Energy transition with variable and intermittent renewable electricity generation

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1 Introduction

In the short/medium term, renewables cannot be deployed at a large scale to replace coal and other fossil fuels in electricity generation, on the one hand because they are on average still more costly than fossil fuels, and on the other hand because they are both variable, which is predictable (night and day, seasons), and intermittent, which is not (cloud cover, etc.). But in the future the production of electricity has to be decarbonized, and producing energy by renewable means seems to be the only possibility to do so[1], before nuclear fusion becomes eventually available.

The literature considering the penetration of renewables in the energy mix consists so far in two rather separate trends.

On the one hand, macro-dynamic models à la Hotelling consider renewable energy as an abundant and steady flow available with certainty, possibly after an investment in capacity has been made, at a higher unit cost than fossil energy[2]. The issue is the cost –otherwise clean renewable energy would replace polluting fossil fuels immediately. Thus standard models of energy transition ignore variability and intermittency and focus on the cost issue. However, the expansion of renewables will probably not be limited by the direct costs of electricity generation in the (near) future. Costs have already been widely reduced, due to technical progress and learning effects in production and installation, and the decrease is expected to continue, until a limit lower bound of the cost is reached. For instance,

[1] Carbon Capture and Storage (CCS) is another option, but it is still expensive and can only offer a partial solution as the potential carbon sinks are of limited capacity. Moreover, CCS has already been extensively studied. See for instance Lafforgue, Magne and Moreaux (2008).

[2] See, for early path-breaking papers, Hoel and Kverndokk (1996), and Tahvonen (1997).

according to the International Energy Agency (2011), solar PV costs have been reduced by 20% for each doubling of the cumulative installed capacity. The Energy Information Administration reports that the US average levelized cost of electricity in 2012 is $/MWh 95.6 for conventional coal, 66.3 for natural gas-fired combined cycle, 80.3 for terrestrial wind, 204.1 for offshore wind, 130 for solar PV, 243 for solar thermal and 84.5 for hydro. Terrestrial wind and hydro (that we do not consider here because the expansion possibilities are very limited in developed countries) are already competitive, and solar PV is rapidly catching up in the US. In sunny and dry countries it is even more so: solar PV has already obtained grid parity[3] in sunny islands, and is expected to reach grid parity very soon for instance in Italy or California. Hence the real obstacle to a non-marginal expansion of renewables is not their cost but their variability, their intermittency, and maybe also their footprint in terms of land use – especially for wind energy.

Another strand of literature is composed of static models not directly interested in energy transition, but in the design of the electric mix (fossil fuels and renewables) when intermittency is taken into account, with or without storage devices. Ambec and Crampes (2012, 2015) are representative of this literature. They study the optimal electricity mix with intermittent renewable sources, and contrast it to the mix chosen by agents in a decentralized economy where the retailing price of electricity does not vary with its availability. They examine the properties of different public policies and their impacts on renewable penetration in the electric mix: carbon tax, feed-in tariffs, renewable portfolio standards, demand-side management policies.

A recent survey on the economics of solar electricity (Baker *et al.,* 2013) emphasizes the lack of economic analysis of a decentralized clean energy provision through renewable sources. We intend to contribute to fill this lack by putting together the two strands of the literature mentioned above, in order to make macro-dynamic models more relevant for the study of the energy transition. Indeed we believe that energy transition is by essence a dynamic problem, which cannot be fully understood through static models. On the other hand, dynamic models are so far unable to take into account properly some crucial features of renewables. We plan to extend to a dynamic setting the static models cited above, taking into account variability and intermittency, in order to study to what extent they actually constitute a serious obstacle to energy transition.

In a first step we tackle the variability issue alone. We build a stylized deterministic dynamic model of the optimal choice of the electricity mix (fossil and renewable), where the fossil energy, coal, is abundant but CO_2-emitting, and the renewable energy, solar, is variable but clean. The originality of the model is that electricity produced when the renewable source of energy (solar) is available, and electricity produced when it is not, are considered two different goods (say day-electricity and night-electricity). At each period of time, consumers

[3]Grid-parity is reached when the cost of electricity generation with the renewable source is roughly equal to the retailing electricity price.

derive utility from the consumption of the two goods. Considering that there are two different goods allows taking into account intra-day variability. Day-electricity can be produced with coal and/or solar. Night-electricity can be produced with coal, or by the release of day-electricity that has been stored to that effect. Storing energy is costly due to the loss of energy during the restoration process. We consider that coal and solar are available at zero variable costs, in order to focus on the variability and intermittency issues. We also make the assumption that at the beginning of the planning horizon coal-fired power plants already exist so that there is no capacity constraint on the production of electricity by the fossil source, but that the existing solar capacity is small so that investments are to be made in order to build up a sizable capacity. We solve the centralized program under the constraint of a carbon budget that cannot be exceeded and derive an optimal succession of regimes. We show that with a low initial solar capacity it is optimal to first use fossil fuels during night and day, then use fossil fuels during night only and finally go for no fossil fuels at all. The optimal dynamics for the capacity of solar power plants is derived, as well as the optimal amount of electricity stored in time. Simulations allow us to analyze the consequences of improvements in the storage and solar power generation technologies and of a more stringent environmental policy on the optimal investment decisions and energy mix.

In a second step, we introduce intermittency[4]. in the model and study the design of the power system enabling to accommodate it. With intermittency, day-electricity generation by solar power plants becomes uncertain. We consider that there is only partial generation if solar radiations are too weak due for instance to the cloud system, which occurs with a given probability. The succession of regimes is now more complex. It depends on the relative values of the efficiency of the storage technology, the weather pattern and whether there is sun at peak or off-peak periods. We perform simulations to compare the variability-only and the variability and intermittency solutions, and show that intermittency does not matter so much, rejoining there the empirical result of Gowrisankaran *et al.* (2016). Simulations also allow us to do a number of comparative dynamics exercises. We show that a lenient climate policy delays both storage and the switch to clean energy, as does a higher probability of bad weather. Off-peak sun, rather than peak sun, increases storage and hinders consumption.

Finally, we go back to the standard literature that ignores variability and intermittency. Joskow (2011) underlines the mistakes that are made when doing so, particularly in the computation of the levelized cost of electricity. Ignoring variability and intermittency and reasoning on average values gives an undue advantage to renewable sources and leads to taking wrong decisions. We compare here the optimal solution and the solution of the model when variability and intermittency are taken into account only on average. Solar panel accumulation is not very sensitive to variability neither, thanks to storage that smoothes electricity provision. Indeed, we show that less solar panels are accumulated without storage.

[4]This means that clean energy is not only variable but intermittent as well

The structure of the paper is the following. Section 2 sets up the framework, solves the model and studies the sensitivity of the solution to the main parameters in the case where variability only is taken into account. Section 3 introduces intermittency. In Section 4 we compare the previous results with the solution in the case where variability and intermittency are not fully taken into account. Section 5 concludes.

2 Variability in renewable electricity generation

One of the novelties of the paper is that day and night electricity are modeled as two different goods that the representative household wants to consume at each point in time. Energy requirements may be satisfied by fossil sources, let's say coal, at day and night. Coal is abundant and carbon-emitting: the issue with coal extraction and consumption is not scarcity but climate change. There are no extraction costs. Climate policy takes the form of a carbon budget that society decides not to exceed, to have a good chance to maintain the temperature increase at an acceptable level -typically 2°C. This carbon budget is consumed when coal is burned. It is also possible to use a renewable source of energy, abundant but clean, provided that a production capacity is built. This energy is *variable* i.e. changes in a predictable way. We consider for the purpose of illustration that it is solar energy, that can be harnessed at day but not at night. Costly investment allows to increase solar capacity. There exists a storage technology that allows to store imperfectly electricity from day to night at no monetary cost but with a physical loss.

2.1 The optimal solution

The social planner seeks to maximize the discounted sum of the net surplus of the economy. Instantaneous net surplus is the difference between the utility of consuming day and night-electricity and the cost of the investment in solar capacity.[5] Day-electricity can be produced by coal-fired power plants and/or solar plants. A fraction of solar electricity can be stored to be released at night.[6] In addition to fossil electricity, night-electricity can be produced by the release of solar electricity stored during the day, with a loss.[7]

[5]For simplicity and to focus on the variability issue, we ignore the extraction cost of coal and the variable cost of using solar panels. We suppose that a large fossil capacity exists at the beginning of the planning horizon but that the initial solar capacity is low.

[6]It does not make sense to store coal electricity since coal-fired power plants can be operated at night as well and there is no capacity constraint.

[7]For instance, according to Yang (2016), the efficiency of pumped hydroelectric storage (defined as the electricity generated divided by the electricity used to pump water) is lower than 60% for old systems, but over 80% for state-of-the-art ones.

The social planner's programme reads:

$$
\begin{aligned}
&\max \int_0^\infty e^{-\rho t}\left[u\left(e_d(t), e_n(t)\right) - C(I(t))\right] dt \\
&e_d(t) = x_d(t) + (1-a(t))Y(t) \\
&e_n(t) = x_n(t) + ka(t)Y(t) \\
&\dot{X}(t) = x_d(t) + x_n(t) \qquad (-\lambda(t)) \\
&\dot{Y}(t) = I(t) \qquad (\mu(t)) \\
&0 \le a(t) \le 1 \qquad (\underline{\omega}_a(t), \overline{\omega}_a(t)) \\
&X(t) \le \overline{X} \qquad (\omega_X(t)) \\
&x_d(t) \ge 0, x_n(t) \ge 0 \qquad (\omega_d(t), \omega_n(t)) \\
&X_0 \ge 0,\ Y_0 \ge 0 \text{ given}
\end{aligned}
$$

where u is the instantaneous utility function, supposed to have the standard properties, e_d and e_n are respectively day and night-electricity consumption, x_d and x_n are fossil-generated electricity consumed respectively at day and night, X is the stock of carbon accumulated into the atmosphere due to fossil fuel combustion, $\overline{X}$ is the carbon budget i.e. the ceiling on the atmospheric carbon concentration, Y is solar capacity, I is the investment in solar capacity, $C(I)$ is the investment cost function, a is the share of solar electricity produced at day that is stored to be released at night. The efficiency of the storage technology is represented by the parameter $k \in [0,1]$ ($1-k$ is the leakage rate of this technology). ρ is the discount rate.

We make the following assumptions on the utility and investment cost functions: utility is logarithmic and investment cost is quadratic (because of adjustment costs):

$$
\begin{aligned}
u\left(e_d, e_n\right) &= \alpha \ln e_d + (1-\alpha)\ln e_n, \qquad 0 < \alpha < 1 \\
C(I) &= c_1 I + \frac{c_2}{2} I^2, \qquad c_1, c_2 > 0
\end{aligned}
$$

With these assumptions we are able to solve the problem analytically. We obtain the following results.

Proposition 1 *In the case where only variability of renewable energy is taken into account and the initial solar capacity is low, the optimal solution consists in 4 phases:*

(1) production of day and night-electricity with fossil fuel-fired power plants complemented at day by solar plants, no storage, investment in solar panels to increase solar capacity (from 0 to $\underline{T}$*);*

(2) production of day-electricity with solar plants only, use of fossil fuel-fired power plants at night while proceeding with the building up of solar capacity, no storage (from $\underline{T}$ *to* T_i*);*

(3) production of day-electricity with solar plants only, use of fossil fuel-fired power plants at night while proceeding with the building up of solar capacity, progressive increase of storage from 0 to its maximal value, which depends on preferences for day and night-electricity (from T_i to $\overline{T}$);

(4) production of day and night-electricity with solar plants only, storage at its maximum value at day to produce night-electricity, and investment in solar panels to increase capacity, up to a steady state (from $\overline{T}$ to ∞). This last phase begins when the carbon budget is exhausted.

Proof. See Appendix A. ■

Proposition 1 shows that it is always optimal to begin installing solar panels immediately and to use them to complement fossil energy at daytime. However, it is never optimal to begin storing immediately. Storage would allow saving fossil energy at night, but at the expense of more fossil at day to compensate for the solar electricity stored; it would also cause a physical loss of electricity. Even if the storage technology is available, as it is the case in the model, storage must only begin after fossil has been abandoned at day because the installed solar capacity has become high enough. Full storage coincides with the final abandonment of fossil.

We show analytically in Appendix A that the 4 phases identified in Proposition 1 are characterized by the following equations.

- Evolution of the shadow value λ of the atmospheric carbon stock (the carbon value) before the carbon budget is exhausted:

$$\lambda(t) = \lambda(0)e^{\rho t} \tag{1}$$

- Evolution of solar capacity over the whole horizon:

$$\dot{Y}(t) = \frac{1}{c_2}(\mu(t) - c_1) \tag{2}$$

- Evolution of the value of solar capacity in each phase:

$$\begin{aligned} &\text{Phase (1)} \qquad \dot{\mu}(t) = \rho\mu(t) - \lambda(t) \\ &\text{Phase (2)} \qquad \dot{\mu}(t) = \rho\mu(t) - \frac{\alpha}{Y(t)} \\ &\text{Phase (3)} \qquad \dot{\mu}(t) = \rho\mu(t) - k\lambda(t) \\ &\text{Phase (4)} \qquad \dot{\mu}(t) = \rho\mu(t) - \frac{1}{Y(t)} \end{aligned} \tag{3}$$

- Fossil fuel use, storage and total electricity consumption in each phase:

$$
\begin{array}{lllll}
\text{Phase (1)} & x_d(t) = \dfrac{\alpha}{\lambda(t)} - Y(t) & x_n(t) = \dfrac{1-\alpha}{\lambda(t)} & a(t) = 0 & \\
 & e_d(t) = \dfrac{\alpha}{\lambda(t)} & e_n(t) = \dfrac{1-\alpha}{\lambda(t)} & & \\
\text{Phase (2)} & x_d(t) = 0 & x_n(t) = \dfrac{1-\alpha}{\lambda(t)} & a(t) = 0 & (4) \\
 & e_d(t) = Y(t) & e_n(t) = \dfrac{1-\alpha}{\lambda(t)} & & \\
\text{Phase (3)} & x_d(t) = 0 & x_n(t) = \dfrac{1}{\lambda(t)} - kY(t) & a(t) = 1 - \dfrac{\alpha}{k\lambda(t)Y(t)} & \\
 & e_d(t) = \dfrac{\alpha}{k\lambda(t)} & e_n(t) = \dfrac{1-\alpha}{\lambda(t)} & & \\
\text{Phase (4)} & x_d(t) = 0 & x_n(t) = 0 & a^* = 1-\alpha & \\
 & e_d(t) = \alpha Y(t) & e_n(t) = (1-\alpha)Y(t) & &
\end{array}
$$

The carbon value follows the Hotelling rule (Eq.(1)).

The joint paths of solar capacity Y and its shadow value μ are determined by the four dynamic systems composed of Eq.(2) and each of the equations in system (4). They are represented on the phase diagram on Figure 1, constructed for a given $\lambda(0)$ (that reflects the stringency of the climate constraint). The last phase is a saddle path leading to a steady state $\mu^* = c_1$, $Y^* = 1/(\rho c_1)$. Moving backward from the steady state along the stable branch, date $\overline{T}$ is reached. The relevant path then corresponds to phase (3) where the dynamics of Y and μ are independent, until date T_i is reached. From T_i on, the saddle path of phase (2), corresponding to the steady state $\mu^* = c_1$, $Y^{**} = \alpha/(\rho c_1)$ is followed until date $\underline{T}$ (the steady state is never reached).[8] After $\underline{T}$ the path followed corresponds to phase (1) where Y and μ are independent, until date 0. The initial shadow value of solar panels, $\mu(0)$ then obtained (as a function of $\lambda(0)$) matches the initial condition $Y(0) = Y_0$.

For a given value of $\lambda(0)$, the joint evolutions of λ and Y trigger the phase switchings, i.e. gives dates $\overline{T}$, T_i and $\underline{T}$ (see Figure 2 and Eqs. (1) and (2)).

The climate constraint then pins down $\lambda(0)$.

In phase (1), $e_d(t) = \alpha/\lambda(t)$ and $e_n(t) = (1-\alpha)/\lambda(t)$: electricity consumptions are only driven by the carbon value, i.e. by climate policy. In phase (2) (when there is no storage), it is still the case for night electricity consumption, but during daytime, electricity only depends on installed solar capacity.[9] In phase (3) storage occurs and no fossil is used

[8] Phase (2) does not exist if there is no loss in the storage technology ($k = 1$).

[9] It implies that there is overcapacity of fossil generation during daytime.

during daytime; but due to storage (driven by the climate constraint), it is again only the climate constraint that determines electricity consumption at each period. In phase (4), when electricity production is totally carbon-free, electricity consumption night and day only depends on installed solar capacity. The amount of electricity stored at day to be consumed at night only depends on the preference for night-electricity: $a^* = 1 - \alpha$. If $\alpha > 1/2$, consumers prefer consuming at day, when there is sun. This means that peak time consumption coincides with the availability of solar electricity. It is obviously the most favorable case. If on the contrary $\alpha < 1/2$, sun is shining at off-peak time. This would correspond to the Californian duck documented by CAISO[10] and recently analyzed by Fowlie[11] and Wolfram[12]

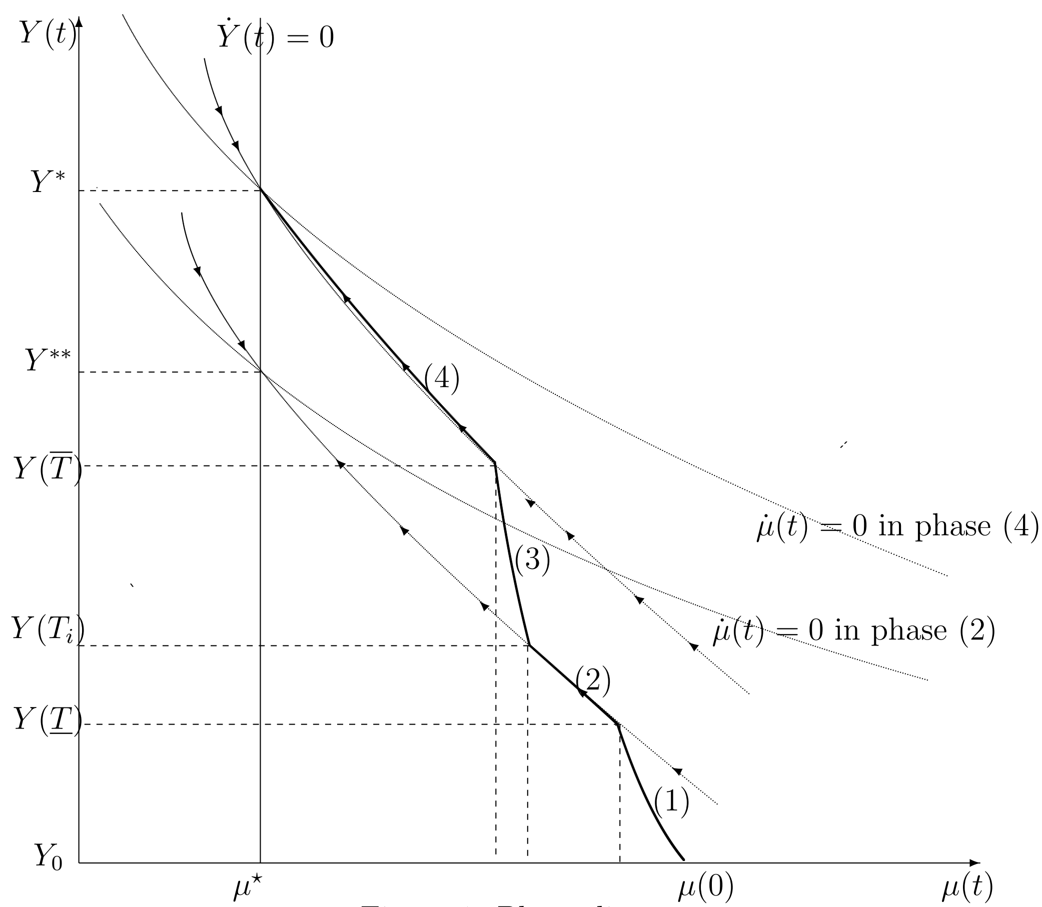


Figure 1: Phase diagram

[10]What the duck curve tells us about managing a green grid, https://www.caiso.com/Documents/FlexibleResourcesHelpRenewables_FastFacts.pdf

[11]See "The duck has landed", the Energy Institute Blog, https://energyathaas.wordpress.com/2016/05/02/the-duck-has-landed/

[12]See "What's the Point of an Electricity Storage Mandate?" https://energyathaas.wordpress.com/2013/07/29/whats-the-point-of-an-electricity-storage-mandate/

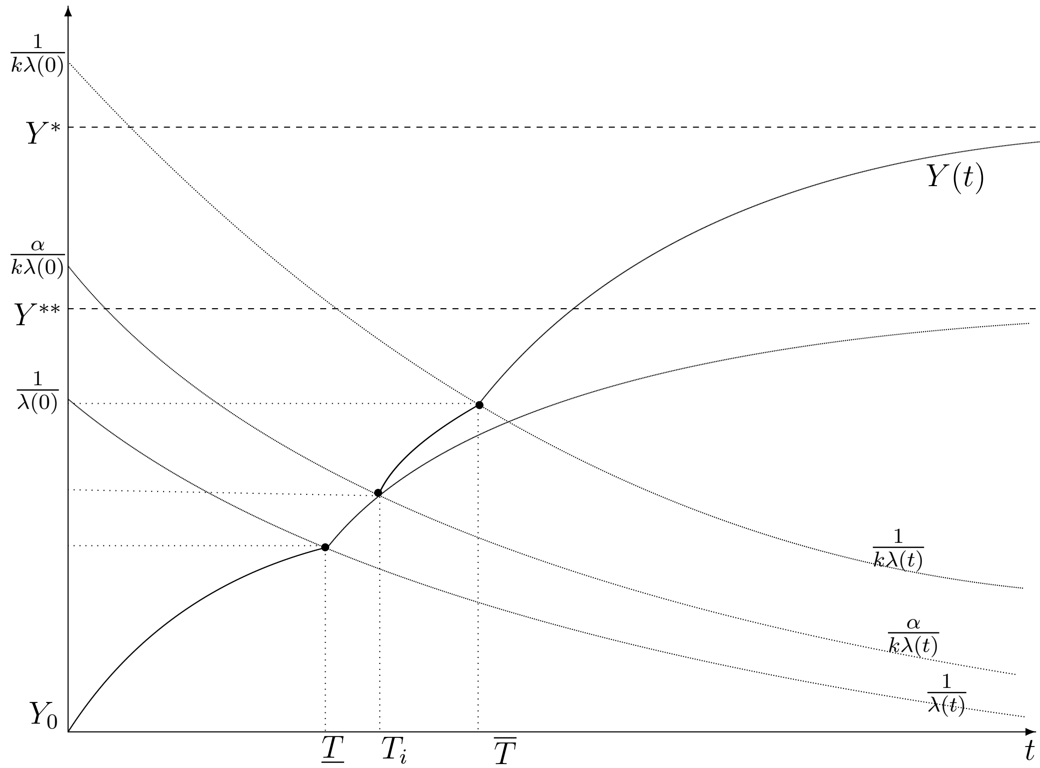


Figure 2: Solar capacity and carbon value before the ceiling

2.2 Numerical illustrations

We now perform some comparative dynamics exercises, to assess the impact of the stringency of climate policy and of the value of the parameters on the level and the time profile of electricity consumption, storage and solar capacity. The parameters of the reference simulation are given in Table 1. They are chosen for illustrative purposes only, without any pretension of realism.

ρ	k	α	c_1	c_2	Y_0	X_0	$\overline{X}$
0.04	0.6	0.8	1	20	0	0	50

Table 1: Parameters in the reference simulation, variability only

In the reference simulation, day-electricity consumption is W-shaped (V-shaped if $k = 1$). It is first decreasing because of the rise of the carbon value (phase (1)); it is then increasing as fossil fuel is abandoned at day and more solar panels are installed (phase (2)); next, storage begins and increases, at the expense of day-electricity consumption, which decreases (phase (3)); finally, the increasing use of solar panels joint with a constant share of day-electricity stored generates a rise in day-electricity consumption (phase (4)). Night-electricity consumption is V-shaped: it is driven by the carbon value i.e. climate policy while fossil energy is used at night, hence decreasing (phases (1)-(3)); then, when fossil fuel is abandoned at night, it increases with the stock of solar panels and the development of storage.

The comparative statics exercises results are represented on figures 3 to 5.

- *Less stringent climate policy* (figure 3):

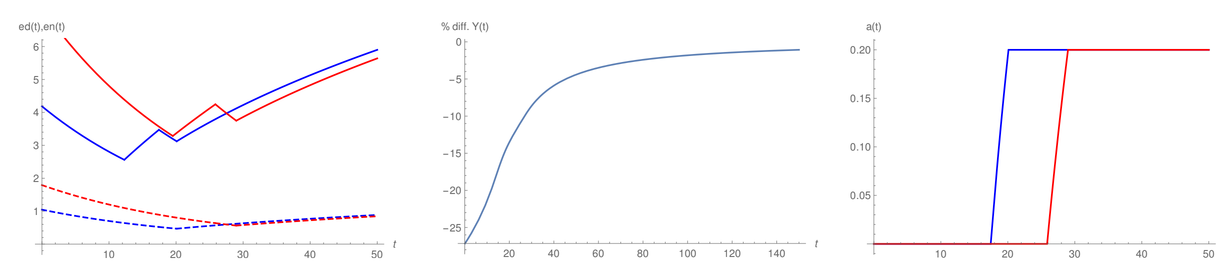
  In the short run energy consumption at day and night are higher than in the reference case, storage occurs later, the switch to clean energy is postponed. In the medium run energy consumption becomes lower than in the reference case, because investment in solar panels has been lower: there is an hysteresis effect. Even in the absence of explicit damages due to climate change, a lenient climate policy has adverse effects in the long run because it delays investment in clean energy.

- *Less efficient storage technology* (figure 4):

  A less efficient storage technology translates in the model in a higher loss rate $1-k$. Then, the date at which storage begins is postponed, which allows to consume more at day in phase (2). Of course night-electricity consumption is smaller. Again an hysteresis effect appears: consumption is lower in the long run, because the development of solar panels has been slower.

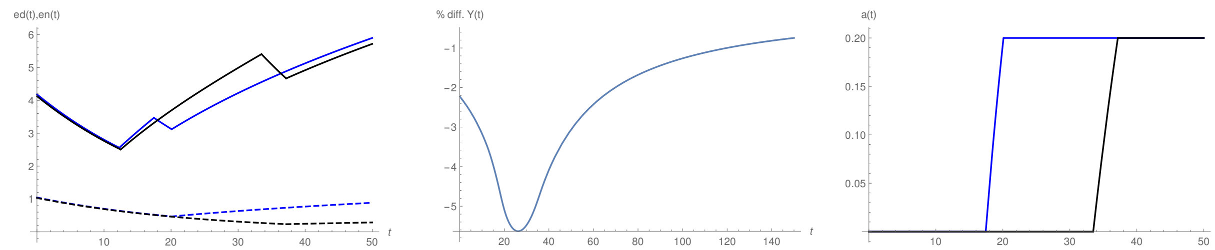
- *Off-peak sun* (figure 5):

  In the reference simulation, consumers prefer to consume electricity when the sun is shining and solar panels can harness its radiation, i.e. when there is sun at peak time. We make in this simulation the opposite assumption: consumers prefer to consume electricity when there is no sun (it corresponds to the Californian duck case). Clearly, the situation is now less favorable. At each date, total electricity consumption (over day and night) is reduced. The date at which fossil is not used at day anymore is brought forward so that fossil consumption at night may be higher, and storage occurs earlier. The long run level of storage has to be higher, which means more overall electricity loss.



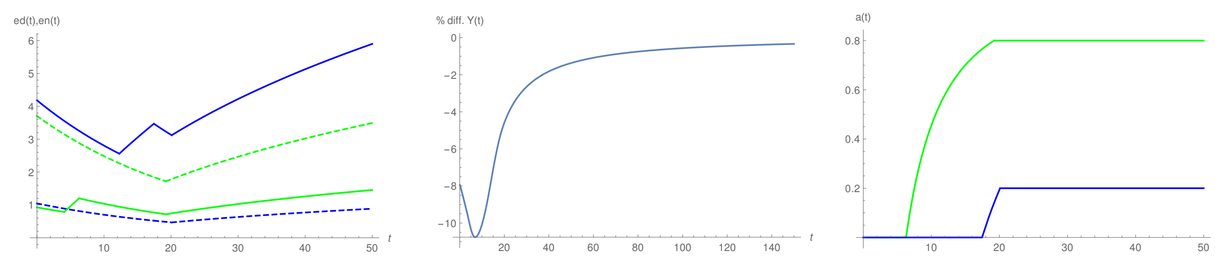
Energy consumption day (plain) and night (dashed) | Solar capacity (% diff.) | Storage

Figure 3. Effect of a less stringent climate policy under variability only ($\overline{X} = 50$ in blue and $\overline{X} = 100$ in red)



Energy consumption day (plain) and night (dashed)	Solar capacity (% diff.)	Storage

Figure 4. Effect of a less efficient storage technology under variability only ($k = 0.6$ in blue and $k = 0.2$ in black)



Energy consumption day (plain) and night (dashed)	Solar capacity (% diff.)	Storage

Figure 5. Effect of a smaller preference in day electricity under variability only ($\alpha = 0.8$ in blue and $\alpha = 0.2$ in green)

3 Intermittency in renewable electricity generation

We now account for the fact that renewable electricity generation is not only variable but *intermittent* as well, i.e. some of its variations are not predictable. For the purpose of illustration, we again consider the example of solar energy. Solar radiations can be fully harnessed during the day if there is sun, but can only be partially harnessed during the day if there are clouds. No harnessing can happen during the night. As only the expected dynamics of pollution is known, the ceiling constraint on the stock of carbon into the atmosphere is now in expectation.

3.1 The optimal solution

During the day, the weather is sunny with a probability q and solar panels are then producing electricity at full capacity Y. With a probability $(1-q)$ the weather is cloudy and solar panel only produce ϕY electricity with $0 < \phi < 1$. As before, there exists a storage technology that allows to store imperfectly electricity from day to night at no monetary cost but with a physical loss. With intermittency, the amount stored depends on the weather: a^u denotes storage when the sun is shining, while storage when there are clouds in noted a^l. Accordingly, fossil-generated electricity consumption and total electricity consumption are denoted with superscripts u and l depending on the weather. The social planner's programme becomes:

$$\max \int_0^\infty e^{-\rho t} \left[qu\left(e_d^u(t), e_n^u(t)\right) + (1-q)u\left(e_d^l(t), e_n^l(t)\right) - C(I(t))\right] dt$$

$$\begin{aligned}
e_d^u(t) &= x_d^u(t) + (1-a^u(t))Y(t),\ \ e_d^l(t) = x_d^l(t) + (1-a^l(t))\phi Y(t) \\
e_n^u(t) &= x_n^u(t) + ka^u(t)Y(t),\ \ e_n^l(t) = x_n^l(t) + ka^l(t)\phi Y(t) \\
E(\dot{X}(t)) &= q\left(x_d^u(t) + x_n^u(t)\right) + (1-q)\left(x_d^l(t) + x_n^l(t)\right) \qquad\qquad (-\lambda(t)) \\
\dot{Y}(t) &= I(t) \qquad\qquad (\mu(t)) \\
0 \le a^u(t) &\le 1,\ \ 0 \le a^l(t) \le 1 \qquad\qquad (\underline{\omega}_a^u(t), \overline{\omega}_a^u(t)),\ \ (\underline{\omega}_a^l(t), \overline{\omega}_a^l(t)) \\
E(X(t)) &\le \overline{X} \qquad\qquad (\omega_X(t)) \\
x_d^u(t) \ge 0, x_n^u(t) &\ge 0, x_d^l(t) \ge 0, x_n^l(t) \ge 0 \qquad\qquad (\omega_d^u(t), \omega_n^u(t), \omega_d^l(t), \omega_n^l(t)) \\
X_0 &\ge 0,\ \ Y_0 \ge 0 \text{ given}
\end{aligned}$$

The characteristics of the optimal solution are described in Proposition 2. The proofs are collected in Appendix B.

Proposition 2 *When the intermittency of renewable energy is taken into account and the initial solar capacity is low, the optimal solution is composed of seven phases (see Figure 6).*

(1) Fossil is used night and day whatever the state of nature, complemented by some solar at day. There is no storage. Investment takes place to build-up solar production capacity.

(2) Solar capacity is large enough for fossil to be dropped at day in the good state of nature but not in the bad state. There is still no storage.

(3) Solar capacity is large enough either to drop fossil at day in the bad state of nature without storing, or to keep fossil at day in the bad state and begin storing in the good state. The first option is chosen when $k < \phi$ i.e. the storage technology is rather poor and the cloud problem not so bad. The second option is adopted when $\phi < k$.

(4) There is storage in the good state of nature while fossil is still used at night in the two states or at night and day in the bad state only. The first option is chosen when $k < \phi/\alpha$

i.e. the storage technology is rather poor, the cloud problem not so bad, and the preference for day-electricity rather low. The second option is adopted when $\phi/\alpha < k$.

(5) If $k < \phi/\alpha$ and $\phi/\alpha > 1$, fossil is used night and day in the bad state and storage begins in the bad state, whereas if $\phi/\alpha < k$ or $k < \phi/\alpha < 1$ fossil is used at night in the bad state only without storing in the bad state.

(6) The complement is done, to get storage in both states and fossil at night only in the bad state only.

(7) Fossil is completely abandoned, and there is storage in both states.

Notice that the date at which this totally clean last phase begins is determined, inter alia, by the product $k\phi$ (see Eq. (66) in Appendix B). Remember that $1-k$ is the leakage rate of the storage technology and $1-\phi$ the loss rate induced by clouds on the solar technology. When the leakage and loss rates are small, the clean phase logically begins early. It is all the more postponed since one of the two rates is high. Moreover, compared to the case where only variability is taken into account, we see that the clean phase is all the more postponed since the loss rate induced by clouds is high, which makes perfect sense.

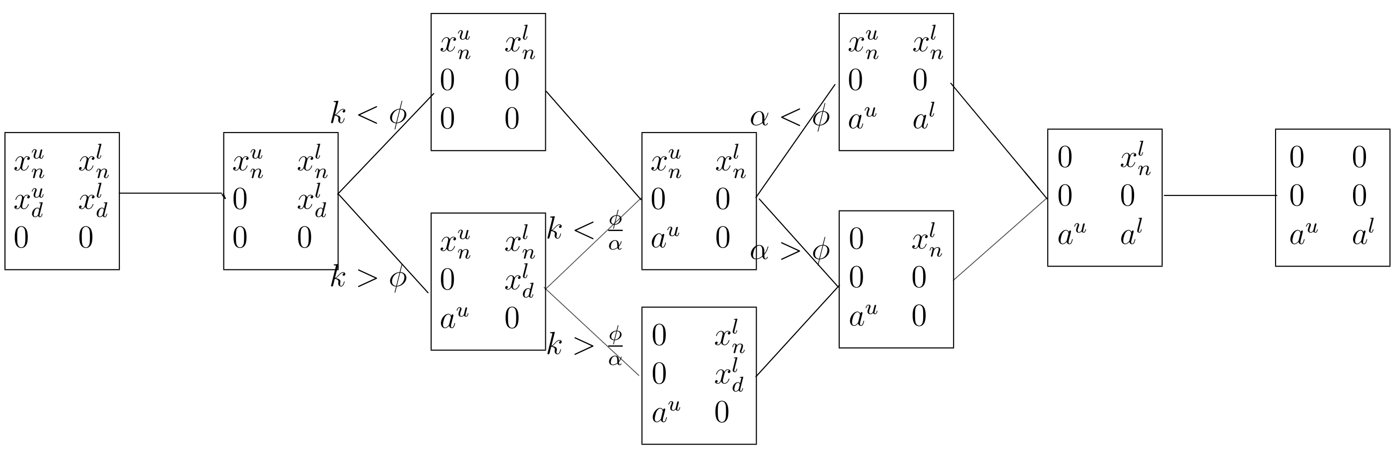


Figure 6: Intermittency: the seven phases, depending on the values of k, ϕ and α

3.2 Numerical illustrations

These simulations are for illustrative purpose only. Parameters are given in Table 2. They are chosen to illustrate the 5 possible different cases identified in the resolution (see Figure 6 and Appendix B):

- Case I: $k < \phi < \phi/\alpha < 1$.
- Case II: $k < \phi < \phi/\alpha$, $\phi/\alpha > 1$.
- Case III: $\phi < k < \phi/\alpha$, $\phi/\alpha > 1$.
- Case IV: $\phi < k < \phi/\alpha$, $\phi/\alpha < 1$.
- Case V: $k > \phi/\alpha$.

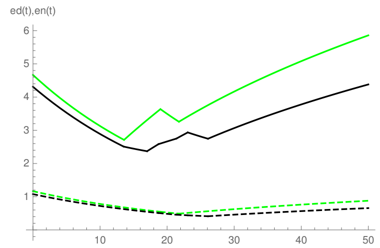
In all cases, we consider that solar is shining at peak time ($\alpha = 0.8 > 1/2$) and that $q = 0.2$. In Cases I and II the inefficiency of the storage technology is worse than the cloud problem ($k < \phi$), whereas it is the contrary in Cases III, IV and V. In Cases I and II and V the preference for day-electricity consumption comes along with a moderate cloud problem ($\phi < \alpha$), whereas it is the contrary in Cases II and III.

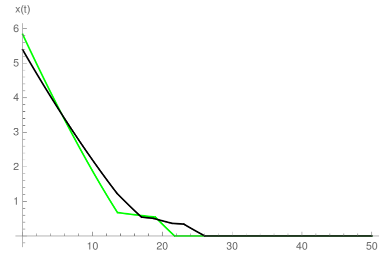
We arbitrarily choose Case I as our reference case. We check that the results are qualitatively the same if any other case is taken as reference.

	k	ϕ
Case I	0.6	0.7
Case II	0.6	**0.9**
Case III	**0.93**	**0.9**
Case IV	**0.8**	0.7
Case V	**0.93**	0.7

Table 2: Parameter values

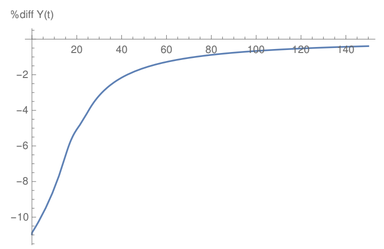
With no surprise, the general result is that intermittency globally makes matters worse (see figure 7). Electricity consumption is always smaller with intermittency than under variability only. The full storage capacity and the no-fossil economy are reached later. Night-electricity consumption begins to increase later. However differences are not so large. This is consistent with the empirical findings in Gowrisankaran and Reynolds (2016). For instance, day-electricity consumption follows a path very close to a W shape as exhibited under variability only.

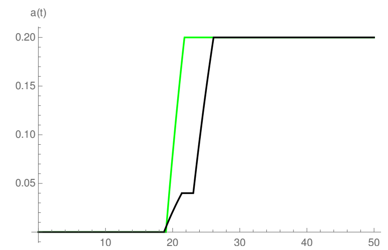




Energy consumption
day (plain) and night (dashed)

Fossil fuel consumption
total day and night





Solar capacity

Storage

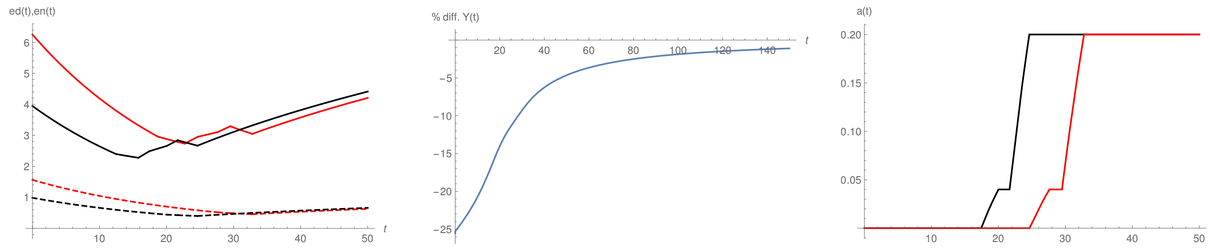
Figure 7. Dynamics of variables of interest under intermittency in Case I (black) and variability only (green)

Figures 8 to 10 present the same comparative dynamics exercices as in the case with variability only.

A less stringent climate policy (figure 8) delays storage and the switch to the no-fossil economy; it increases electricity consumption both night and day in the short and medium run but decreases them in the long run. As expected, phases during which fossil is used exhibit the larger differences in consumption. The effects are qualitatively the same as under variability only (figure 3).

A less efficient storage technology has very few effect in the short run since storage is not used, and it transfers energy use from night to day in the medium run. In the long run both day and night energy consumption are smaller. Again, the effects are qualitatively the same as under variability only (see figures 9 and 4).

In general, when peak consumption occurs when there is sun (therefore solar electricity generation), the whole daytime energy consumption path is moved up, and the reverse happens for the night energy consumption. Again, this is in accordance with what has been obtained without intermittency (see figures 10 and 5).

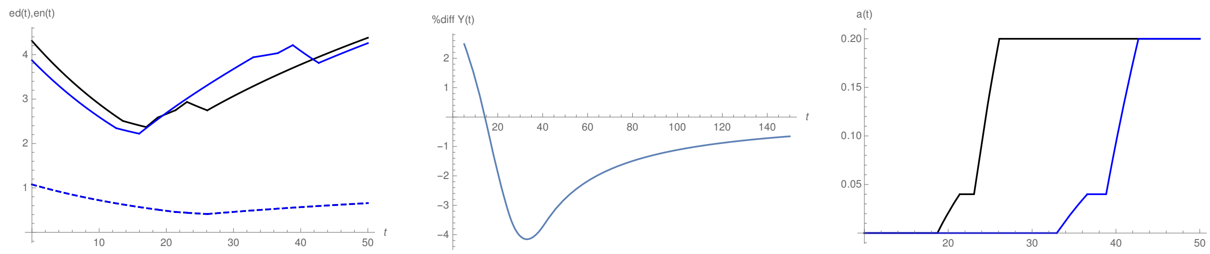


Energy consumption
day (plain) and night (dashed)

Solar capacity
(% diff.)

Storage

Figure 8. Effect of a less stringent climate policy in Case I ($\overline{X} = 50$ in blue and $\overline{X} = 100$ in red)

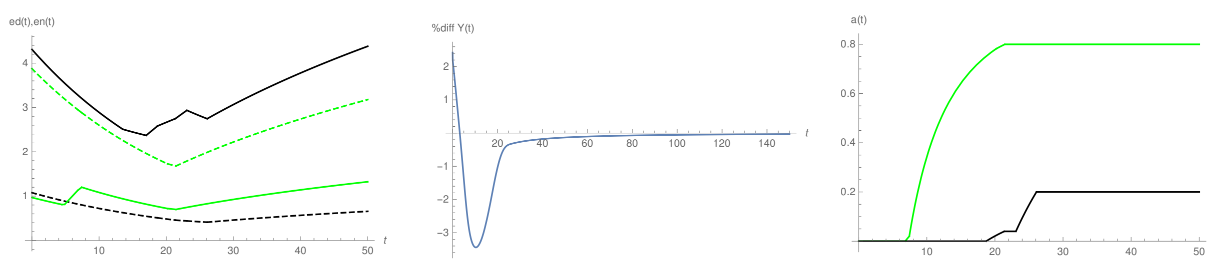


Energy consumption
day (plain) and night (dashed)

Solar capacity
(% diff.)

Storage

Figure 9. Effect of a less efficient storage technology in Case I ($k = 0.6$ in black and $k = 0.2$ in blue)



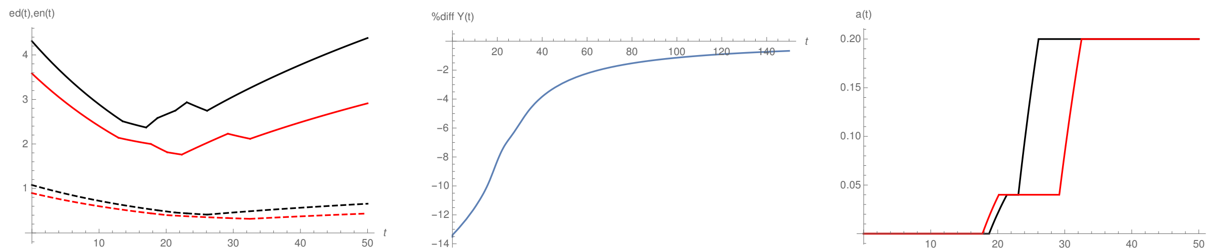
Energy consumption
day (plain) and night (dashed)

Solar capacity
(% diff.)

Storage

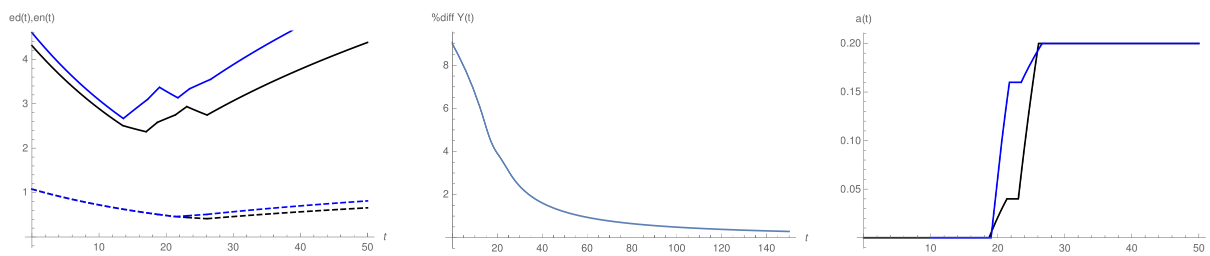
Figure 10. Effect of a smaller preference in day electricity (Case I with $\alpha = 0.8$ in black and Case II with $\alpha = 0.2$ in green)

Figures 11 and 12 present new exercises: the effect of a larger cloud problem the the effect of a higher probability of sunny weather. A larger cloud problem (figure 11) delays storage and the switch to the no-fossil economy and leads to less energy consumption, in particular during the day in the medium and long run. Such a case is relevant for less sunny countries. A higher probability of sunny weather (figure 12) has the opposite effect.



Energy consumption day (plain) and night (dashed)	Solar capacity (% diff.)	Storage

Figure 11. Effect of a larger cloud problem (Case I with $\phi = 0.7$ in black and Case V with $\phi = 0.4$ in red)



Energy consumption day (plain) and night (dashed)	Solar capacity (% diff.)	Storage

Figure 12. Effect of a higher probability of sunny weather in Case I ($q = 0.2$ in black and $q = 0.8$ in blue)

4 Appraising the effect of variability and storage

To appraise whether renewables are profitable, variability and intermittency are most of the time ignored and it is considered that renewables generate electricity in a continuous way, but at less than full capacity. In order to appraise the consequences of this simplification, we consider a special case of our general model where there is constant electricity per unit generation, that is, when an installed solar capacity Y generates an amount of electricity κY day and night, where κ is constant. Other assumptions remain the same as in the general model.

First, we compare solar panels accumulation, fossil fuel use, and electricity consumption paths under constant per unit generation, with those obtained under variability and intermittency (as derived in the previous sections). As storage is suspected to play a major role in tackling intermittency and variability, we also compare the model under constant electricity generation with the one under variability but without storage.[13]

Under constant per unit generation the social planner program reads:

$$\max \int_0^\infty e^{-\rho t} \left[u\left(e_d(t), e_n(t)\right) - C(I(t)) \right] dt$$

$$\begin{aligned}
e_d(t) &= x_d(t) + \kappa Y(t) \\
e_n(t) &= x_n(t) + \kappa Y(t) \\
\dot{X}(t) &= x_d(t) + x_n(t) && (-\lambda(t)) \\
\dot{Y}(t) &= I(t) && (\mu(t)) \\
X(t) &\leq \overline{X} && (\omega_X(t)) \\
x_d(t) &\geq 0, x_n(t) \geq 0 && (\omega_d(t), \omega_n(t)) \\
X_0 &\geq 0,\ Y_0 \geq 0 \text{ given}
\end{aligned}$$

Only three phases appear. In Phase (1), fossil fuels are used night and day. They provide electricity only during day in Phase (2) if $\alpha > 0.5$, which we assume (in case $\alpha < 0.5$, they are used only during night) and finally, electricity is generated with only solar generation in Phase (3).

During all these phases, the shadow value λ of the atmospheric carbon stock follows the Hotelling rule as in the general model. The dynamic equation driving solar capacity accumulation over the whole horizon is the same as in the general model as well. The evolution

[13] Intermittency could be accounted for as well, but it would complicate the presentation without altering the qualitative results, as it has been shown in the previous section.) We also appraise the importance of storage by comparing the general model with the one without storage possibilities.

of the shadow value of solar capacity in each phase is:

$$\begin{aligned}
&\text{Phase (1)} \qquad \dot{\mu}(t) = \rho\mu(t) - 2\kappa\lambda(t) \\
&\text{Phase (2)} \qquad \dot{\mu}(t) = \rho\mu(t) - \kappa\lambda(t) - \frac{1-\alpha}{Y(t)} \\
&\text{Phase (3)} \qquad \dot{\mu}(t) = \rho\mu(t) - \frac{1}{Y(t)}
\end{aligned}$$

Fossil fuel use and total electricity consumption in each phase are:

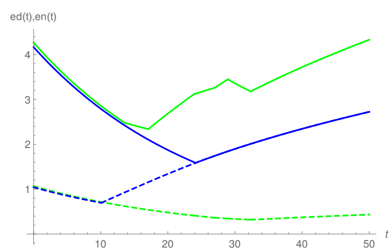
$$\begin{array}{llll}
\text{Phase (1)} & x_d(t) = \dfrac{\alpha}{\lambda(t)} - \kappa Y(t) & & x_n(t) = \dfrac{1-\alpha}{\lambda(t)} - \kappa Y(t) \\
 & e_d(t) = \dfrac{\alpha}{\lambda(t)} & & e_n(t) = \dfrac{1-\alpha}{\lambda(t)} \\
\text{Phase (2)} & x_d(t) = \dfrac{\alpha}{\lambda(t)} - \kappa Y(t) & & x_n(t) = 0 \\
 & e_d(t) = \dfrac{\alpha}{\lambda(t)} & & e_n(t) = \kappa Y(t) \\
\text{Phase (3)} & x_d(t) = 0 & & x_n(t) = 0 \\
 & e_d(t) = \kappa Y(t) & & e_n(t) = \kappa Y(t)
\end{array}$$

Only Phase (3) is a saddle path. During this phase, electricity generation is completely carbon-free as solar generation provides the whole electricity and consumptions night and day only depend on the installed solar capacity. It leads to a steady state $\mu^* = c_1$, $Y^* = 1/(\rho c_1)$. Moving backward from the steady state along the stable branch, a date is reached when Phase (2) starts. In this second phase, the dynamics of Y and μ are independent and fossil fuels are only used during daytime (for $\alpha > 0.5$). It implies overcapacity of fossil generation during night, which strikingly differs from what happens in the general model where such an overcapacity occurs during daytime. Electricity consumption is determined by climate policy during daytime and by solar capacity during nighttime. Eventually, Phase (1) is reached, during which Y and μ are independent again, until date 0. Electricity consumptions night and day require fossil fuels and are determined by the carbon value.[14]

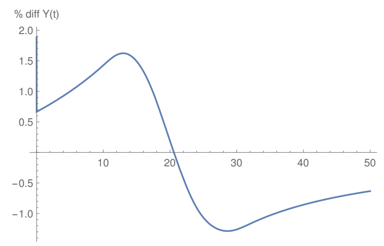
Numerical simulations allows us to go further into the comparison of the general case and the constant generation case. First, we consider that constant electricity generation is equal to the mean of electricity generation under variability and intermittency i.e.

[14] As in the general model, the initial shadow value of solar panels $\mu(0)$ (as a function of $\lambda(0)$) matches the initial condition Y_0. For a given value of $\lambda(0)$ the joint evolutions of λ and Y trigger the phase switchings and the climate constraint then pins down $\lambda(0)$.

$\kappa = 0.5(q + (1 - q)\phi)$. Comparisons with the paths obtained with the general model are provided in figure 13. As expected, electricity consumptions (that are fictive since solar generation during night cannot happen in reality) are close to an average of those obtained under variability and intermittency. It is striking that solar panel accumulation is nearly insensitive to the account of variability. This comes from the role of storage that provides a way to transfer solar generated electricity from day to night. We also consider an alternative assumption for the constant electricity generation, such that the planner is over-optimistic with respect to κ and, for instance $\kappa = 0.5(q + (1 - q)\phi)$. In this case, solar panel accumulation is quicker when variability is ignored. It must be kept in mind, however, that the steady state level of the solar panel capacity is the same whether or not variability is accounted for and regardless of the assumption on κ.



Energy consumption
day (plain) and night (dashed),



Solar capacity
(% diff)

Figure 13. Comparison of the general case (intermittency in Case I, in green) and the constant generation case (in blue)

As storage seems to play a large role in smoothing the consequences of variability and intermittency, we now compare the constant generation model with a model with variability but no storage. In the latter, the social planner program reads:

$$\max \int_0^\infty e^{-\rho t} \left[u\left(e_d(t), e_n(t)\right) - C(I(t)) \right] dt$$

$$\begin{aligned}
&e_d(t) = x_d(t) + Y(t) \\
&e_n(t) = x_n(t) \\
&\dot{X}(t) = x_d(t) + x_n(t) && (-\lambda(t)) \\
&\dot{Y}(t) = I(t) && (\mu(t)) \\
&X(t) \leq \overline{X} && (\omega_X(t)) \\
&x_d(t) \geq 0, x_n(t) \geq 0 && (\omega_d(t), \omega_n(t)) \\
&\quad X_0 \geq 0,\ Y_0 \geq 0 \text{ given}
\end{aligned}$$

Only two phases appear. In Phase (1), fossil fuels are used night and day, but they are used only during night in Phase (2). During these two phases, the shadow value λ of the atmospheric carbon stock still follows Hotelling rule. The dynamic equation driving solar capacity accumulation over the whole horizon is the same as in the general model as well. The evolution of the value of solar capacity in each phase is:

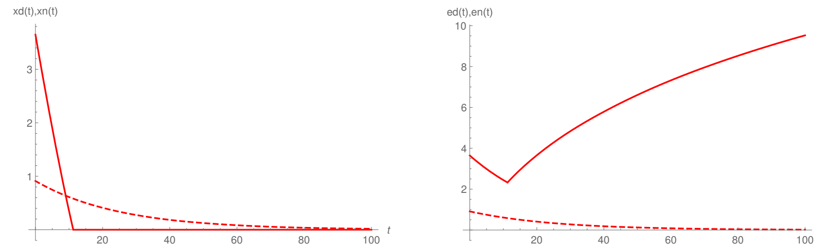
$$\begin{aligned} &\text{Phase (1)} \qquad \dot{\mu}(t) = \rho\mu(t) - \lambda(t) \\ &\text{Phase (2)} \qquad \dot{\mu}(t) = \rho\mu(t) - \frac{\alpha}{Y(t)} \end{aligned}$$

Fossil fuels use and total electricity consumption in each phase are:

$$\begin{aligned} &\text{Phase (1)} \qquad x_d(t) = \frac{\alpha}{\lambda(t)} - Y(t) \qquad && x_n(t) = \frac{1-\alpha}{\lambda(t)} \\ &\qquad\qquad\quad\; e_d(t) = \frac{\alpha}{\lambda(t)} && e_n(t) = \frac{1-\alpha}{\lambda(t)} \\ &\text{Phase (2)} \qquad x_d(t) = 0 && x_n(t) = \frac{1-\alpha}{\lambda(t)} \\ &\qquad\qquad\quad\; e_d(t) = Y(t) && e_n(t) = \frac{1-\alpha}{\lambda(t)} \end{aligned}$$

Dynamics for fossil fuels use and electricity consumption are shown in figure 14. Phase (2) is a saddle path leading to a steady state $\mu^* = c_1$, $Y^{**} = \alpha/(\rho c_1)$. Along this path, daytime electricity consumption is determined by solar capacity, but night electricity consumption still uses fossil fuels as there exists no mean to transfer daytime solar generation towards the night. Therefore electricity generation is never carbon-free and night consumption (that is equal to fossil fuels use) asymptotically tends toward zero as it is driven by the climate constraint. It implies overcapacity of fossil generation during day, as in the general model (but opposite to what happens if variability is ignored). Moving backward from the steady state along the stable branch, a date is reached when there is a switch to Phase (1). In this phase, the dynamics of Y and μ are independent; electricity consumptions night and day use fossil fuels and are determined by the climate policy.[15]

[15] As in the previous models, the initial value of solar panels, $\mu(0)$ then provided matches the initial condition Y_0. For a given value of $\lambda(0)$ the joint evolutions of λ and Y trigger the phase switchings and the climate constraint then pins down $\lambda(0)$.



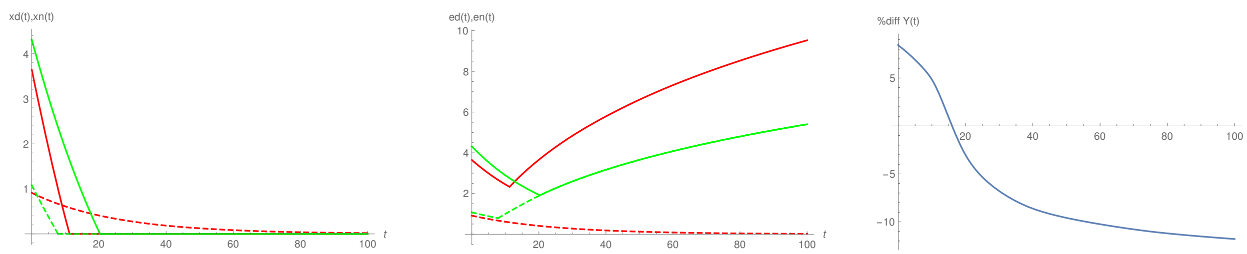
Fossil fuels use
day (plain) and night (dashed)

Energy consumption
day (plain) and night (dashed)

Figure 14. No storage

Comparisons between the model under variability with no storage and the model under constant generation is provided in figure 15.

The absence of storage prevents from using solar generated electricity to be used at night. On the contrary, constant generation allows generation during night as well. Therefore the benefits of solar panels are larger in the later case. This explains that the steady state level of the solar capacity is higher when variability and intermittency are ignored ($Y^* = 1/(\rho c_1)$) than without storage ($Y^{**} = \alpha/(\rho c_1)$). This result can be observed in figure 15, even if, during the first periods, accumulation is quicker in the absence of storage. The figure also shows that, without storage, daytime fossil fuels consumption stops while nighttime fossil fuels use asymptotically goes to zero, while under constant generation it is nighttime consumption of fossil fuels that stops first, later followed by a complete stop during daytime as well.



Fossil fuel use
day (plain) and night (dashed)

energy consumption
days (plain) and night (dashed)

Solar capacity
(% diff)

Figure 15. The role of storage (no storage in red and constant generation in green)

Another way to appraise the importance of storage is to compare the model without storage and the general model (with variability, intermittency and storage). Thanks to storage, it is possible to transfer electricity generated at day using solar capacity towards the night. Benefits from solar generation are therefore higher with storage, which explains that solar capacity at the steady state is higher with storage ($Y^* = 1/(\rho c_1)$ compared to $Y^{**} = \alpha/(\rho c_1)$ without storage). Finally, the value of storage can be appraised by computing the welfares in the two models.

5 Conclusion

In this paper, we build a stylized dynamic model of the optimal choice of the electricity mix, where the fossil energy, coal, is abundant but CO_2-emitting, and the renewable energy, solar, is variable and intermittent but clean. We solve the centralized program under the constraint of a carbon budget that cannot be exceeded and derive an optimal succession of regimes. The optimal dynamics for the capacity of solar power plants is derived, as well as the optimal amount of electricity stored in time. Simulations allow us to show that a lenient climate policy delays both storage and the switch to clean energy, as does a higher probability of bad weather. Off-peak sun, rather than peak sun, increases storage and hinders consumption. We also obtain that, compared with variability only, intermittency worsens the situation although not so significantly. Solar panel accumulation is not very sensitive to variability neither, thanks to storage that smoothes electricity provision. Indeed, we show that less solar panels are accumulated without storage.

This work can be considered as a first step in the study of energy transition under variability and intermittency of the clean sources. Next steps should concern the account of an endogenous capacity for the fossil fuel generation. In addition, a decentralized version of the model would be interesting and challenging as the energy market exhibits several peculiar features, and it would allow designing policy instruments. Finally, R&D investments improving storage technologies, and learning effects allowing a decrease in the cost of solar energy may also be considered.

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A Variability

The current value Hamiltonian associated to the social planner's programme reads:

$$\mathcal{H} = u\left(x_d + (1-a)Y, x_n + kaY\right) - C(I) - \lambda\left(x_d + x_n\right) + \mu I$$

and the Lagrangian is:

$$\mathcal{L} = \mathcal{H} + \underline{\omega}_a a + \overline{\omega}_a(1-a) + \omega_d x_d + \omega_n x_n + \omega_X\left(\overline{X} - X\right)$$

The first order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_d} &= u_1 - \lambda + \omega_d = 0 \\
\frac{\partial \mathcal{L}}{\partial x_n} &= u_2 - \lambda + \omega_n = 0 \\
\frac{\partial \mathcal{L}}{\partial a} &= -Yu_1 + kYu_2 + \underline{\omega}_a - \overline{\omega}_a = 0 \\
\frac{\partial \mathcal{L}}{\partial I} &= -C'(I) + \mu = 0 \\
-\frac{\partial \mathcal{L}}{\partial X} &= \omega_X = -\dot{\lambda} + \rho\lambda \\
-\frac{\partial \mathcal{L}}{\partial Y} &= -(1-a)u_1 - kau_2 = \dot{\mu} - \rho\mu
\end{aligned}$$

i.e.

$$u_1 = \lambda - \omega_d \tag{5}$$

$$u_2 = \lambda - \omega_n \tag{6}$$

$$Y\left(u_1 - ku_2\right) = \underline{\omega}_a - \overline{\omega}_a \tag{7}$$

$$C'(I) = \mu \tag{8}$$

$$-\omega_X = \dot{\lambda} - \rho\lambda \tag{9}$$

$$-(1-a)u_1 - kau_2 = \dot{\mu} - \rho\mu \tag{10}$$

The complementarity slackness conditions read:

$$\underline{\omega}_a a = 0, \underline{\omega}_a \geq 0, a \geq 0$$
$$\overline{\omega}_a (1-a) = 0, \overline{\omega}_a \geq 0, 1-a \geq 0$$
$$\omega_d x_d = 0, \omega_d \geq 0, x_d \geq 0$$
$$\omega_n x_n = 0, \omega_n \geq 0, x_n \geq 0$$
$$\omega_X \left(\overline{X} - X\right) = 0, \omega_X \geq 0, \overline{X} - X \geq 0$$

Before the ceiling, $X < \overline{X}$ and $\omega_X = 0$. Then FOC (9) reads $\dot{\lambda}/\lambda = \rho$, i.e.:

$$\lambda(t) = \lambda(0)e^{\rho t} \tag{11}$$

The shadow price of carbon concentration follows a Hotelling rule before the ceiling, as long as fossil fuel is used.

A.1 No fossil

This phase is necessarily the last one, and it always exists. Let $\overline{T} \geq 0$ be the date at which this last phase begins. $\overline{T}$ is also the date at which the ceiling is reached.

We have $x_d = 0$, $x_n = 0$, $\omega_d > 0$, $\omega_n > 0$ $\forall t \geq \overline{T}$.

The FOC then read:

$$\frac{\alpha}{(1-a)Y} = \lambda - \omega_d \tag{12}$$
$$\frac{1-\alpha}{kaY} = \lambda - \omega_n \tag{13}$$
$$\frac{a-(1-\alpha)}{a(1-a)} = \underline{\omega}_a - \overline{\omega}_a \tag{14}$$
$$c_1 + c_2 I = \mu \tag{15}$$
$$\dot{\lambda} - \rho\lambda = -\omega_X \tag{16}$$
$$\dot{\mu} - \rho\mu = -\frac{1}{Y} \tag{17}$$

The no storage ($a = 0$) or complete storage ($a = 1$) cases cannot occur because the marginal utility of consumption at night or day would become infinite. Hence the solution is an interior solution on a. Eq. (14) yields:

$$a^* = 1 - \alpha \tag{18}$$

There is a constant rate of storage all along this phase, depending on the weight of night-electricity in utility.

Eq. (15) and (17) together with $\dot{Y} = I$ yield a dynamic system in (μ, Y):

$$\begin{cases} \dot{\mu} = \rho\mu - \frac{1}{Y} \\ \dot{Y} = \frac{1}{c_2}\left(\mu - c_1\right) \end{cases}$$

Saddle-point. See the phase diagram on Figure 1. The values of μ and Y at the steady state are:

$$\mu^* = c_1 \tag{19}$$

$$Y^* = \frac{1}{\rho c_1} \tag{20}$$

Finally, Eq. (12) and (13) imply:

$$\frac{1}{Y} \leq \lambda \text{ and } \frac{1}{kY} \leq \lambda$$

The second condition is more stringent than the first one. It therefore provides the value of λ at the beginning of this phase:

$$\lambda(\overline{T}) = \frac{1}{kY(\overline{T})} \tag{21}$$

The solution in this phase is to store a fraction $1-\alpha$ of the electricity produced at day to use it at night, and to invest in solar panels to increase capacity from $Y(\overline{T})$ (to be determined) to $Y^* = 1/\rho c_1$.

A.2 Fossil night and day

This phase is necessarily the first one, if it exists. We denote by $\underline{T}$ the date at which it ends. We have $x_d > 0$, $x_n > 0$, $\omega_d = 0$, $\omega_n = 0$. The FOC read:

$$\frac{\alpha}{x_d + (1-a)Y} = \lambda \tag{22}$$

$$\frac{1-\alpha}{x_n + kaY} = \lambda \tag{23}$$

$$(1-k)\lambda Y = \underline{\omega}_a - \overline{\omega}_a \tag{24}$$

$$c_1 + c_2 I = \mu \tag{25}$$

$$-\left((1-a) + ka\right)\lambda = \dot{\mu} - \rho\mu \tag{26}$$

to which we add (11).

The left-hand side member of (24) is necessarily positive. Hence the case $\underline{\omega}_a = 0$ and $\overline{\omega}_a > 0$, i.e. $a = 1$ (full storage) is excluded. The interior case $\underline{\omega}_a = 0$ and $\overline{\omega}_a = 0$, i.e. $0 < a < 1$, is possible iff $Y = 0$, which means that no intermittent source of energy is used. But it does not make sense to have a positive capacity of storage absent any intermittent source of energy. This case is also excluded. The only possibility is thus $\underline{\omega}_a > 0$ and $\overline{\omega}_a = 0$, i.e. $a = 0$: no storage.

With no storage, Eq. (26) simplifies into:

$$\dot{\mu} - \rho\mu = -\lambda \tag{27}$$

Using (11), this equation can be integrated, to obtain $\mu(t)$ as a function of $\mu(0)$, $\lambda(0)$ and time. It allows us to obtain $I(t)$ and, by integration, $Y(t)$ as functions of the same variables. We suppose that Y_0 is low enough, s.t. $I(0) > 0$, which requires $\mu(0) > c_1$.

Eq. (22) and (23) now read:

$$\frac{\alpha}{x_d + Y} = \lambda \tag{28}$$

$$\frac{1-\alpha}{x_n} = \lambda \tag{29}$$

Eq. (29) and (28) allow to compute x_n and x_d:

$$x_n(t) = \frac{1-\alpha}{\lambda(t)} \tag{30}$$

$$x_d(t) = \frac{\alpha}{\lambda(t)} - Y(t) \tag{31}$$

At the end of this phase, $x_d(\underline{T}) = 0$ ($x_n(\underline{T}) = 0$ is impossible, since it would require $\lambda(\underline{T}) = +\infty$). Hence:

$$\lambda(\underline{T}) = \frac{\alpha}{Y(\underline{T})} \tag{32}$$

$$x_n(\underline{T}) = \frac{1-\alpha}{\lambda(\underline{T})} \tag{33}$$

The solution in this phase is to use fossil fuel-fired power plants night and day but less and less, not to store, and to build up solar capacity, from Y_0 to $Y(\underline{T})$ to be determined.

A.3 Fossil at night only

This or these phases are intermediate. We have here $x_d = 0$, $x_n > 0$, $\omega_d > 0$, $\omega_n = 0$ and the FOC read:

$$\frac{\alpha}{(1-a)Y} = \lambda - \omega_d \tag{34}$$

$$\frac{1-\alpha}{x_n + kaY} = \lambda \tag{35}$$

$$\frac{\alpha}{1-a} - \frac{(1-\alpha)kY}{x_n + kaY} = \underline{\omega}_a - \overline{\omega}_a \tag{36}$$

$$c_1 + c_2 I = \mu \tag{37}$$

$$-\frac{\alpha}{Y} - \frac{(1-\alpha)ka}{x_n + kaY} = \dot{\mu} - \rho\mu \tag{38}$$

to which we add (11).

- In the case of no storage, the FOC become:

$$\frac{\alpha}{Y} = \lambda - \omega_d \tag{39}$$

$$\frac{1-\alpha}{x_n} = \lambda \tag{40}$$

$$\alpha - \lambda kY = \underline{\omega}_a \tag{41}$$

$$c_1 + c_2 I = \mu \tag{42}$$

$$-\frac{\alpha}{Y} = \dot{\mu} - \rho\mu \tag{43}$$

Eq. (42) and (43) together with $\dot{Y} = I$ yield the following dynamic system in (μ, Y):

$$\begin{cases} \dot{\mu} = \rho\mu - \frac{\alpha}{Y} \\ \dot{Y} = \frac{1}{c_2}(\mu - c_1) \end{cases}$$

Saddle-point. See the phase diagram on Figure 1. The steady state values of μ and Y are:

$$\mu^* = c_1 \tag{44}$$

$$Y^{**} = \frac{\alpha}{\rho c_1} \tag{45}$$

According to (39) and (41), we must have $\frac{\alpha}{Y} \leq \lambda \leq \frac{\alpha}{kY}$. Hence this phase begins at $\underline{T}$

(see Eq. (32)), and it ends at T_i defined by:

$$\lambda(T_i) = \frac{\alpha}{kY(T_i)} \tag{46}$$

Moreover, according to (40), we have:

$$x_n(\underline{T}) = \frac{1-\alpha}{\alpha} Y(\underline{T})$$
$$x_n(T_i) = \frac{1-\alpha}{\alpha} kY(T_i)$$

- In the case of an interior solution on a, Eq. (36) reads:

$$x_n = k\frac{(1-\alpha)-a}{\alpha} Y \tag{47}$$

Eqs. (47) and (35) yield:

$$\frac{\alpha}{k(1-a)Y} = \lambda \tag{48}$$

which implies:

$$a = 1 - \frac{\alpha}{k\lambda Y}$$

Then $a \geq 0$ requires $\frac{\alpha}{kY} \leq \lambda$, which shows that this phase begins at T_i (see Eq (46)). The date at which this phase ends is given by the fact that fossil fuel consumption at night becomes nil. Then Eq. (47) shows that $a = 1-\alpha$ at the end of this phase, and Eq. (48) that $\lambda = \frac{1}{kY}$, showing that this date is $\overline{T}$ (see Eq. (21)).

Eq. (38) reads:

$$-k\lambda = \dot{\mu} - \rho\mu \tag{49}$$

This equation can be integrated to obtain $\mu(t)$ as a function of $\mu(T_i)$, T_i, $\lambda(0)$ and time, which allows us to obtain $I(t)$ and, by integration, $Y(t)$ as a function of the same variables and $Y(T_i)$.

- Case of full storage: impossible.

The solution is to use fossil fuel at night only while proceeding with the building of solar capacity, first without storing and then with storage, progressively increasing from 0 to its maximal value a^*.

A.4 Respect of the carbon budget

The model is completed by specifying that the total quantity of fossil fuel burned cannot exceed the carbon budget.

- Between 0 and $\underline{T}$, Eqs (30) and (31) yield:

$$x_n(t) + x_d(t) = \frac{1}{\lambda(t)} - Y(t)$$

- Between $\underline{T}$ and T_i, $x_d(t) = 0$ and $x_n(t)$ is given by Eq. (40):

$$x_n(t) = \frac{1-\alpha}{\lambda(t)}$$

- Between T_i and $\overline{T}$, $x_d(t) = 0$ and $x_n(t)$ is given by Eqs (35) and (48), which allows us to eliminate $a(t)$:

$$x_n(t) = \frac{1}{\lambda(t)} - kY(t)$$

Hence:

$$\begin{aligned}\overline{X} &= \int_0^{\underline{T}} (x_n(t) + x_d(t))dt + \int_{\underline{T}}^{T_i} x_n(t)dt + \int_{T_i}^{\overline{T}} x_n(t)dt \\ &= \int_0^{\overline{T}} \frac{1}{\lambda(t)}dt - \alpha \int_{\underline{T}}^{T_i} \frac{1}{\lambda(t)}dt - \int_0^{\underline{T}} Y(t)dt - k \int_{T_i}^{\overline{T}} Y(t)dt\end{aligned}$$

B Intermittency

$$\mathcal{H} = qu\left(e_d^u, e_n^u\right) + (1-q)u\left(e_d^l, e_n^l\right) - C(I) - \lambda\left(q\left(x_d^u + x_n^u\right) + (1-q)\left(x_d^l + x_n^l\right)\right) + \mu I$$

$$\mathcal{L} = \mathcal{H} + \underline{\omega}_a^u a^u + \overline{\omega}_a^u(1-a^u) + \omega_d^u x_d^u + \omega_n^u x_n^u + + \underline{\omega}_a^l a^l + \overline{\omega}_a^l(1-a^l) + \omega_d^l x_d^l + \omega_n^l x_n^l + \omega_X\left(\overline{X} - E(X)\right)$$

FOC:

$$\frac{\alpha}{e_d^u} = \lambda - \frac{\omega_d^u}{q},\ \frac{\alpha}{e_d^l} = \lambda - \frac{\omega_d^l}{1-q} \tag{50}$$

$$\frac{1-\alpha}{e_n^u} = \lambda - \frac{\omega_n^u}{q},\ \frac{1-\alpha}{e_n^l} = \lambda - \frac{\omega_n^l}{1-q} \tag{51}$$

$$Y\left(\frac{\alpha}{e_d^u} - k\frac{1-\alpha}{e_n^u}\right) = \frac{\underline{\omega}_a^u - \overline{\omega}_a^u}{q},\ Y\left(\frac{\alpha}{e_d^l} - k\frac{1-\alpha}{e_n^l}\right) = \frac{\underline{\omega}_a^l - \overline{\omega}_a^l}{\phi(1-q)} \tag{52}$$

$$c_1 + c_2 I = \mu \tag{53}$$

$$\dot{\lambda} - \rho\lambda = -\omega_X \tag{54}$$

$$\dot{\mu} - \rho\mu = -q\left[(1-a^u)\frac{\alpha}{e_d^u} + ka^u\frac{1-\alpha}{e_n^u}\right] - (1-q)\phi\left[(1-a^l)\frac{\alpha}{e_d^l} + ka^l\frac{1-\alpha}{e_n^l}\right] \tag{55}$$

to which we add the complementarity slackness conditions.

Before the ceiling, $E(X) < \overline{X}$ and $\omega_X = 0$. Then $\dot{\lambda}/\lambda = \rho$, i.e.:

$$\lambda(t) = \lambda(0)e^{\rho t} \tag{56}$$

B.1 No fossil

This phase is necessarily the last one, and it always exists. Let $\overline{T} \geq 0$ be the date at which this last phase begins. $\overline{T}$ is also the date at which $E(X) = \overline{X}$.

We have $x_d^u = x_d^l = 0$, $x_n^u = x_n^l = 0$, $\omega_d^u > 0$, $\omega_d^l > 0$, $\omega_n^u > 0$, $\omega_n^l > 0$, $\forall t \geq \overline{T}$.

The FOC then read:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q},\ \frac{\alpha}{(1-a^l)\phi Y} = \lambda - \frac{\omega_d^l}{1-q} \tag{57}$$

$$\frac{1-\alpha}{ka^uY} = \lambda - \frac{\omega_n^u}{q},\ \frac{1-\alpha}{ka^l\phi Y} = \lambda - \frac{\omega_n^l}{1-q} \tag{58}$$

$$\frac{a^u - (1-\alpha)}{a^u(1-a^u)} = \frac{\underline{\omega}_a^u - \overline{\omega}_a^u}{q},\ \frac{a^l - (1-\alpha)}{a^l(1-a^l)} = \frac{\underline{\omega}_a^l - \overline{\omega}_a^l}{1-q} \tag{59}$$

$$c_1 + c_2 I = \mu \tag{60}$$

$$\dot{\lambda} - \rho\lambda = -\omega_X \tag{61}$$

$$\dot{\mu} - \rho\mu = -\frac{1}{Y} \tag{62}$$

No storage or complete storage impossible because the marginal utility of consumption would become infinite. Hence the solution is an interior solution on a^u and a^l:

$$a^{*u} = a^{*l} = 1 - \alpha \tag{63}$$

Investment in renewables:

$$\begin{cases} \dot{\mu} = \rho\mu - \frac{1}{Y} \\ \dot{Y} = \frac{1}{c_2}(\mu - c_1) \end{cases}$$

Dynamic system in (μ, Y). Saddle-point.

$$\mu^* = c_1 \tag{64}$$

$$Y^* = \frac{1}{\rho c_1} \tag{65}$$

Finally,

$$\frac{1}{Y} \leq \lambda, \ \frac{1}{\phi Y} \leq \lambda, \ \frac{1}{kY} \leq \lambda \text{ and } \frac{1}{k\phi Y} \leq \lambda$$

The last condition is more stringent than the other ones. It therefore provides

$$\lambda(\overline{T}) = \frac{1}{k\phi Y(\overline{T})} \tag{66}$$

Hence the solution in this phase is to store a fraction $1 - \alpha$ of the electricity produced at day time to use it at night, whatever the state of nature, and to invest in solar panels to increase capacity from $Y(\overline{T})$ (to be determined) to $Y^* = 1/\rho c_1$.

B.2 Fossil night and day in the two states of nature

This phase is necessarily the first one, if it exists.

We have $x_d^u > 0$, $x_n^u > 0$, $\omega_d^u = 0$, $\omega_n^u = 0$, $x_d^l > 0$, $x_n^l > 0$, $\omega_d^l = 0$, $\omega_n^l = 0$. The FOC read:

$$\frac{\alpha}{x_d^u + (1 - a^u)Y} = \lambda, \ \frac{\alpha}{x_d^l + (1 - a^l)\phi Y} = \lambda \tag{67}$$

$$\frac{1 - \alpha}{x_n^u + ka^u Y} = \lambda, \ \frac{1 - \alpha}{x_n^l + ka^l \phi Y} = \lambda \tag{68}$$

$$(1 - k)\,\lambda Y = \frac{\underline{\omega}_a^u - \overline{\omega}_a^u}{q}, \ \ (1 - k)\,\lambda Y = \frac{\underline{\omega}_a^l - \overline{\omega}_a^l}{\phi(1 - q)} \tag{69}$$

$$c_1 + c_2 I = \mu \tag{70}$$

$$\dot{\mu} - \rho\mu = -q\left[(1 - a^u) + ka^u\right]\lambda - (1 - q)\phi\left[(1 - a^l) + ka^l\right]\lambda \tag{71}$$

With the same arguments than in the variability case, we conclude that there is no storage in this phase whatever the state of nature: $a^u = a^l = 0$. Then the FOC simplify into:

$$\frac{\alpha}{x_d^u + Y} = \frac{\alpha}{x_d^l + \phi Y} = \lambda \Longrightarrow x_d^l = x_d^u + (1-\phi)Y \text{ with } x_d^u = \frac{\alpha}{\lambda} - Y \tag{72}$$

$$\frac{1-\alpha}{x_n^u} = \frac{1-\alpha}{x_n^l} = \lambda \Longrightarrow x_n^u = x_n^l = \frac{1-\alpha}{\lambda} \tag{73}$$

$$c_1 + c_2 I = \mu \tag{74}$$

$$\dot{\mu} - \rho\mu = -\left[q + (1-q)\phi\right]\lambda \tag{75}$$

The two last equations give:

$$\begin{cases} \dot{Y} = \frac{1}{c_2}\left(\mu - c_1\right) \\ \dot{\mu} = \rho\mu - \left[q + (1-q)\phi\right]\lambda(0)e^{\rho t} \end{cases}$$

They can be integrated, to get $\mu(t)$ and $Y(t)$.

This phase begins at date 0 and ends at date $\underline{T}$.

At the end of this phase,

$$x_d^u(\underline{T}) = 0 \text{ and } x_d^l(\underline{T}) = (1-\phi)Y(\underline{T}) \tag{76}$$

Hence:

$$\lambda(\underline{T}) = \frac{\alpha}{Y(\underline{T})} \tag{77}$$

and

$$x_n^u(\underline{T}) = x_n^l(\underline{T}) = \frac{1-\alpha}{\alpha}Y(\underline{T}) \tag{78}$$

The solution in this phase is to use fossil fuel-fired power plants night and day whatever the state of nature, but less and less, not to store, and to build up solar capacity, from Y_0 to $Y(\underline{T})$ (to be determined). At the end of this phase, fossil is not used anymore at day in the good state of nature, but it is still used in the bad state to complement solar.

B.3 Fossil at night whatever the state of nature, and at day in the bad state only

We have $x_d^u = 0$, $x_n^u > 0$, $\omega_d^u > 0$, $\omega_n^u = 0$, $x_d^l > 0$, $x_n^l > 0$, $\omega_d^l = 0$, $\omega_n^l = 0$. The FOC read:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q}, \ \frac{\alpha}{x_d^l + (1-a^l)\phi Y} = \lambda \tag{79}$$

$$\frac{1-\alpha}{x_n^u + ka^u Y} = \lambda, \ \frac{1-\alpha}{x_n^l + ka^l \phi Y} = \lambda \tag{80}$$

$$Y\left(\frac{\alpha}{(1-a^u)Y} - k\lambda\right) = \frac{\underline{\omega}_a^u - \overline{\omega}_a^u}{q}, \ (1-k)\,\lambda Y = \frac{\underline{\omega}_a^l - \overline{\omega}_a^l}{\phi(1-q)} \tag{81}$$

$$c_1 + c_2 I = \mu \tag{82}$$

$$\dot{\mu} - \rho\mu = -q\left[\frac{\alpha}{Y} + ka^u\lambda\right] - (1-q)\phi\left[(1-a^l)\lambda + ka^l\lambda\right] \tag{83}$$

- In the case of no storage the FOC become:

$$\frac{\alpha}{Y} = \lambda - \frac{\omega_d^u}{q}, \ \frac{\alpha}{x_d^l + \phi Y} = \lambda \Longrightarrow x_d^l = \frac{\alpha}{\lambda} - \phi Y \tag{84}$$

$$\frac{1-\alpha}{x_n^u} = \frac{1-\alpha}{x_n^l} = \lambda \Longrightarrow x_n^u = x_n^l = \frac{1-\alpha}{\lambda} \tag{85}$$

$$\alpha - k\lambda Y = \frac{\underline{\omega}_a^u}{q}, \ (1-k)\lambda Y = \frac{\underline{\omega}_a^l}{\phi(1-q)} \tag{86}$$

$$c_1 + c_2 I = \mu \tag{87}$$

$$\dot{\mu} - \rho\mu = -q\frac{\alpha}{Y} - (1-q)\phi\lambda \tag{88}$$

Hence

$$\begin{cases} \dot{Y} = \frac{1}{c_2}(\mu - c_1) \\ \dot{\mu} = \rho\mu - q\frac{\alpha}{Y} - (1-q)\phi\lambda(0)e^{\rho t} \end{cases}$$

To the best of our knowledge there is no analytical solution to this system of differential equations. We will resort to numerical simulations.

We have $\frac{\alpha}{Y} \le \lambda$, $x_d^l \ge 0 \Leftrightarrow \frac{\alpha}{\lambda} - \phi Y \ge 0$ i.e. $\frac{\alpha}{\phi Y} \ge \lambda$, $\alpha - k\lambda Y \ge 0 \Leftrightarrow \lambda \le \frac{\alpha}{kY}$ i.e., taking into account the most stringent inequalities:

$$\frac{\alpha}{Y} \le \lambda \le \begin{cases} \frac{\alpha}{\phi Y} \text{ iff } k < \phi \\ \frac{\alpha}{kY} \text{ iff } \phi < k \end{cases}$$

  - Boundaries of this phase in the case $k < \phi$: this phase begins at $\underline{T}$ and ends at $T_{\phi 1}$ such that:

$$\lambda(T_{\phi 1}) = \frac{\alpha}{\phi Y(T_{\phi 1})} \tag{89}$$

We also have:

$$x_d^l(T_{\phi 1}) = 0, \text{ and } x_n^u(T_{\phi 1}) = x_n^l(T_{\phi 1}) = \frac{1-\alpha}{\alpha}\phi Y(T_{\phi 1}) \tag{90}$$

– Boundaries of this phase in the case $\phi < k$: this phase begins at $\underline{T}$ and ends at T_{k2} such that:

$$\lambda(T_{k2}) = \frac{\alpha}{kY(T_{k2})} \tag{91}$$

We also have:

$$x_d^l(T_{k2}) = (k-\phi)Y(T_{k2}), \text{ and } x_n^u(T_{k2}) = x_n^l(T_{k2}) = \frac{1-\alpha}{\alpha}kY(T_{k2}) \tag{92}$$

- If there is interior storage in the good state of nature, the FOC read:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q}, \ \frac{\alpha}{x_d^l + \phi Y} = \lambda \Longrightarrow x_d^l = \frac{\alpha}{\lambda} - \phi Y \tag{93}$$

$$\frac{1-\alpha}{x_n^u + ka^u Y} = \frac{1-\alpha}{x_n^l} = \lambda \Longrightarrow x_n^l = x_n^u + ka^u Y = \frac{1-\alpha}{\lambda} \tag{94}$$

$$k\lambda Y = \frac{\alpha}{1-a^u}, \ (1-k)\lambda Y = \frac{\omega_a^l}{\phi(1-q)} \tag{95}$$

$$c_1 + c_2 I = \mu \tag{96}$$

$$\dot{\mu} - \rho\mu = -\left[qk + (1-q)\phi\right]\lambda \tag{97}$$

Hence

$$\begin{cases} \dot{Y} = \frac{1}{c_2}(\mu - c_1) \\ \dot{\mu} = \rho\mu - \left[qk + (1-q)\phi\right]\lambda(0)e^{\rho t} \end{cases}$$

which can be integrated.

We have $\frac{\alpha}{(1-a^u)Y} \leq \lambda$ i.e. $k\lambda \leq \lambda$, $a^u \geq 0$ iff $\frac{\alpha}{kY} \leq \lambda$, $x_d^l \geq 0$ iff $\lambda \leq \frac{\alpha}{\phi Y}$, and $x_n^u \geq 0$ iff $\lambda \leq \frac{1}{kY}$. The relevant boundaries for this phase are then:

$$\frac{\alpha}{kY} \leq \lambda \leq \begin{cases} \frac{\alpha}{\phi Y} \text{ iff } k < \frac{\phi}{\alpha} \\ \frac{1}{kY} \text{ iff } \frac{\phi}{\alpha} < k \end{cases}$$

– Boundaries of this phase in the case $k < \frac{\phi}{\alpha}$:

$$\frac{\alpha}{kY} \leq \lambda \leq \frac{\alpha}{\phi Y}$$

Hence the existence of this phase in the case $k < \frac{\phi}{\alpha}$ requires that $\phi < k$. When it exists, this phase begins at date T_{k2} defined above, and ends at date $T_{\phi 2}$ s.t.

$$\lambda(T_{\phi 2}) = \frac{\alpha}{\phi Y(T_{\phi 2})} \tag{98}$$

We have:

$$\begin{aligned}
&a^u(T_{\phi 2}) = 1 - \frac{\phi}{k} \\
&x_d^l(T_{\phi 2}) = 0 \\
&x_n^u(T_{\phi 2}) = \left(\frac{\phi}{\alpha} - k\right) Y(T_{\phi 2}),\ x_n^l(T_{\phi 2}) = \frac{1-\alpha}{\alpha}\phi Y(T_{\phi 2})
\end{aligned}$$

– Boundaries of this phase in the case $\frac{\phi}{\alpha} < k$:

$$\frac{\alpha}{kY} \leq \lambda \leq \frac{1}{kY}$$

It begins at date T_{k2} defined above, and ends at date $\widehat{T}_2$ s.t.

$$\lambda(\widehat{T}_2) = \frac{1}{kY(\widehat{T}_2)} \tag{99}$$

We have:

$$\begin{aligned}
&a^u(\widehat{T}_2) = 1 - \alpha \\
&x_d^l(\widehat{T}_2) = (\alpha k - \phi) Y(\widehat{T}_2) \\
&x_n^u(\widehat{T}_2) = 0,\ x_n^l(\widehat{T}_2) = (1-\alpha) k Y(\widehat{T}_2)
\end{aligned}$$

- Interior storage in both states of nature: impossible.

To sum up, this phase consists in using fossil fuel-fired power plants at night whatever the state of nature, and at day only in the bad state of nature, to complement solar, first without storage and then, if $\phi < k \leq \frac{\phi}{\alpha}$, with storage at day in the good state of nature, up to $1 - \frac{\phi}{k}$, while building solar capacity proceeds.

B.4 Fossil night and day in the bad state of nature only

We have here $x_d^u = x_n^u = 0$, $x_d^l > 0$, $x_n^l > 0$, $\omega_d^u > 0$, $\omega_n^u > 0$, $\omega_d^l = \omega_n^l = 0$, and the FOC read:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q}, \ \frac{\alpha}{x_d^l + (1-a^l)\phi Y} = \lambda \tag{100}$$

$$\frac{1-\alpha}{ka^uY} = \lambda - \frac{\omega_n^u}{q}, \ \frac{1-\alpha}{x_n^l + ka^l\phi Y} = \lambda \tag{101}$$

$$Y\left(\frac{\alpha}{(1-a^u)Y} - k\frac{1-\alpha}{ka^uY}\right) = \frac{\underline{\omega}_a^u - \overline{\omega}_a^u}{q}, \ (1-k)\,\lambda Y = \frac{\underline{\omega}_a^l - \overline{\omega}_a^l}{\phi(1-q)} \tag{102}$$

$$c_1 + c_2 I = \mu \tag{103}$$

$$\dot{\mu} - \rho\mu = -q\frac{1}{Y} - (1-q)\phi\left[(1-a^l) + ka^l\right]\lambda \tag{104}$$

- No storage: impossible.
- Case of an interior solution on a^u only:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q}, \ \frac{\alpha}{x_d^l + \phi Y} = \lambda \Rightarrow x_d^l = \frac{\alpha}{\lambda} - \phi Y \tag{105}$$

$$\frac{1-\alpha}{ka^uY} = \lambda - \frac{\omega_n^u}{q}, \ \frac{1-\alpha}{x_n^l} = \lambda \Rightarrow x_n^l = \frac{1-\alpha}{\lambda} \tag{106}$$

$$Y\left(\frac{\alpha}{(1-a^u)Y} - k\frac{1-\alpha}{ka^uY}\right) = 0 \Rightarrow a^u = 1-\alpha, \ (1-k)\,\lambda Y = \frac{\underline{\omega}_a^l}{\phi(1-q)} \tag{107}$$

$$c_1 + c_2 I = \mu \tag{108}$$

$$\dot{\mu} - \rho\mu = -q\frac{1}{Y} - (1-q)\phi\lambda \tag{109}$$

Hence

$$\begin{cases} \dot{Y} = \frac{1}{c_2}(\mu - c_1) \\ \dot{\mu} = \rho\mu - q\frac{1}{Y} - (1-q)\phi\lambda(0)e^{\rho t} \end{cases}$$

To the best of our knowledge there is no analytical solution to this system of differential equations. We will resort to numerical simulations.

$\frac{1}{Y} \leq \lambda$, $\frac{1}{kY} \leq \lambda$, $x_d^l \geq 0 \iff \lambda \leq \frac{\alpha}{\phi Y}$. Hence this phase is only relevant in the case $k \geq \frac{\phi}{\alpha}$. Then, the boundaries of this phase are:

$$\frac{1}{kY} \leq \lambda \leq \frac{\alpha}{\phi Y}$$

Hence this phase begins at $\widehat{T}_2$ and ends at $T_{\phi 3}$ s.t.

$$\lambda(T_{\phi 3}) = \frac{\alpha}{\phi Y(T_{\phi 3})} \tag{110}$$

We have:

$$\begin{aligned} x_d^l(T_{\phi 3}) &= 0 \\ x_n^l(T_{\phi 3}) &= \frac{1-\alpha}{\alpha}\phi Y(T_{\phi 3}) \end{aligned}$$

- Case of an interior solution on a^u and a^l : impossible.

B.5 Fossil at night only

We have here $x_d^u = x_d^l = 0$, $x_n^u > 0$, $x_n^l > 0$, $\omega_d^u > 0$, $\omega_d^l > 0$, $\omega_n^u = \omega_n^l = 0$, and the FOC read:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q},\ \frac{\alpha}{(1-a^l)\phi Y} = \lambda - \frac{\omega_d^l}{1-q} \tag{111}$$

$$\frac{1-\alpha}{x_n^u + ka^uY} = \frac{1-\alpha}{x_n^l + ka^l\phi Y} = \lambda \tag{112}$$

$$Y\left(\frac{\alpha}{(1-a^u)Y} - k\lambda\right) = \frac{\underline{\omega}_a^u - \overline{\omega}_a^u}{q},\ Y\left(\frac{\alpha}{(1-a^l)\phi Y} - k\lambda\right) = \frac{\underline{\omega}_a^l - \overline{\omega}_a^l}{\phi(1-q)} \tag{113}$$

$$c_1 + c_2 I = \mu \tag{114}$$

$$\dot{\mu} - \rho\mu = -\frac{\alpha}{Y} - q\left[a^u + (1-q)\phi a^l\right]k\lambda \tag{115}$$

- In the case of no storage, the FOC become:

$$\frac{\alpha}{Y} = \lambda - \frac{\omega_d^u}{q},\ \frac{\alpha}{\phi Y} = \lambda - \frac{\omega_d^l}{1-q} \tag{116}$$

$$\frac{1-\alpha}{x_n^u} = \frac{1-\alpha}{x_n^l} = \lambda \Longrightarrow x_n^u = x_n^l = \frac{1-\alpha}{\lambda} \tag{117}$$

$$Y\left(\frac{\alpha}{Y} - k\lambda\right) = \frac{\underline{\omega}_a^u}{q},\ Y\left(\frac{\alpha}{\phi Y} - k\lambda\right) = \frac{\underline{\omega}_a^l}{\phi(1-q)} \tag{118}$$

$$c_1 + c_2 I = \mu \tag{119}$$

$$\dot{\mu} - \rho\mu = -\frac{\alpha}{Y} \tag{120}$$

Hence:

$$\begin{cases} \dot{\mu} = \rho\mu - \frac{\alpha}{Y} \\ \dot{Y} = \frac{1}{c_2}(\mu - c_1) \end{cases}$$

Dynamic system in (μ, Y). Saddle-point. Steady state:

$$\mu^* = c_1 \tag{121}$$
$$Y^{**} = \frac{\alpha}{\rho c_1} \tag{122}$$

Moreover, $\frac{\alpha}{Y} \leq \lambda$, $\frac{\alpha}{\phi Y} \leq \lambda$, $\lambda \leq \frac{\alpha}{kY}$ and $\lambda \leq \frac{\alpha}{k\phi Y}$. Taking the more stringent of these conditions we get the boundaries of this phase:

$$\frac{\alpha}{\phi Y} \leq \lambda \leq \frac{\alpha}{kY}$$

Hence the existence of this phase requires that $k < \phi$. When this condition is satisfied, this phase begins at $T_{\phi 1}$ and ends at T_{k1} s.t.

$$\lambda(T_{k1}) = \frac{\alpha}{kY(T_{k1})} \tag{123}$$

We have:

$$x_n^u(T_{k1}) = x_n^l(T_{k1}) = \frac{1-\alpha}{\alpha} kY(T_{k1}) \tag{124}$$

- Case of an interior solution on a^u only:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q}, \; \frac{\alpha}{\phi Y} = \lambda - \frac{\omega_d^l}{1-q} \tag{125}$$
$$\frac{1-\alpha}{x_n^u + ka^uY} = \frac{1-\alpha}{x_n^l} = \lambda \Longrightarrow x_n^l = x_n^u + ka^uY = \frac{1-\alpha}{\lambda} \tag{126}$$
$$\frac{\alpha}{(1-a^u)Y} = k\lambda \Longrightarrow a^u = 1 - \frac{\alpha}{k\lambda Y}, \; \frac{\alpha}{\phi} - k\lambda Y = \frac{\omega_a^l}{\phi(1-q)} \tag{127}$$
$$c_1 + c_2 I = \mu \tag{128}$$
$$\dot{\mu} - \rho\mu = -(1-q)\frac{\alpha}{Y} - qk\lambda \tag{129}$$

Hence

$$\begin{cases} \dot{Y} = \frac{1}{c_2}(\mu - c_1) \\ \dot{\mu} = \rho\mu - (1-q)\frac{\alpha}{Y} - qk\lambda(0)e^{\rho t} \end{cases}$$

To the best of our knowledge there is no analytical solution to this system of differential equations. We will resort to numerical simulations.

$\frac{\alpha}{\phi Y} \leq \lambda$, $a^u \geq 0 \Longleftrightarrow \frac{\alpha}{kY} \leq \lambda$, $x_n^u \geq 0 \Longleftrightarrow \lambda \leq \frac{1}{kY}$, $\frac{\alpha}{\phi} - k\lambda Y \geq 0 \Longleftrightarrow \lambda \leq \frac{\alpha}{k\phi Y}$. Hence the boundaries of this phase are:

$$\begin{cases} \frac{\alpha}{kY} \text{ if } k < \phi \\ \frac{\alpha}{\phi Y} \text{ if } \phi < k \end{cases} \leq \lambda \leq \begin{cases} \frac{\alpha}{k\phi Y} \text{ if } \alpha < \phi \\ \frac{1}{kY} \text{ if } \phi < \alpha \end{cases}$$

– In the case $k < \phi$ and $\alpha < \phi$, this phase begins at T_{k1} and ends at $T_{k\phi 1}$ s.t.

$$\frac{\alpha}{k\phi Y(T_{k\phi 1})} = \lambda(T_{k\phi 1})$$

– In the case $k < \phi < \alpha$, this phase begins at T_{k1} and ends at $\widehat{T}_1$ s.t.

$$\frac{1}{kY(\widehat{T}_1)} = \lambda(\widehat{T}_1)$$

– *In the case $\phi/\alpha < k$, this phase does not exist*
In the case $\alpha < \phi < k < \phi/\alpha$, this phase begins at $T_{\phi 2}$ and ends at $T_{k\phi 1}$

– *In the case $\phi < k < \phi/\alpha$ and $\phi < \alpha$, this phase begins at $T_{\phi 2}$ and ends at $\widehat{T}_1$*

- In the case of an interior solution on a^u and a^l, the FOC read:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q},\ \frac{\alpha}{(1-a^l)\phi Y} = \lambda - \frac{\omega_d^l}{1-q} \tag{130}$$

$$\frac{1-\alpha}{x_n^u + ka^u Y} = \frac{1-\alpha}{x_n^l + ka^l\phi Y} = \lambda \Longrightarrow x_n^l = x_n^u + k(a^u - a^l\phi)Y \text{ and } x_n^u = \frac{1-\alpha}{\lambda} - ka^u Y \tag{131}$$

$$\frac{\alpha}{(1-a^u)Y} = \frac{\alpha}{(1-a^l)\phi Y} = k\lambda \Longrightarrow a^u = 1 - \frac{\alpha}{k\lambda Y} \text{ and } a^l = \frac{1}{\phi}\left[a^u - (1-\phi)\right] = 1 - \frac{\alpha}{k\phi\lambda Y} \tag{132}$$

$$c_1 + c_2 I = \mu \tag{133}$$

$$\dot{\mu} - \rho\mu = -\left[q + (1-q)\phi\right]k\lambda \tag{134}$$

Hence

$$\begin{cases} \dot{Y} = \frac{1}{c_2}(\mu - c_1) \\ \dot{\mu} = \rho\mu - \left[q + (1-q)\phi\right]k\lambda(0)e^{\rho t} \end{cases}$$

which can be integrated.

With the values of a^u and a^l, the second equation yields:

$$x_n^l = x_n^u + k(1-\phi)Y = \frac{1}{\lambda} - k\phi Y \text{ and } x_n^u = \frac{1}{\lambda} - kY \tag{135}$$

We must have $a^u, a^l \geq 0$. Hence $\frac{\alpha}{kY} \leq \lambda$ and $\frac{\alpha}{k\phi Y} \leq \lambda$. Similarly, we must have $x_n^u, x_n^l \geq 0$. Hence $\lambda \leq \frac{1}{kY}$ and $\lambda \leq \frac{1}{k\phi Y}$. Taking the most stringent of these conditions, we get the boundaries of this phase:

$$\frac{\alpha}{k\phi Y} \leq \lambda \leq \frac{1}{kY}$$

The condition of existence of this phase is therefore $\alpha < \phi$. This phase begins at $T_{k\phi 1}$ and ends at $\widehat{T}_3$ s.t.

$$\frac{1}{kY(\widehat{T}_3)} = \lambda(\widehat{T}_3)$$

We have:

$$a^u(\widehat{T}_3) = 1 - \alpha \text{ and } a^l(\widehat{T}_3) = 1 - \frac{\alpha}{\phi}$$
$$x_n^u(\widehat{T}_3) = 0, \ x_n^l = k(1-\phi)Y(\widehat{T}_3)$$

- Case of full storage: impossible.

B.6 Fossil at night only, in the bad state only

We have here $x_d^u = x_d^l = 0$, $x_n^u = 0$, $x_n^l > 0$, $\omega_d^u > 0$, $\omega_d^l > 0$, $\omega_n^u > 0$, $\omega_n^l = 0$, and the FOC read:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q}, \ \frac{\alpha}{(1-a^l)\phi Y} = \lambda - \frac{\omega_d^l}{1-q} \quad (136)$$
$$\frac{1-\alpha}{ka^uY} = \lambda - \frac{\omega_n^u}{q}, \ \frac{1-\alpha}{x_n^l + ka^l\phi Y} = \lambda \quad (137)$$
$$\frac{a^u - (1-\alpha)}{a^u(1-a^u)} = \frac{\underline{\omega}_a^u - \overline{\omega}_a^u}{q}, \ Y\left(\frac{\alpha}{(1-a^l)\phi Y} - k\lambda\right) = \frac{\underline{\omega}_a^l - \overline{\omega}_a^l}{\phi(1-q)} \quad (138)$$
$$c_1 + c_2 I = \mu \quad (139)$$
$$\dot{\mu} - \rho\mu = -q\frac{1}{Y} - (1-q)\frac{\alpha}{Y} - (1-q)\phi k a^l \lambda \quad (140)$$

- No storage at day in the good state of nature: impossible.

- Storage in the good state, no storage in the bad one:

$$\frac{\alpha}{(1-a^u)Y} = \lambda - \frac{\omega_d^u}{q}, \ \frac{\alpha}{\phi Y} = \lambda - \frac{\omega_d^l}{1-q} \quad (141)$$
$$\frac{1-\alpha}{ka^uY} = \lambda - \frac{\omega_n^u}{q}, \ \frac{1-\alpha}{x_n^l} = \lambda \quad (142)$$
$$a^u = 1-\alpha, \ \frac{\alpha}{\phi} - k\lambda Y = \frac{\underline{\omega}_a^l}{\phi(1-q)} \quad (143)$$
$$c_1 + c_2 I = \mu \quad (144)$$
$$\dot{\mu} - \rho\mu = -\left[q + (1-q)\alpha\right]\frac{1}{Y} \quad (145)$$

Hence

$$\left\{\begin{array}{l}\dot{Y}=\frac{1}{c_2}\left(\mu-c_1\right)\\ \dot{\mu}=\rho\mu-\left[q+(1-q)\alpha\right]\frac{1}{Y}\end{array}\right.$$

which can be integrated. Saddle-point.....

Boundaries of this phase: $\frac{1}{Y}\le\lambda$, $\frac{\alpha}{\phi Y}\le\lambda$, $\frac{1}{kY}\le\lambda$, $\frac{\alpha}{k\phi Y}\ge\lambda$, i.e.

$$\left\{\begin{array}{l}\frac{1}{kY}\text{ if }k<\frac{\phi}{\alpha}\\ \frac{\alpha}{\phi Y}\text{ if }\frac{\phi}{\alpha}<k\end{array}\right.\le\lambda\le\frac{\alpha}{k\phi Y}$$

Hence a necessary condition of existence of this phase, when $k>\frac{\phi}{\alpha}$, is $\phi<\alpha$. This phase begins at $\widehat{T}_1$ or $T_{\phi 3}$ and ends at $T_{k\phi 2}$ s.t.

$$\frac{\alpha}{k\phi Y(T_{k\phi 2})}=\lambda(T_{k\phi 2})$$

We have:

$$a^u(T_{k\phi 2})=1-\alpha\text{ and }a^l(T_{k\phi 2})=0$$
$$x_n^u(T_{k\phi 2})=0,\ x_n^l(T_{k\phi 2})=\frac{1-\alpha}{\alpha}k\phi Y(T_{k\phi 2})$$

- Interior storage whatever the state of nature:

$$\frac{\alpha}{(1-a^u)Y}=\lambda-\frac{\omega_d^u}{q},\ \frac{\alpha}{(1-a^l)\phi Y}=\lambda-\frac{\omega_d^l}{1-q} \tag{146}$$
$$\frac{1-\alpha}{ka^uY}=\lambda-\frac{\omega_n^u}{q},\ \frac{1-\alpha}{x_n^l+ka^l\phi Y}=\lambda \tag{147}$$
$$a^u=1-\alpha,\ \frac{\alpha}{(1-a^l)\phi Y}=k\lambda\Longrightarrow a^l=1-\frac{\alpha}{k\phi\lambda Y} \tag{148}$$
$$c_1+c_2I=\mu \tag{149}$$
$$\dot{\mu}-\rho\mu=-q\frac{1}{Y}-(1-q)\phi k\lambda \tag{150}$$

Hence

$$\left\{\begin{array}{l}\dot{Y}=\frac{1}{c_2}\left(\mu-c_1\right)\\ \dot{\mu}=\rho\mu-q\frac{1}{Y}-(1-q)\phi k\lambda(0)e^{\rho t}\end{array}\right.$$

To the best of our knowledge there is no analytical solution to this system of differential equations. We will resort to numerical simulations.

Boundaries for this phase: $\frac{1}{Y}\le\lambda$, $\frac{1}{kY}\le\lambda$, $a^l\ge 0\Longleftrightarrow\lambda\ge\frac{\alpha}{k\phi Y}$, $x_n^l=\frac{1-\alpha}{\lambda}-ka^l\phi Y=$

$\frac{1}{\lambda} - k\phi Y \geq 0 \Longleftrightarrow \frac{1}{k\phi Y} \geq \lambda$. Hence

$$\left\{ \begin{array}{l} \frac{1}{kY} \text{ if } \alpha < \phi \\ \frac{\alpha}{k\phi Y} \text{ if } \phi < \alpha \end{array} \right. \leq \lambda \leq \frac{1}{k\phi Y}$$

– In the case $\alpha < \phi$, this phase begins at $\widehat{T}_3$ and ends at $\overline{T}$.

– In the case $\phi < \alpha$, this phase begins at $\widehat{T}_{k\phi 2}$ and ends at $\overline{T}$.