Empowerment of social norms on water consumption

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Abstract

This study develops a model of water extraction using endogenous social norms. Many users are connected by a unique shared resource that can become scarce in the case of over-exploitation. Preferences of individuals are guided by their extraction values and their taste for conformity to social norms which provide incentives to follow others. As the main result of this study, the uniqueness of the Nash equilibrium is established under a sufficient condition. Subsequently, some comparative statics analysis shows the effects of changes in individual heterogeneous parameters, conformism, and network density on the global quantity extracted. Welfare and social optimum properties are established to avoid the tragedy of the commons and suboptimal consumption of water. Finally, this theoretical framework is completed by extensions to discuss anticonformist behaviours, levers of water preservation, and awareness of consumers.

Keywords: Comparative statics, Conformism, Nash equilibrium, Network theory, Social norms, Water extraction.

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1 Introduction

In our daily lives, behaviours are oriented by social norms[1] in diverse situations through consumption norms, regulation of the use of money, reciprocity or cooperation, and even work norms (Elster, 1989). To cite only a few examples, normative effects appear in working hours, dress code, courtesy rules, waste sorting, and mealtimes. According to Kreps (1997), the fundamentals of these norms are multiple, including peer pressure effects, coordination between agents, and lack of costs. Azar (2004) suggests norms avoiding over-exploitation of the commons.

As norms can sustain vicious or virtuous cycles on environmental issues with coordination for more eco-friendly solutions (Nyborg, 2020), they may be useful in order to avoid depletion of the commons, especially for scarce resources such as water. Heterogeneous spatio-temporal repartition of water and conflicts of use, escalating with climate change (Ambec and Dinar, 2010), concern the actual use of water and generate new challenges. Therefore, to avoid transboundary conflicts or more local distortions, water must be efficiently shared between users and without waste. Game theory researchers have been exploring this issue by focusing on various types of consumers (farmers, industries, and households) and territories (Madani, 2010). The main objective of this theoretical framework is to limit suboptimal extraction, whether in cross-border flowing rivers (Ambec and Sprumont, 2002; Ambec and Ehlers, 2008b; Ambec et al., 2013), or in case of multiple water sources (İlkiliç, 2011).

Many instruments such as taxes, quotas, or even laws have been implemented to save water, but they are often not efficient enough to prevent overconsumption and the tragedy of the commons. Barnes et al. (2013) show that sometimes people subject to regulatory instruments suffer not only from an aversion of responsibility and lack of knowledge on regulative goals but also high resistance to enforced regulation. To correct these market failures, some authors (Barnes et al., 2013; Schubert, 2017) focus on the flourishing concept of nudge that can appeal to normative messages and people's inclination to follow the crowd. Both empirical and theoretical studies have emphasised the effects of informational

[1] Many definitions of social norms exist in the literature (see, for example, Elster (1989) and Kreps (1997)). In our study, we consider the social norm as the average action of neighbours, as in the approach of Ushchev and Zenou (2020).

social norms imposed by a regulator on water consumption (see, for example, Datta et al. (2015)'s study on the influence of neighbours' consumption information on domestic water, Chabe-Ferret et al. (2019)'s study on farmers with social comparison through smart grid consumption, Earnhart et al. (2020)'s study on social comparison in wastewater treatment facilities, and Ouvrard and Stenger (2020)'s study on a formalisation of informational social norm incentives). In addition, Bénabou and Tirole (2006) note that behaviours can be guided by not only intrinsic and extrinsic, but also reputational motivations, which can backfire. Rewards can be low or even negative reinforcers when they exert hidden social costs (Bénabou and Tirole, 2006). Moreover, economic incentives can reduce the effects of normative messages (Pellerano et al., 2017). For example, Chabe-Ferret et al. (2019) observe a "boomerang" effect with an increase in consumption by low-water consumers. This is undesirable for preserving the resource.

To avoid the limitations of the regulative approach raised in the previous paragraph, this study aims to offer a theoretical framework for endogenous social norms in water extraction games. Let us start with a realistic example to obtain an intuition. Internalised norms can play a strong role in refining the preferences of water users. Imagine a group of farmers whose farms are close to each other, who endure the same periods of drought or abundance of the resource, who know each other, and discuss their crops and irrigation practices. A farmer who waters without measurements during a drought will be singled out by others. Such a farmer may be exposed to the characteristics of disapproval from others defined by Elster (1989), such as shame, low self-esteem, embarrassment, and guilt. Preferences of water users are often influenced by the way people regard each other, the coordination between users, and the desire to make an effort if others do likewise. This echoed the quote by Gintis (2003) when the author said, *"internalised norms are accepted not as instruments towards and constraints upon achieving other ends, but rather as arguments in the preference function that the individual maximises"* (p. 156).

To the best of our knowledge, the effect of endogenous social norms on water extraction has not been adequately discussed theoretically in the literature. To address this research gap, we bridge three academic frameworks: social norms, water extraction games, and network theory. While we have already introduced the first two, we now add a few comments on the last one. First, network theory has been widely used in the contribution and provision of public goods (Allouch, 2015; Bramoullé et al., 2014). Second, as shown by Ballester et al. (2006), some agents can play a crucial role in the behaviour of others and can, in our case, significantly influence the water extraction in the network of water users. Third, this literature uses the linear complementarity problem as a tool to study network games, including games with cross-influences (Ballester and Calvó-Armengol, 2010). That is, both substitutabilities and complementarities that appear in water extraction with social norms can be considered. Finally, Kyriakopoulou and Xepapadeas (2021) use the theory of social networks to analyse the role of interactions between resource users and the structure of networks in the exploitation of natural resources. In their approach, local interaction is studied to obtain more efficient payoffs from natural resource extraction. Our study differs from theirs mainly in that we consider it a norm of consumption.

More formally, we consider a group of heterogeneous agents in a connected network with no self-loop links, sharing one common water resource. As in İlkiliç (2011), agents receive a concave benefit from their extraction, such that the first units of water are essential, but as in Ambec and Ehlers (2008b), they are also satiable. Additionally, we rely on İlkiliç (2011), who assume that agents endorse a convex cost from extraction. This cost varies with the consumption of the others. It introduces substitutabilities between agents because when one user extracts more, water becomes scarcer and less affordable for the others, who consequently consume less. The converse is true. Substitutabilities are sometimes balanced by complementarities arising from normative effects. When an agent increases (decreases) their consumption of water, neighbours will follow this trend by conformist transmission[2] and also increase (decrease) their extractions. Note that we consider a descriptive type of norm[3] because, as in the work of Ushchev and Zenou (2020), norms are induced by the network of relations in itself and generate externalities on agents who deviate from it.

To our knowledge, this is the first study to bridge social norms, water extraction games, and network theory to analyse the effect of endogenous social norms on the extraction

[2]Conformism in this study follows the definition of Azar (2004), who states that *"conformist transmission is a tendency to copy the most frequent behaviour in the population, using the popularity of a choice as an indirect measure of its worth"* (p. 50).

[3]In 1990, Cialdini et al. introduced the distinction between injunctive (what ought to be done) and descriptive norms (what is done); our study considers the second ones.

of water. In this context, the next section introduces the model of water extraction with endogenous social norms. The main result of this study, presented in Section 3, is to establish the uniqueness of the Nash equilibrium[4]. This section also characterises the equilibrium in terms of network structure, thus highlighting some diffusion effects[5]. Subsequently, comparative statics are provided on the relationship between individual parameters and the global quantity extracted, and the network's density. We discuss these effects with different types of networks of water users, and some apparent results appear. A variation in extraction benefit can positively influence total water consumption, while the cost and density of the network can have a direct negative effect on it. Section 5 discusses the social optimum properties such that water users consider the diffusion of their actions in the entire network. Thus, we consider social welfare[6] and provide a condition for the Nash equilibrium to be socially optimal. To avoid suboptimal water extractions, we discuss the tragedy of the commons when individual extractions at equilibrium exceed the socially optimum ones. Section 6 extends this model by discussing anti-conformism, consumer awareness, public implications, and regulatory interventions. We conclude with the main contributions and limitations of this study. The proofs are provided in the appendix.

2 A model of water extraction

Consider a territory comprising n agents located around a unique common water pool. The set of agents, denoted by $N = \{1, \dots, n\}$, shares Q units of water, which is the total amount of water extracted from this source (lake, river, ...). Each agent i extracts q_i such that the total quantity of retrieved water is the aggregate of individual consumption, that is,

$$Q = \sum_{i \in N} q_i.$$

We denote Q_{-i} as the total consumption of all agents except i.

[4]We focus on the uniqueness of the equilibrium, but it could be interesting in another approach to study the possible occurrence of multiple equilibria.

[5]At equilibrium, we show that the behaviour of one agent influences their neighbours. This affects, in its turn, their neighbours' behaviour, and so on within the network.

[6]This definition is commonly accepted in the literature (Lange, 1942) and has been widely used in water extraction processes (Ambec and Sprumont, 2002).

As agents share a common pool, they can interact and influence each other in water allocation. These interactions comprise the set of links between agents (with no self-loop links), denoted by L. Agent i and agent j are connected if $ij \in L$ exists. More formally, the undirected and unweighted network $g = \{N, L\}$ represents social interactions between agents during the water extraction process[7]. The network includes both a disjoint set of nodes formed by N agents and a set of links L between them.

Realistically, an agent does not necessarily interact with all others. However, because they share a common resource, the network is connected, and there are no isolated individuals. Given the interaction structure, let N_i be the set of neighbours of agent i, or i's neighbourhood, that is,

$$N_i = \{j \in N \text{ such that } ij \in L\}.$$

We denote ν_i as the cardinal of N_i; that is, the number of agents that i interacts with, such that $\nu_i \geq 1$ for all $i \in N$. Moreover, we write $\bar{Q}_i$, the social norm associated with the quantity extracted by i's neighbours, such that

$$\bar{Q}_i = \sum_{j \in N_i} \frac{q_j}{\nu_i} = \frac{Q_{N_i}}{\nu_i}.$$

Many definitions of social norms exist but this model focus on a standard one that consist in the mean value of neighbours' consumption inspired by Ushchev and Zenou (2020), empirical references (Datta et al., 2015; Chabe-Ferret et al., 2019) and the literature on public goods (Brekke et al., 2003).

Each agent i has a utility function $U_i : \mathbb{R}^n_+ \to \mathbb{R}$, given by:

$$U_i = \alpha_i q_i - \frac{1}{2} q_i^2 - \gamma q_i Q - \frac{\delta_i}{2} (q_i - \bar{Q}_i)^2,$$

where α_i, γ, and δ_i are strictly positive parameters. Note that agents' preferences are heterogeneous because water users do not necessarily have the same needs and will for the resource. This function is composed of three parts, where the first two follow the

[7] In this model, we consider a non-ponderated network where only the quantity of links and number of connections matter. The intensity and direction of links are not studied as long as a link is necessarily established between two agents bilaterally, and agents only obtain perfect information about others' consumption when they are connected.

water extraction game of İlkiliç (2011), but contrary to his approach, we focus on a single source. This characterisation follows the standard convex cost and concave benefit functions, which are widely used in natural resources (Smith, 1968). The third part is a social norm inspired by the work of Ushchev and Zenou (2020).

First, $\alpha_i q_i - \frac{1}{2}q_i^2 : \mathbb{R}_+ \rightarrow \mathbb{R}$ represents i's concave benefit associated with the value of water extraction. The marginal value of extraction is defined by the amplitude of the benefit α_i and its depreciating slope.

Second, $\gamma q_i Q : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a strictly convex cost function of extraction that relies on the total amount of consumed water and reflects the scarcity of the resource. The marginal cost γ is defined such that the price of extraction for an additional unit of water is always costlier than the previous one. However, this parameter is sufficiently low to maintain an affordable cost of water for agents. Realistically, the first water units extracted benefit from direct accessibility, better quality, abundance of resources, and proximity. Conversely, the more consumption increases, the more scarce and expensive the resource is due to the lack of accessibility and proximity, transportation costs, leaks of conveyance, and poor quality. The convex cost function is dissuasive and implicitly limits the global amount of water as if there were a maximum stock of water because the cost becomes prohibitive after an extraction level and users will not be incentivised to extract more.

Third, the term $\frac{\delta_i}{2}(q_i - \bar{Q}_i)^2$ represents the effect of the endogenised social norm and, consequently, the influence of the neighbourhood's water consumption on the extraction of agent i. As in Ushchev and Zenou (2020) and Brekke et al. (2003), agents seek to minimise the distance with their social group regarding water extraction because a difference, both in lower or higher directions, negatively impacts their utility. Water users assume a disutility induced by moral cost to deviating from the norm; they are encouraged to follow the others. The parameter δ_i represents the taste for the conformity of agent i such that $\delta_i > 0$ and $\bar{Q}_i$ is the endogenous social norm that varies according to the structure of the network. The higher δ_i is, the more agent i is a conformist and has a moral constraint to follow the others.

3 Equilibrium properties

In the following section, we introduce the equilibrium properties of the water extraction game presented in the previous section.

3.1 Existence and uniqueness of Nash equilibrium

Each water user chooses to maximise U_i by taking the network structure of relations and extractions of other agents on the common water source as given. All of them face the following optimisation problem:

$$\max_{q_i} \alpha_i q_i - \frac{1}{2}q_i^2 - \gamma q_i Q - \frac{\delta_i}{2}(q_i - \bar{Q}_i)^2$$

$$s.t. \quad q_i \geq 0, \forall i \in N.$$

Under the Nash assumption, Q_{-i} (quantity of water consumed by all agents except i) and $\bar{Q}_i$ are treated exogenously.

In the appendix, we show that the above maximisation program is associated with a linear complementarity problem (LCP). Widely used to determine the property of uniqueness and in games with cross-influences in the actions of individuals (Ballester et al., 2006; İlkiliç, 2011), the methodology of the LCP fits with our model. We obtain the following result.

Theorem 1. *Assume that the following condition holds:*

$$\frac{1}{\gamma} > n - 3 \quad \textit{for all } i \in N. \tag{1}$$

Then, the water extraction game admits a unique Nash equilibrium.

Several comments on Theorem 1 are in order. First, we do not fully generalise the results of İlkiliç (2011)[8], and we find a condition different from that of Kyriakopoulou

[8]Following the proof of Theorem 1, we find another sufficient condition for the uniqueness of the Nash equilibrium, based on the positive definiteness of the matrix of interactions. Let assume that $\frac{\delta_i}{\nu_i} = \frac{\delta_j}{\nu_j}$ for all $i, j \in N$. Then, the water extraction game admits a unique Nash equilibrium. Although more restrictive than Condition (1), this statement shows two important facts. First, our model extends İlkiliç (2011)'s model only in the case of a single source. Second, equilibrium uniqueness may not be uncommon in our model.

and Xepapadeas (2021). One major interest of this condition is that we consider both positive and negative externalities (complementarities and substitutabilities, respectively) between agents, following the work of Ballester and Calvó-Armengol (2010).

Second, keeping γ lower if the number of extractors increases, this condition holds. The larger the size of the network, the more scarce and costly the resource becomes. Thus, if we still want agents to consume water, the ponderation of cost should be low enough to ensure access to the resource for all agents.

Third, Condition (1) is less restrictive than it seems to be because the parameter γ is low. It must be sufficiently small to avoid the unaffordable cost of water. Thus, if this cost parameter is sufficiently low, $\frac{1}{\gamma}$ tends towards a high value:

$$\lim_{\gamma \to 0} \frac{1}{\gamma} = \infty.$$

In this case, the condition of uniqueness is thus easily satisfied because $\frac{1}{\gamma}$ is high and easily exceeds $n - 3$. It can even occur in very large networks of many agents.

Fourth, this condition is sufficient, but not necessary, so that the uniqueness of the Nash equilibrium is not guaranteed only under it. The unique equilibrium is characterised in the following section.

3.2 Characterisation of Nash equilibrium

Following the work of Ambec and Ehlers (2008a) and Ambec and Ehlers (2008b), we assume that all agents consume at least a subsistence amount of water because the marginal benefit tends to infinity when q_i tends to be null. Thus, we investigate the characterisation of the interior pure strategy Nash equilibrium when all agents consume at least a minimum vital level of water.

The first-order condition of utility maximisation for agent i with respect to q_i is given by

$$\frac{\partial U_i}{\partial q_i} = \alpha_i - q_i - \gamma(q_i + Q) - \delta_i(q_i - \bar{Q}_i) + \mu_i = 0$$

$$\text{with } \mu_i \geq 0 \text{ and } \mu_i q_i = 0$$

where μ_i is the Karush-Kuhn-Tucker multiplier associated with the positivity constraint on water extraction quantities[9]. Note that the implications induced by social norms in the model are reflected in first-order conditions, such that

$$-\delta_i(q_i - \bar{Q}_i) \lesseqgtr 0 \iff \bar{Q}_i \lesseqgtr q_i.$$

Thus, to maximise utility, an agent can be in three diverse situations. If $(q_i - \bar{Q}_i) = 0$, it is similar to a standard maximisation program without social norm and marginal cost equal to marginal benefit. If $(q_i - \bar{Q}_i) > 0$, then the benefit has to compensate both the cost and the disutility of the social norm induced by the overconsumption of water. When $(q_i - \bar{Q}_i) < 0$, the benefit and social norm externality must compensate for the cost following a trend of not consuming a lot.

By computing the first-order condition of agent i with respect to q_i, we can express the best-reply function for each water user as follows:

$$q_i = \frac{\alpha_i - \gamma Q_{-i} + \delta_i \bar{Q}_i}{1 + 2\gamma + \delta_i}$$

or equivalently, written in matrix form[10]:

$$\mathbf{q} = \mathbf{a} - \mathbf{B}\mathbf{q} + \mathbf{C}\mathbf{q}$$

where $\mathbf{a} = [\alpha_i/(1 + 2\gamma + \delta_i)]_{n\times 1}$, $\mathbf{B} = [b_{i,j}]_{n\times n}$ is such that

$$b_{i,j} = \begin{cases} 0 & \text{for } i = j \\ \dfrac{\gamma}{1 + 2\gamma + \delta_i} & \text{for } i \neq j \end{cases}$$

[9]Following Ambec and Ehlers (2008b), we assume that the marginal benefit for the first units of water is high enough to avoid corner solutions. The first-order condition for utility maximisation can be rewritten as $\alpha_i - q_i - 2\gamma q_i - \gamma Q_{-i} - \delta_i(q_i - \bar{Q}_i) = 0$, which induces $\alpha_i - \gamma Q_{-i} + \delta_i \bar{Q}_i > 0$ when q_i tends to zero. More precisely, we obtain a relative condition on the parameters $\alpha_i > \gamma Q_{-i} - \delta_i \bar{Q}_i$ $\forall i \in N$ such that the benefit is high enough to compensate for the cost of extraction which relies on the consumption of others and the disutility induced by the social normative effect.

[10]In this study, matrices are written in the upper case and boldface, whereas the vectors are in the lower case and boldface. A matrix to the power $\intercal$ denotes its transpose, and $\mathbf{I}$ is the notation for the identity matrix.

and $\mathbf{C} = [c_{i,j}]_{n\times n}$ is such that

$$c_{i,j} = \begin{cases} 0 & \text{for } i = j \text{ or for } (i \neq j \text{ and } j \notin N_i) \\ \dfrac{\delta_i}{1+2\gamma+\delta_i} & \text{for } i \neq j \text{ and } j \in N_i. \end{cases}$$

Hence, the matrix $\mathbf{B}$ represents substitutabilities and $\mathbf{C}$ represents the neighbourhood's complementarities. A substitutability effect is induced by the cost of water extraction, which increases for agent j when i consumes more and vice versa. Conversely, when individuals influence each other through peer effects, social norms act as a complementarity effect. By conformity, if individual i increases (or decreases) their consumption, their neighbour j will be encouraged to do likewise. These effects are widely studied in the literature (Ballester et al., 2006; Ballester and Calvó-Armengol, 2010; Bramoullé et al., 2007; Kyriakopoulou and Xepapadeas, 2021), and the sign of interactions among agents is positive through complementarities and negative through substitutabilities. Thus, the vector of individual extracted quantities $\mathbf{q}$ is given as follows:

Fact 1. *Assume that condition (1) holds, and let $q_i^* > 0$ for all $i = 1, \ldots, n$. Then, the unique Nash equilibrium is given by:*

$$\mathbf{q}^* = [\mathbf{I} - (\mathbf{C} - \mathbf{B})]^{-1}\mathbf{a}.$$

In the following proposition, $\mathbf{C} > \mathbf{B}$ implies that at least one entry of matrix $\mathbf{C}$ is superior to its equivalent entry in $\mathbf{B}$, and that all other entries are at least equal. Let ρ be the spectral radius[11] of a matrix. We denote l as the length of a path between two agents in the network.

Proposition 1. *Assume condition (1) holds and let $q_i^* > 0$ for all $i = 1, \ldots, n$. Then,*

1. *If $\mathbf{C} > \mathbf{B}$ and $\rho(\mathbf{C} - \mathbf{B}) < 1$, the unique Nash equilibrium is given by*

$$\mathbf{q}^* = \sum_{l=0}^{\infty} (\mathbf{C} - \mathbf{B})^l \mathbf{a}.$$

[11] Let us consider an arbitrary matrix $\mathbf{M}$; the spectral radius of this matrix, denoted by $\rho(\mathbf{M})$, is given by the largest modulus of its eigenvalues (see e.g., Ballester et al. (2006)).

2. If $\mathbf{C} < \mathbf{B}$ *and* $\rho(\mathbf{B} - \mathbf{C}) < 1$, *the unique Nash equilibrium is given by*

$$\mathbf{q}^* = \left[\sum_{l=0}^{\infty} (\mathbf{B} - \mathbf{C})^{2l} - \sum_{l=0}^{\infty} (\mathbf{B} - \mathbf{C})^{2l+1} \right] \mathbf{a}.$$

3. If $\mathbf{C} = \mathbf{B}$, *i.e., if* $\delta_i = \gamma$ *for all* $i = 1, ...n$, *the unique Nash equilibrium is given by*

$$\mathbf{q}^* = \mathbf{a}.$$

A few comments on Proposition 1 are in order. The first case is specific as long as it concerns only complete networks such that all of the out-of-diagonal terms in the matrix comprise social norms and costs. This happens when society is composed of strongly conformist agents and when social norms take the lead on cost effects. For both even and odd paths between agents, the effects on water extraction are positive, and complementarities introduced by norms exceed the costs. This situation is more plausible in small networks when everybody knows and talks to each other. In this case, water users are more likely to influence their neighbours' consumption and create spillover effects. These spillover effects come from the fact that the water consumption of a user will influence his neighbours' consumption, which will, in turn, influence their neighbours' consumption.

The second case corresponds to a weakly conformist society in which the costs assumed by agents are predominant compared to social norms. As $\mathbf{C} < \mathbf{B}$, the positive sign associated with the first sum implies that the equilibrium extraction from a link is negatively related to the even links that start from it. These strategic substitutabilities arise from costs. Conversely, the negative sign behind the second sum for odd links induces complementarity effects between nodes that come from normative conformism effects. Thus, complementarities are overtaken by the substitutabilities induced by costs. Depending on the structure of the links, either the substitutabilities or complementarities take over. Neighbours connected by an even number of links are influenced by strategic substitutabilities, while agents connected by an odd number of links are more influenced by the social norm, which implies strong complementarities. This result highlights the role of intermediary agents who can balance the effects between non-neighbours. Here, there is no general effect, and equilibrium is guided more by individual characteristics. For instance, if an agent is characterised by a very strong social norm influence, the cost effect will be exceeded. Thus, the equilibrium quantities of extraction given by $\mathbf{q}$ rely on individual heterogeneous parameters and the positioning of agents in the network.

The last case corresponds to a complete network where substitutabilities are exactly balanced by complementarities such that agents are guided only by their own benefit from extraction. Agents behave as if the network structure of water consumers does not impact their choice of consumption.

Following Ballester and Calvó-Armengol (2010), the spectral radius is an increasing function of network density. Proposition 1 requires that the spectral radius of matrices $(\mathbf{C} - \mathbf{B})$ and $(\mathbf{B} - \mathbf{C})$ (respectively, for cases 1 and 2) to be lower than 1[12]. This implies that when $\mathbf{C} > \mathbf{B}$, the difference between complementarities and substitutabilities is sufficiently low, and complementarities overcompensate the cost. When $\mathbf{C} < \mathbf{B}$, the values of substitutabilities are not sufficiently low to be compensated by complementarities, but the difference between the two remains small. In the first case, all the out-of-diagonal terms are composed of both complementarities and substitutabilities. Each agent is connected to others to make the network dense and regular, such that all water users interact with each other. In contrast, in the second case, it is not required that all water users know each other (no complete network is required) and the network is most likely to be less dense than in the first case.

Remark 1. If we cannot conclude which effect between substitutabilities or complementarities underpasses the other one, then we cannot give a global characterisation of $\mathbf{q}$. Indeed, in this case, only some elements of the matrix are non-negative.

In conclusion, the effects of norms on water extraction have complex implications. To avoid suboptimal consumption and over-exploitation of resources, it is necessary to avoid the destructive effects of norms, which leads to an increase in water consumption and tragedy of the commons. The following section determines the effects of individual parameters and the network's influence on the extracted global quantity.

[12]See the Perron-Frobenius theorem in the case of a non-negative matrix and the Gershgorin theorem on how to bound the eigenvalues of a square matrix.

4 Comparative statics analysis

We differ from Kyriakopoulou and Xepapadeas (2021) by providing comparative statics analyses to understand the properties of a model with local interactions on natural resources. To do so, we study the effects of parameters on total water consumption. By totally differentiating the best-reply functions, we obtain the variation in total consumption with respect to the parameters of water extraction:

$$dQ = \underbrace{\frac{h}{1+\gamma+\delta_i}}_{\text{benefit effect}} d\alpha_i + \underbrace{h\left(\frac{-Q^* - q_i^*}{1+\gamma+\delta_i}\right)}_{\text{cost effect}} d\gamma_i + \underbrace{h\left(\frac{\bar{Q}_i^* - q_i^*}{1+\gamma+\delta_i}\right)}_{\text{conformism effect}} d\delta_i + \underbrace{h\left(\frac{-\delta_i \bar{Q}_{N_i}^*/\nu_i}{1+\gamma+\delta_i}\right)}_{\text{density effect}} d\nu_i + \underbrace{h\sum_{i\in N}\frac{e_i}{\nu_i}}_{\text{network effect}} dQ_{N_i}.$$

The first four effects represent the change in total water consumption when individual parameters vary, such as the benefit and cost from water extraction, taste for conformity of agents, and density of the network. The last term represents the endogenous network effect that can be positive or negative and can accentuate or balance the effects of individual parameters. As this last effect is unknown, general results only hold for some types of networks.

As in the analysis of Ushchev and Zenou (2020), we keep different benefits α_i for consumers such that all extractors of water do not obtain the same satisfaction from extraction. It is realistic as long as some consumers have a higher willingness to consume the resource than others, and it maintains heterogeneity in the preferences of water consumers. In the following propositions, we consider the term $e_i = \frac{\delta_i}{1+\gamma+\delta_i}$, which denotes agent i's moral motivation since it increases with the taste for the conformity of agents. More precisely, this parameter pertains to $]0,1[$, and a value close to one indicates a highly conformist behaviour under strong moral constraint, while a value close to zero indicates a low conformist behaviour.

Relying on comparative statics analysis of the Nash equilibrium, this section aims to understand the properties of the model through the effects of heterogeneous individual parameters, conformism, and network density on global water extraction.

In the following propositions, a regular network is a network in which every agent has the same number of neighbours. Examples include the cycle and complete networks. A bipartite network is a network in which agents can be partitioned into two independent sets V_1 and V_2, such that no two agents within the same set are neighbours. A bipartite network is complete if every pair of agents from different independent sets are neighbours. Examples include star networks. Bipartite networks are commonly used in network economics (İlkiliç, 2011). A useful property of complete bipartite networks is that agents belonging to the same independent set have the same set of neighbours. Let ν^1 (resp. ν^2) denote the number of neighbours of agents belonging to V_1 (resp. V_2).

4.1 How individual parameters influence water extraction

Let us start with the benefit α_i from water extraction.

Proposition 2. *Assume condition (1) holds and let $q_i^* > 0$ for all $i = 1, \ldots, n$. Suppose one of the following conditions also holds:*

(i) *The network is regular and $\delta_1 = \ldots = \delta_n$;*

(ii) *The network is complete bipartite and $\frac{1}{\nu^1} \sum_{i \in V_1} e_i = \frac{1}{\nu^2} \sum_{i \in V_2} e_i$.*

Then, the change in total water consumption resulting from a change in parameter α_i for any agent i is given by

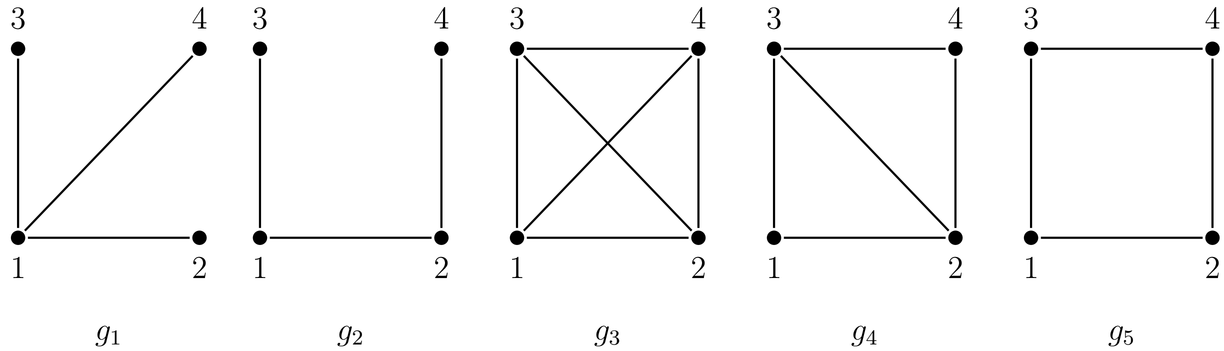
$$dQ = \sigma_i d\alpha_i$$

where $\sigma_i > 0$. Otherwise, if neither (i) *nor* (ii) *is satisfied, and without requiring additional conditions on the δ_i's, the result is ambiguous.*

In cases (i) and (ii) of the previous proposition, considering the individual benefit, we observe a direct positive effect of a change in parameter α_i on the change in total water consumption. This result was apparent and intuitive. An increase in the benefit amplitude for an agent induces an increase in water consumption and consequently raises total water extraction.

Otherwise, we cannot conclude because a negative network effect can sometimes underpass the positive effect of benefit on total water consumption. To illustrate this result, we provide in Figure 1 below some examples of networks with four agents, their neighbourhood type, and the condition for the results given in Proposition 2 to hold.

Now, we look at the impact of the cost effect on individual and global water extraction.



Network	Type of structure	$dQ/d\alpha_i$ for any $i = 1, \ldots 4$
g_1	Star (complete bipartite)	Positive if $\frac{1}{3}e_1 = e_2 + e_3 + e_4$ (ambiguous otherwise)
g_2	Line (neither regular nor complete bipartite)	Ambiguous (without more restrictions on the δ_i's)
g_3	Complete	Positive if $\delta_1 = \ldots = \delta_4$ (ambiguous otherwise)
g_4	Neither regular nor complete bipartite	Ambiguous (without more restrictions on the δ_i's)
g_5	Cycle (complete bipartite)	Positive if $e_1 + e_4 = e_2 + e_3$ (ambiguous otherwise)

Figure 1: Example of comparative statics analysis on networks with four water users

Proposition 3. *Assume condition (1) holds and let $q_i^* > 0$ for all $i = 1, \ldots, n$. Suppose one of the following conditions also holds:*

(i) *The network is regular and $\delta_1 = \ldots = \delta_n$;*

(ii) *The network is complete bipartite and $\frac{1}{\nu^1}\sum_{i \in V_1} e_i = \frac{1}{\nu^2}\sum_{i \in V_2} e_i$.*

Then, the change in total water consumption resulting from a change of the slope of marginal cost for any agent i is given by

$$dQ = -\sigma_i \left(Q^* + q_i^*\right) d\gamma_i$$

where $\sigma_i > 0$. Otherwise, if neither (i) *nor* (ii) *is satisfied, and without requiring additional conditions on the δ_i's, the result is ambiguous.*

In cases (i) and (ii) of the previous proposition, the direct price effect of a small change in the slope of the marginal cost (γ_i) negatively impacts the total water consumption. This result seems logical insofar as a variation in the slope of the marginal cost will have an impact on the direct benefit derived by agent i from water consumption. The higher the cost of an additional unit of water, the less incentive an individual will have to extract water. Furthermore, this change in total water extraction is ponderated by the quantity extracted by agent i and the total water consumption at equilibrium. The cost of the first units of water is lower because it benefits from direct accessibility, proximity, and availability of resources. Thus, if the individual quantity of any agent i and the general quantity extracted at equilibrium increase, it amplifies the negative direct effect of a variation in the slope of marginal benefit on the total water extraction, leading to a direct negative impact of a change in the slope of the marginal cost on the change in the total water extraction.

Otherwise, we cannot generally determine this effect. Indeed, if the network effect is positive and underpasses the negative impact of cost, the resulting change in total consumption can be positive. One possible explanation is a more complex mechanism of interactions, which can arise and can be decomposed into the following steps:

- A direct negative impact of an increase in the price for one agent decreases their consumption and, consequently, the global quantity.
- Neighbours of this agent have an incentive to follow this line and also decrease their water consumption because of conformity to the norm.
- However, if many agents decrease their consumption, water will be more accessible and cost less.
- This reduction of cost and water accessibility encourages agents, even the first agent previously impacted by the cost effect, to increase their consumption.

Thus, depending on the predominant effect, a price increase can also lead to increased consumption. This effect of cost should be treated cautiously depending on the structure of the network and the parameters of taste for conformity.

4.2 Does a conformist society extract more water?

This subsection focuses on the eagerness of taste for conformity δ_i on the individual and global outcomes of water extraction.

Proposition 4. *Assume condition (1) holds and let $q_i^* > 0$ for all $i = 1, \dots, n$. Suppose one of the following conditions also holds:*

(i) *The network is regular and $\delta_1 = \ldots = \delta_n$;*

(ii) *The network is complete bipartite and $\frac{1}{\nu^1} \sum_{i \in V_1} e_i = \frac{1}{\nu^2} \sum_{i \in V_2} e_i$.*

Then, the change in total water consumption resulting from a change of the taste for conformity for any agent i is given by

$$dQ = \sigma_i \left(\bar{Q}_i^* - q_i^* \right) d\delta_i$$

where $\sigma_i > 0$. Otherwise, if neither (i) *nor* (ii) *is satisfied, and without requiring additional conditions on the δ_i's, the result is ambiguous.*

In cases (i) and (ii) of the previous proposition, the change in taste for conformity for any agent i impacts the change in the total water consumption in two ways that induce an ambiguous effect. This direct effect is positively related to the value of the social norm of agent i. A change in taste for conformity — for instance, an individual i is more conformist — induces a positive change in the total water extraction that is amplified through the value of their social norm. A high social norm incentivises individual i to increase their consumption. With peer effects, the total quantity of consumed water will increase. Conversely, the change in taste for conformity induces a negative direct effect of the total water consumption directly related to the individual extraction of agent i at equilibrium. The higher the individual extraction of agent i, the higher the negative impact of a change in their taste for conformity on total water extraction. Thus, the ambiguous effect of taste for conformity offers two configurations. The first occurs if the social norm of agent i exceeds the agent's consumption at equilibrium ($\bar{Q_i}^* - q_i^* > 0$). A change in the taste for conformity induces a positive change in the total water extraction. Agents want to conform more to the norm because of the variation in the taste for conformity and imitation of others, thus raising the total consumption. The second configuration occurs if the individual consumption of agent i exceeds their social norm at equilibrium ($\bar{Q}_i^* - q_i^* < 0$) and induces a negative change in the total water extraction. Here, the change in taste for conformity negatively affects the total water extraction because user i is a huge water extractor. If the agent increases their taste for conformity, they will follow others, thus reducing their extraction and consequently the global one.

Otherwise, we cannot determine the effect of change in taste for conformity for one agent on total water consumption as long as it also depends on the network effect. Therefore, it can be either positive or negative.

4.3 Do users extract more water in denser networks?

In this section, we investigate how the creation or deletion of a link between two network agents influences water extraction.

Proposition 5. *Assume condition (1) holds and let $q_i^* > 0$ for all $i = 1, \ldots, n$. Suppose one of the following conditions also holds:*

(i) *The network is regular and $\delta_1 = \ldots = \delta_n$;*

(ii) *The network is complete bipartite and $\frac{1}{\nu^1} \sum_{i \in V_1} e_i = \frac{1}{\nu^2} \sum_{i \in V_2} e_i$.*

Then, the change in total water consumption resulting from the addition or the deletion of a link between any two agents i and j is given by

$$dQ = -\sigma \left(\frac{e_i}{\nu_i} \bar{Q}_i^* d\nu_i + \frac{e_j}{\nu_j} \bar{Q}_j^* d\nu_j \right)$$

where $\sigma > 0$. Otherwise, if neither (i) *nor* (ii) *is satisfied, and without requiring additional conditions on the δ_i's, the result is ambiguous.*

In cases (i) and (ii) of the previous proposition, this proposition shows that in any network, we can observe a negative effect of a change in the network's density on the total water consumption. This negative effect increases with the respective moral motivations of agents i and j, denoted by e_i and e_j, but also by their respective social norm values at equilibrium. The more their neighbours extract the resource, and they have a moral motivation to follow them, the more the direct negative effect of a change in density on total water extraction is important. In contrast, if the number of neighbours for i and j is high, this direct negative effect would be less important. This is understandable because if there are already many links in the network, the creation or deletion of one link will only have a slight effect on the total water extraction.

Otherwise, we cannot determine the effect of changes in the density of the network that can be either positive or negative depending on the value of the endogenous network effect. Thus, there is an ambiguous outcome, sometimes positive or negative, on the change in total water consumption.

We studied the effects of individual parameter variation, conformism strength, and network density on total water consumption. Some intuitive results appear and are in line with standard models of water consumption. However, by introducing the social norm, it is plausible that the network effect takes the lead on the standard effect. Thus, in some network structures, the effect of changes in parameters on total water consumption is ambiguous. The following section provides more details on how agents can reach a social optimum configuration.

5 Welfare and social optimum properties

We now analyse the social optimum properties in the case of interior solutions. In this study, we consider social welfare denoted by W as the sum of individual utilities given by

$$W = \sum_{i=1,2,\dots,n} U_i.$$

Here, social welfare represents the aggregated satisfaction of agents coming from their extraction of water. Thus, the maximisation problem of society's welfare from water extraction is given by:

$$\max_{q_i} \sum_{i=1}^{n} \left[\alpha_i q_i - \frac{1}{2} q_i^2 - \gamma q_i Q - \frac{\delta_i}{2} (q_i - \bar{Q}_i)^2 \right]$$

$$\text{s.t. } q_i > 0, \text{ for all } i \text{ in } N.$$

The next proposition introduces a characterisation of the first-best extraction of water. It also provides a condition for the Nash equilibrium to be the first best.

Proposition 6 (First best)**.** *Let $q_i^o > 0$ for all $i = 1, \ldots, n$. Then,*

1. *For each agent i, the first best extraction of water q_i^o is a solution to*

$$q_i = \frac{\alpha_i - \gamma Q_{-i} + \delta_i \bar{Q}_i - \sum_{j \neq i} \gamma q_j + \sum_{k \in N_i} \frac{\delta_k}{\nu_k}(q_k - \bar{Q}_k)}{1 + 2\gamma + \delta_i}$$

or, in a matrix form,

$$\mathbf{q} = \mathbf{a} - \mathbf{B}\mathbf{q} + \mathbf{C}\mathbf{q} - \mathbf{N}\mathbf{q}.$$

2. *If condition (1) holds and $q_i^* > 0$ for all $i = \ldots, n$, the unique Nash equilibrium is socially optimal, i.e., $\mathbf{q}^* = \mathbf{q}^o$, if and only if the following condition holds:*

$$\mathbf{N}\left[\mathbf{I} - (\mathbf{C} - \mathbf{B})\right]^{-1} \mathbf{a} = \mathbf{0}.$$

Part 1 of this proposition highlights the difference between the Nash equilibrium and optimum best answer. Compared to the Nash equilibrium, this first-best answer has two additional terms, also represented by the addition of the $\mathbf{N}$ matrix[13]. At social optimum, agents care about the diffusion of their influence in the network on the choices of others. The first additional term, denoted by $\sum_{j \neq i} \gamma q_j$ represents the negative impact of the cost induced by others' extraction from the common pool. The higher the sum of individual costs assumed by others, the lesser the amount of water individual i extracts at the social optimum. The second additional term, denoted by $\sum_{k \in N_i} \frac{\delta_k}{\nu_k}(q_k - \bar{Q}_k)$, corresponds to the social norm deviations of all neighbours of agent i. For each agent k, which is a neighbour of agent i, it sums the deviation between k's extraction and their respective social norm, ponderated by their taste for conformity. Thus, two configurations were observed. First, if $(q_k - \bar{Q}_k)$ is positive, for instance, agent k extracts more than the mean consumption of their neighbours, the quantity extracted by agent i is positively impacted. As k is part of i's neighbourhood, if their consumption is high, agent i will have an incentive to do so. Second, if $(q_k - \bar{Q}_k)$ is negative, for instance, agent k extracts

[13] A formal definition of $\mathbf{N}$ is given in the appendix.

less than the mean consumption of their neighbours, it will negatively impact the quantity extracted by i at the social optimum. Agent i will get closer to their neighbours and thus decrease consumption to follow this line. The last term is the sum of $\frac{\delta_k}{\nu_k}(q_k - \bar{Q}_k)$ across all neighbours of agent i. Thus, some neighbours can be in the first configuration and others in the second configuration. One effect of this social norm prevails on the other and positively or negatively influences the first-best extraction at optimum. We also observe a snowball effect from the indirect social norm because this effect relies on the social norms of neighbours. Specifically, agent i is influenced by neighbours of agent k, through k's social norm.

In conclusion, at the Nash equilibrium, when agents decide their best level of water extraction, they do not consider the positive or negative externalities induced by their extraction on others' satisfaction. In contrast, at the social optimum's first best, agents consider costs assumed by others and their influence through direct and indirect social norms. The focus on society's welfare implies that each individual's choice of water consumption depends on their impact on the rest of the water extractors through consumption costs and the direct and indirect norms of the others. For instance, in a group of domestic consumers or farmers, it implies that people will care about others and show altruism to ensure that everybody can afford some water and care about other-regarding preferences and self-image.

The second part of Proposition 6 provides a condition such that the extraction quantities at the Nash equilibrium are identical to those extracted at the social optimum. This condition relies on the matrix $\mathbf{N}$, which introduces the consideration of the utility of others in the maximisation problem. It needs precise parameter adequacy and could, thus, be uncommon to hold. Still, water extractors can reach the vector of individual extracted quantities at the social optimum $\mathbf{q}^o$, given by the following Fact 2. It not only relies on both complementarities $\mathbf{C}$ and substitutabilities $\mathbf{B}$ but also on $\mathbf{N}$, which represents interactions induced by the consideration of society's welfare.

Fact 2. *Let $q_i^o > 0$ for all $i = 1, \ldots, n$. Then, the social optimum is given by*

$$\mathbf{q}^o = [\mathbf{I} - (\mathbf{C} - \mathbf{B}) + \mathbf{N}]^{-1}\mathbf{a}.$$

To respect individual social welfare and implement a fair division of resources, over-exploitation by some water users must be avoided. Otherwise, as they all extract on a single shared resource, it can lead to a tragedy of the commons that deteriorates the water resource. The definition of the tragedy of the commons introduced by Hardin (1968) is taken in its strong sense, meaning that all agents over-extract from the common water pool. Thus, for all agents i, the individual extraction at equilibrium exceeds that at the social optimum. The following proposition states a condition for the tragedy of the commons to hold.

Proposition 7 (Tragedy of the commons)**.** *Assume that Condition (1) holds. Let* $q_i^* > 0$ *and* $q_i^o > 0$ *for all* $i = 1, \dots, n$. *If* $\mathbf{C} > \mathbf{B}$, $\rho(\mathbf{C} - \mathbf{B}) < 1$ *and the following condition holds:*

$$\sum_{j \in N \setminus \{i\}} \gamma q_j^o - \sum_{k \in N_i} \frac{\delta_k}{\nu_k} \left(q_k^o - \bar{Q}_k^o \right) > 0 \quad \text{for all } i \in N, \tag{2}$$

then, at equilibrium, each agent overconsumes water compared to the first best.

Remark 2. If Condition (2), in the previous proposition, is reversed such that the difference between the two sums is negative, then, at equilibrium, each agent underconsumes water compared to the first best.

This proposal considers all agents that form the extraction network, with positive extractions at (unique) equilibrium and social optimum. We observe a tragedy of the commons when Condition (2) applies to all of them. This condition requires that for each agent i, the difference between the sum of costs assumed by all agents except i ponderated by their optimal individual quantity of extraction and the weighted sum of the differences for each of their neighbours between their equilibrium quantity and their social norm is positive. In this case, agent i overconsumes. By doing the same for all agents i, we find that all of them overconsume at an individual scale, and thus a tragedy of the commons in a strong sense occurs. The tragedy of the commons is a typical outcome of water extraction games and natural resources (Hardin, 1968; İlkiliç, 2011). However, Proposition 7 requires that complementarities underpass substitutabilities, which can happen only in complete networks. Now that we have discussed welfare and consumption optimality, the following section extends our model.

6 Extensions

6.1 Anti-conformism

This extension follows the model settings of Ushchev and Zenou (2020), in which the social norm is ponderated by the taste for conformity of agents. We now consider the anti-conformist behaviours of water extractors such that $\delta_i < 0$ represents the taste for non-conformity. The amplitude of this parameter indicates the will of an agent to distinguish themselves from others. For instance, this can happen when individuals have strong, anchored habits in water consumption or even when maintaining a good self-image when they do not consume a lot. Instead of complementarities, the social norm here acts as substitutabilities. Thus, when a neighbour of agent i increases their consumption of water, agent i has an incentive to decrease their consumption and deviate from the norm.

Proposition 8. *Assume that the following condition holds:*

$$\frac{1+2\delta_i}{\gamma} > (n-3) \quad \textit{for all } i \in N$$

Then, the water extraction game admits a unique Nash equilibrium.

As long as agents are not too anti-conformists, our model with a unique Nash equilibrium can be extended to the case of non-conformity, as in Ushchev and Zenou (2020). Thus, we observe higher differences between individual extractions, as conformism is no longer the rule. The relative value of taste for non-conformity (δ_i) must be sufficiently low. In the case of slightly non-conformist agents, most of the equilibrium analysis still holds, but this implies new interpretations of the results. For instance, the equilibrium's best-reply function (given in section 3.2) states that agent i's consumption relies positively on the amplitude of individual benefit, which is now balanced by both cost and social norms. When others extract more, i will extract less to deviate from others and avoid unaffordable costs.

One major concern of this extension compared to the approach of Ushchev and Zenou (2020) is that the complementarities induced by cost effects included in individual decisions are accentuated with these anti-conformist behaviours. Nonconformity acts as a reinforcer of cost-effectiveness. When an agent i increases their consumption, their neighbour j will be doubly influenced to decrease their consumption. This is because the cost of water increases due to scarcity.

Generally, the degree of conformism can have a considerably strong influence on water consumption. To discuss this further, we can distinguish four main situations and make a parallel with the approach of Schultz et al. (2007), who distinguish constructive, destructive, and reconstructive effects of norms. The first occurs when an individual i, a less-water-consumer, increases their extraction to get closer to others, weakening the water resource. This is defined by Schultz et al. (2007) as the destructive effect of norms. Another situation occurs when individual i is a conformist and a high consumer among the low ones. In this situation, we observe a constructive effect such that the agent will decrease their extraction to get close to the others, preserving more of the resource. The third situation considers an anti-conformist agent i in a high consumer group that will have an incentive to consume less water to deviate from the others. This effect is constructive. The last situation occurs when a non-conformist agent is among less-water-consumers and is incentivised to increase their consumption as a free-rider behaviour. In this situation, i enjoys affordability and availability of the resource, given that others do not extract a lot on the common resource. This elicitation of various situations and the effects of conformism show that it could play a strong role in the preservation of water resources.

Remark 3. In the case of heterogeneous agents with both conformist and anticonformist agents, the matrix of interactions remains undefined and the results of the model do not generally hold.

6.2 Aware consumption of water

Our model focuses on what is done by others (descriptive norm), but not on what must be done (injunctive norm). However, the literature shows that it is interesting to combine both of them (Le Coent et al., 2021). People know that water is an important and scarce resource that must be preserved. Thus, it seems realistic to assume that agents may be incentivised to diminish their consumption of water. In the context of natural resource depletion, consumers are aware of environmental issues. To consider such a situation, this extension develops a case in which individuals obtain a positive utility when they extract less water than their neighbours, thus contributing to saving water. Conversely, they obtain a disutility or penalty when they extract more than their neighbours. To do so, we follow Falkinger (1996) and modify the utility function to measure the difference between the consumption of an agent and the average of their neighbours. Thus, for each agent i, $U_i : \mathbb{R}^n_+ \to \mathbb{R}$ is given by

$$U_i = \alpha_i q_i - \frac{1}{2}q_i^2 - \gamma q_i Q - \frac{\delta_i}{2}(q_i - \bar{Q}_i).$$

The last term is related to social pressure through consumption norms. This term is linear and negative if $q_i > \bar{Q}_i$, and positive if $q_i < \bar{Q}_i$. If agents extract more than their neighbours because of environmental concerns and consumer awareness, they suffer disutility. Conversely, if individuals consume less than their neighbours, they receive a social reward that positively impacts their utility. In this case, the first order conditions are given by

$$\frac{\partial U_i}{\partial q_i} = \alpha_i - q_i - \gamma(Q + q_i) - \frac{\delta_i}{2}, \quad \text{for all } i \in N,$$

and the best reply of each agent is given by

$$q_i^{**} = \frac{\alpha_i - \gamma Q_{-i} - \frac{\delta_i}{2}}{1 + 2\gamma}.$$

This result shows that the quantity decreases when the moral pressure associated with the taste for conformity increases. When agents care about what others think, they have an incentive to preserve their resources.

Proposition 9. *The water extraction game with aware consumption admits a unique Nash equilibrium.*

It is interesting to note that when agents get a positive utility when they extract less than their neighbours, but get a negative utility when they extract more, no condition is needed to guarantee the uniqueness of a Nash equilibrium. However, in contrast to Falkinger (1996)'s study, the introduction of such incentive to the model does not allow to restore the social optimum.

7 Concluding comments

This study analyses the impacts of social norms in a model of water extraction where heterogeneous agents share a single common resource. As proposed by İlkiliç (2011), individual utility functions are composed of a concave benefit of extraction and a convex cost, which relies on others' consumption. To refine these preferences, we add social norms and inclination of users to follow the others using the term "taste for conformity" inspired by Ushchev and Zenou (2020) and local interactions between agents on a common resource, as in Kyriakopoulou and Xepapadeas (2021). The main result of this study is to establish the uniqueness of the Nash equilibrium. As in Ushchev and Zenou (2020), this result also holds when agents are slightly anti-conformist. The results allow us to consider various situations. Conformism occurs when agents care about peer pressure effects, fairness of water sharing, imitation, and trends effect. It also allows considering small deviations from the norm because of the anchored habits of consumption, self-image, or even free-riding behaviours. Thus, this model offers an operational framework for studying equilibrium water consumption.

The study then provides a comparative statics analysis to understand the effects of individual parameters and the global consumption of water. Some intuitive conclusions include a direct positive effect of an increase in extraction benefit on the global extraction or a direct negative effect from an increase in the cost of water. However, some effects concerning the taste for conformity are ambiguous. It also depends on the structure of the relationship between water users. This echoes the literature on social norms, which highlights constructive, reconstructive, or destructive effects (Schultz et al., 2007). More specifically, in the case of water, ambiguity can also arise from geographical delimitation, where proximity often encourages the collective reduction of water consumption (Datta et al., 2015).

As water is indispensable for an agent, this study also offers insights into social welfare and optimal water consumption. Water users consider the impacts of their consumption on others' satisfaction and the spillover effects of norms. Additionally, we provide a condition for the Nash equilibrium to be socially optimal and avoid the tragedy of the commons.

As the effects of the norm and peer pressure can strongly impact people's behaviour, the last part offers extensions of this model to discuss various situations, such as when some individuals turn to become anti-conformists or free riders. In addition, norm incentives have been widely seen as mean values of neighbours' consumption in the academic literature (Ushchev and Zenou, 2020) and also in experimental fields (Bernedo et al., 2014; Datta et al., 2015). Here, we deliberately focus on endogenous social norms, as they are inadequately studied elsewhere in water theoretical frameworks. Moreover, a second extension discusses consumers' awareness and social reward (penalty) for those who extract less (more) than their neighbours.

This study raises interesting research questions. First, the endogenous structure of the network stems from the consideration of water resources and the domestic extraction process. In real life, people do not choose their living place or farming area depending on the water extraction of their neighbours, but mainly on other criteria. Thus, the network in itself is already imposed on people and, consequently, at least partly, on social norms. However, external regulation from public authorities or water firms can play a crucial role in generating links to raise collective awareness among water users. An additional regulatory intervention could influence the network structure with incentives, taxes, and connections to avoid suboptimal consumption.

This first research perspective involves the analysis of multiple equilibria. A good starting point would be to find the conditions under which multiple equilibria of water extraction occur. It would then be interesting to find general rules to select the better one to preserve the resource.

Another perspective relates to the formalisation of norms. In this paper, we focus on social norms, defined as the mean value of local neighbours. However, this measurement has limitations. First, only direct neighbours with a one-degree connection enter the social norms of an agent. Second, we observe smoothing of neighbours' consumption. This point is briefly discussed in Ushchev and Zenou (2020). The mean value of the norm does not reflect the variations between consumers and provides smoothed incentives to the agents[14]. Third, the mean value norm points out only on large consumers, while it can be interesting to consider relative performance to target them all (Brent et al., 2020).

To address these issues, we may investigate other norm specifications. One could rely on variance to catch variations between individual consumption, or on the popularity of agents that increase their influence on others' consumption. For instance, the definition of social norms could be based on the concept of strength of weak ties (Granovetter, 1973), to focus on intermediary agents and the central positions of individuals. Torres and Carlsson (2018) show that direct effects of social information on water savings are coupled with spill-over effects on untargeted agents and that there is a strong diffusion of social incentives among people. We could also consider a norm based on proximity and closeness between agents, thus relying on the paper of Datta et al. (2015), which shows that city comparison involves fewer effects than neighbourhood comparison. Our study focuses on descriptive norms, but injunctive norms can also be appropriate.

This goes hand-in-hand with further research on the complementarity between normative incentives and other regulative tools. Hence, our work may be a good starting point for natural experiments and empirical studies. It would be interesting to apply our model to empirical simulations and agent-based models to provide new insights into the role of social norms in networks.

[14]For instance, in France, people use approximately 150 liters of tap water each day. However, this mean value could be composed of consumers who extract 130 and 170 liters, or 100 and 200 liters. The last two situations provide the same social norm, while the reality of consumption is very different.

8 Appendix

The first order conditions define the following linear complementarity problem (Cottle et al., 2009). For all $i = 1, \ldots, n$, the problem is to find an extraction $q_i \geq 0$ which satisfies the system

$$\begin{cases} q_i \geq 0 \\ \alpha_i - q_i - \gamma(q_i + Q) - \delta_i(q_i - \bar{Q}_i) \leq 0 \\ \left[\alpha_i - q_i - \gamma(q_i + Q) - \delta_i(q_i - \bar{Q}_i)\right] q_i = 0 \end{cases}$$

or equivalently, find a vector $\mathbf{q} \in \mathbb{R}^n_+$ which satisfies the system

$$\begin{cases} \mathbf{q} \geq \mathbf{0} \\ -\boldsymbol{\alpha} + \mathbf{M}\mathbf{q} \geq \mathbf{0} \\ \mathbf{q}^\mathsf{T}(-\boldsymbol{\alpha} + \mathbf{M}\mathbf{q}) = 0 \end{cases}$$

where $\boldsymbol{\alpha} = [\alpha_i]_{n \times 1} \in \mathbb{R}^n_+$ and $\mathbf{M} = [m_{i,j}]_{n \times n}$ is such that

$$m_{i,j} = -\frac{\partial U_i}{\partial q_i \partial q_j} = \begin{cases} 1 + 2\gamma + \delta_i & \text{for } i = j \\ \gamma - \dfrac{\delta_i}{\nu_i} & \text{for } i \neq j \text{ and } j \in N_i \\ \gamma & \text{for } j \neq i \text{ and } j \notin N_i. \end{cases}$$

Let LCP$(-\boldsymbol{\alpha}, \mathbf{M})$ denote the above linear complementarity problem.

Proof of Theorem 1. Following Cottle et al. (2009, Theorem 3.3.7), the LCP$(-\boldsymbol{\alpha}, \mathbf{M})$ admits a unique solution if $\mathbf{M}$ is a P-matrix. A sufficient condition is that $\mathbf{M}$ be a strictly diagonally dominant matrix with positive diagonal entries (Berman and Plemmons, 1994, Theorem 2.3, p.134). The matrix $\mathbf{M}$ is said to be strictly diagonally dominant if

$$m_{i,i} > \sum_{j \in N \setminus \{i\}}^{n} |m_{i,j}| \quad \text{for all } i \in N.$$

Since $\frac{1}{\gamma} > n - 3$, it holds that

$$1 + 2\gamma + \delta_i > (n-1)\,|\gamma| + \nu_i \left| -\frac{\delta_i}{\nu_i} \right| \quad \text{for all } i \in N.$$

By the triangle inequality property of the absolute value, it holds that

$$\begin{aligned} & |\gamma| + \left|-\frac{\delta_i}{\nu_i}\right| \geq \left|\gamma - \frac{\delta_i}{\nu_i}\right| \\ \iff\ & \nu_i\,|\gamma| + \nu_i\left|-\frac{\delta_i}{\nu_i}\right| \geq \nu_i\left|\gamma - \frac{\delta_i}{\nu_i}\right| \\ \iff\ & \nu_i\left|-\frac{\delta_i}{\nu_i}\right| \geq \nu_i\left|\gamma - \frac{\delta_i}{\nu_i}\right| - \nu_i\,|\gamma| \quad \text{for all } i \in N. \end{aligned}$$

It follows that

$$\begin{aligned} 1 + 2\gamma + \delta_i &> (n-1)\,|\gamma| + \nu_i\left|\gamma - \frac{\delta_i}{\nu_i}\right| - \nu_i\,|\gamma| \\ &= (n - \nu_i - 1)\,|\gamma| + \nu_i\left|\gamma - \frac{\delta_i}{\nu_i}\right| \quad \text{for all } i \in N. \end{aligned}$$

Thus, $\mathbf{M}$ is a strictly diagonally dominant matrix with positive diagonal entries, and uniqueness is established. □

Proof of statement in footnote 8. Following the proof of Theorem 1, we need to show that $\mathbf{M}$ is a P-matrix. Observe that $\mathbf{M}$ can be decomposed as

$$\mathbf{M} = \mathbf{D} + \mathbf{F} + \mathbf{G}$$

where $\mathbf{D}$ is a $n \times n$ diagonal matrix such that $\mathbf{D} = \text{diag}(1+\gamma, \ldots, 1+\gamma)$, $\mathbf{F} = [f_{i,j}]_{n\times n}$ where $f_{i,j} = \gamma$ for all $i, j \in N$ and $\mathbf{G} = [g_{i,j}]_{n\times n}$ is such that

$$g_{i,j} = \begin{cases} \delta_i & \text{for } i = j \\ -\dfrac{\delta_i}{\nu_i} & \text{for } i \neq j \text{ and } j \in N_i \\ 0 & \text{for } i \neq j \text{ and } j \notin N_i \end{cases}$$

Being a diagonal matrix with positive diagonal entries, $\mathbf{D}$ is a positive definite matrix. Moreover, $\mathbf{F}$ is a matrix of ones multiplied by a positive scalar, so $\mathbf{F}$ is a positive semi-definite matrix. Let $\frac{\delta_i}{\nu_i} = \frac{\delta_j}{\nu_j}$ for all $i, j \in N$. Then $\mathbf{G}$, which is a strictly diagonally dominant matrix with positive diagonal entries, is symmetric. Hence, $\mathbf{G}$ is a positive semi-definite matrix. It follows that $\mathbf{M}$ is the sum of a positive definite matrix and two positive semi-definite matrices. Thus, $\mathbf{M}$ is a positive definite matrix. Then, $\mathbf{M}$ is a P-matrix, and uniqueness is established. □

Proof of Fact 1. Since $q_i^* > 0$ for all $i = 1, \ldots, n$, the $LCP(-\boldsymbol{\alpha}, \mathbf{M})$ reduces to

$$-\boldsymbol{\alpha} + \mathbf{M}\mathbf{q} = \mathbf{0} \iff \mathbf{q} = \mathbf{M}^{-1}\boldsymbol{\alpha}$$

where $\mathbf{M}^{-1}$ exists since $\mathbf{M}$ is a P-matrix. Hence, the first order conditions yield

$$\frac{\partial U_i}{\partial q_i} = \alpha_i - q_i - \gamma(q_i + Q) - \delta_i(q_i - \bar{Q}_i) = 0.$$

Then,

$$\alpha_i - q_i - \gamma(2q_i + Q_{-i}) - \delta_i(q_i - \bar{Q}_i) = 0$$

Rearranging, we get

$$q_i = \frac{\alpha_i - \gamma Q_{-i} + \delta_i \bar{Q}_i}{1 + 2\gamma + \delta_i} \quad \text{for all } i \in N,$$

or equivalently, in matrix notation,

$$\mathbf{q} = \mathbf{a} - \mathbf{B}\mathbf{q} + \mathbf{C}\mathbf{q} \iff \mathbf{q}^* = [\mathbf{I} - (\mathbf{C} - \mathbf{B})]^{-1}\mathbf{a}.$$

□

Proof of Proposition 1. Part 1. Since $\mathbf{C} - \mathbf{B}$ is nonnegative and $\rho(\mathbf{C} - \mathbf{B}) < 1$, it holds that $\mathbf{C} - \mathbf{B}$ is convergent (Berman and Plemmons, 1994, Lemma 2.1, p.133). Hence, $[\mathbf{I} - (\mathbf{C} - \mathbf{B})]^{-1}$ exists and

$$\mathbf{q}^* = [\mathbf{I} - (\mathbf{C} - \mathbf{B})]^{-1}\mathbf{a} = \sum_{l=0}^{\infty} (\mathbf{C} - \mathbf{B})^l \mathbf{a}.$$

Part 2. Since $\mathbf{B} - \mathbf{C}$ is nonnegative and $\rho(\mathbf{B} - \mathbf{C}) < 1$, it holds that $\mathbf{B} - \mathbf{C}$ is convergent, so $(\mathbf{B} - \mathbf{C})^2$ is also convergent.[15] Hence, $[\mathbf{I} - (\mathbf{B} - \mathbf{C})^2]^{-1}$ exists and

$$\left[\mathbf{I} - (\mathbf{B} - \mathbf{C})^2\right]^{-1} = \sum_{l=0}^{\infty} (\mathbf{B} - \mathbf{C})^{2l}.$$

[15] By Gelfand's Formula, it holds that $\rho((\mathbf{B} - \mathbf{C})^2) \leq \rho(\mathbf{B} - \mathbf{C})\rho(\mathbf{B} - \mathbf{C}) < 1$.

Furthermore, it holds that

$$\begin{aligned} [\mathbf{I}+(\mathbf{B}-\mathbf{C})]\,[\mathbf{I}-(\mathbf{B}-\mathbf{C})] &= [\mathbf{I}-(\mathbf{B}-\mathbf{C})^2] \\ \iff \qquad \mathbf{I}+(\mathbf{B}-\mathbf{C}) &= [\mathbf{I}-(\mathbf{B}-\mathbf{C})^2]\,[\mathbf{I}-(\mathbf{B}-\mathbf{C})]^{-1} \\ \iff \qquad [\mathbf{I}+(\mathbf{B}-\mathbf{C})]^{-1} &= [\mathbf{I}-(\mathbf{B}-\mathbf{C})^2]^{-1}\,[\mathbf{I}-(\mathbf{B}-\mathbf{C})]\,. \end{aligned}$$

Hence,

$$\mathbf{q}^* = [\mathbf{I}+(\mathbf{B}-\mathbf{C})]^{-1}\,\mathbf{a} = \sum_{l=0}^{\infty}(\mathbf{B}-\mathbf{C})^{2l}\,[\mathbf{I}-(\mathbf{B}-\mathbf{C})]\,\mathbf{a},$$

that is,

$$\mathbf{q}^* = \left[\sum_{l=0}^{\infty}(\mathbf{B}-\mathbf{C})^{2l} - \sum_{l=0}^{\infty}(\mathbf{B}-\mathbf{C})^{2l+1}\right]\mathbf{a}.$$

Part 3. Since $\mathbf{C}=\mathbf{B}$, and using Fact 1, it holds that

$$\mathbf{q}^* = [\mathbf{I}-(\mathbf{C}-\mathbf{B})]^{-1}\mathbf{a} = \mathbf{I}^{-1}\mathbf{a} = \mathbf{a}.$$

□

Proof of Proposition 2. Totally differentiating i's best-response function (while keeping $d\gamma_i = d\delta_i = d\nu_i = 0$) yields

$$\begin{aligned} dq_i &= \frac{1}{1+2\gamma+\delta_i}d\alpha_i - \frac{\gamma}{1+2\gamma+\delta_i}dQ_{-i} + \frac{\delta_i/\nu_i}{1+2\gamma+\delta_i}dQ_{N_i} \\ &= \frac{1}{1+\gamma+\delta_i}d\alpha_i - \frac{\gamma}{1+\gamma+\delta_i}dQ + \frac{\delta_i/\nu_i}{1+\gamma+\delta_i}dQ_{N_i} \\ &= \frac{1}{1+\gamma+\delta_i}d\alpha_i - \frac{\gamma}{1+\gamma+\delta_i}dQ + \frac{e_i}{\nu_i}dQ_{N_i}. \end{aligned}$$

Then, summing across all i,

$$\begin{aligned} dQ &= \sum_{i\in N}\left\{\frac{1}{1+\gamma+\delta_i}d\alpha_i - \frac{\gamma}{1+\gamma+\delta_i}dQ + \frac{e_i}{\nu_i}dQ_{N_i}\right\} \\ &= h\sum_{i\in N}\left\{\frac{1}{1+\gamma+\delta_i}d\alpha_i + \frac{e_i}{\nu_i}dQ_{N_i}\right\} \end{aligned}$$

where

$$h = \left(1+\sum_{i\in N}\frac{\gamma}{1+\gamma+\delta_i}\right)^{-1} \in (0,1).$$

Let $d\alpha_i \neq 0$ for one agent i and $d\alpha_j = 0$ for all other agent $j \neq i$. It follows that

$$dQ = \frac{h}{1+\gamma+\delta_i} d\alpha_i + h \sum_{i \in N} \frac{e_i}{\nu_i} dQ_{N_i},$$

where

$$\sum_{i \in N} \frac{e_i}{\nu_i} dQ_{N_i} = \frac{e_1}{\nu_1} dQ_{N_1} + \ldots + \frac{e_n}{\nu_n} dQ_{N_n} = \sum_{i \in N_1} \frac{e_i}{\nu_i} dq_1 + \ldots + \sum_{i \in N_n} \frac{e_i}{\nu_i} dq_n.$$

(i) Since the network is regular, it holds that $\nu_1 = \ldots = \nu_n = \nu \geq 1$. Moreover, since $\delta_1 = \ldots = \delta_n$, it holds that $e_1 = \ldots = e_n = e \in (0,1)$. It follows that

$$\sum_{i \in N} \frac{e_i}{\nu_i} dQ_{N_i} = e(dq_1 + \ldots + dq_n) = edQ.$$

Thus, we get

$$dQ = \frac{h}{1+\gamma+\delta_i} d\alpha_i + hedQ \iff dQ = \sigma_i d\alpha_i$$

where

$$\sigma_i = \frac{h}{(1-he)(1+\gamma+\delta_i)} > 0.$$

(ii) Since the network is complete bipartite, it holds that agents belonging to the same independent set have the same set of neighbors. Let $\frac{1}{\nu^1} \sum_{i \in V_1} e_i = e$. Then, since $\frac{1}{\nu^1} \sum_{i \in V_1} e_i = \frac{1}{\nu^2} \sum_{i \in V_2} e_i = e \in (0,1)$, it follows that

$$\sum_{i \in N} \frac{e_i}{\nu_i} dQ_{N_i} = \frac{dQ_{V_1}}{\nu^1} \sum_{i \in V_1} e_i + \frac{dQ_{V_2}}{\nu^2} \sum_{i \in V_2} e_i = e(dQ_{V_1} + dQ_{V_2}) = edQ,$$

where $dQ_{V_k} = \sum_{j \notin V_k} dq_j$ for all $k = 1, 2$. Thus, we get

$$dQ = \frac{h}{1+\gamma+\delta_i} d\alpha_i + hedQ \iff dQ = \sigma_i d\alpha_i$$

where

$$\sigma_i = \frac{h}{(1-he)(1+\gamma+\delta_i)} > 0.$$

Finally, if neither condition (i) nor condition (ii) is satisfied, it holds that

$$dQ = \frac{h}{1+\gamma+\delta_i} d\alpha_i + h \left(\sum_{i \in N_1} \frac{e_i}{\nu_i} dq_1 + \ldots + \sum_{i \in N_n} \frac{e_i}{\nu_i} dq_n \right),$$

which cannot be further simplified without requiring additional conditions on the δ_i's. Hence, in this case, the result is ambiguous. □

Proof of Proposition 3. Totally differentiating i's best-response function (while keeping $d\alpha_i = d\delta_i = d\nu_i = 0$) yields

$$\begin{aligned} dq_i = & \frac{-Q^*_{-i}(1+2\gamma+\delta_i) - 2(\alpha_i - \gamma Q^*_{-i} + \delta_i \bar{Q}^*_i)}{(1+2\gamma+\delta_i)^2} d\gamma_i - \frac{\gamma}{1+2\gamma+\delta_i} dQ_{-i} + \frac{\delta_i/\nu_i}{1+2\gamma+\delta_i} dQ_{N_i} \\ = & \frac{-Q^*_{-i} - 2q^*_i}{1+2\gamma+\delta_i} d\gamma_i - \frac{\gamma}{1+2\gamma+\delta_i} dQ_{-i} + \frac{\delta_i/\nu_i}{1+2\gamma+\delta_i} dQ_{N_i} \\ = & \frac{-Q^* - q^*_i}{1+\gamma+\delta_i} d\gamma_i - \frac{\gamma}{1+\gamma+\delta_i} dQ + \frac{\delta_i/\nu_i}{1+\gamma+\delta_i} dQ_{N_i} \\ = & \frac{-Q^* - q^*_i}{1+\gamma+\delta_i} d\gamma_i - \frac{\gamma}{1+\gamma+\delta_i} dQ + \frac{e_i}{\nu_i} dQ_{N_i}. \end{aligned}$$

Then, summing across all i yields

$$\begin{aligned} dQ = & \sum_{i \in N} \left\{ \frac{-Q^* - q^*_i}{1+\gamma+\delta_i} d\gamma_i - \frac{\gamma}{1+\gamma+\delta_i} dQ + \frac{e_i}{\nu_i} dQ_{N_i} \right\} \\ = & h \sum_{i \in N} \left\{ \frac{-Q^* - q^*_i}{1+\gamma+\delta_i} d\gamma_i + \frac{e_i}{\nu_i} dQ_{N_i} \right\} \end{aligned}$$

where h is the same parameter than the one in the proof of Proposition 2. Let $d\gamma_i \neq 0$ for one agent i and $d\gamma_j = 0$ for all other agent $j \neq i$. The rest of the proof follows the same lines as that of Proposition 2. □

Proof of Proposition 4. Totally differentiating i's best-response function (while keeping $d\alpha_i = d\gamma_i = d\nu_i = 0$) yields

$$\begin{aligned} dq_i = & \frac{\bar{Q}^*_i(1+2\gamma+\delta_i) - (\alpha_i - \gamma Q^*_{-i} + \delta_i \bar{Q}^*_i)}{(1+2\gamma+\delta_i)^2} d\delta_i - \frac{\gamma}{1+2\gamma+\delta_i} dQ_{-i} + \frac{\delta_i/\nu_i}{1+2\gamma+\delta_i} dQ_{N_i} \\ = & \frac{\bar{Q}^*_i - q^*_i}{1+2\gamma+\delta_i} d\delta_i - \frac{\gamma}{1+2\gamma+\delta_i} dQ_{-i} + \frac{\delta_i/\nu_i}{1+2\gamma+\delta_i} dQ_{N_i} \\ = & \frac{\bar{Q}^*_i - q^*_i}{1+\gamma+\delta_i} d\delta_i - \frac{\gamma}{1+\gamma+\delta_i} dQ + \frac{\delta_i/\nu_i}{1+\gamma+\delta_i} dQ_{N_i} \\ = & \frac{\bar{Q}^*_i - q^*_i}{1+\gamma+\delta_i} d\delta_i - \frac{\gamma}{1+\gamma+\delta_i} dQ + \frac{e_i}{\nu_i} dQ_{N_i}. \end{aligned}$$

Then, summing across all i yields

$$
\begin{aligned}
dQ = & \sum_{i\in N}\left\{\frac{\bar{Q}_i^*-q_i^*}{1+\gamma+\delta_i}d\delta_i-\frac{\gamma}{1+\gamma+\delta_i}dQ+\frac{e_i}{\nu_i}dQ_{N_i}\right\} \\
= & \; h\sum_{i\in N}\left\{\frac{\bar{Q}_i^*-q_i^*}{1+\gamma+\delta_i}d\delta_i+\frac{e_i}{\nu_i}dQ_{N_i}\right\}
\end{aligned}
$$

where h is the same parameter than the one in the proof of Proposition 2. Let $d\delta_i \neq 0$ for one agent i and $d\delta_j = 0$ for all other agent $j \neq i$. The rest of the proof follows the same lines as that of Proposition 2. □

Proof of Proposition 5. Totally differentiating i's best-response function (while keeping $d\alpha_i = d\gamma_i = d\delta_i = 0$) yields

$$
\begin{aligned}
dq_i = & \frac{-\delta_i Q_{N_i}^*/(\nu_i)^2}{1+2\gamma+\delta_i}d\nu_i-\frac{\gamma}{1+2\gamma+\delta_i}dQ_{-i}+\frac{\delta_i/\nu_i}{1+2\gamma+\delta_i}dQ_{N_i} \\
= & \frac{-\delta_i \bar{Q}_{N_i}^*/\nu_i}{1+2\gamma+\delta_i}d\nu_i-\frac{\gamma}{1+2\gamma+\delta_i}dQ_{-i}+\frac{\delta_i/\nu_i}{1+2\gamma+\delta_i}dQ_{N_i} \\
= & \frac{-\delta_i \bar{Q}_{N_i}^*/\nu_i}{1+\gamma+\delta_i}d\nu_i-\frac{\gamma}{1+\gamma+\delta_i}dQ+\frac{\delta_i/\nu_i}{1+\gamma+\delta_i}dQ_{N_i} \\
= & \frac{-\delta_i \bar{Q}_{N_i}^*/\nu_i}{1+\gamma+\delta_i}d\nu_i-\frac{\gamma}{1+\gamma+\delta_i}dQ+\frac{e_i}{\nu_i}dQ_{N_i}.
\end{aligned}
$$

Then, summing across all i yields

$$
\begin{aligned}
dQ = & \sum_{i\in N}\left\{\frac{-\delta_i \bar{Q}_{N_i}^*/\nu_i}{1+\gamma+\delta_i}d\nu_i-\frac{\gamma}{1+\gamma+\delta_i}dQ+\frac{e_i}{\nu_i}dQ_{N_i}\right\} \\
= & \; h\sum_{i\in N}\left\{\frac{-\delta_i \bar{Q}_{N_i}^*/\nu_i}{1+\gamma+\delta_i}d\nu_i+\frac{e_i}{\nu_i}dQ_{N_i}\right\}
\end{aligned}
$$

where h is the same parameter than the one in the proof of Proposition 2. Let $d\nu_i = d\nu_j = \pm 1$ for two agents i and j, and $d\delta_k = 0$ for all other agent $k \neq i, j$. The rest of the proof follows the same lines as that of Proposition 2. □

Proof of Proposition 6. Part 1. Since $q_i^o > 0$ for all $i = 1, \ldots, n$, the first order condition of total welfare maximization with respect to q_i is given by

$$
\frac{\partial W}{\partial q_i} = \alpha_i - q_i - \gamma_i\left(q_i + Q\right) - \delta_i\left(q_i - \bar{Q}_i\right) - \sum_{j\in N\setminus\{i\}}\gamma_j q_j + \sum_{k\in N_i}\frac{\delta_k}{\nu_k}\left(q_k - \bar{Q}_k\right) = 0.
$$

Hence, it holds that

$$q_i = \frac{\alpha_i - \gamma_i Q_{-i} + \delta_i \bar{Q}_i - \sum_{j \neq i} \gamma_j q_j + \sum_{k \in N_i} \frac{\delta_k}{\nu_k} \left(q_k - \bar{Q}_k \right)}{1 + 2\gamma_i + \delta_i} \quad \text{for all } i \in N.$$

Let $N_i^2 = \{k \in N \text{ such that } k \in N_j \text{ for all } j \in N_i,\ k \neq i\}$ denote the set of neighbours (except i) of i's neighbours. Then, in matrix notation, it holds that

$$\mathbf{q} = \mathbf{a} - \mathbf{B}\mathbf{q} + \mathbf{C}\mathbf{q} - \mathbf{N}\mathbf{q}$$

where $\mathbf{N} = [\eta_{i,j}]_{n \times n}$ is such that

$$\eta_{i,j} = \begin{cases} \dfrac{\sum_{k \in N_i} \frac{\delta_k}{(\nu_k)^2}}{1 + 2\gamma_i + \delta_i} & \text{for } i = j \\ \dfrac{\gamma_j - \frac{\delta_j}{\nu_j} + \sum_{k \in N_i \cap N_j} \frac{\delta_k}{(\nu_k)^2}}{1 + 2\gamma_i + \delta_i} & \text{for } i \neq j \text{ s.t. } j \in N_i \text{ and } j \in N_i^2 \\ \dfrac{\gamma_j - \frac{\delta_j}{\nu_j}}{1 + 2\gamma_i + \delta_i} & \text{for } i \neq j \text{ s.t. } j \in N_i \text{ and } j \notin N_i^2 \\ \dfrac{\gamma_j + \sum_{k \in N_i \cap N_j} \frac{\delta_k}{(\nu_k)^2}}{1 + 2\gamma_i + \delta_i} & \text{for } i \neq j \text{ s.t. } j \notin N_i \text{ and } j \in N_i^2 \\ \dfrac{\gamma_j}{1 + 2\gamma_i + \delta_i} & \text{for } i \neq j \text{ s.t. } j \notin N_i \text{ and } j \notin N_i^2. \end{cases}$$

Part 2. Comparing the equilibrium profile (Fact 1) to the socially optimal profile (Part 1 above), we find that $\mathbf{q}^* = \mathbf{q}^o$ if and only if the following condition holds:

$$\mathbf{N}\mathbf{q}^* = \mathbf{0}.$$

Using Fact 1, this is equivalent to

$$\mathbf{N} \left[\mathbf{I} - (\mathbf{C} - \mathbf{B}) \right]^{-1} \mathbf{a} = \mathbf{0}.$$

□

Proof of Fact 2. Since $q_i^o > 0$ for all $i = 1, \ldots, n$, the first order conditions of total welfare maximisation yield

$$\mathbf{q} = \mathbf{a} - \mathbf{B}\mathbf{q} + \mathbf{C}\mathbf{q} - \mathbf{N}\mathbf{q} \Longleftrightarrow \mathbf{q}^o = \left[\mathbf{I} - (\mathbf{C} - \mathbf{B}) + \mathbf{N} \right]^{-1} \mathbf{a}.$$

□

Proof of Proposition 7. In equilibrium, the first order conditions are

$$\alpha_i - q_i^* - \gamma(q_i^* + Q^*) - \delta_i(q_i^* - \bar{Q}_i^*) = 0 \quad \text{for all } i \in N,$$

since $q_i^* > 0$ for all $i = 1, \ldots, n$. Hence, in matrix notation, we obtain

$$\mathbf{q}^* = \mathbf{a} - \mathbf{B}\mathbf{q}^* + \mathbf{C}\mathbf{q}^* \Longleftrightarrow [\mathbf{I} - (\mathbf{C} - \mathbf{B})]\, \mathbf{q}^* = \mathbf{a}.$$

Moreover, since $q_i^o > 0$ for all $i = 1, \ldots, n$, the first order conditions for the efficient profile are

$$\alpha_i - q_i^o - \gamma\,(q_i^o + Q^o) - \delta_i\left(q_i^o - \bar{Q}_i^o\right) - \left[\sum_{j \in N\setminus\{i\}} \gamma_j q_j^o - \sum_{k \in N_i} \frac{\delta_k}{\nu_k}\left(q_k^o - \bar{Q}_k^o\right)\right] = 0 \quad \text{for all } i \in N.$$

Under condition (2) it follows that

$$\alpha_i - q_i^o - \gamma\,(q_i^o + Q^o) - \delta_i\left(q_i^o - \bar{Q}_i^o\right) > 0, \quad \text{for all } i \in N.$$

Hence, in matrix notation, we obtain

$$\mathbf{q}^o < \mathbf{a} - \mathbf{B}\mathbf{q}^o + \mathbf{C}\mathbf{q}^o \Longleftrightarrow [\mathbf{I} - (\mathbf{C} - \mathbf{B})]\, \mathbf{q}^o < \mathbf{a}$$

Then,

$$[\mathbf{I} - (\mathbf{C} - \mathbf{B})]\, \mathbf{q}^* = \mathbf{a} > [\mathbf{I} - (\mathbf{C} - \mathbf{B})]\, \mathbf{q}^o$$

$$[\mathbf{I} - (\mathbf{C} - \mathbf{B})]^{-1} [\mathbf{I} - (\mathbf{C} - \mathbf{B})]\, \mathbf{q}^* > [\mathbf{I} - (\mathbf{C} - \mathbf{B})]^{-1} [\mathbf{I} - (\mathbf{C} - \mathbf{B})]\, \mathbf{q}^o$$

$$\mathbf{q}^* > \mathbf{q}^o$$

Since $\mathbf{C} > \mathbf{B}$ and $\rho(\mathbf{C} - \mathbf{B}) < 1$, so $[\mathbf{I} - (\mathbf{C} - \mathbf{B})]^{-1} \geq \mathbf{I}$. □

Proof of Proposition 8. Following the proof of Theorem 1, we need to show that $\mathbf{M}$ is a

P-matrix. Since $\delta_i < 0$ and $\frac{1+2\delta_i}{\gamma} > (n-3)$, it holds that

$$\begin{aligned}
&\iff \quad 1+2\gamma+\delta_i > (n-1)\gamma - \delta_i \\
&\iff \quad 1+2\gamma+\delta_i > (n-\nu_i-1)\gamma + \nu_i(\gamma - \tfrac{\delta_i}{\nu_i}) \\
&\iff \quad 1+2\gamma+\delta_i > (n-\nu_i-1)\,|\gamma| + \nu_i \left|\gamma - \tfrac{\delta_i}{\nu_i}\right| > 0 \quad \text{for all } i \in N.
\end{aligned}$$

Thus, $\mathbf{M}$ is a strictly diagonally dominant matrix with positive diagonal entries. So $\mathbf{M}$ is a P-matrix, and uniqueness is established. □

Proof of Proposition 9. The first order conditions define the following linear compremen-tarity problem. For all $i = 1, \ldots, n$, the problem is to find an extraction $q_i \geq 0$ which satisfies the system

$$\begin{cases} q_i \geq 0 \\ \alpha_i - q_i - \gamma(Q+q_i) - \frac{\delta_i}{2} \leq 0 \\ \left[\alpha_i - q_i - \gamma(Q+q_i) - \frac{\delta_i}{2}\right] q_i = 0 \end{cases}$$

or equivalently, find a vector $\mathbf{q} \in \mathbb{R}^n_+$ which satisfies the system

$$\begin{cases} \mathbf{q} \geq \mathbf{0} \\ -\tilde{\boldsymbol{\alpha}} + \tilde{\mathbf{M}}\mathbf{q} \geq \mathbf{0} \\ \mathbf{q}^{\mathsf{T}}(-\tilde{\boldsymbol{\alpha}} + \tilde{\mathbf{M}}\mathbf{q}) = 0 \end{cases}$$

where $\tilde{\boldsymbol{\alpha}} = \left[\alpha_i - \frac{\delta_i}{2}\right]_{n\times 1} \in \mathbb{R}^n_+$ and $\tilde{\mathbf{M}} = [\tilde{m}_{i,j}]_{n\times n}$ is such that

$$\tilde{m}_{i,j} = -\frac{\partial U_i}{\partial q_i \partial q_j} = \begin{cases} 1+2\gamma & \text{for } i = j \\ \gamma & \text{for } i \neq j. \end{cases}$$

Let LCP$(-\tilde{\boldsymbol{\alpha}}, \tilde{\mathbf{M}})$ denote the above complementarity problem. Following Cottle et al. (2009, Theorem 3.3.7), the LCP$(-\tilde{\boldsymbol{\alpha}}, \tilde{\mathbf{M}})$ admits a unique solution if $\tilde{\mathbf{M}}$ is a P-matrix. Observe that $\tilde{\mathbf{M}}$ is a strictly diagonally dominant matrix with positive diagonal entries. So $\tilde{\mathbf{M}}$ is a P-matrix, and uniqueness is established. □

References

AMBEC S., SPRUMONT Y. (2002), “ Sharing a river ”, *Journal of Economic Theory*, 107(2), 453-462.

AMBEC S., EHLERS L. (2008), “ Cooperation and equity in the river sharing problem ”, *Game Theory and Policy Making in Natural Resources and the Environment*, 112-131.

AMBEC S., EHLERS L. (2008), “ Sharing a river among satiable agents ”, *Games and Economic Behavior*, 64(1), 35-50.

AMBEC S., DINAR A. (2010), “ Hot stuff: Would climate change alter transboundary water sharing treaties? ”, *TSE Working Paper n. 10-216*.

AMBEC S., DINAR A., MCKINNEY D. (2013), “ Water sharing agreements sustainable to reduced flows ”, *Journal of Environmental Economics and Management*, 66(3), 639-655.

ALLOUCH N. (2015), “ On the private provision of public goods on networks ”, *Journal of Economic Theory*, 157, 527-552.

AZAR O.H. (2004), “ What sustains social norms and how they evolve? The case of tipping ”, *Journal of Economic Behavior & Organization*, 54(1), 49-64.

BALLESTER C., CALVÓ-ARMENGOL A.(2010), “ Interactions with hidden complementarities ”, *Regional Science and Urban Economics*, 40(6), 397-406.

BALLESTER C., CALVÓ-ARMENGOL A., ZENOU Y. (2006), “ Who’s who in networks. Wanted: the key player ”, *Econometrica*, 74(5), 1403-1417.

BARNES A.P., TOMA L., WILLOCK J., HALL C. (2013), “ Comparing a “ budge ” to a “ nudge ”: Farmer responses to voluntary and compulsory compliance in a water quality management regime ”, *Journal of Rural Studies*, 32, 448-459.

BÉNABOU R., TIROLE J. (2006), “ Incentives and prosocial behavior ”, *American Economic Review*, 96(5), 1652-1678.

BERMAN A., PLEMMONS R.J. (1994), *Nonnegative matrices in the mathematical sciences*, SIAM, Philadelphia.

Bernedo M., Ferraro P.J., Price M. (2014), " The persistent impacts of norm-based messaging and their implications for water conservation ", *Journal of Consumer Policy*, 37(3), 437-452.

Bramoullé Y., Kranton R. (2007), " Public goods in networks ", *Journal of Economic Theory*, 135(1), 478-494.

Bramoullé Y., Kranton R., D'amours M. (2014), " Strategic interaction and networks ", *American Economic Review*, 104(3), 898-930.

Brekke K.A., Kverndokk S., Nyborg K. (2003), " An economic model of moral motivation ", *Journal of Public Economics*, 87(9-10), 1967-1983.

Brent D.A., Lott C., Taylor M., Cook J., Rollins K., Stoddard S. (2020), " What Causes Heterogeneous Responses to Social Comparison Messages for Water Conservation? ", *Environmental and Resource Economics*, 77(3), 503-537.

Chabe-Ferret S., Le Coent P., Reynaud A., Subervie J., Lepercq D. (2019), " Can we nudge farmers into saving water? Evidence from a randomised experiment ", *European Review of Agricultural Economics*, 46(3), 393-416.

Cialdini R.B., Reno R.R., Kallgren C.A. (1990), " A focus theory of normative conduct: recycling the concept of norms to reduce littering in public places ", *Journal of Personality and Social Psychology*, 58(6), 1015.

Cottle R.W., Pang J.S., Stone R.E. (2009), *The linear complementarity problem*, SIAM, Philadelphia.

Datta S., Miranda J.J., Zoratto L., Calvo-González O., Darling M., Lorenzana K. (2015), " A behavioral approach to water conservation: Evidence from Costa Rica ", *World Bank Policy Research Working Paper 7283.*

Earnhart D., Ferraro P.J. (2020), " The effect of peer comparisons on polluters: a randomized field experiment among wastewater dischargers ", *Environmental and Resource Economics*, 1-26.

Elster J. (1989), “ Social norms and economic theory ”, *Journal of Economic Perspectives*, 3(4), 99-117.

Falkinger J. (1996), “ Efficient private provision of public goods by rewarding deviations from average ”, *Journal of Public Economics*, 62(3), 413-422.

Gintis H. (2003), “ Solving the puzzle of prosociality ”, *Rationality and Society*, 15(2), 155-187.

Granovetter M.S. (1973), “ The strength of weak ties ”, *American journal of sociology*, 78(6), 1360-1380.

Hardin G. (1968), “ The tragedy of the commons ”, *Science*, 162, 1243-1248.

İlkiliç R. (2011), “ Networks of common property resources ”, *Economic Theory*, 47(1), 105-134.

Kyriakopoulou E., Xepapadeas A. (2021), “ Natural resource management: a network perspective ”, *Environmental and Resource Economics* , 80, 221256.

Kreps D.M. (1997), “ Intrinsic motivation and extrinsic incentives ”, *American Economic Review, Papers and Proceedings*, 87(2), 359-364.

Lange O. (1942), “ The foundations of welfare economics ”, *Econometrica*, 215-228.

Le Coent P., Préget R., Thoyer S. (2021), “ Farmers follow the herd: a theoretical model on social norms and payments for environmental services ”, *Environmental and Resource Economics*, 78(2), 287-306.

Madani K. (2010), “ Game theory and water resources ”, *Journal of Hydrology*, 381(3-4), 225-238.

Nyborg K. (2020), “ No man is an island: social coordination and the environment ”, *Environmental and Resource Economics*, 76(1), 177-193.

Ouvrard B., Stenger A. (2020), “ Informational nudges and public goods in networks ”, *Journal of Environmental Economics and Policy*, 9(3), 281-303.

Pellerano J.A., Price M.K., Puller S.L., Sánchez G.E. (2017), " Do extrinsic incentives undermine social norms? Evidence from a field experiment in energy conservation ", *Environmental and resource Economics*, 67(3), 413-428.

Schubert C. (2017), " Green nudges: Do they work? Are they ethical? ", *Ecological Economics*, 132, 329-342.

Schultz P.W., Nolan J.M, Cialdini R.B., Goldstein N.J. , Griskevicius V. (2007), " The constructive, destructive, and reconstructive power of social norms ", *Psychological Science*, 18(5), 429-434.

Smith V. L. (1968), " Economics of production from natural resources ", *American Economic Review*, 409-431.

Torres, M.M.J., Carlsson F. (2018), " Direct and spillover effects of a social information campaign on residential water-savings ", *Journal of Environmental Economics and Management*, 92, 222-243.

Ushchev P., Zenou Y. (2020), " Social norms in networks ", *Journal of Economic Theory*, 185, 104969.