is then said to be non-viable. *If* $P(x,0)=0$, then when $s=0$, each firm will produce zero output, and the trivial equilibrium becomes self-fulfilling, i.e., $q_n(0)=0$. More generally, The following observation characterizes the trivial equilibrium.

Lemma 1 *The trivial outcome is a RECE if and only if $xP(x,0)\leq cx$ for all $x\in[0,K]$. Hence, the trivial outcome is a RECE for n firms if and only if it is a RECE for $(n+1)$ firms.*

A useful direct consequence of Lemma 1 is now noted. It holds that if for some n the trivial equilibrium is not an equilibrium for an n-firm industry (i.e., if $q_n(0)>0$), it will not be an equilibrium for the same industry with any number of firms.

Corollary 1 *If $q_n(0)=0$ for some n, then we must have $q_n(0)=0$ for all n.*

In light of Lemma 1, Theorem 1 is a priori of potentially limited interest, in that the equilibium that is shown to exist in a non-constructive manner may well be the trivial one. A natural question then is, what conditions on the primitives ensure the existence of a non-trivial equilibrium, i.e., one with strictly positive industry output. Theorem 2 answers this question.

Theorem 2 *A non-trivial symmetric equilibrium exists if at least one of the following conditions holds:*
(i) $q_n(0)>0$ for some n, i.e., zero is not a RECE for some n (or $xP(x,0)>cx$ for some $x\in(0,K]$);
(ii) $q_n(0)=0$ and $(n+1)P_1(0,0)+P_2(0,0)>0$; or
(iii) $q_n(0)=0,(n+1)P_1(0,0)+P_2(0,0)<0$, and $P(z,s)+\frac{z}{n}P_1(z,s)\geq c$ for some $s\in(0,K]$ and for all $z\leq ns$.

The result in Theorem 2 part (i) is immediate from Corollary 1 and Theorem 1, because we know that at least one symmetric equilibrium always exists.

The most useful condition for the existence of a non-trivial equilibrium is probably that given in part (ii). As will become clear in the proofs, the role of the condition, $(n+1)P_1(0,0)+P_2(0,0)>0$, is to lead to the key property $q_n'(0)>1$, thereby ensuring that the map $q_n(s)$ starts above the 45° at 0 and therefore that it possesses a strictly positive fixed point.

As to part (iii), the condition $(n+1)P_1(0,0)+P_2(0,0)<0$ implies that $q_n(s)$ starts below the $45°$ at 0, but the role of the third condition in part (iii) is to guarantee that the graph of $q_n(s)$ lies above the $45°$ for some $s>0$, and this in itself implies the existence of a strictly positive fixed point for $q_n(s)$, or a non-trivial RECE.

We now provide a systematic comparison between the two models (with and without compatibility) in terms of industry viability.[13]

2.3 Compatibility and viability

In this subsection, we compare the scope for industry viability for the two types of oligopoly, with compatible networks (as in Amir and Lazzati, 2011) and with firm-specific networks (as in the present paper).

In order to formally define the notion of viability, we consider the dynamic process given by the following expectations/network size recursion, starting from any initial $s_0 \geq 0$,

$$s_{t+1} = q_n\left(s_t\right),\ t = 0, 1, ... \tag{1}$$

An industry is said to be uniformly viable, or just viable for short, if the process (1) converges to a strictly positive equilibrium, or fixed-point of $q_n\left(\cdot\right)$, from any (sufficiently large) initial point $s_0 > 0$. An industry is said to be conditionally viable if the same convergence takes place from any sufficiently large initial point, i.e. for $s_0 \geq \overline{s}$, for some $\overline{s} > 0$. The minimal such $\overline{s}$ is called the *critical mass*.

As such, both notions of viability require the existence of a non-trivial equilibrium. An industry without one is called non-viable. From the results in the previous subsection, it follows that an industry is non-viable if and only if the trivial outcome is its unique equilibrium.

These definitions are adapted from Amir and Lazzati (2011), with the important difference that the relevant dynamic process (1) they used is, in the present notation,

$$s_{t+1} = nq_n\left(s_t\right), t = 0, 1, ... \tag{2}$$

It is important to observe that the issue of viability does not arise in the early work of Katz and Shapiro (1985), due to their simplifying assumption of additively separable inverse demand there,

[13] It is also of interest to investigate the effects of technological progress on industry viability.

namely that $P(z,s) = p(z) + g(s)$ for some functions p and g. Under this formulation, $P(z,0) > 0$, and the viability problem is identical to its counterpart for regular (non-network) industries. It would only arise as a result of low demand, relative to production costs. As we shall see below, the viability problem for general network industries has a much wider scope, as indeed reflected in the case studies by Rohlfs (2003).

The condition in Theorem 2 part (ii) sheds some light on the role of market structure in determining viability, in the form of an upper bound on the number of firms that yields viability.

Corollary 2 *A sufficient condition for viability for oligopoly without compatibility is*

$$n < \frac{P_2(0,0)}{-P_1(0,0)} - 1. \tag{3}$$

It can be seen by inspection that this condition is restrictive for large values of n, and will tend to hold only for small values of n in general. In particular, for any given inverse demand function P, there will be some $\overline{n}$ such that (3) fails for all $n \geq \overline{n}$ (though viability may then still hold via part (iii)).

This condition can be interpreted as requiring that network effects around the origin be sufficiently strong, in that $P_2(0,0)$ must be large enough for (3) to hold.

The following result on how market structure affects viability is a key conclusion of this paper.

Proposition 1 *As n increases, the viability of an industry with firm-specific networks decreases.*

This result is a major reversal from the analogous result in [Amir and Lazzati, 2011, Theorem 7], which holds the opposite, namely that a higher number of firms always enhances viability in the case of a single-network. In light of the latter result, it certainly appears counter-intuitive in the present context that viability is maximal for a monopoly, out of all market structures.

As in Amir and Lazzati (2011), the effect of exogenous technological progress on viability is positive, which is a highly intuitive fact (being similar to the previous one, the proof is omitted).

Proposition 2 *As c decreases, the viability of an industry with firm-specific networks increases.*

The comparison of the viability of the models is stated next.

Proposition 3 *If an industry with incompatible networks is viable (conditionally viable) for n firms, for some $n \geq 1$, then the same industry with compatible networks is viable (conditionally viable) for a monopolist, and thus for any number of firms.*

The reverse of Proposition 3 clearly does not hold in general, the one exception being the special case of monopoly for which the two models are fully equivalent. With the benchmark of monopoly as the common starting point, as more firms enter the market, viability increases for the single-network model whereas it decreases for the firm-specific network model.

Let us revisit Example 1 in Amir and Lazzati (2011) to illustrate Proposition 3.

Example 1. Consider the inverse demand function $P(z,s) = \exp(-\frac{2z}{\exp(1-1/s)})$ and no production costs. With compatible networks, Amir and Lazzati (2011) show that, for any given s, every firm has a dominant strategy given by $x(y,s) = (1/2)\exp(1-1/s)$, and that the industry emerges if and only if there are two or more firms in the market. A monopoly would choose not to produce since it is not profitable to do so. In other words, the function $q_1(s) = (1/2)\exp(1-1/s)$ has a unique fixed point at 0, while the function $\exp(1-1/s)$ has 2 fixed points, namely 0 and 1, with 1 being a tangency point.

However, with incompatible networks, this industry will not emerge for any number of firms. Since every firm has its own network and their strategy is dominant (independent of the other players' choices), every single firm behaves as a monopolist in the compatible networks world, thus, every potential firm will decide not to enter the market. Formally, this equilibrium is given by the fixed point of $q_n(s) = (1/2)\exp(1-1/s)$, which is unique and equal to 0. $\|$

In terms of policy implications, the results of this section provide an instructive complement to the viability results in the single-network model (Amir and Lazzati, 2011). Upon examining the birth and development of a number of different network industries, Rohlfs (2003) concludes that the single most important determinant of whether a newly launched network industry will survive is what he calls "inter-connection", which is synonymous with complete compatibility in this paper.[14] As a particular case study, he covers in detail the history of the fax industry, starting with a failed initial launch in the mid-nineteenth century, a failure he unambiguously attributes to two reasons: Firm-specific networks (i.e., no inter-connection), and a poor initial production technology. In covering future steps in the evolution of this industry, he concludes that the much later successful launch of the fax industry in the 1980's is mostly due to government-mandated full compatibility,

[14]He also discusses the importance of technological progress in the production of network goods over time, but the role of this factor is more intuitive, and in line with regular (non-network) industries. The role of exogenous technological progress is captured by Proposition 2.

and in addition to improvements in production technology. The results of this section, taken in conjunction with those of Amir and Lazzati (2011), provide a very neat theoretical under-pinning for Rohlf's case studies and policy analysis.

A key implication of the above results is that, in industries where inter-connection is difficult or overly costly to achieve,[15] on account of the important issue of viability, monopoly might be socially preferable to any other market structure. This applies in cases where monopoly is the only viable market structure, or in cases where it is the only conditionally viable market structure with a sufficiently small critical mass (defined above as the smallest value of s_0 for which the adjustment dynamics converges to a strictly positive RECE; see also Amir and Lazzati, 2011). Interestingly, this provides a novel motivation for what might be termed *natural monopoly*, although the underlying reasons are altogether different from the production efficiency considerations that underpin the classical notion. Clearly, in case of industries with firm-specific networks that can be viable only as monopolies, the reference to *natural monopoly* is unambiguously justified.

The next section investigates the comparative statics effects of market performance with respect to exogenous entry, i.e, how equilibrium variables of interest respond to more competition.

2.4 Market structure and performance

In this section, we analyze the effects of exogenously increasing the number of firms in the market on the symmetric equilibria, i.e., we look at the changes in the equilibrium variables of interest, prices, outputs, profit, consumer surplus and social welfare, as competition increases. The results hold unambigously for the maximal and minimal equilibria, Milgrom and Roberts (1990, 1994).[16] The results are listed for the maximal equilibria, denoted by the corresponding equilibrium variable (sub-indexed by n) with an upper bar. These results are valid for the minimal equilibria (although this is vacuous when 0 is an equilibrium), and for regular equilbria as well (as defined in Amir et. al. 2014).

[15]Inter-connection may fail to be implemented because of a variety of reasons, including historical factors, inability of firms to compromise on adopting a rival's standard, high costs sunk into firms' current standards, perceived threat to firms' prestige in case of abandonment of one's standard, etc... A frequent reason is perception by an industry leader that competition with firm-specific networks will end up driving its rivals out of the market.

[16]The results will also hold for regular equilibria, as defined in Amir et. al (2014). However, due to the well known Correspondence Principle, they cannot hold for all equilbria (Samuelson, 1947 and Echenique, 2002).

For the next result, we introduce the following assumption

(A4) $\Delta_1(z,s) \triangleq P(z,s)P_{12}(z,s) - P_1(z,s)P_2(z,s) > 0$ for all (z,s).

Assumption (A4) means that the inverse demand function is strictly log-supermodular in (z,s), which implies that every selection of $z(y,s)$ is increasing in s, for every y. The latter result follows because (A4) guarantees that the alternative profit function $\log \tilde{\pi}(z,y,s)$ has strict increasing differences in (z,s), Topkis (1998). Assumption (A4) is quite broadly satisfied since it imposes only a restriction on how negative $P_{12}(z,s)$ can be. In addition, (A4) has an exact and very natural economic interpretation: It holds that the price elasticity of demand is increasing in the expected network size. As such, (A4) reflects the demand-side scale economies associated with network industries (see Amir and Lazzati, 2011 for more discussion).

Before proceeding to the main comparative statics result, we first note the main implication behind Assumption (A4), which will be useful in proving some of the main results.

Lemma 2 *Under Assumptions (A1)-(A4), $q_n(s)$ is increasing in s.*

Under Assumptions (A1)-(A4), we now show that with firm-specific networks, price, per-firm output and profit decrease with more firms, while total output might go either way.

Proposition 4 *As n increases, we have, provided the maximal equilibrium is interior,*

(a) $\bar{x}_{n+1} \leq \bar{x}_n$.

(b) $\bar{z}_{n+1} \geq (\leq) \bar{z}_n$ *if*

$$\Delta_2(z) \triangleq P_1(z,z/n) + P_2(z,z/n) + \frac{z}{n}P_{12}(z,z/n) \leq (\geq) 0.$$

(c) $\bar{P}_{n+1} \leq \bar{P}_n$ *if either (i)* $\bar{z}_{n+1} \geq \bar{z}_n$, *or (ii) with all the P terms evaluated at* $(z,z/n)$,

$$\Delta_3(z) \triangleq P_1^2 + \frac{z}{n}(P_1P_{21} - P_2P_{11}) \geq 0$$

(d) $\bar{\pi}_{n+1} \leq \bar{\pi}_n$ *if* $\bar{P}_{n+1} \leq \bar{P}_n$.

Part (a) is a direct consequence of the fact that $q_n(s)$ is decreasing in n, which follows from Assumption (A3), Amir and Lambson (2000). Indeed as the largest fixed point of $q_n(s)$, $\bar{x}_n$ must be decreasing in n. Part (b) holds that the conventional result that total output increases with entry here requires the condition $\Delta_2(z) \leq 0$, which holds that marginal revenue along the fulfilled expectation path $s = z/n$ is decreasing in z. This condition may be interpreted as setting a bound on how strong network effects may be. For $\Delta_2(z) \geq 0$, it is necessary to have strong network effects (i.e., large $P_2(z, z/n)$ or $P_{12}(z, z/n)$ or both.) It can be seen by inspection that $\Delta_2(z) \geq 0$ is not necessarily a very restrictive condition, as illustrated in a simple example below.

In light of the absence of a clear-cut result on industry output, the same holds for industry price, but the conventional wisdom on price holds more broadly. Indeed, the condition $\Delta_3(z) \geq 0$ imposes very little restriction on inverse demand. The same can be said a fortiori of per-firm profit, which may be seen as declining with more competition virtually always, as a direct consequence of parts (a) and (c). (We could not find any specific example with declining per-firm profit.)[17]

Yet, overall, these comparative statics results cannot be said to be particularly intuitive, in light of the known results on single-network industries. Indeed, Amir and Lazzati (2011) show that in Cournot oligopoly with a single-network, total output always increases, and per-firm profit may well increase or decrease, in n. In addition, a single-network may well lead to per-firm output and price increasing in the number of firms, even with a log-concave inverse demand. Therefore, for the present model, Proposition 4 shows substantial departures from the single-network model, as far as the comparative statics of exogenous entry is concerned.

On the other hand, recall that in standard Cournot oligopoly under our assumptions, industry price and per-firm profit are decreasing in n; per-firm output also decreases if the inverse demand is log-concave (Amir and Lambson, 2000). Since a log-concave inverse demand function is a very general assumption, our model is closer to standard Cournot oligopoly than to oligopoly with a single-network when looking at the change in per-firm output, price and profit driven by a change in n. The one significant disagreement is on the comparative statics of industry output.

As an instructive illustration of Proposition 4, Example 2 shows that if the inverse demand function is linear in both output and expected size of the network and production is costless (as in Katz and Shapiro, 1985), industry output may increase or decrease with entry.

[17] It can be shown that the sign of $d\bar{\pi}_n/dn$ is the same as the sign of

$$-2P_1^2 - \frac{z}{n}(P_1 P_{21} - P_2 P_{11}) - \frac{z}{n} P_1 P_{11}$$

which will be generally negative. The same can said about dP_n/dn.

Example 2. Consider an oligopoly with inverse demand function $P(z,s) = \max\{a+4s-z, 0\}$, with $0 \leq a \leq K$ and $c = 0$ (costless production).

The first order condition reduces to

$$a + 4s - 2x - y = 0 \text{ or } q_n(s) = \frac{a+4s}{n+1}.$$

The reader can easily verify that the symmetric equilibrium is given by

$$x_n = \begin{cases} K \text{ if } n \leq 3, \\ \frac{a}{n-3} \text{ if } n \geq 4, \end{cases} \quad z_n = \begin{cases} nK \text{ if } n \leq 3, \\ \frac{na}{n-3} \text{ if } n \geq 4, \end{cases} \quad P_n = \begin{cases} a + (4-n)K \text{ if } n \leq 3, \\ a \text{ if } n \geq 4, \end{cases}$$

$$\pi_n = \begin{cases} aK + (4-n)K^2 \text{ if } n \leq 3, \\ \frac{a^2}{n-3} \text{ if } n \geq 4. \end{cases}$$

It is easy to verify that

(i) Per-firm output x_n is constant in n up to $n = 3$ and then decreases in n from there on.

(ii) Industry output z_n is increasing in n up to $n = 3$. In line with the facts that we move to an interior equilibrium and $\Delta_2(z) = 3$, industry output decreases in n for $n \geq 4$.[18]

(iii) Industry price P_n decreases in n up to $n = 4$ and is then constant in n from there on.

(iv) Per-firm profit π_n is decreasing for all n (in line with Proposition 4 (d) and part (iii) of this example). ||

As a final item in this section, we turn to analyze how equilibrium industry profit, consumer surplus and welfare change with exogenous entry. It is worth recalling here that industries with network effects share an unusual feature, that, by changing the equilibrium outputs, entry necessarily triggers a change in the demand function itself, thereby making welfare comparisons more complex than in regular industries.

Consumer surplus with total output z and expected size of each network s is defined by

$$CS(z,s) = \int_0^z P(t,s)dt - zP(z,s),$$

and social welfare is given by (with all the firms producing the same output level z/n)

$$W(z,s) = \int_0^z P(t,s)dt - cz.$$

[18] Total output increases when $n \leq 3$ because we are in a corner solution. We cannot tell what happens when we move from 3 to 4 firms, if $3K > 4a$, total output decreases, it increases if the inequality reverses and stays the same otherwise.

Proposition 5 *At the highest equilibrium output, when the number of firms increases*

(a) Industry profit decreases, $(n+1)\bar{\pi}_{n+1} \leq n\bar{\pi}_n$ *if either* $\bar{z}_{n+1} \geq \bar{z}_n$ *or* $\bar{z}_{n+1} \leq \bar{z}_n$ *and* $\bar{P}_{n+1} \leq \bar{P}_n$.

(b) Consumer surplus increases if total output increases (i.e., $\bar{z}_{n+1} \geq \bar{z}_n$*) and* $P_{12}(z,s) \geq 0$ *for all* z, s.

(c) Social welfare increases if total output increases, i.e, $\bar{W}_{n+1} \geq \bar{W}_n$ *if* $\bar{z}_{n+1} \geq \bar{z}_n$.

This result confirms that the one divergence between the present model and the standard Cournot oligopoly has to do with the effect of entry on industry output. If one assumes that this result goes in the intuitive direction (more entry leads to a higher industry output), then the effects of entry essentially coincide with those of the standard Cournot oligopoly. The added assumption for consumer surplus $P_{12}(z,s) \geq 0$ is quite compatible with Assumption (A4), and forms a minor strengthening in this context. On the other hand, parts (a) and (b) reflect major divergences with the case of single-network oligopoly (Amir and Lazzati, 2011).

It is instructive to relate this Proposition to the issue of viability, in light of the apparent contradiction with Proposition 1. In industries where viability holds for n firms but not for $n+1$ firms, welfare will necessarily decrease to zero as we move from n to $n+1$ firms. Though it may appear otherwise, this is actually compatible with Proposition 5 (c) since the latter relies on industry output increasing as we move from n to $n+1$ firms, which would not hold if viability fails with $n+1$ firms.

As exposited in Amir and Lazzati (2011), the RECE concept for industries with network effects amounts to a nice illustration of the notion of co-opetition in the single-network case (see e.g., Brandenburger and Nalebuff, 1996). Indeed, the concept treats firms as partners in jointly building a common network of consumers, but at the same time as Cournot-style rivals in serving that shared network and competing for its consumers. The present results indicate that, in contrast to the single-network model, for the case of firm-specific networks, the RECE concept boils down essentially to a notion of rivalry (with a key role for expectations). In particular, industry viability cannot possibly increase with more firms, and per-firm profit is virtually always decreasing in the number of firms, clear-cut reversals relative to the single-network case that make the model closer to the standard Cournot oligopoly.

2.5 Relaxing the log-concavity of inverse demand

The log-concavity of the inverse demand function, Assumption (A3), plays an important role in our analysis. In particular, it leads to the uniqueness of the symmetric Cournot equilibrium for all n given s. As a consequence, the per-firm equilibrium output $q_n(s)$ is a single-valued continuous function.[19] For standard Cournot oligopoly, the log-concavity of inverse demand is a ubiquitous assumption, since it ensures the characteristic property of strategic substitutes (Amir, 1996). It is therefore only natural to continue to make such an assumption for Cournot oligopoly with network effects. Nevertheless, we now present an example that illustrates the effects of such an assumption in the present setting: Without Assumption (A3), total output and per-firm output output need not be monotonic in n. The inverse demand function in Example 3 is globally log-convex in z.

Example 3. Let $P(z,s) = s/z^2$. Then, a firm's profit is given by $\pi(x,y,s) = x\frac{s}{(x+y)^2} - cx$.

Since P is not defined at $z = 0$, this example does not fit our Assumption (A1). We keep it due to its analytical convenience for a log-convex inverse demand (thus globally violating Assumption (A3)), and familiarity (iso-elastic demands have often been used in industrial organization).

The FOC, $\partial\pi(x,y,s)/\partial x = 0$, reduces to $s(y-x) = c(x+y)^3$. To solve for the symmetric Cournot equilibrium given s, and then for a RECE, we consider two separate cases.

If $n \geq 3$, we get

$$q_n(s) = \sqrt{\frac{(n-2)s}{cn^3}}$$

Hence, the unique RECE has per-firm output, industry output, price and per-firm profit respectively given by, for $n \geq 3$,

$$x_n = \frac{n-2}{cn^3}, \quad z_n = \frac{n-2}{cn^2}, \; P_n = \frac{cn}{n-2} \text{ and } \pi_n = \frac{2}{n^3}.$$

Instead, for $n = 1, 2$, we get

$$x_1 = x_2 = 0, \; P_1 = P_2 = \infty, \text{ and } \pi_1 = 1, \pi_2 = 1/4.$$

Thus, for all n, the industry is viable. As n increases, we have the following effects:

(i) x_n increases for $n \leq 3$ and decreases for $n \geq 3$;

[19] As shown in the appendix, $q_n(s)$ is even a C^1 function, a key property that is fully exploited in many of our comparative statics results to simplify the analysis.

(ii) z_n increases for $n \leq 4$ and decreases for $n \geq 4$;

(iii) P_n and π_n decrease globally in n. $\|$

To recapitulate, due to this inverse demand violating Assumption (A3), per-firm output x_n is not globally decreasing in n.[20] Again, this outcome takes place just in the same way in regular Cournot oligopoly. In order to avoid adding another layer of complexity to an already rich set of possible outcomes, we have decided to impose (A3), in line with much of oligopoly theory.

3 Compatibility versus incompatibility

In this section, we provide a thorough comparison of the market performances of the two types of oligopoly when the exogenously given number of firms is the same in both markets.[21] We conduct an equilibrium comparison of market variables such as output, prices, profits, and welfare. Here, we refer tacitly to the extremal (largest or smallest) variables.

The main results, that we justify later on, are that the industry will produce more when the networks are compatible and this scenario is more desirable in terms of social social welfare, since it maximizes it.

We need to define another expression whose sign plays a key role below

$$\Delta_4(z) \triangleq P_1(z, z) + P_2(z, z),$$

which denotes the change in the market price when industry output changes along the fulfilled expectation path for the single-network model.

In order to distinguish the equilibrium variables between the two models under study, we add to each a superscript C for the case with compatibility and I for incompatibility.

The first result deals with the output and price comparisons across the two models. This is a generalization of the result by Katz and Shapiro (1985) for linear and additively separable inverse

[20]In addition, as inverse demand is unbounded at zero output, thus violating Assumption (A1), the results for $n = 1, 2$ are not fully compatible with our conclusions (this is a familiar deviation when using iso-elastic demand functions.)

[21]In light of our viability results, the comparison with equal numbers of firms is not necessarily the most relevant. Indeed, viability in the case of firm-specific networks typically would not involve many firms. The opposite holds for the single network case, at least from a normative perspective.

demand function.

Proposition 6 *At the highest (symmetric) equilibrium output and for every $n \in N$*

(i) $\bar{x}_n^C \geq \bar{x}_n^I$ *and* $\bar{z}_n^C \geq \bar{z}_n^I$

(ii) $P_n^C \geq P_n^I$ *if* $\Delta_4(\cdot) \geq 0$ *on* $[n\bar{x}_n^I, n\bar{x}_n^C]$.

Proposition 6 part (i) is quite intuitive. Consumers have a higher willingness to pay under a single, larger, network, so the latter leads to a much larger demand, which calls for more output per firm, and therefore for the industry too. As can be seen in the proof, part (i) actually invokes Assumption (A3) in a critical manner.

Proposition 6 part (ii) says that if the network effect is sufficiently strong to offset the law of demand along the fulfilled expectation path, as reflected by $\Delta_4(\cdot) \geq 0$, the network effect becomes relatively more pronounced in the single-network case, and this results in a higher equilibrium price for this case.

As a limit case of Proposition 6 part (i), the following example shows that equilibrium outputs can be the same for the two types of oligopoly. Nonetheless, the resulting price is higher in the compatible world, which is to be expected since the network is larger and consumers' willingness to pay increases with the size of the network.

Example 4. Let $P(z,s) = se^{-z}$ and $c = 0$. A firm's profit function is $\pi(x,y,s) = xse^{-(x+y)}$. The first-order condition is then $se^{-(x+y)}(1-x) = 0$. Therefore, the symmetric RECE when the firms are incompatible is

$$x_n = 1, \quad z_n = n, \text{ and } P_n = \pi_n = e^{-n}.$$

On the other hand, if there is only one common network, the symmetric RECE is

$$x_n = 1, \quad z_n = n, \text{ and } P_n = \pi_n = ne^{-n}.$$

Hence, per-firm output is independent of the number of firms in the industry, and is the same for the two models. Price and per-firm profit are higher in the single-network case. ‖

The next result deals with the profit and welfare comparisons between the two cases.

Proposition 7 *At the highest equilibrium outputs and for every $n \in N$*

(i) $\pi_n^C \geq \pi_n^I$ *if* $\Delta_4(\cdot) \geq 0$.

(ii) $CS_n^C \geq CS_n^I$ *if* $P_n^I \geq P_n^C$ *or* $P_{12}(z,s) \leq 0$, *and*

(iii) $W_n^C \geq W_n^I$.

Notice that society is always better off under complete compatibility, by Proposition 7 part (iii), but we cannot conclude in general that the firms or the consumers always prefer one kind of industry. Nevertheless, we can conclude that complete compatibility always benefits at least one group, consumers or firms.

Proposition 7 part (ii) relates to Church and Gandal (1992) in the sense that consumer surplus cannot be compared in a global sense. Which setting gives the consumers a higher surplus relies on the characteristics of the market, specifically, of the demand.

To show that consumer surplus can have opposite comparative statics under compatibility and incompatibility, and be larger for the latter, we can simply extend Example 3 in Amir and Lazzati (2011) as follows.

Example 5. Let $P(z,s) = \max\{a - z/s^3, 0\}$, with $a \geq n/K^2$ and $K > 1$, and assume that firms that face zero production costs. The reaction function of a firm is given by

$$x(y,s) = \begin{cases} \max\{(as^3 - y)/2, 0\}, & \text{if } (as^3 - y)/2 < K, \\ K & \text{if } (as^3 - y)/2 \geq K. \end{cases}$$

With compatible networks, the RECE industry output set is $z_n^C = \{0, \sqrt{(n+1)/na}, nK\}$. Then, for the highest equilibrium $\overline{z}_n^C = nK$, the consumer surplus is $CS_n^C = 1/(2nK)$, which is decreasing in n.

With incompatible networks, the RECE industry output set is given by $z_n^I = \{0, n\sqrt{(n+1)/a}, nK\}$. Then, for the highest RECE $\overline{z}_n^I = nK$, $CS_n^I = n^2/(2K)$, which is increasing in n, in contrast to the single-network case.

Importantly, for this specification, RECE consumer surplus is higher under incompatibility than under compatibility.

In addition, $P_n^C = a - 1/(nK)^2$ is greater than $P_n^I = a - n/K^2$ and $P_{12}(z,s) = 3/s^4 > 0$. Hence, the two sufficient conditions of Proposition 7 part (ii) are both violated.

Finally, we have that $\pi_n^C = K(a - 1/(nK)^2) \geq K(a - n/K^2) = \pi_n^I$ and $W_n^C = anK - 1/(2nK) \geq W_n^I = anK - n^2/(2K)$, as Proposition 7 parts (i) and (iii) predict, given that $\Delta_4(z) = P_1(z,z) + P_2(z,z) = 2/z^3 > 0$. $\|$

From this example, it appears that a sufficient condition for the consumers to prefer incompatible networks is that the incompatible firms produce at their maximum capacity when the inverse demand function exhibits increasing differences in (z, s). By Proposition 6 part (i), this implies that the compatible firms also produce at their maximum capacity. As a more general result, it can be shown that if both industries produce exactly the same output, consumers will prefer incompatibility. The reason is that the market output is the same, a smaller network decreases the price and by supermodularity of the inverse demand function, the competitive effect is reinforced. This result is summarized next. It is valid for any symmetric equilibrium, not only for the extremal ones.

Proposition 8 *For every $n \in N$, if $x_n^I = x_n^C$ and $P_{12}(z, s) \geq 0$, then $CS_n^I \geq CS_n^C$.*

Under the hypotheses of Proposition 8, and by Proposition 7 part (iii), one may conclude for the extremal equilibria that industry profits are higher under compatibility, since social welfare is higher for compatible networks.

In terms of policy implications, the conclusions of this section tend to unambiguously reinforce the policy analysis and conclusions reported in Rohlfs (2003) in his overview of the history of network industries, as already discussed as part of the viability problem above.

4 Conclusions

This paper has provided a thorough study of the symmetric equilibria in oligopolies with identical firms and firm-specific network effects, under a general formulation. Since the setting nests pure network goods as a special case, one needs to go beyond a general existence result for symmetric Cournot equilibrium with rational expectations (as defined by Katz and Shapiro, 1985) and provide a separate existence argument for equilibria with strictly positive output, following the approach of Amir and Lazzati (2011). This allows us to tackle the key issue of industry viability. The main result is that an increase in the number of firms operating in the industry actually lowers its viability. Therefore, monopoly leads to the highest prospects for viability, in sharp contrast to the case of single-network industries (Amir and Lazzati, 2011). These two opposite results provide a complete theoretical foundation for the key conclusions on viability reached by Rohlfs (2003) through his multiple case studies: firm-specific networks is unambiguously inferior to the interconnected (or single-network) case, except in the case of monopoly for which the two models fully coincide.

As to the effects of exogenous entry on market performance, we find that industry output may go either way in response to entry, while profit, per-firm output and social welfare follow the analogous effects in regular Cournot oligopoly under constant returns to scale (Amir and Lambson, 2000), rather than the single-network case (Amir and Lazzati, 2011). This suggests that the rational expectations Cournot equilibrium concept reflects mostly a sense or rivalry in industries with firm-specific networks, in contrast to single-network industries where it captures a clear sense of co-opetition. This constitutes a major differrence beetween the two types of network industries.

As a final issue, the paper addresses the comparison of market performance between the two different types of network industries. The main results again confirm Rohlfs' (2003) primary conclusions, namely that with the same market structure, the single-network case is unambiguously superior in overall performance to the case of firm-specific networks.

In terms of policy prescriptions, the main conclusion, derived from the combination of results of the present paper and those of Amir and Lazzati (2011), is that Rohlfs' call for government-mandated interconnection as a blanket policy tool is clearly justified, both on grounds of enhanced prospects for viability for new network industries, and of superior market performance for more mature industries (see also Shapiro and Varian, 1998). Another conclusion of interest is that in industries where interconnection is difficult to achieve and viability is likely to be an issue, monopoly may well be an appropriate policy goal. This provides possible new grounds for natural monopoly.

5 Appendix

Proof of Theorem 1

(a) Let the expected size of the network be s for each firm. From Amir and Lambson (2000), we know that, due to (A3), a unique and symmetric Cournot equilibrium exists for all n and s. Let the corresponding per-firm equilibrium output be $q_n(s)$. By the upper hemi-continuity of the equilibrium correspondence for strategic games (e.g., Fudenberg and Tirole, 1991), $q_n(s)$ is upper hemi-continuous as a correspondence. Since it is also single-valued, it must be a continuous function. Finally, by the (smooth) Implicit Function Theorem and the fact that P is C^2, $q_n(s)$ is C^1 in s.

(b) Since the set of RECE coincides with the set of fixed points of $q_n(s)$, the existence of a RECE follows directly from Brouwer's fixed point theorem applied to the function $q_n(s)$, which is defined from $[0, K]$ to itself. $\square$

Proof of Lemma 1 By definition, an individual (and hence, industry) output of 0 is a symmetric RECE if $0 \in x(0,0)$. This holds if and only if $\pi(0,0,0) \geq \pi(x,0,0) \forall x \in [0,K]$, i.e., $0 \geq xP(x,0) - cx \forall x \in [0,K]$. Moreover, $q_n(0) = 0$ if and only if $\pi(0,0,0) \geq \pi(x,0,0) \forall x \in [0,K]$ if and only if $q_{n+1}(0) = 0$. $\square$

The following lemmas will be useful to prove Theorem 2.

Lemma 3 *The function $q_n(s)$ is differentiable in s, and, if $q_n(s) \in (0,K)$ for $s > 0$, one has*

$$\frac{\partial q_n(s)}{\partial s} = -\frac{q_n P_{12}(nq_n(s), s) + P_2(nq_n(s), s)}{(n+1)P_1(nq_n(s), s) + nq_n P_{11}(nq_n(s), s)}. \tag{4}$$

In particular, if $q_n(0) = 0$,

$$\frac{\partial q_n(0)}{\partial s} = -\frac{P_2(0,0)}{(n+1)P_1(0,0)}. \tag{5}$$

Proof of Lemma 3 If $q_n(s)$ is interior, it satisfies the first order condition

$$P(nq_n(s), s) + q_n(s)P_1(nq_n(s), s) - c = 0. \tag{6}$$

Since $q_n(s)$ is C^1, one can differentiate both sides of equation (6) with respect to s. Reordering terms yields (4). Evaluating at $s = 0$ and $q_n(0) = 0$ yields (5). $\square$

Let $\Pi(z,s) \triangleq \frac{n-1}{n}\left[\int_0^z P(t,s)dt - cz\right] + \frac{1}{n}[zP(z,s) - cz]$, a weighted average of welfare and industry profits when s is the same for all firms. Similar to Amir and Lazzati (2011), we have the following result relating, for given s, argmax's of $\Pi(z,s)$ and symmetric Cournot equilibria.

Lemma 4 *If $z^* \in \arg\max\{\Pi(z,s), 0 \leq z \leq nK\}$, then, $x^* \equiv \frac{z^*}{n} \in q_n(s)$, for all $n \in N$ and $s \in [0,K]$.*

Proof of Lemma 4 Since the cost function is linear, it is convex. By Amir and Lazzati (2011), Lemma 14, for any $n \in N$ and $s \in [0,K]$, if $z^* \in \arg\max\{\Pi(z,s), 0 \leq z \leq nK\}$, then, $z^* \in Q_n(s)$, where $Q_n(s)$ is the total output equilibrium correspondence for a given s. Then, by symmetry,

$z^* \in Q_n(s)$ implies that $x^* \equiv \frac{z^*}{n} \in q_n(s)$. $\square$

Proof of Theorem 2

(i) If the trivial outcome is not part of the equilibrium set, Theorem 1 guarantees there is a symmetric RECE with strictly positive individual output.

(ii) Parts (ii) and (iii) use the following argument. By the proof of Theorem 1, $q_n(s)$ is C^1 and maps [0,K] into itself. In addition, suppose that there exists $s' \in (0, K)$ such that $q_n(s') > s'$, then, by Brouwer's fixed point theorem, it exists at least one fixed point, say s'', such that $s'' > s'$ and hence, $s'' > 0$, i.e., there exists a non-trivial symmetric RECE, s''. Therefore, we only need to show that such s' exists; to this end, it suffices to have $q_n'(0) > 1$, since it implies that there is a small $\epsilon > 0$ for which $q_n(\epsilon) > \epsilon$. By hypothesis, $q_n(0) = 0$ and by Lemma 3, $q_n'(0) > 1$ if $(n+1)P_1(0,0) + P_2(0,0) > 0$, which proves our result.

(iii) The third condition in this part implies that $\Pi_1(z, s) \geq 0$ for some $s \in (0, K]$ and for all $z \leq ns$, i.e., there exist $s \in (0, K]$ and $z' \geq ns$ such that $\Pi(z', s) \geq \Pi(z, s)$ for all $z \leq ns$. Hence, the largest argmax of $\Pi(z, s)$, say z^*, must be greater than or equal to ns, i.e., $z^* \geq ns$ and $z^*/n \geq s$. By Lemma 4, $z^*/n \in q_n(s)$ so there is an $s \in (0, K]$ such that an element of $q_n(s)$ is greater or equal than s. By the argument in part (ii), it follows that a non-trivial symmetric equilibrium exists for n firms. $\square$

Proof of Proposition 1 From Amir and Lambson (2000), Theorem 2.3, $q_n(s)$ is decreasing in n for each fixed s when P log-concave (this is just the condition for the regular Cournot game to be of strategic substitutes, for each fixed s). Consequently, the viability of the industry decreases in n (since the critical mass increases). $\square$

Proof of Proposition 3 By Proposition 1, if an industry with n incompatible networks is viable, the industry with one incompatible network is too. In other words, $\bar{q}_1(s)$ has a fixed point different than zero which is also a non-trivial RECE for the monopolist with complete compatibility. Hence, the viability of the n-oligopoly with incompatible networks implies the viability of a monopolist in the single-network model. By Amir and Lazzati (2011) Theorem 7, the viability of the industry with complete compatibility increases in n, which completes our proof. $\square$

Proof of Lemma 2 By Lemma 1 in Amir and Lazzati (2011), and (A1)-(A4), every selection of the best-response correspondence $z(y,s)$ increases in both y and s. Then, the correspondence

$$B_s : [0,(n-1)K] \to 2^{[0,(n-1)K]},$$

$$y \to \frac{n-1}{n} z(y,s)$$

has a unique fixed point, which corresponds to the symmetric Cournot equilibrium (Amir and Lambson, 2000). By Milgrom and Roberts (1990), such fixed point, say $y_n(s)$, increases in s. Hence, by symmetry and $z(y,s)$ increasing in s, the function (by Proof of Theorem 1) $q_n : [0,K] \longrightarrow [0,K]$, $q_n(s) = y_n(s)/(n-1)$, is increasing in s. $\square$

Proof of Proposition 4

(a) Since $q_n(s)$ decreases in n (proof of Prop. 1), its largest fixed point $\bar{x}_n$ is decreasing in n.

(b) At any interior equilibrium, $\bar{z}_n$ must satisfy the first order condition

$$P(\bar{z}_n, \bar{z}_n/n) + \frac{\bar{z}_n}{n} P_1(\bar{z}_n, \bar{z}_n/n) - c = 0, \tag{7}$$

which implies that $\bar{z}_n$ is the maximal zero of the function

$$F(z;n) \triangleq P(z, z/n) + \frac{z}{n} P_1(z, z/n) - c.$$

Treating n as a real variable for simplicity, it is easy to see that $\frac{\partial F(z;n)}{\partial n} = -\frac{z}{n^2}\Delta_2(z)$, which also holds when n is discrete. Hence, the direction of change in $\bar{z}_n$ as n increases is given by the sign of $-\Delta_2(z)$.

(c) (i) Consider

$$\begin{aligned} P(\bar{z}_n, \bar{x}_n) &\geq P(\bar{z}_{n+1}, \bar{x}_n) \text{ since } \bar{z}_{n+1} \geq \bar{z}_n \\ &\geq P(\bar{z}_{n+1}, \bar{x}_{n+1}) \text{ since } \bar{x}_n \geq \bar{x}_{n+1.} \end{aligned}$$

(ii) Differentiating (7) w.r.t. n and collecting terms yields (with all the P terms evaluated at $(\bar{z}_n, \bar{z}_n/n)$)

$$\frac{d\bar{z}_n}{dn} = \bar{x}_n \frac{P_1 + P_2 + \bar{x}_n P_{12}}{(n+1)P_1 + P_2 + n\bar{x}_n P_{11} + \bar{x}_n P_{12}}. \tag{8}$$

Substituting (8) in $d\bar{P}_n/dn = P_1(\bar{z}_n, \bar{z}_n/n)\frac{d\bar{z}_n}{dn} - P_2(\bar{z}_n, \bar{z}_n/n)(\frac{nd\bar{z}_n/dn - \bar{z}_n}{n^2})$ and collecting terms yields

$$\frac{d\bar{P}_n}{dn} = \frac{\bar{x}_n[P_1^2 + \bar{x}_n(P_1P_{21} - P_2P_{11})]}{(n+1)P_1 + P_2 + n\bar{x}_nP_{11} + \bar{x}_nP_{12}}. \tag{9}$$

If interior, the largest equilibrium is stable (in the sense of best reply Cournot dynamics), so that the denominator in (9) is < 0. It follows that $\frac{dP_n}{dn}$ has the opposite sign from the sign of the numerator, or of $P_1^2 + \bar{x}_n(P_1P_{21} - P_2P_{11})$, as claimed.

(d) Consider

$$\begin{aligned}
\bar{\pi}_n &= \bar{x}_n(\bar{P}_n - c) \\
&\geq \bar{x}_n(\bar{P}_{n+1} - c) \text{ since } \bar{P}_n \geq \bar{P}_{n+1} \\
&\geq \bar{x}_{n+1}(\bar{P}_{n+1} - c) = \bar{\pi}_{n+1} \text{ since } \bar{x}_n \geq \bar{x}_{n+1}. \ \square
\end{aligned}$$

Proof of Proposition 5

(a) If $\bar{P}_n \geq \bar{P}_{n+1}$ and $\bar{z}_n \geq \bar{z}_{n+1}$, then

$$\begin{aligned}
n\bar{\pi}_n &= \bar{z}_n\left\{\bar{P}_n - c\right\} \\
&\geq \bar{z}_{n+1}\left\{\bar{P}_{n+1} - c\right\} \\
&= (n+1)\bar{\pi}_{n+1}.
\end{aligned}$$

Now assume that $\bar{z}_n \leq \bar{z}_{n+1}$, and consider the following steps

$$\begin{aligned}
\bar{\pi}_n &= \bar{x}_n\left\{P(n\bar{x}_n, \bar{x}_n) - c\right\} \\
&\geq \frac{n+1}{n}\bar{x}_{n+1}\left\{P\left[\frac{n+1}{n}\bar{x}_{n+1} + (n-1)\bar{x}_n, \bar{x}_n\right] - c\right\} \\
&\geq \frac{n+1}{n}\bar{x}_{n+1}\left\{P\left[\frac{n+1}{n}\bar{x}_{n+1} + (n-1)\frac{n+1}{n}\bar{x}_{n+1}, \bar{x}_n\right] - c\right\} \\
&= \frac{n+1}{n}\bar{x}_{n+1}\left\{P\left[(n+1)\bar{x}_{n+1}, \bar{x}_n\right] - c\right\} \\
&\geq \frac{n+1}{n}\bar{x}_{n+1}\left\{P\left[(n+1)\bar{x}_{n+1}, \bar{x}_{n+1}\right] - c\right\} \\
&= \frac{n+1}{n}\bar{\pi}_{n+1}.
\end{aligned}$$

The first inequality follows by the Cournot property. The second one, by the assumptions $\bar{z}_{n+1} = (n+1)\bar{x}_{n+1} \geq n\bar{x}_n = \bar{z}_n$ and (A1), $P_1(z, s) < 0$. The last inequality holds by Proposition 4 (a) and $P_2(z, s) > 0$.

(b) Consider

$$\begin{aligned} CS_{n+1} - CS_n &= \int_0^{\bar{z}_{n+1}} \{P(t, \bar{x}_{n+1}) - P(\bar{z}_{n+1}, \bar{x}_{n+1})\}dt - \int_0^{\bar{z}_n} \{P(t, \bar{x}_n) - P(\bar{z}_n, \bar{x}_n)\}dt \\ &\geq \int_0^{\bar{z}_n} \{P(t, \bar{x}_{n+1}) - P(\bar{z}_{n+1}, \bar{x}_{n+1}) - P(t, \bar{x}_n) + P(\bar{z}_n, \bar{x}_n)\}dt \\ &\geq \int_0^{\bar{z}_n} \{P(t, \bar{x}_{n+1}) - P(\bar{z}_n, \bar{x}_{n+1}) - P(t, \bar{x}_n) + P(\bar{z}_n, \bar{x}_n)\}dt \\ &\geq 0. \end{aligned}$$

The first and second inequalities follow from $P_1(z, s) < 0$ and $\bar{z}_{n+1} \geq \bar{z}_n$, and the last inequality from $\bar{x}_{n+1} \leq \bar{x}_n$ and $P_{12}(z, s) \geq 0$ for all z, s, which implies that $P(\bar{z}_n, \bar{x}_n) - P(t, \bar{x}_n) \geq P(\bar{z}_n, \bar{x}_{n+1}) - P(t, \bar{x}_{n+1})$ for all $t \leq \bar{z}_n$.

(c) Consider

$$\begin{aligned} \bar{W}_{n+1} - \bar{W}_n &= \int_0^{\bar{z}_{n+1}} \{P(t, \bar{x}_{n+1}) - c\}dt - \int_0^{\bar{z}_n} \{P(t, \bar{x}_n) - c\}dt \\ &\geq \int_0^{\bar{z}_n} \{P(t, \bar{x}_{n+1}) - c\}dt - \int_0^{\bar{z}_n} \{P(t, \bar{x}_n) - c\}dt \\ &= \int_0^{\bar{z}_n} \{P(t, \bar{x}_n) - P(t, \bar{x}_{n+1})\}dt \\ &\geq 0 \end{aligned}$$

The first inequality is due to $\bar{z}_{n+1} \geq \bar{z}_n$, and the last one to Proposition 4 (a), $\bar{x}_{n+1} \leq \bar{x}_n$, and $P_2(z, s) > 0$.□

Proof of Proposition 6

(i) The (largest) RECE of the oligopoly with incompatible networks is the (largest) fixed point of $\bar{q}_n(s)$, say $\bar{q}_n(s') = s'$. Hence, $\overline{x}_n^I = s'$.

On the other hand, the (largest) RECE of the oligopoly with a single-network maybe seen as the (largest) intersection point of $\bar{q}_n(s)$ with the line (through the origin) s/n, say $\bar{q}_n(s'') = s''/n$. Hence, $\overline{x}_n^C = \bar{q}_n(s'')$.

Since $\bar{q}_n(s)$ is increasing in s, it is easy to see that $\overline{x}_n^C = \bar{q}_n(s'') = s''/n \geq \overline{x}_n^I = \bar{q}_n(s') = s'$.

It follows that industry output goes the same way, i.e., $\overline{z}_n^C = n\overline{x}_n^C \geq \overline{z}_n^I = n\overline{x}_n^I$.

(ii) Consider the following inequalities

$P_n^C = P(n\overline{x}_n^C, n\overline{x}_n^C)$

$\geq P(n\bar{x}_n^I, n\bar{x}_n^I)$

$\geq P(n\bar{x}_n^I, \bar{x}_n^I)$.

The first inequality follows by the assumption that $\Delta_3(\cdot) \geq 0$ and Proposition 6 part (i); the second one, by the assumption that $P_2(z,s) > 0$. $\square$

Proof of Proposition 7

(i) Consider the following inequalities

$$\begin{aligned}
\pi_n^C &= \bar{x}_n^C P(\bar{x}_n^C + \bar{y}_n^C, \bar{x}_n^C + \bar{y}_n^C) - c\bar{x}_n^C \\
&\geq \bar{x}_n^I P(\bar{x}_n^I + \bar{y}_n^C, \bar{x}_n^C + \bar{y}_n^C) - c\bar{x}_n^I \\
&\geq \bar{x}_n^I P(\bar{x}_n^C + \bar{y}_n^C, \bar{x}_n^C + \bar{y}_n^C) - c\bar{x}_n^I \\
&\geq \bar{x}_n^I P(\bar{x}_n^I + \bar{y}_n^I, \bar{x}_n^I + \bar{y}_n^I) - c\bar{x}_n^I \\
&\geq \bar{x}_n^I P(\bar{x}_n^I + \bar{y}_n^I, \bar{x}_n^I) - c\bar{x}_n^I \\
&= \pi_n^I.
\end{aligned}$$

The first inequality follows by Cournot property; the second one, by (A1), $P_1(z,s) < 0$, and Proposition 6 part (i), $\bar{x}_n^C \geq \bar{x}_n^I$; the third one is true by the assumption that $\Delta_3(\cdot) \geq 0$ and $\bar{x}_n^C \geq \bar{x}_n^I$, and the last one, by (A1), $P_2(z,s) > 0$.

(ii) Consider the following inequalities

$$\begin{aligned}
CS_n^C - CS_n^I &= \int_0^{n\bar{x}_n^C} [P(t, n\bar{x}_n^C) - P(n\bar{x}_n^C, n\bar{x}_n^C)]\ dt - \int_0^{n\bar{x}_n^I} [P(t, \bar{x}_n^I) - P(n\bar{x}_n^I, \bar{x}_n^I)]dt \\
&\geq \int_0^{n\bar{x}_n^I} [P(t, n\bar{x}_n^C) - P(n\bar{x}_n^C, n\bar{x}_n^C)]\ dt - \int_0^{n\bar{x}_n^I} [P(t, \bar{x}_n^I) - P(n\bar{x}_n^I, \bar{x}_n^I)]\ dt \\
&\geq \int_0^{n\bar{x}_n^I} [P(t, n\bar{x}_n^C) - P(n\bar{x}_n^C, n\bar{x}_n^C)]\ dt - \int_0^{n\bar{x}_n^I} [P(t, \bar{x}_n^I) - P(n\bar{x}_n^C, \bar{x}_n^I)]\ dt \geq 0.
\end{aligned}$$

The first inequality follows by Proposition 6 part (i) and $P_1(z,s) < 0$.

Under the assumption that $P_n^I \geq P_n^C$ at the highest equilibrium, the second line becomes positive given that $n\bar{x}_n^C \geq \bar{x}_n^I$ and $P_2(z,s) > 0$, which proves the first part of the result.

For the second part, notice that the second inequality follows from $n\bar{x}_n^C \geq n\bar{x}_n^I$ and $P_1(z,s) < 0$, and the last one by the assumption that $P_{12}(z,s) \leq 0$. To see this, notice that $t \in [0, n\bar{x}_n^I]$ and

$n\bar{x}_n^C \geq n\bar{x}_n^I$ imply that $n\bar{x}_n^C \geq t$, thus, adding the assumption that $P_{12}(z,s) \leq 0$ and the result that $n\bar{x}_n^C \geq \bar{x}_n^I$ imply that $P(t, n\bar{x}_n^C) - P(t, \bar{x}_n^I) \geq P(n\bar{x}_n^C, n\bar{x}_n^C) - P(n\bar{x}_n^C, \bar{x}_n^I)$ for all $t \in [0, n\bar{x}_n^I]$, which gives us the result.

(iii) First notice that the social welfare function at any production level x and expected size of the network s, when there are n symmetric firms is given by

$$V_n(x,s) = \int_0^{nx} P(t,s)\, dt - ncx,$$

which is a concave function with respect to x since $\frac{\partial^2 V_n(x,s)}{\partial x^2} = n^2 P_1(nx,s) < 0$, by (A1). Then, we have that at the highest equilibria

$$\begin{aligned}
W_n^C - W_n^I &= \left\{ \int_0^{n\bar{x}_n^C} P(t, n\bar{x}_n^C)\, dt - nc\bar{x}_n^C \right\} - \left\{ \int_0^{n\bar{x}_n^I} P(t, \bar{x}_n^I)\, dt - nc\bar{x}_n^I \right\} \\
&\geq \left\{ \int_0^{n\bar{x}_n^C} P(t, n\bar{x}_n^C)\, dt - nc\bar{x}_n^C \right\} - \left\{ \int_0^{n\bar{x}_n^I} P(t, n\bar{x}_n^C)\, dt - nc\bar{x}_n^I \right\} \\
&= V_n(\bar{x}_n^C, n\bar{x}_n^C) - V_n(\bar{x}_n^I, n\bar{x}_n^C) \\
&\geq \frac{\partial V_n(\bar{x}_n^C, n\bar{x}_n^C)}{\partial x} (\bar{x}_n^C - \bar{x}_n^I) \\
&= n[P(n\bar{x}_n^C, n\bar{x}_n^C) - c](\bar{x}_n^C - \bar{x}_n^I) \geq 0.
\end{aligned}$$

The first inequality follows by $n\bar{x}_n^C \geq \bar{x}_n^I$ and $P_2(z,s) > 0$; the second one, by concavity of $V_n(\cdot, s)$, and the last one, because $P(n\bar{x}_n^C, n\bar{x}_n^C) \geq c$ and $\bar{x}_n^C \geq \bar{x}_n^I$, by Proposition 6 part (i). □

Proof of Proposition 8 Let us define $x_n \equiv x_n^C = x_n^I$ and $z_n = nx_n$; let $0 \leq t \leq z_n$. Then,

$$\begin{aligned}
CS_n^I - CS_n^C &= \int_0^{z_n} [P(t, x_n) - P(z_n, x_n)] dt - \int_0^{z_n} [P(t, z_n) - P(z_n, z_n)] dt \\
&= \int_0^{z_n} [P(t, x_n) - P(t, z_n) - P(z_n, x_n) + P(z_n, z_n)] dt \\
&\geq 0, \text{ by the assumption that } P_{12}(z,s) \geq 0. \ \square
\end{aligned}$$

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