Expected representative-utility sufficientarianism: a characterization

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Abstract. We develop an ex post approach to sufficientarianism under risk that is based on the notion of expected representative utility. Sufficientarianism is a theory of distributive justice that grants absolute priority to individuals below a sufficiency threshold. While Fleurbaey (2010) characterizes expected representative-utility criteria and related orderings using weak Pareto for equal risk, this axiom conflicts with the absolute priority principle central to sufficientarianism. However, by restricting weak Pareto for equal risk to apply separately within the subdomains above and below the threshold, and by weakening continuity accordingly, we obtain a characterization of the expected representative-utility sufficientarian orderings. Our main theorem establishes that these orderings are characterized by ex post absolute priority, weak Pareto for no risk, restricted versions of weak Pareto for equal risk, statewise dominance, and restricted continuity axioms. We further characterize a subclass based on quasiarithmetic means by strengthening weak Pareto for no risk to its strong counterpart, and adding anonymity and outcome independence of the unconcerned to the list of axioms.

1 Introduction

One of the central recent developments in both welfare economics and normative ethics concerns how to evaluate welfare in situations involving risk. Under risk, what is compared are not the resulting distributions of welfare produced by social actions or public policies, but the prospects, which include the probability distributions over such outcomes. This issue traces back to John Harsanyi's classical aggregation theorem, which shows that in the standard framework with risk, any ordering that is rational (in the sense of conforming to the expected utility theorem) and satisfies the Pareto indifference principle based on individuals' ex ante welfare evaluations must be (possibly weighted) utilitarian (Harsanyi, 1955; Blackorby, Donaldson, and Weymark, 1999, 2008). While this provides a powerful justification for utilitarian approaches under risk, it also highlights the difficulty of incorporating equity considerations into welfare evaluation under risk; see, for example, Diamond (1967).

One simple response is to pursue equality of ex ante individual welfare. This can be done, for example, by applying the maximin principle—or, less extremely, a prioritarian weighting—to the individual expected utilities. Approaches of this nature satisfy ex ante weak Pareto, but they depart from expected utility theory at the social level. To be precise, social judgments violate the von Neumann–Morgenstern axioms (von Neumann and Morgenstern, 1944; 1947). This violation can, in turn, lead to serious difficulties, including the threat of dynamic inconsistency.

The most common alternative proposed in the literature is the ex post approach. Instead of aggregating individuals' expected utilities directly, the ex post approach first evaluates the welfare distribution that arises in each state of the world and then takes its expectation. Concretely, one might apply the maximin criterion across individuals within each state and then take the expectation across states, or apply prioritarian weights within each state before taking the expectation. The latter approach, known as ex post prioritarianism, has attracted considerable attention; see Adler (2018, 2019), Adler and Holtug (2019), Holtug (2019), Adler and Norheim (2022), and Adler, Bossert, Cato, and Kamaga (2025). The ex post approach allows equity considerations to be incorporated by achieving ex post equality of welfare. However, the approach violates ex ante weak Pareto.

Fleurbaey (2010) introduces a novel perspective with the axiom of weak Pareto for equal risk. This axiom requires applying ex ante weak Pareto when comparing prospects in which all individuals face the same risk. Existing ex post approaches not only violate ex ante weak Pareto but also fail to satisfy weak Pareto for equal risk. To address this problem, Fleurbaey (2010) characterizes a new class of ex post evaluation methods using the equally distributed equivalent level of welfare, which we call representative utility in this paper, while ensuring compliance with weak Pareto for equal risk. In cases where all individuals face the same risk, the expected value of representative utility coincides with personal expected utility, thereby satisfying weak Pareto for equal risk.

The purpose of this paper is to extend Fleurbaey's (2010) approach to sufficientarianism. Sufficientarianism is a theory of distributive justice based on welfare thresholds: individuals whose welfare equals or exceeds a given threshold are considered sufficiently well off, while those below the threshold are deemed insufficiently well off. This idea was first proposed by Frankfurt (1987), and in recent years, axiomatic analyses have further developed the framework within welfare economics; see, for example, Roemer (2004), Alcantud, Mariotti, and Veneziani (2022), Bossert, Cato, and Kamaga (2022, 2023), and Chambers and Ye (2024). Frankfurt's approach aimed to maximize the headcount of those above the threshold, but such a method is incompatible with

the weak Pareto principle, whether ex ante or ex post. Moreover, it violates the Pigou-Dalton transfer principle, introduced by Pigou (1912) and Dalton (1920), although Frankfurt himself intended to challenge the ceteris paribus preference for equality implied by the Pigou-Dalton transfer principle. Since Crisp (2003), sufficientarian theories that respect both weak Pareto and the Pigou-Dalton transfer principle have been explored; see also Casal (2007) for a different perspective on sufficientarianism. In particular, Brown (2005), Hirose (2016), Bossert, Cato, and Kamaga (2022, 2023), and others have proposed approaches that first apply priority-based considerations to the group below the threshold. Only when the aggregated welfare of this group shows no difference do utilitarian or prioritarian considerations extend to the group above the threshold. These approaches capture the core of sufficientarianism by granting absolute priority to those below the threshold.

Unlike prioritarianism, sufficientarianism has received little attention in relation to prospects involving risk. Adler, Bossert, Cato, and Kamaga (2025) examine how sufficientarianism can be extended to welfare evaluations in the presence of risk. In it, ex post sufficientarianism is developed as a natural extension of ex post prioritarianism, and is axiomatized by combining the ex post absolute priority axiom and an independence property called prospect independence of the unconcerned with other standard principles. However, since ex post sufficientarianism extends ex post prioritarianism, it fails to satisfy weak Pareto for equal risk. This raises the question of whether there is an ex post sufficientarian approach that does satisfy weak Pareto for equal risk.

The answer is no. The axiom of ex post absolute priority, which assigns unequivocal priority to welfare levels of those below the threshold, conflicts with weak Pareto for equal risk (Adler, Bossert, Cato, and Kamaga, 2025, Theorem 7). Hence, if one adopts sufficientarianism, weak Pareto for equal risk cannot be fully maintained. This, however, does not rule out the applicability of Fleurbaey's representative-utility approach. A possibility emerges if the weak Pareto requirement is restricted to apply only within the domain above the threshold and within the domain below the threshold. This paper investigates how Fleurbaey's evaluation criteria can be adapted to sufficientarianism under such restricted versions of weak Pareto for equal risk.

Our main theorem states that expected representative-utility sufficientarian orderings are characterized by the axioms described in the following paragraph, in addition to ex post absolute priority and weak Pareto for equal risk above and below the threshold.

Statewise dominance is a well-established property in decision theory, according to which one prospect is better than another if, in every state, its outcome is better than that of the other. This is a mild condition for social rationality. The axioms of continuity above the threshold and continuity below the threshold require that small changes in distributions within these subdomains do not lead to sudden jumps in welfare evaluation. These axioms, used by Bossert, Cato, and Kamaga (2022, 2023), are restricted versions of the standard continuity property. Weak Pareto for no risk above the threshold and weak Pareto for no risk below the threshold require that the weak Pareto principle applies if there is no risk within the restricted domains.

Let us clarify the distinction between expected representative-utility sufficientarianism and ex post sufficientarianism. The difference between the orderings in these two classes broadly parallels that between expected representative-utility prioritarianism and ex post prioritarianism. However, as this paper makes clear, the axiomatic foundation of expected representative-utility prioritarianism does not run fully in parallel with that of expected representative-utility sufficientarianism; rather, sufficientarianism requires its own distinctive axiomatic treatment—see Theorems 3 and 4. As shown by Adler, Bossert, Cato, and Kamaga (2025), the defining property of ex post sufficien-

tarianism is prospect independence of the unconcerned, which requires that the relative ranking of prospects does not depend on individuals whose ex post utilities are unaffected in any state. In general, this property is not satisfied by expected representative-utility sufficientarianism. In contrast, as noted above, expected representative-utility sufficientarianism satisfies restricted versions of weak Pareto for equal risk, which are not satisfied by ex post sufficientarianism, in general. This raises the question of whether an ordering can be both expected representative-utility sufficientarian and ex post sufficientarian—that is, whether it can satisfy both prospect independence of the unconcerned and restricted versions of weak Pareto for equal risk. This is possible only in an extremely limited case—specifically, when the transformation function in the formulation of ex post sufficientarianism is affine. The resulting ex post sufficientarian ordering coincides with what we call the arithmetic expected representative-utility sufficientarian ordering, a special case of expected representative-utility sufficientarianism. A characterization of the arithmetic expected representative-utility sufficientarian ordering is provided by additionally using prospect independence of the unconcerned (Theorem 6).

In Section 2, we introduce our setting and basic definitions. Our axioms are presented in Section 3. Section 4 contains our main characterization result and a special case based on quasiarithmetic means. The Pigou-Dalton transfer principle is discussed in Section 5. In Section 6, we explain why it is essential to use censored prospects when deriving representative utilities. Section 7 concludes. Appendix A provides a characterization of the expected representative-utility criteria. The utilitarian ordering, a special case, is also characterized. The independence of the axioms used in our main result is established in Appendix B.

2 Preliminaries

Let $\mathbb{N}$ be the set of all positive integers and $\mathbb{R}$ (respectively $\mathbb{R}_{++}$) be the set of all (respectively all positive) real numbers. For $r \in \mathbb{N}$, $\mathbf{1}_r$ is the r-dimensional vector composed of r ones.

We consider a welfarist framework of social evaluation of state-contingent alternatives. Let $S = \{1, \ldots, m\}$ be the finite set of states, and we assume that $m \geq 2$. Let $(\pi^s)_{s \in S}$ be an exogenously given fixed probability distribution over the states $s \in S$. We assume that $\pi^s > 0$ for all $s \in S$ and $\sum_{s \in S} \pi^s = 1$. This assumption involves no loss of generality as long as there are at least two states with positive probabilities.

The set of individuals $N = \{1, \ldots, n\}$ is fixed, and we assume that $n \geq 2$. Let u_i^s denote the ex post utility level of individual $i \in N$ in state $s \in S$. Social alternatives to be evaluated are given by prospects. A prospect is a profile of individual utilities in all states and is represented by an $n \times m$ matrix $u = (u_i^s)_{i \in N, s \in S}$. The set of all prospects is $\mathcal{D} = \mathbb{R}^{n \times m}$. For any prospect $u = (u_i^s)_{i \in N, s \in S}$, let $u^s = (u_1^s, \ldots, u_i^s, \ldots, u_n^s) \in \mathbb{R}^n$ be the prospect in state $s \in S$. Similarly, $u_i = (u_i^1, \ldots, u_i^s, \ldots, u_i^m) \in \mathbb{R}^m$ represents individual i's prospect. For all $u \in \mathcal{D}$ and for all $i \in N$, let $E(u_i)$ be the expected value $E(u_i) = \sum_{s \in S} \pi^s u_i^s$ of individual i's ex post utilities.

A prospect $u \in \mathcal{D}$ is riskless if $u^s = u^{s'}$ for all $s, s' \in S$. Let $\mathcal{D}^c$ denote the set of all riskless prospects. For all $s \in S$ and for all $u^s \in \mathbb{R}^n$, we write $[u^s]$ as $[u^s] = (u^s, \ldots, u^s) \in \mathcal{D}^c$; that is, $[u^s]$ denotes a riskless prospect such that u^s occurs in each possible state in S. By definition, for each riskless prospect $u \in \mathcal{D}^c$, there exists $u^s \in \mathbb{R}^n$ such that $[u^s] = u$.

A prospect $u \in \mathcal{D}$ is egalitarian if $u_i = u_j$ for all $i, j \in N$. Let $\mathcal{D}^e$ be the set of all egalitarian prospects. Note that, if $u \in \mathcal{D}^e$, then $E(u_i) = E(u_j)$ for all $i, j \in N$. Analogously, if $u \in \mathcal{D}^c$, then $E(u_i) = u_i^s$ for all $s \in S$ and for all $i \in N$.

An important parameter in sufficientarian theories is given by an exogenous threshold level of utility, denoted by $\theta \in \mathbb{R}$. This threshold is the same for all states $s \in S$, and it applies to the ex post utility levels in all states. The interpretation of the threshold θ is that, for each state $s \in S$, those who experience ex post utility levels of at least θ are deemed to have enough.

Let $\mathbb{R}^n_{LE} := (-\infty, \theta]^n$ and $\mathbb{R}^n_{HE} := [\theta, \infty)^n$. For $n = 1$, we write $\mathbb{R}_{LE}$ and $\mathbb{R}_{HE}$ instead of $\mathbb{R}^1_{LE}$ and $\mathbb{R}^1_{HE}$, respectively. In addition to $\mathcal{D}^c$ and $\mathcal{D}^e$, two more subdomains of $\mathcal{D}$ are considered in our analysis. Define $\mathcal{D}_L$ and $\mathcal{D}_H$ by

$$\mathcal{D}_L = \{u \in \mathcal{D} \mid u_i^s \leq \theta \text{ for all } i \in N \text{ and for all } s \in S\}$$

and

$$\mathcal{D}_H = \{u \in \mathcal{D} \mid u_i^s \geq \theta \text{ for all } i \in N \text{ and for all } s \in S\}.$$

Prospects that are censored at the level of the threshold θ are of crucial importance in this paper; see also Takayama (1979) for a discussion of censored distributions. The following two types of censored prospects are used. For each $u \in \mathcal{D}$, let

$$u_L = (\min\{u_i^s, \theta\})_{i \in N, s \in S}$$

and

$$u_H = (\max\{u_i^s, \theta\})_{i \in N, s \in S}.$$

For a given prospect u, the prospect u_L (u_H) is such that all utility levels above (below) θ in u are replaced with the threshold θ. By definition, for all $u \in \mathcal{D}$, it follows that $u_L \in \mathcal{D}_L$ and $u_L^s \in \mathbb{R}^n_{LE}$ for all $s \in S$. Analogously, $u_H \in \mathcal{D}_H$ and $u_H^s \in \mathbb{R}^n_{HE}$ for all $s \in S$.

A social ordering of prospects is a reflexive, complete, and transitive relation R on $\mathcal{D}$. For two prospects $u, v \in \mathcal{D}$, we write uRv instead of $(u, v) \in R$ to indicate that u is at least as good as v. The asymmetric and symmetric parts of R are denoted by P and I.

We examine an ex post approach to sufficientarianism that is based on the notion of expected representative utility. Given an ordering R on $\mathcal{D}$, the representative-utility function $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ corresponding to R is implicitly defined by letting $\Xi(u^s) \in \mathbb{R}$ be equal to the utility level that satisfies

$$[u^s] I [\Xi(u^s)\mathbf{1}_n] \tag{1}$$

for each prospect $u^s \in \mathbb{R}^n$. If an ordering R satisfies suitable monotonicity and continuity properties, the utility level $\Xi(u^s)$ satisfying (1) uniquely exists, so that the function Ξ is well-defined. Following Blackorby and Donaldson (1984), we refer to $\Xi(u^s)$ as the representative utility for u^s. The level of representative utility is analogous to the equally distributed equivalent income familiar from ethical approaches to the measurement of income inequality. Fleurbaey (2010) refers to Ξ as an equally distributed equivalent function in the presence of continuity.

Fleurbaey (2010) uses the expected value of the representative utility level to define what he refers to as expected equally distributed equivalent criteria. An ordering R is an expected equally distributed equivalent criterion if there exists a function $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ such that Ξ satisfies (1) on $\mathbb{R}^n$ and, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s \Xi(u^s) \geq \sum_{s \in S} \pi^s \Xi(v^s).$$

In plain words, equally distributed equivalent criteria use the expected value of representative utility levels obtained from each state. Note that expected equally distributed equivalent criteria

allow for the possibility that Ξ is not increasing. In this paper, we explicitly introduce what we call expected representative-utility criteria in order to focus, as suggested by Fleurbaey (2010, p. 658), on a well-behaved subclass of equally distributed equivalent criteria. To do so, let us define increasingness and strict increasingness. A function $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ is increasing if, for all $u^s, v^s \in \mathbb{R}^n$,

$$[u_i^s > v_i^s \text{ for all } i \in N] \;\Rightarrow\; \Xi(u^s) > \Xi(v^s),$$

and strictly increasing if, for all $u^s, v^s \in \mathbb{R}^n$,

$$[u_i^s \geq u_i^s \text{ for all } i \in N \text{ and } \exists j \in N \text{ such that } u_j^s > v_j^s] \;\Rightarrow\; \Xi(u^s) > \Xi(v^s).$$

An ordering R is an expected representative-utility criterion if there exists an increasing and continuous function $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ such that Ξ satisfies (1) on $\mathbb{R}^n$ and, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \Xi(u^s) \geq \sum_{s\in S} \pi^s \Xi(v^s).$$

We define a notion of ex post sufficientarianism that is based on expected representative utilities. Let $\mathcal{F}$ be the set of all functions $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ that are increasing and continuous on $\mathbb{R}^n_{LE} \cup \mathbb{R}^n_{HE}$. Similarly, let $\mathcal{F}^*$ be the set of all functions $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ that are strictly increasing and continuous on $\mathbb{R}^n_{LE} \cup \mathbb{R}^n_{HE}$. An ordering R is expected representative-utility sufficientarian if there exists a function $\Xi \in \mathcal{F}$ such that Ξ satisfies (1) on both $\mathbb{R}^n_{LE}$ and $\mathbb{R}^n_{HE}$, and, for all $u, v \in \mathcal{D}$,

$$\begin{aligned} uRv \;\Leftrightarrow\; & \sum_{s\in S} \pi^s \Xi(u_L^s) > \sum_{s\in S} \pi^s \Xi(v_L^s) \;\text{ or} \\ & \left[\sum_{s\in S} \pi^s \Xi(u_L^s) = \sum_{s\in S} \pi^s \Xi(v_L^s) \;\text{ and }\; \sum_{s\in S} \pi^s \Xi(u_H^s) \geq \sum_{s\in S} \pi^s \Xi(v_H^s)\right]. \end{aligned}$$

An ordering R is strict expected representative-utility sufficientarian if it is an expected representative-utility sufficientarian ordering with a function Ξ that belongs to $\mathcal{F}^*$.

3 Axioms

We now present the axioms that play a role in this paper. As alluded to above, a crucial property for a sufficientarian ordering is absolute priority below the threshold. It requires that the distribution of those above the threshold is used as a tie-breaking device only, in case the distribution of those below the threshold are equally good. Bossert, Cato, and Kamaga (2023) formulate a variant of this axiom in a variable population setting.

Ex post absolute priority: For all $u, v \in \mathcal{D}$,

$$u_L P v_L \;\Rightarrow\; uPv,$$

and

$$u_L I v_L \;\Rightarrow\; [u_H R v_H \;\Leftrightarrow\; uRv].$$

Next, we introduce a series of Paretian axioms. The first of these is a natural translation of the weak Pareto principle for outcomes. It applies in situations without risk.

Weak Pareto for no risk: For all $u, v \in \mathcal{D}^c$, if $u_i^s > v_i^s$ for all $i \in N$, then uPv.

We also employ the two restricted versions of weak Pareto for no risk. These two axioms apply weak Pareto for no risk to the two subdomains $\mathcal{D}_H$ and $\mathcal{D}_L$.

Weak Pareto for no risk above the threshold: For all $u, v \in \mathcal{D}^c \cap \mathcal{D}_H$, if $u_i^s > v_i^s$ for all $i \in N$, then uPv.

Weak Pareto for no risk below the threshold: For all $u, v \in \mathcal{D}^c \cap \mathcal{D}_L$, if $u_i^s > v_i^s$ for all $i \in N$, then uPv.

The strong counterpart of weak Pareto for no risk is defined as follows.

Strong Pareto for no risk: For all $u, v \in \mathcal{D}^c$, if $u_i^s \geq v_i^s$ for all $i \in N$, then uRv and, if at least one of these inequalities is strict, then uPv.

Another unanimity-motivated axiom is the ex ante weak Pareto principle, which requires that if every individual experiences a higher expected utility under u, then u is better. It is well-known that this property is in conflict with the traditional social expected utility hypothesis combined with the Pigou-Dalton transfer principle, even when the principle is restricted to riskless prospects, because the ex ante weak Pareto principle together with the social expected utility hypothesis implies that the evaluation must be done by the comparison of weighted sums of individuals' expected utilities (Blackorby, Donaldson, and Weymark, 1999, 2008); see also Fleurbaey (2009) and Adler (2019, pp. 284–285). Fleurbaey (2010) weakens the ex ante weak Pareto principle by limiting its scope to the subdomain $\mathcal{D}^e$ of egalitarian prospects.

Weak Pareto for equal risk: For all $u, v \in \mathcal{D}^e$, if $E(u_i) > E(v_i)$ for all $i \in N$, then uPv.

Adler, Bossert, Cato, and Kamaga (2025, Theorem 7) show that this axiom is not compatible with ex post absolute priority. However, this observation does not necessarily mean that weak Pareto for equal risk should be abandoned entirely. We now formulate two weaker versions of the property. They restrict the requirement to egalitarian prospects such that no one is below or at (above or at) the threshold in each state.

Weak Pareto for equal risk above the threshold: For all $u, v \in \mathcal{D}^e \cap \mathcal{D}_H$, if $E(u_i) > E(v_i)$ for all $i \in N$, then uPv.

Weak Pareto for equal risk below the threshold: For all $u, v \in \mathcal{D}^e \cap \mathcal{D}_L$, if $E(u_i) > E(v_i)$ for all $i \in N$, then uPv.

These two restricted versions are implied by weak Pareto for equal risk, but the conjunction of the two does not imply weak Pareto for equal risk.

The following axiom requires that if state s in prospect u is better than state s in prospect v for all states $s \in S$, then prospect u is better than v. This is a plausible axiom of social rationality.

Statewise dominance: For all $u, v \in \mathcal{D}$, if $[u^s]P[v^s]$ for all $s \in S$, then uPv.

As pointed out by Roemer (2004), suitably defined sufficientarian orderings cannot satisfy continuity, defined as follows.

Continuity: For all $u \in \mathcal{D}$, the sets $\{v \in \mathcal{D} \mid vRu\}$ and $\{v \in \mathcal{D} \mid uRv\}$ are closed in $\mathcal{D}$.

Bossert, Cato, and Kamaga (2022, 2023) formulate continuity axioms that apply only within the subdomains in which all individuals are above or below the threshold. The following axioms are their extension to social orderings on prospects; see also Adler, Bossert, Cato, and Kamaga (2025).

Continuity above the threshold: For all $u \in \mathcal{D}_H$, the sets $\{v \in \mathcal{D}_H \mid vRu\}$ and $\{v \in \mathcal{D}_H \mid uRv\}$ are closed in $\mathcal{D}_H$.

Continuity below the threshold: For all $u \in \mathcal{D}_L$, the sets $\{v \in \mathcal{D}_L \mid vRu\}$ and $\{v \in \mathcal{D}_L \mid uRv\}$ are closed in $\mathcal{D}_L$.

For all non-empty strict subsets M of N and for all $u \in \mathcal{D}$, we define $u_M = (u_i)_{i \in M}$ and $u_{N \setminus M} = (u_i)_{i \in N \setminus M}$. The axiom of prospect independence of the unconcerned employed by Adler, Bossert, Cato, and Kamaga (2025, p. 1375) requires that the social ordering R be independent of the ex post utilities of those who are unconcerned in every state; see Adler, Bossert, Cato, and Kamaga (2025) for a formal definition and a detailed discussion of this property. The following axiom weakens this requirement by restricting its scope to riskless prospects.

Outcome independence of the unconcerned: For all $u, u', v, v' \in \mathcal{D}^c$, and for all non-empty $M \subset N$,

$$(u_M, v_{N \setminus M}) R (u'_M, v_{N \setminus M}) \Leftrightarrow (u_M, v'_{N \setminus M}) R (u'_M, v'_{N \setminus M}).$$

Anonymity is the standard impartiality axiom according to which people's identities are irrelevant for social evaluation.

Anonymity: For all $u, v \in \mathcal{D}$ and all bijections $\rho \colon N \to N$, if $v_i = u_{\rho(i)}$ for all $i \in N$, then uIv.

4 Expected representative-utility sufficientarianism

As alluded to in the Introduction, there exists no social ordering R that satisfies weak Pareto for equal risk and ex post absolute priority; see Adler, Bossert, Cato, and Kamaga (2025, Theorem 7). This suggests that weak Pareto for equal risk must be restricted in scope. Accordingly, we impose weak Pareto for equal risk above the threshold and below the threshold. The key question is whether these restricted versions remain compatible with continuity when ex post absolute priority is imposed. The following result shows that they do not: even the restricted forms of weak Pareto for equal risk are incompatible with continuity under ex post absolute priority.

Theorem 1. There exists no social ordering R that satisfies ex post absolute priority, weak Pareto for equal risk above the threshold, weak Pareto for equal risk below the threshold, and continuity.

Proof. For simplicity, we assume that $N = \{1, 2\}$. Let u^k be such that

$$u_i^k = (\theta - 1/k, \theta + 10)$$

for all $i \in N$. Let v^k be such that

$$v_i^k = (\theta - 1/k, \theta)$$

for all $i \in N$. By weak Pareto for equal risk below the threshold,

$$wPv^k$$

where $w_i = (\theta, \theta)$ for all $i \in N$. Ex post absolute priority implies that

$$wPu^k.$$

This is true for all $k \in \mathbb{N}$. Thus, continuity implies that

$$wRw',$$

where $w'_i = (\theta, \theta + 10)$ for all $i \in N$. However, weak Pareto for equal risk above the threshold implies that

$$w'Pw.$$

This is a contradiction. ■

This theorem suggests that it is necessary to relax continuity as long as weak Pareto for equal risk above and below the threshold and ex post absolute priority are simultaneously imposed.

We note that ex post absolute priority by itself does not necessarily conflict with continuity. If we drop weak Pareto for equal risk above the threshold and weak Pareto for equal risk below the threshold, it is compatible with continuity even in the presence of statewise dominance and weak Pareto for no risk.

Theorem 2. There exists a social ordering R that satisfies ex post absolute priority, weak Pareto for no risk, statewise dominance, and continuity.

Proof. Define R^m by letting

$$uR^m v \;\Leftrightarrow\; \min u \geq \min v$$

where

$$\min u = \min\{u_i^s\}_{s \in S, i \in N}.$$

This satisfies all properties in the theorem statement. ■

We now present a characterization result for the class of expected representative-utility sufficientarian orderings.

Theorem 3. A social ordering R satisfies ex post absolute priority, weak Pareto for no risk above the threshold, weak Pareto for no risk below the threshold, weak Pareto for equal risk above the threshold, weak Pareto for equal risk below the threshold, statewise dominance, continuity above the threshold, and continuity below the threshold if and only if it is expected representative-utility sufficientarian.

The axioms in the theorem statement are logically independent; see Appendix B.

The following three lemmas are used in the proof of Theorem 3.

Lemma 1. Assume that a social ordering R satisfies statewise dominance and continuity below the threshold (continuity above the threshold, respectively). For all $u, v \in \mathcal{D}_L$ ($u, v \in \mathcal{D}_H$, respectively), if $[u^s]I[v^s]$ for all $s \in S$, then uIv.

Proof. We prove the statement of the lemma for the case in which continuity below the threshold is satisfied; the proof for continuity above the threshold is analogous.

Assume that $[u^s]I[v^s]$ for all $s \in S$, and let $u(k), v(k) \in \mathcal{D}_L$ be such that

$$u_i^s(k) = u_i^s - \frac{1}{k};$$
$$v_i^s(k) = v_i^s - \frac{1}{k}.$$

Since $[u^s]P[u^s(k)]$ for all $s \in S$ and for all $k \in \mathbb{N}$, transitivity implies that $[v^s]P[u^s(k)]$ for all $s \in S$ and for all $k \in \mathbb{N}$. Therefore, statewise dominance implies that

$$vPu(k) \text{ for all } k \in \mathbb{N}.$$

Analogously, it follows that

$$uPv(k) \text{ for all } k \in \mathbb{N},$$

and continuity below the threshold implies that uRv. Combining these two observations, it follows that uIv. ■

Lemma 2. If a social ordering R satisfies weak Pareto for no risk below the threshold, weak Pareto for equal risk below the threshold, statewise dominance, and continuity below the threshold, then there exists a function $\Xi_L \colon \mathbb{R}^n_{LE} \to \mathbb{R}$ such that Ξ_L satisfies (1) and

$$uRv \;\Leftrightarrow\; \sum_{s \in S} \pi^s \Xi_L(u^s) \geq \sum_{s \in S} \pi^s \Xi_L(v^s)$$

for all $u, v \in \mathcal{D}_L$.

Proof. First, we show that the representative utility level is well-defined on $\mathcal{D}_L$. That is, we prove that, for each prospect $u \in \mathcal{D}_L$ and for all $s \in S$, there exists a unique utility level $\Xi_L(u^s)$ such that

$$[u^s]I[\Xi_L(u^s)\mathbf{1}_n].$$

We follow the proof strategy that is employed in Theorem 4.1 of Blackorby, Bossert, and Donaldson (2005, p. 94). The conjunction of weak Pareto for no risk and continuity implies that, for all $u, v \in \mathcal{D}^c$, if $u_i^s \geq v_i^s$ for all $i \in N$ and $u_i^s > v_i^s$ for some $i \in N$, then uRv. This observation and reflexivity together imply

$$[a\mathbf{1}_n]R[u^s]$$

for all $a \in [\max u_i^s, \theta]$. Note that $[a\mathbf{1}_n] \in \mathcal{D}_L$. Analogously, we have

$$[u^s]R[b\mathbf{1}_n]$$

for $b \leq \min u_i^s$. Since the restriction of R_L on $\mathcal{D}_L \cap \mathcal{D}^c$ is continuous, there exists $\xi \in [\min u_i^s, \max u_i^s]$ such that $[\xi\mathbf{1}_n]I[u^s]$. Weak Pareto for no risk guarantees the uniqueness of ξ. Let $\Xi_L(u^s) = \xi$. By definition, $\Xi_L(x\mathbf{1}_n) = x$.

Now assume that, for $u, v \in \mathcal{D}_L$,

$$\sum_{s \in S} \pi^s \Xi_L(u^s) > \sum_{s \in S} \pi^s \Xi_L(v^s).$$

Note that $[\Xi_L(u^s)]\mathbf{1}_n I[u_s]$ and $[\Xi_L(v^s)]\mathbf{1}_n I[v_s]$. Define $\hat{u}, \hat{v}$ by letting

$$\hat{u}_i^s = \Xi_L(u^s) \text{ for all } i \in N \text{ and for all } s \in S;$$
$$\hat{v}_i^s = \Xi_L(v^s) \text{ for all } i \in N \text{ and for all } s \in S.$$

Individual i's expected utility levels in $\hat{u}$ and in $\hat{v}$ are given by $\sum_{s\in S} \pi^s \Xi_L(u^s)$ and $\sum_{s\in S} \pi^s \Xi_L(v^s)$, respectively. By construction,

$$[u^s] I [\hat{u}^s] \text{ and } [v^s] I [\hat{v}^s]$$

for all $s \in S$. Lemma 1 implies

$$u I \hat{u} \text{ and } v I \hat{v}.$$

Since $\hat{u}, \hat{v} \in \mathcal{D}_L \cap \mathcal{D}^e$, weak Pareto for equal risk below the threshold can be applied. Because $\sum_{s\in S} \pi^s \Xi_L(u^s) > \sum_{s\in S} \pi^s \Xi_L(v^s)$, we obtain

$$\hat{u} P \hat{v}.$$

Transitivity implies that uPv.

It remains to show that, for all $u, v \in \mathcal{D}_L$, if

$$\sum_{s\in S} \pi^s \Xi_L(u^s) = \sum_{s\in S} \pi^s \Xi_L(v^s),$$

then uIv. Define $\tilde{u}(k), \tilde{v}(k)$ by letting

$$\tilde{u}_i^s(k) = \Xi_L(u^s) - \frac{1}{k} \text{ for all } i \in N;$$
$$\tilde{v}_i^s(k) = \Xi_L(v^s) - \frac{1}{k} \text{ for all } i \in N.$$

Since $\sum_{s\in S} \pi^s \Xi_L(u^s) > \sum_{s\in S} \pi^s \Xi_L(\tilde{v}^s(k))$ for all $k \in \mathbb{N}$, it follows that

$$u P \tilde{v}(k)$$

for all $k \in \mathbb{N}$. Continuity below the threshold implies that uRv. Analogously, it follows that

$$v P \tilde{u}(k)$$

and, thus, we obtain vRu and therefore uIv. ■

Lemma 3. If a social ordering R satisfies weak Pareto for no risk above the threshold, weak Pareto for equal risk above the threshold, statewise dominance, and continuity above the threshold, then there exists a function $\Xi_H \colon \mathbb{R}^n_{HE} \to \mathbb{R}$ such that Ξ_H satisfies (1) and

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \Xi_H(u^s) \geq \sum_{s\in S} \pi^s \Xi_H(v^s)$$

for all $u, v \in \mathcal{D}_H$.

The proof of Lemma 3 is analogous to the proof of Lemma 2.

Proof of Theorem 3. 'If.' Let R be an expected representative-utility sufficientarian ordering. By definition, there exists a function $\Xi \in \mathcal{F}$ such that Ξ satisfies (1) on both $\mathbb{R}^n_{LE}$ and $\mathbb{R}^n_{HE}$, and, for all $u, v \in \mathcal{D}$,

$$uRv \Leftrightarrow \sum_{s\in S} \pi^s \Xi(u^s_L) > \sum_{s\in S} \pi^s \Xi(v^s_L) \text{ or } \left[\sum_{s\in S} \pi^s \Xi(u^s_L) = \sum_{s\in S} \pi^s \Xi(v^s_L) \text{ and } \sum_{s\in S} \pi^s \Xi(u^s_H) \geq \sum_{s\in S} \pi^s \Xi(v^s_H)\right].$$

That R satisfies ex post absolute priority follows by definition.

Next, we show that weak Pareto for no risk below the threshold is satisfied. Let $u, v \in \mathcal{D}^c \cap \mathcal{D}_L$ be such that $u^s_i > v^s_i$ for all $i \in N$. It follows that

$$(u^s_L)_i > (v^s_L)_i \text{ for all } i \in N.$$

Increasingness implies that

$$\Xi(u^s_L) > \Xi(v^s_L).$$

Therefore, uPv, and weak Pareto for no risk below the threshold is satisfied.

That weak Pareto for no risk above the threshold is satisfied follows analogously.

We now show that weak Pareto for equal risk below the threshold is satisfied. Let $u, v \in \mathcal{D}^e \cap \mathcal{D}_L$ be such that $E(u_i) > E(v_i)$ for all $i \in N$. By definition, $u_i = u_j$ for all $i, j \in N$, and $v_i = v_j$ for all $i, j \in N$. We have

$$\Xi(u^s) = u^s_i \text{ and } \Xi(v^s) = v^s_i$$

and, thus,

$$E(u_i) = \sum_{s\in S} \pi^s \Xi(u^s) > \sum_{s\in S} \pi^s \Xi(v^s) = E(v_i).$$

By the definition of an expected representative-utility sufficientarian ordering, we obtain uPv. Thus, weak Pareto for equal risk below the threshold is satisfied.

That weak Pareto for equal risk above the threshold is satisfied follows analogously.

We now prove that statewise dominance is satisfied. Let $u, v \in \mathcal{D}$ and assume that $[u^s]P[v^s]$ for all $s \in S$. By definition,

$$\Xi(u^s_L) > \Xi(v^s_L) \text{ or } [\Xi(u^s_L) = \Xi(v^s_L) \text{ and } \Xi(u^s_H) > \Xi(v^s_H)] \text{ for all } s \in S.$$

We obtain

$$\sum_{s\in S} \pi^s \Xi(u^s_L) > \sum_{s\in S} \pi^s \Xi(v^s_L)$$

or

$$\left[\sum_{s\in S} \pi^s \Xi(u^s_L) = \sum_{s\in S} \pi^s \Xi(v^s_L) \text{ and } \sum_{s\in S} \pi^s \Xi(u^s_H) > \sum_{s\in S} \pi^s \Xi(v^s_H)\right].$$

This implies uPv and, therefore, statewise dominance is satisfied.

It is immediate that the two continuity axioms are satisfied.

'Only if.' Let R be a social ordering that satisfies the axioms listed in the theorem. By Lemmas 2 and 3, there are $\Xi_L\colon \mathbb{R}^n_{LE} \to \mathbb{R}$ and $\Xi_H\colon \mathbb{R}^n_{HE} \to \mathbb{R}$ with the requisite properties. Define $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ by

$$\begin{aligned}
&\Xi(u^s) = \Xi_L(u^s) \text{ for all } u^s \in \mathbb{R}^n_{LE};\\
&\Xi(u^s) = \Xi_H(u^s) \text{ for all } u^s \in \mathbb{R}^n_{HE};\\
&\Xi(u^s) = 0 \text{ for all } u^s \notin \mathbb{R}^n_{LE} \cup \mathbb{R}^n_{HE}.
\end{aligned}$$

Note that Ξ is continuous and increasing on $\mathbb{R}^n_{LE} \cup \mathbb{R}^n_{HE}$ because $\Xi_L(\theta \mathbf{1}_n) = \Xi_H(\theta \mathbf{1}_n)$. Moreover, by construction, Ξ satisfies (1) on $\mathbb{R}^n_{LE}$ and $\mathbb{R}^n_{HE}$.

Assume that

$$\sum_{s\in S} \pi^s \Xi(u^s_L) > \sum_{s\in S} \pi^s \Xi(v^s_L)$$

or

$$\left[\sum_{s\in S} \pi^s \Xi(u^s_L) = \sum_{s\in S} \pi^s \Xi(v^s_L) \text{ and } \sum_{s\in S} \pi^s \Xi(u^s_H) > \sum_{s\in S} \pi^s \Xi(v^s_H)\right].$$

If $\sum_{s\in S} \pi^s \Xi(u^s_L) > \sum_{s\in S} \pi^s \Xi(v^s_L)$, then $\sum_{s\in S} \pi^s \Xi_L(u^s_L) > \sum_{s\in S} \pi^s \Xi_L(v^s_L)$. Lemma 2 implies that $u_L P v_L$. By ex post absolute priority, we obtain uPv.

If $\sum_{s\in S} \pi^s \Xi(u^s_L) = \sum_{s\in S} \pi^s \Xi(v^s_L)$ and $\sum_{s\in S} \pi^s \Xi(u^s_H) > \sum_{s\in S} \pi^s \Xi(v^s_H)$, then $\sum_{s\in S} \pi^s \Xi_L(u^s_L) = \sum_{s\in S} \pi^s \Xi_L(v^s_L)$ and $\sum_{s\in S} \pi^s \Xi_H(u^s_H) > \sum_{s\in S} \pi^s \Xi_H(v^s_H)$. Lemmas 2 and 3 imply that $u_L I v_L$ and $u_H P v_H$. It follows from ex post absolute priority that uPv.

Ex post absolute priority implies that uIv whenever

$$\sum_{s\in S} \pi^s \Xi(u^s_L) = \sum_{s\in S} \pi^s \Xi(v^s_L) \text{ and } \sum_{s\in S} \pi^s \Xi(u^s_H) = \sum_{s\in S} \pi^s \Xi(v^s_H).$$

This completes the proof. ■

An expected representative-utility sufficientarian ordering does not necessarily satisfy weak Pareto for no risk. Consider the case where

$$\Xi(u^s) = \max u^s \text{ for all } u^s \in \mathbb{R}^n_{LE}$$

and

$$\Xi(u^s) = \min u^s \text{ for all } u^s \in \mathbb{R}^n_{HE}.$$

Note that equation (1) is satisfied on $\mathbb{R}^n_{LE}$ and $\mathbb{R}^n_{HE}$. The resulting ordering is given by

$$\begin{aligned}
uRv \;\Leftrightarrow\; &\sum_{s\in S} \pi^s \max u^s_L > \sum_{s\in S} \pi^s \max v^s_L \text{ or}\\
&\left[\sum_{s\in S} \pi^s \max u^s_L = \sum_{s\in S} \pi^s \max v^s_L \text{ and } \sum_{s\in S} \pi^s \min u^s_H \geq \sum_{s\in S} \pi^s \min v^s_H\right].
\end{aligned}$$

This ordering satisfies all axioms in Theorem 3, but it does not satisfy weak Pareto for no risk. To see the violation of weak Pareto for no risk, consider the riskless prospects $u, v \in \mathcal{D}^c$ given by

$$u = \begin{bmatrix} 1 & 1 \\ 12 & 12 \end{bmatrix} \text{ and } v = \begin{bmatrix} 2 & 2 \\ 100 & 100 \end{bmatrix}$$

where $\theta = 10$. Note that

$$\max u_L^s = 10 \text{ and } \max v_L^s = 10$$

and

$$\min u_H^s = 10 \text{ and } \min v_H^s = 10.$$

Hence, the two prospects are indifferent.

In contrast to expected representative-utility sufficientarian orderings, strict expected representative-utility sufficientarian orderings satisfy strong Pareto for no risk. Thus, the following characterization result can be obtained.

Theorem 4. A social ordering R satisfies ex post absolute priority, strong Pareto for no risk, weak Pareto for equal risk above the threshold, weak Pareto for equal risk below the threshold, statewise dominance, continuity above the threshold, and continuity below the threshold if and only if it is strict expected representative-utility sufficientarian.

Proof of Theorem 4. 'If.' The key to this part is to show that strong Pareto for no risk is satisfied. Let $u, v \in \mathcal{D}^c$ be such that $u_i^s \geq v_i^s$ for all $i \in N$ and there exists $j \in N$ such that $u_j^s > v_j^s$. It follows that

$$[(u_L^s)_i \geq (v_L^s)_i \text{ and } (u_H^s)_i \geq (v_H^s)_i] \text{ for all } i \in N$$

and, moreover,

$$[(u_L^s)_j > (v_L^s)_j \text{ or } (u_H^s)_j > (v_H^s)_j] \text{ for some } j \in N.$$

Strict increasingness implies that

$$\Xi(u_L^s) > \Xi(v_L^s) \text{ or } [\Xi(u_L^s) = \Xi(v_L^s) \text{ and } \Xi(u_H^s) > \Xi(v_H^s)].$$

Therefore, uPv, and strong Pareto for no risk is satisfied.

'Only if.' This part is essentially the same as that of Theorem 3. Strict increasingness of Ξ on $\mathbb{R}_{HE}^n$ and $\mathbb{R}_{LE}^n$ follows from strong Pareto for no risk. ■

An interesting special case of expected representative-utility sufficientarianism is obtained if R is based on a quasiarithmetic mean. Suppose that there is an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $[u^s], [v^s] \in \mathcal{D}^c$,

$$[u^s] R [v^s] \Leftrightarrow \sum_{i \in N} g(u_i^s) \geq \sum_{i \in N} g(v_i^s).$$

The corresponding representative utility is given by

$$\Xi(u^s) = g^{-1}\Big(\frac{\sum_{i \in N} g(u_i^s)}{n}\Big)$$

for all $u^s \in \mathbb{R}^n$. A social ordering R is quasiarithmetic expected representative-utility sufficientarian if there exists an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$\begin{aligned} uRv \Leftrightarrow & \sum_{s \in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i \in N} g(u_{Li}^s)}{n}\Big)\Big) > \sum_{s \in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i \in N} g(v_{Li}^s)}{n}\Big)\Big) \text{ or} \\ & \Big[\sum_{s \in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i \in N} g(u_{Li}^s)}{n}\Big)\Big) = \sum_{s \in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i \in N} g(v_{Li}^s)}{n}\Big)\Big) \text{ and} \\ & \sum_{s \in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i \in N} g(u_{Hi}^s)}{n}\Big)\Big) \geq \sum_{s \in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i \in N} g(v_{Hi}^s)}{n}\Big)\Big)\Big]. \end{aligned}$$

If a quasiarithmetic expected representative-utility sufficientarian ordering is associated with a function g that is affine both on $\mathbb{R}_{LE}$ and $\mathbb{R}_{HE}$, we call it the arithmetic expected representative-utility sufficientarian ordering; note that all functions g satisfying the requisite properties generate a single identical social ordering.

The class of quasiarithmetic expected representative-utility sufficientarian orderings is characterized in the following theorem.

Theorem 5. Assume that $n \geq 3$. A social ordering R satisfies ex post absolute priority, strong Pareto for no risk, weak Pareto for equal risk above the threshold, weak Pareto for equal risk below the threshold, statewise dominance, continuity above the threshold, continuity below the threshold, outcome independence of the unconcerned, and anonymity if and only if it is quasiarithmetic expected representative-utility sufficientarian.

Lemma 4. Assume that $n \geq 3$. If a social ordering R satisfies strong Pareto for no risk, continuity above the threshold, outcome independence of the unconcerned, and anonymity, then there exists an increasing and continuous function $g_H \colon \mathbb{R} \to \mathbb{R}$ such that, for all $[u^s], [v^s] \in \mathcal{D}^c \cap \mathcal{D}_H$,

$$[u^s] R [v^s] \;\Leftrightarrow\; \sum_{i \in N} g_H(u_i^s) \geq \sum_{i \in N} g_H(v_i^s).$$

Proof. This is a variant of Lemma 1 of Adler, Bossert, Cato, and Kamaga (2025). The restriction of R on $\mathcal{D}^c \cap \mathcal{D}_H$ gives us the ordering R_H^s on $\{u^s \in \mathbb{R}^n \mid u_i^s \geq \theta\}$. Note that R_H^s satisfies strong Pareto, anonymity, continuity, and independence of the unconcerned because R satisfies strong Pareto for no risk, continuity above the threshold, outcome independence of the unconcerned, and anonymity. Since $n \geq 3$, the results of Debreu (1959, pp. 56–59) and Blackorby, Bossert, and Donaldson (2005, Theorem 4.7) imply that there exists an increasing and continuous function $g_H \colon \mathbb{R} \to \mathbb{R}$ such that

$$[u^s] R^s [v^s] \;\Leftrightarrow\; \sum_{i \in N} g_H(u_i^s) \geq \sum_{i \in N} g_H(v_i^s). \blacksquare$$

Lemma 5. Assume that $n \geq 3$. If a social ordering R satisfies strong Pareto for no risk, continuity below the threshold, outcome independence of the unconcerned, and anonymity, then there exists an increasing and continuous function $g_L \colon \mathbb{R} \to \mathbb{R}$ such that, for all $[u^s], [v^s] \in \mathcal{D}^c \cap \mathcal{D}_L$,

$$[u^s] R [v^s] \;\Leftrightarrow\; \sum_{i \in N} g_L(u_i^s) \geq \sum_{i \in N} g_L(v_i^s).$$

Proof. The proof of this lemma is similar to that of Lemma 4. $\blacksquare$

Proof of Theorem 5. 'If.' It is straightforward to show that the quasiarithmetic expected representative-utility sufficientarian orderings satisfy the axioms in the theorem statement.

'Only if.' By Lemmas 4 and 5, there are $g_H \colon \mathbb{R}_{HE} \to \mathbb{R}$ and $g_L \colon \mathbb{R}_{LE} \to \mathbb{R}$ with the requisite properties. Define

$$g(t) = \begin{cases} g_L(t) - g_L(\theta) + g_H(\theta) \text{ for } t \in (-\infty, \theta]; \\ g_H(t) \text{ for } t \in (\theta, +\infty). \end{cases}$$

By construction, this is an increasing and continuous function. Note that the corresponding representative utility is given by

$$\Xi(u^s) = g^{-1}\Big(\frac{\sum_{i \in N} g(u_i^s)}{n}\Big).$$

This applies to both $\mathcal{D}_H$ and $\mathcal{D}_L$. Theorem 4 implies that R is an expected representative-utility sufficientarian ordering with $\Xi(u^s) = g^{-1}\left(\frac{\sum_{i\in N} g(u_i^s)}{n}\right)$. This establishes the 'only if' part of the theorem. ■

The members of the class of quasiarithmetic expected representative-utility sufficientarian orderings respect a concern for distributive fairness in the sense that it assigns absolute moral priority to the welfare levels of individuals below the threshold over those above it. However, it remains (potentially) insensitive to distributive inequalities among individuals below the threshold (and likewise among those above it). To incorporate these additional concerns and to require full compliance with standard fairness properties, it becomes necessary to introduce a variant of the Pigou-Dalton transfer axiom that is restricted to riskless prospects within the domain below (above) the threshold. For further discussion, see Bossert, Cato, and Kamaga (2022, 2023). Typically, the transformation function g is required to be strictly concave on $\mathbb{R}_{LE}$ and $\mathbb{R}_{HE}$. Note that it is not necessary for g to be concave on the entire domain. A formal observation of the equity property that expected representative-utility sufficientarian orderings can possess is provided in the next section.

When associated with a transformation function g that is strictly concave on $\mathbb{R}_{LE}$ and $\mathbb{R}_{HE}$, the corresponding quasiarithmetic expected representative-utility sufficientarian orderings become distinct from ex post sufficientarianism characterized by Adler, Bossert, Cato, and Kamaga (2025). For all $u \in \mathcal{D}$ and for all $s \in S$, let $L(u^s) = \{i \in N \mid u_i^s < \theta\}$ and $H(u^s) = \{i \in N \mid u_i^s > \theta\}$, that is, the sets of those individuals whose utility is lower than and higher than the threshold in state s. A social ordering R is ex post sufficientarian if there exists an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$, uRv if and only if

$$\sum_{s\in S} \pi^s \sum_{i\in L(u^s)} [g(u_i^s) - g(\theta)] > \sum_{s\in S} \pi^s \sum_{i\in L(v^s)} [g(v_i^s) - g(\theta)]$$

or

$$\sum_{s\in S} \pi^s \sum_{i\in L(u^s)} [g(u_i^s) - g(\theta)] = \sum_{s\in S} \pi^s \sum_{i\in L(v^s)} [g(v_i^s) - g(\theta)] \text{ and}$$
$$\sum_{s\in S} \pi^s \sum_{i\in H(u^s)} [g(u_i^s) - g(\theta)] \geq \sum_{s\in S} \pi^s \sum_{i\in H(v^s)} [g(v_i^s) - g(\theta)].$$

The arithmetic expected representative-utility sufficientarian ordering coincides with the ex post sufficientarian ordering associated with g that is affine both on $\mathbb{R}_{LE}$ and $\mathbb{R}_{HE}$ because, if g is affine both on $\mathbb{R}_{LE}$ and $\mathbb{R}_{HE}$, it follows that, for all $u, v \in \mathcal{D}$,

$$\sum_{s\in S} \pi^s \sum_{i\in L(u^s)} [g(u_i^s) - g(\theta)] \geq \sum_{s\in S} \pi^s \sum_{i\in L(v^s)} [g(v_i^s) - g(\theta)]$$

if and only if

$$\sum_{s\in S} \pi^s \left(\frac{\sum_{i\in N} u_{Li}^s}{n}\right) \geq \sum_{s\in S} \pi^s \left(\frac{\sum_{i\in N} v_{Li}^s}{n}\right),$$

and

$$\sum_{s\in S} \pi^s \sum_{i\in H(u^s)} [g(u_i^s) - g(\theta)] \geq \sum_{s\in S} \pi^s \sum_{i\in H(v^s)} [g(v_i^s) - g(\theta)]$$

if and only if

$$\sum_{s\in S}\pi^s\left(\frac{\sum_{i\in N}u^s_{Hi}}{n}\right)\geq\sum_{s\in S}\pi^s\left(\frac{\sum_{i\in N}v^s_{Hi}}{n}\right).$$

Adler, Bossert, Cato, and Kamaga (2025, Corollary 1) characterize the class of ex post sufficientarian orderings by adding the restricted social expected utility hypothesis and prospect independence of the unconcerned to the axioms of strong Pareto for no risk, continuity above the threshold, continuity below the threshold, and anonymity; they also assume that $n\geq 3$.

The restricted social expected utility hypothesis is a weak form of the social expected utility hypothesis that applies only within the domain below the threshold and within the domain above the threshold.

Restricted social expected utility hypothesis: There exists a function $W\colon\mathbb{R}^n\to\mathbb{R}$ such that, for all $u,v\in\mathcal{D}_L$,

$$uRv\ \Leftrightarrow\ \sum_{s\in S}\pi^sW(u^s)\geq\sum_{s\in S}\pi^sW(v^s)$$

and, for all $u,v\in\mathcal{D}_H$,

$$uRv\ \Leftrightarrow\ \sum_{s\in S}\pi^sW(u^s)\geq\sum_{s\in S}\pi^sW(v^s).$$

Combined with ex post absolute priority, the restricted social expected utility hypothesis implies statewise dominance. Moreover, all expected representative-utility sufficientarian orderings satisfy the restricted social expected utility hypothesis. Statewise dominance becomes logically equivalent to the restricted social expected utility hypothesis when combined with the other axioms of Theorem 3.

Prospect independence of the unconcerned extends the requirement of outcome independence of the unconcerned to the entire domain.

Prospect independence of the unconcerned: For all $u,u',v,v'\in\mathcal{D}$, and for all non-empty $M\subset N$,

$$(u_M,v_{N\setminus M})R(u'_M,v_{N\setminus M})\ \Leftrightarrow\ (u_M,v'_{N\setminus M})R(u'_M,v'_{N\setminus M}).$$

Since the arithmetic expected representative-utility sufficientarian ordering is ex post sufficientarian, it satisfies prospect independence of the unconcerned. Furthermore, as we show in the following theorem, assuming $n\geq 3$, only the arithmetic expected representative-utility sufficientarian ordering satisfies prospect independence of the unconcerned among the anonymous strict expected representative-utility sufficientarian orderings.

Theorem 6. Assume that $n\geq 3$. A social ordering R satisfies ex post absolute priority, strong Pareto for no risk, weak Pareto for equal risk above the threshold, weak Pareto for equal risk below the threshold, statewise dominance, continuity above the threshold, continuity below the threshold, prospect independence of the unconcerned, and anonymity if and only if it is the arithmetic expected representative-utility sufficientarian ordering.

Proof. 'If.' This part of the proof follows from Theorem 5 and the characterization of the class of ex post sufficientarian orderings established by Adler, Bossert, Cato, and Kamaga (2025, Corollary 1).

'Only if.' From Theorem 5, there exists an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$ such that R is the quasiarithmetic expected representative-utility sufficientarian ordering associated with g. Furthermore, R satisfies the restricted social expected utility hypothesis. From Corollary 1 of Adler, Bossert, Cato, and Kamaga (2025), there exists an increasing and continuous function $h\colon \mathbb{R} \to \mathbb{R}$ such that R is the ex post sufficientarian ordering associated with h.

We show that h is affine on $\mathbb{R}_{LE}$. Let $a, c \in \mathbb{R}_{LE}$ and $b = \pi^1 a + (1-\pi^1)c$, and consider $u, v \in \mathcal{D}^e \cap \mathcal{D}_L$ defined by, for all $i \in N$,

$$u_i = (a, c, \ldots, c) \text{ and } v_i = (b, \ldots, b). \tag{2}$$

Note that $u = u_L$ and $v = v_L$. We obtain

$$\sum_{s \in S} \pi^s \left(g^{-1} \left(\frac{1}{n} \sum_{i \in N} g(u_i^s) \right) \right) = \pi^1 a + (1-\pi^1)c$$

and

$$\sum_{s \in S} \pi^s \left(g^{-1} \left(\frac{1}{n} \sum_{i \in N} g(v_i^s) \right) \right) = b.$$

Since $\pi^1 a + (1-\pi^1)c = b$ and R is the quasiarithmetic expected representative-utility sufficientarian ordering associated with g, we obtain that uIv. This implies that

$$\sum_{s \in S} \pi^s \sum_{i \in N} h(u_i^s) = \sum_{s \in S} \pi^s \sum_{i \in N} h(v_i^s)$$

because R is the ex post sufficientarian ordering associated with h. By the construction of u and v, we obtain that

$$\pi^1 h(a) + (1-\pi^1)h(c) = h(\pi^1 a + (1-\pi^1)c).$$

Since a and c were arbitrarily chosen from $\mathbb{R}_{LE}$, the function h satisfies this weighted variant of Jensen's functional equation for all $a, c \in \mathbb{R}_{LE}$. Define $f\colon \mathbb{R}_+ \to \mathbb{R}$ by

$$f(x) = h(\theta - x).$$

The function f is continuous on $\mathbb{R}_+$ because h is continuous on $\mathbb{R}_{LE}$. Furthermore, f satisfies the weighted variant of Jensen's functional equation on $\mathbb{R}_+$ because, for all $x, y \in \mathbb{R}_+$,

$$\begin{aligned} f(\pi^1 x + (1-\pi^1)y) &= h(\pi^1(\theta - x) + (1-\pi^1)(\theta - y)) \\ &= \pi^1 h(\theta - x) + (1-\pi^1)h(\theta - y) \\ &= \pi^1 f(x) + (1-\pi^1)f(y). \end{aligned}$$

Now, we define $f_0\colon \mathbb{R}_+ \to \mathbb{R}$ by $f_0(x) = f(x) - f(0)$. Then, applying the argument provided by Aczél (1966, p. 66; 1969, p. 14), it follows that, for all $x, y \in \mathbb{R}_+$,

$$f_0(x+y) = f_0(x) + f_0(y),$$

that is, f_0 satisfies Cauchy's functional equation. Since f is continuous on $\mathbb{R}_+$, so is the function f_0. Thus, there exists $\alpha_L \in \mathbb{R}$ such that $f_0(x) = \alpha_L x$. Consequently, there exists $\beta \in \mathbb{R}$ such that $f(x) = f_0(x) - f(0) = \alpha_L x + \beta$. By the definition of f, we obtain that, for all $z \in \mathbb{R}_{LE}$

$$h(z) = f(\theta - z) = \alpha_L(\theta - z) + \beta = \alpha_L z + \alpha_L \theta + \beta.$$

Letting $\beta_L = \alpha_L\theta + \beta$, we obtain that $h(z) = \alpha_L z + \beta_L$ for all $z \in \mathbb{R}_{LE}$. Since h is increasing on $\mathbb{R}_{LE}$, $\alpha_L > 0$.

Next, we show that h is affine on $\mathbb{R}_{HE}$. Letting $a, b, c \in \mathbb{R}_{HE}$ with $b = \pi^1 a + (1-\pi^1)c$, define $u, v \in \mathcal{D}^e \cap \mathcal{D}_H$ as in (2). Then, applying the same argument as that we used for $u, v \in \mathcal{D}^e \cap \mathcal{D}_L$, it follows that h satisfies the variant of Jensen's functional equation for all $a, c \in \mathbb{R}_{HE}$. Now, define $f\colon \mathbb{R}_+ \to \mathbb{R}$ by

$$f(x) = h(\theta + x).$$

Then, applying the same argument as above, we can show that there exist $\alpha_H \in \mathbb{R}_{++}$ and $\beta_H \in \mathbb{R}$ such that $h(z) = \alpha_H z + \beta_H$. Since h is continuous on $\mathbb{R}$, $\alpha_L\theta + \beta_L = \alpha_H\theta + \beta_H$ must hold. ■

5 Incorporating the Pigou-Dalton principle

The Pigou-Dalton principle is not explicitly incorporated in our axiomatic characterizations. As discussed in the introduction, it plays a central role in the evaluation of welfare under risk. Its formulation is as follows.

Pigou-Dalton transfer principle for no risk: For all $u, v \in \mathcal{D}^c$, if, for all $s \in S$, there exist $i, j \in N$ and $\delta \in \mathbb{R}_{++}$ such that $v_i^s = u_i^s - \delta \geq u_j^s + \delta = v_j^s$ and $u_k^s = v_k^s$ for all $k \in N \setminus \{i, j\}$, then vPu.

The axiom states that, for riskless prospects, welfare transfers from relatively advantaged to relatively disadvantaged individuals are desirable, provided that their relative ranking remains unchanged and the well-being of others is unaffected. This property is widely regarded as a key ingredient of prioritarianism. Indeed, strong Pareto for no risk, outcome independence of the unconcerned, continuity, and the Pigou-Dalton transfer principle for no risk are sufficient to obtain a prioritarian ordering over outcomes; see Adler, Bossert, Cato, and Kamaga (2025). Note that, in the case of such prioritarian orderings, g must be strictly concave on the entire domain $\mathbb{R}$ to satisfy the Pigou-Dalton transfer principle for no risk.

The Pigou-Dalton transfer principle is naturally satisfied under sufficientarianism, provided that cases violating strong Pareto, such as the headcount approach, are excluded; see Bossert, Cato, and Kamaga (2022, 2023).

Let us consider a quasiarithmetic sufficientarian social ordering such that there exists an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}^c$,

$$\begin{aligned} uRv \;\Leftrightarrow\; & \sum_{i\in N} g(u_{Li}^s) > g(v_{Li}^s) \;\text{ or} \\ & \sum_{i\in N} g(u_{Li}^s) = g(v_{Li}^s) \;\text{ and }\; \sum_{i\in N} g(u_{Hi}^s) \geq \sum_{i\in N} g(v_{Hi}^s). \end{aligned}$$

For such an ordering to satisfy the Pigou-Dalton transfer principle for no risk, g need not be strictly concave on the entire domain. The requisite property of g is strict concavity below and above the threshold; see Bossert, Cato, and Kamaga (2022, 2023). To be precise, a quasiarithmetic expected representative-utility sufficientarian ordering satisfies the Pigou-Dalton transfer principle for no risk if and only if the corresponding g is strictly concave on $\mathbb{R}_{LE}$ and on $\mathbb{R}_{HE}$.

Applying the standard argument based on Schur-concavity and majorization (Hardy, Littlewood, and Pólya, 1934; Marshall and Olkin, 1979), this observation extends to expected

representative-utility sufficientarian orderings. In this setting, the Schur-concavity of the associated function Ξ on $\mathbb{R}^n_{LE}$ and $\mathbb{R}^n_{HE}$ is crucial for the Pigou-Dalton transfer principle in the absence of risk. Specifically, an expected representative-utility sufficientarian ordering satisfies anonymity and the Pigou-Dalton transfer principle for no risk if and only if the corresponding function Ξ is strictly Schur-concave on both $\mathbb{R}^n_{LE}$ and $\mathbb{R}^n_{HE}$.

6 Why censored prospects are needed

A central feature of expected representative-utility sufficientarianism is its effective use of censored prospects. Specifically, the representative utility of the individuals below the threshold is derived from their censored prospect, and the same is true for the representative utility of those above the threshold. The purpose of this section is to explain why it is essential to use these censored prospects.

To illustrate, suppose that we have a threshold given by $\theta = 10$, and that we employ a quasiarithmetic expected representative-utility sufficientarian ordering associated with an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$. For simplicity, we use an example that involves two individuals only; clearly, this can be generalized. Consider the riskless prospects $u, v \in \mathcal{D}^c$ given by

$$u = \begin{bmatrix} 1 & 1 \\ 8 & 8 \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} 1 & 1 \\ 100 & 100 \end{bmatrix}.$$

Because $\theta = 10$, it follows that

$$u_L = \begin{bmatrix} 1 & 1 \\ 8 & 8 \end{bmatrix} \quad \text{and} \quad v_L = \begin{bmatrix} 1 & 1 \\ 10 & 10 \end{bmatrix}$$

and, moreover,

$$u_H = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} \quad \text{and} \quad v_H = \begin{bmatrix} 10 & 10 \\ 100 & 100 \end{bmatrix}.$$

According to the corresponding expected representative-utility sufficientarian ordering, v is better than u because

$$g^{-1}\Big(\frac{g(1) + g(10)}{2}\Big) > g^{-1}\Big(\frac{g(1) + g(8)}{2}\Big).$$

This is a plausible implication; actually, any ordering that satisfies strong Pareto for no risk arrives at this judgment.

Now suppose that we apply a modification of expected representative-utility sufficientarianism without suitably censoring these prospects. To be precise, we calculate the representative utilities by considering only those who are actually below the threshold and apply the lexicographic procedure inherent in our sufficientarian orderings. In u, there are two individuals who are below the threshold. Thus, its representative utility remains the same—that is, the representative utility of u for those below the threshold is

$$g^{-1}\Big(\frac{g(1) + g(8)}{2}\Big).$$

In the case of v, there is only one individual who is below the threshold. As a consequence, the representative utility for v now assumes the value one because we do not censor the requisite

prospects. Because

$$g^{-1}\Big(\frac{g(1)+g(8)}{2}\Big) > 1,$$

u is now ranked above v, which contradicts strong Pareto for no risk. Hence, using censored prospects is crucial in order to avoid violations of strong Pareto for no risk, a highly plausible property for any theory of distributive justice.

7 Concluding remarks

This paper provides an axiomatic characterization of expected representative-utility sufficientarianism. In welfare evaluation under risk, the treatment of prospects with equal risk is central to the representative-utility approach. However, sufficientarianism cannot fully endorse weak Pareto for equal risk. It is therefore necessary to restrict weak Pareto for equal risk to the subdomains above and below the threshold. By combining Fleurbaey's (2010) argument with the approach to sufficientarianism developed by Adler, Bossert, Cato, and Kamaga (2025), we obtain a characterization of expected representative-utility sufficientarianism. From a sufficientarian perspective, an important question then arises—namely, whether expected representative-utility sufficientarianism is based on a more compelling foundation than ex post sufficientarianism as advocated in Adler, Bossert, Cato, and Kamaga (2025).

For the sake of keeping our exposition simple, we do not explicitly impose any inequality-aversion requirements—such as variants of the Pigou-Dalton transfer principle—in our characterization results. However, as we have discussed, it is not difficult to incorporate properties of this nature. As is usually the case, this enrichment of our setup would endow the requisite functions with a curvature property, such as (strict) concavity on $\mathbb{R}_{LE} \cup \mathbb{R}_{HE}$ of the transformation function g used in quasiarithmetic representative-utility sufficientarianism.

This paper does not address variable-population considerations. The necessity of extending welfare analyses to environments in which population may vary is demonstrated forcefully by Blackorby and Donaldson (1984) and by Blackorby, Bossert, and Donaldson (1998, 2005). Bossert, Cato, and Kamaga (2022, 2023) show that sufficientarianism can be extended naturally in this context, at least when comparisons are made under certainty. In contrast, how to extend expected representative-utility sufficientarianism to variable-population settings requires careful examination. Changes in population size introduce some new complexities because expected representative-utility sufficientarianism relies on censored prospects that are defined relative to a fixed population for both those below and above the threshold. Developing a framework that accommodates varying populations remains an important task for future investigations.

Appendix A: Characterizations of the expected representative-utility criteria

Fleurbaey (2010) does not provide a characterization of the class of expected equally distributed equivalent criteria. This appendix provides an axiomatization by focusing on the class of expected representative-utility criteria.

Theorem 7. A social ordering R satisfies weak Pareto for no risk, weak Pareto for equal risk, statewise dominance, and continuity if and only if it is an expected representative-utility criterion.

Proof. 'If.' Assume that R is an expected representative-utility criterion. There exists an increasing and continuous function $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ such that Ξ satisfies (1) on $\mathbb{R}^n$ and, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \Xi(u^s) \geq \sum_{s\in S} \pi^s \Xi(v^s). \tag{3}$$

To show that weak Pareto for no risk is satisfied, let $u, v \in \mathcal{D}^c$ be such that $u_i^s > v_i^s$ for all $i \in N$. Since Ξ is increasing, $\Xi(u^s) > \Xi(v^s)$. Since $u, v \in \mathcal{D}^c$,

$$uPv \;\Leftrightarrow\; \Xi(u^s) > \Xi(v^s).$$

Thus, uPv.

To show that weak Pareto for equal risk is satisfied, let $u, v \in \mathcal{D}^e$ be such that $E(u_i) > E(v_i)$ for all $i \in N$. Since $u, v \in \mathcal{D}^e$, $u_i = u_j$ for all $i, j \in N$, and $v_i = v_j$ for all $i, j \in N$. Hence, we obtain

$$\Xi(u^s) = u_i^s \;\text{ and }\; \Xi(v^s) = v_i^s.$$

This implies that

$$E(u_i) = \sum_{s\in S} \pi^s \Xi(u^s) > \sum_{s\in S} \pi^s \Xi(v^s) = E(v_i).$$

(3) implies that uPv.

It is obvious that statewise dominance is satisfied. Continuity follows from the continuity of Ξ.

'Only if.' This part is essentially established by Fleurbaey (2010). As in the case of Lemma 1, statewise dominance and continuity imply that for all $u, v \in \mathcal{D}$, if $[u^s]I[v^s]$ for all $s \in S$, then uIv. Moreover, the representative utility level is well-defined on $\mathcal{D}$. This establishes the existence of the function $\Xi\colon \mathbb{R}^n \to \mathbb{R}$. This function is increasing and continuous because of weak Pareto for no risk and continuity. Using a strategy similar to that employed in the proof of Lemma 2 and observing that weak Pareto for equal risk and continuity are satisfied, it follows that

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \Xi(u^s) \geq \sum_{s\in S} \pi^s \Xi(v^s)$$

for all $u, v \in \mathcal{D}$. ■

A special case of the expected representative-utility criteria is the utilitarian ordering that is obtained if the function Ξ is given by the arithmetic mean; that is, if

$$\Xi(u^s) = \frac{\sum_{i\in N} u_i^s}{n}$$

for all $u^s \in \mathbb{R}^n$.

In analogy to the characterization of the arithmetic expected representative-utility sufficientarian ordering in Theorem 6, a characterization of the utilitarian ordering is provided by adding strong Pareto for no risk, anonymity, and prospect independence of the unconcerned to the set of axioms of Theorem 7.

Theorem 8. Assume that $n \geq 3$. A social ordering R satisfies strong Pareto for no risk, weak Pareto for equal risk, statewise dominance, continuity, anonymity, and prospect independence of the unconcerned if and only if it is the utilitarian ordering.

Proof. 'If.' From Theorem 4, R satisfies strong Pareto for no risk, weak Pareto for equal risk, statewise dominance, and continuity. It is straightforward that the utilitarian ordering satisfies anonymity and prospect independence of the unconcerned.

'Only if.' From Theorem 7, there exists an increasing and continuous function $\Xi\colon \mathbb{R}^n \to \mathbb{R}$ such that Ξ satisfies (1) on $\mathbb{R}^n$ and, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \Xi(u^s) \geq \sum_{s\in S} \pi^s \Xi(v^s).$$

Thus, R satisfies the requirement of the restricted social expected utility hypothesis on the entire domain and, therefore, the social expected utility hypothesis is satisfied. From Theorem 1 of Adler, Bossert, Cato, and Kamaga (2025), it follows that R is an ex post generalized utilitarian ordering. This means that there exists an increasing and continuous function $g\colon \mathbb{R} \to \mathbb{R}$ such that, for all $u, v \in \mathcal{D}$,

$$uRv \;\Leftrightarrow\; \sum_{s\in S} \pi^s \sum_{i\in N} g(u_i^s) \geq \sum_{s\in S} \pi^s \sum_{i\in N} g(v_i^s).$$

Let $a, c \in \mathbb{R}$ and $b = \pi^1 a + (1-\pi^1)c$, and define $u, v \in \mathcal{D}$ by

$$u_i = (a, c, \ldots, c) \;\text{ and }\; v_i = (b, \ldots, b)$$

for all $i \in N$. Then, we obtain uIv because

$$\sum_{s\in S} \pi^s \Xi(u^s) = \pi^1 a + (1-\pi^1)c = b = \sum_{s\in S} \pi^s \Xi(v^s).$$

Since R is the ex post generalized utilitarian ordering associated with g, it follows that

$$\pi^1 g(a) + (1-\pi^1) g(c) = g(\pi^1 a + (1-\pi^1)c)$$

for all $a, c, \in \mathbb{R}$. Thus, applying the same argument as in the proof of Theorem 6 to the function defined by $g_0(x) = g(x) - g(0)$ for all $x \in \mathbb{R}$, we conclude that there exist $\alpha \in \mathbb{R}_{++}$ and $\beta \in \mathbb{R}$ such that $g(x) = \alpha x + \beta$ for all $x \in \mathbb{R}$. Thus, R is the utilitarian ordering. ■

Appendix B: Independence of the axioms in Theorem 3

Consider an increasing and strictly concave function $g\colon \mathbb{R} \to \mathbb{R}$. Note that g is also continuous.

Define R^1 by letting, for all $u, v \in \mathcal{D}$, uR^1v if and only if

$$\sum_{s\in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i\in N} g(u_i^s)}{n} \Big) \Big) > \sum_{s\in S} \pi^s \Big(g^{-1}\Big(\frac{\sum_{i\in N} g(v_i^s)}{n} \Big) \Big).$$

The ordering R^1 satisfies all axioms other than ex post absolute priority.

Let $k \in N$, and define R^2 by letting, for all $u, v \in \mathcal{D}$, uR^2v if and only if

$$\begin{gathered}\textstyle\sum_{s\in S}\pi^s\sum_{i\in N}u^s_{Li} > \sum_{s\in S}\pi^s\sum_{i\in N}v^s_{Li} \text{ or}\\ \Big[\textstyle\sum_{s\in S}\pi^s\sum_{i\in N}u^s_{Li} = \sum_{s\in S}\pi^s\sum_{i\in N}v^s_{Li} \text{ and}\\ \textstyle\sum_{s\in S}\pi^s\left(\sum_{i\neq k}u^s_{Hi} - u^s_{Hk}/2\right) \geq \sum_{s\in S}\pi^s\left(\sum_{i\neq k}v^s_{Hi} - v^s_{Hk}/2\right)\Big].\end{gathered}$$

The ordering R^2 satisfies all axioms other than weak Pareto for no risk above the threshold.

Let $k \in N$, and define R^3 by letting, for all $u, v \in \mathcal{D}$, uR^3v if and only if

$$\begin{gathered}\textstyle\sum_{s\in S}\pi^s\left(\sum_{i\neq k}u^s_{Li} - u^s_{Lk}/2\right) > \sum_{s\in S}\pi^s\left(\sum_{i\neq k}v^s_{Li} - v^s_{Lk}/2\right) \text{ or}\\ \Big[\textstyle\sum_{s\in S}\pi^s\left(\sum_{i\neq k}u^s_{Li} - u^s_{Lk}/2\right) = \sum_{s\in S}\pi^s\left(\sum_{i\neq k}v^s_{Li} - v^s_{Lk}/2\right) \text{ and}\\ \textstyle\sum_{s\in S}\pi^s\sum_{i\in N}u^s_{Hi} \geq \sum_{s\in S}\pi^s\sum_{i\in N}v^s_{Hi}\Big].\end{gathered}$$

The ordering R^3 satisfies all axioms other than weak Pareto for no risk below the threshold.

Let

$$\Xi(u^s) = g^{-1}\Big(\frac{\sum_{i\in N}g(u^s_i)}{n}\Big)$$

for all $u^s \in \mathbb{R}^n$. That is, Ξ is the representative-utility function associated with the prioritarian ordering identified by g. Define R^4 by letting, for all $u, v \in \mathcal{D}$, uR^4v if and only if

$$\begin{gathered}\textstyle\sum_{s\in S}\pi^s\Xi(u^s_L) > \sum_{s\in S}\pi^s\Xi(v^s_L) \text{ or}\\ \big[\textstyle\sum_{s\in S}\pi^s\Xi(u^s_L) = \sum_{s\in S}\pi^s\Xi(v^s_L) \text{ and } \sum_{s\in S}\pi^s\sum_{i\in N}g(u^s_{Hi}) \geq \sum_{s\in S}\pi^s\sum_{i\in N}g(v^s_{Hi})\big].\end{gathered}$$

The ordering R^4 satisfies all axioms other than weak Pareto for equal risk above the threshold. Note that weak Pareto for equal risk above the threshold is violated because ex-post prioritarianism applies to the subdomain above the threshold.

Define R^5 by letting, for all $u, v \in \mathcal{D}$, uR^5v if and only if

$$\begin{gathered}\textstyle\sum_{s\in S}\pi^s\sum_{i\in N}g(u^s_{Li}) > \sum_{s\in S}\pi^s\sum_{i\in N}g(v^s_{Li}) \text{ or}\\ \big[\textstyle\sum_{s\in S}\pi^s\Xi(u^s_L) = \sum_{s\in S}\pi^s\Xi(v^s_L) \text{ and } \sum_{s\in S}\pi^s\Xi(u^s_H) \geq \sum_{s\in S}\pi^s\Xi(v^s_H)\big].\end{gathered}$$

The ordering R^5 satisfies all axioms other than weak Pareto for equal risk below the threshold.

Define the ordering R^6 by letting, for all $u, v \in \mathcal{D}$, uR^6v if and only if

$$\begin{gathered}\min_{i\in N}E(u_{Li}) > \min_{i\in N}E(v_{Li}) \text{ or}\\ [\min_{i\in N}E(u_{Li}) = \min_{i\in N}E(v_{Li}) \text{ and } \min_{i\in N}E(u_{Hi}) > \min_{i\in N}E(v_{Hi})].\end{gathered}$$

The ordering R^6 satisfies all axioms other than statewise dominance.

Define R^7 by letting, for all $u, v \in \mathcal{D}$, uR^7v if and only if

$$\begin{gathered}\textstyle\sum_{s\in S}\pi^s\Xi(u^s_L) > \sum_{s\in S}\pi^s\Xi(v^s_L) \text{ or}\\ \big[\textstyle\sum_{s\in S}\pi^s\Xi(u^s_L) = \sum_{s\in S}\pi^s\Xi(v^s_L) \text{ and } \sum_{s\in S}\pi^s\Xi(u^s_H) > \sum_{s\in S}\pi^s\Xi(v^s_H)\big] \text{ or}\\ \big[\textstyle\sum_{s\in S}\pi^s\Xi(u^s_L) = \sum_{s\in S}\pi^s\Xi(v^s_L) \text{ and } \sum_{s\in S}\pi^s\Xi(u^s_H) = \sum_{s\in S}\pi^s\Xi(v^s_H) \text{ and } u^1_{H1} \geq v^1_{H1}\big].\end{gathered}$$

The ordering R^7 satisfies all axioms other than continuity above the threshold.

Define R^8 by letting, for all $u, v \in \mathcal{D}$, uR^8v if and only if

$$\sum_{s\in S} \pi^s \Xi(u_L^s) > \sum_{s\in S} \pi^s \Xi(v_L^s) \text{ or}$$
$$\left[\sum_{s\in S} \pi^s \Xi(u_L^s) = \sum_{s\in S} \pi^s \Xi(v_L^s) \text{ and } u_{L1}^1 > v_{L1}^1\right] \text{ or}$$
$$\left[\sum_{s\in S} \pi^s \Xi(u_L^s) = \sum_{s\in S} \pi^s \Xi(v_L^s) \text{ and } u_{L1}^1 = v_{L1}^1 \text{ and } \sum_{s\in S} \pi^s \Xi(u_H^s) \geq \sum_{s\in S} \pi^s \Xi(v_H^s)\right].$$

The ordering R^8 satisfies all axioms other than continuity below the threshold.

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