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Temporary Natural Resource Cartels[1]

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Abstract

We analyze the behavior of a nonrenewable resource cartel that anticipates being forced, at some date in the future, to break-up into an oligopolistic market in which its members will then have to compete as rivals. Under reasonable assumptions about the value function of the individual firms in the oligopolistic equilibrium that follows the break-up, we show that the cartel will then produce more over the same interval of time than it would if there were no threat of dissolution, and that its rate of extraction is a decreasing function of the cartel's life; that there are circumstances under which the cartel will attach a negative marginal value to the resource stocks, in which case the rate of depletion will be increasing over time during the cartel phase; that, for a given date of dissolution, the equilibrium stocks allocated to the post-cartel phase will increase as a function of the total initial stocks, whereas those allocated to the cartel phase will increase at first, but begin decreasing beyond some level of the total initial stocks.

Keywords: cartels, dissolution, nonrenewable natural resources

JEL classification: Q3, L13

1 Introduction

The static theory of cartels teaches us that a cartel is formed to restrict output relative to that of a perfectly competitive or oligopolistic industry, thus raising prices and profits. This result extends to the case of a cartel that faces an intertemporal problem, such as natural resource cartels: it will, as for a perfectly competitive or oligopolistic industry, follow an output path that decreases over time until exhaustion, but the exercise of its monopoly power will generally result in the resource stocks being depleted more slowly than it would in a perfectly competitive or oligopolistic industry (see Hotelling, 1931, Sweeney, 1977, Pindyck, 1978, or Stiglitz and Dasgupta, 1982).

The theoretical literature on resource cartels has mainly concentrated on the study of the output and pricing paths of a partially cartelized industry with a competitive fringe. Some have taken a Nash-Cournot approach to the problem (Salant, 1976; Lewis and Schmalensee, 1979; Ulph and Folie, 1980); others have taken a Stackelberg approach, with the cartel acting as a leader and with much attention being payed to the problem of time inconsistency of the open-loop equilibrium in such a case (Gilbert, 1978; Newbery, 1981; Ulph, 1982; Groot, Withagen and de Zeeuw, 1992, 2003).

In all of this literature, the cartel has been assumed to last until the resource stocks under its control are exhausted. But there are various reasons why this may not be the case: regulators may force a cartel to break up, or cartel members may defect to free ride on the output restrictions of the members that remain loyal to its goal. The question then arises as to what are the consequences for the cartel's optimal output path of knowing that at some time in the future it will have to break up into an oligopolistic industry? In the case of a static cartel, there are clearly no consequences: it will simply produce during its existence the same monopoly output as it would if no break-up was envisaged, and the industry will revert to the static oligopoly equilibrium output afterwards. But when the cartel faces an intertemporal problem, the answer is not so simple, since the initial state faced by the oligopolist after the break-up then depends on the cartel's decisions during its existence.

We will focus on the cartelization of an industry extracting a nonrenewable resource. There are N identical firms, each owning a stock of the resource. The initial stocks are assumed to be of equal size. A cartel formed by these N firms anticipates that at some future time it will have to break up. After the break-up, all firms become Cournot rivals. It would seem that under such circumstances, the cartel may want to choose remaining stocks in order to reduce their rivalry from that date on. The aim of this paper is to study the implications of this for the cartel's extraction path.

We will show that a cartel that anticipates a break-up before the resource stocks are exhausted will not only choose to produce during its existence at a higher rate than it would otherwise, with this rate being a decreasing function of the cartel's life, but it may even want to deplete the resource at an increasing rate, contrary to what is usually expected of a non-renewable resource industry under concave profit functions, be it cartelized or not. We will also show that up to some level of the initial stocks, the greater the initial stocks, the greater the quantity of stocks depleted during the cartel phase, but that beyond this level of initial stocks, the relationship is reversed: more initial stocks will imply less stocks depleted during the cartel phase and more left over for the post-cartel phase.

The model is developed in the Section 2. In Section 3 we characterize of the temporary cartel equilibrium. Section 4 is devoted to some concluding remarks.

2 The Model

We will assume that the cartel must maintain the initial symmetry, by allocating equal quotas to its members during the cartel phase[1], and, for ease of exposition, will restrict

[1] This begs the issue of optimal asymmetry, which we leave for future research. For even if we have at the beginning perfect symmetry, the cartel may have an interest in allocating different quotas to different members, *if all it cares for is total industry profit*. Such asymmetric quotas would result in an asymmetric Cournot oligopoly at the time of cartel dissolution. We can expect that for a given aggregate industry stock at the dissolution time, the more asymmetric the distribution, the greater will be the industry profit. For, at the extreme, if at dissolution time only one firm has a positive stock, we have a monopoly. Industry profit under monopoly would be greater than under oligopoly with the same stock evenly divided among firms. The feasibility of such asymmetric behavior on the part of the cartel obviously requires some form of profit redistribution.

attention to symmetric equilibria in the oligopoly phase. Let q denote the individual rate of extraction of the typical firm and $Q = Nq$ the industry's rate of extraction. The inverse demand function will be denoted $P(Q)$ and will satisfy the following assumptions:

A1 $P(Q) > 0$ for $Q \in [0, \xi)$ and $P(Q) = 0$ for $Q \in [\xi, \infty)$, for some $\xi \in (0, \infty)$;

A2 $P(Q)$ is twice continuously differentiable and $P'(Q) < 0$;

A3 $P'(Q) + qP''(Q) < 0$ for all $q \in [0, Q], Q \in [0, \xi)$;

A4 $P(Nq)q$ attains a unique maximum at some $q \in (0, \xi)$.

The extraction cost function, $C(q)$, will be assumed to satisfy:

A5 $C(q)$ is twice continuously differentiable, with $C'(q) > 0$ and $C''(q) \geq 0$.

Notice that these assumptions allow for marginal revenue and marginal profit to become negative beyond some level of output. They thus exclude, for instance, a constant elasticity demand function.

Suppose the cartel knows that it will be dissolved after some exogenously given period of time τ. Using superscripts c and o to denote respectively the cartel and the oligopoly that follows and given the perfect symmetry that holds throughout amongst the N firms in the industry, the cartel's problem can be formulated as:

$$\max_{\{q^c(t)\}} \int_0^{\tau} N[P(Nq^c(t))q^c(t) - C(q^c(t))]e^{-rt}dt + Ne^{-r\tau}V_i^o(\vec{x}^{\,c}(\tau))$$

subject to:

$$\dot{x}_i(t) = -q^c(t) \tag{1}$$

$$x_i(0) = x_0 \tag{2}$$

where

$$\vec{x}^{\,c}(t) = (x_1^c(t), x_2^c(t), \ldots, x_N^c(t)), \tag{3}$$

and r is the rate of discount.

$V_i^o(\vec{x}^{\,c}(\tau))$ is the value function of each individual firm in the oligopoly game that begins at τ with initial stocks $\vec{x}^{\,c}(\tau)$. We will assume it to be differentiable. Under symmetry, at τ, we have $x_i(\tau) = x_D$ for all i, where x_D denotes the stock left to each firm at dissolution. Hence:

$$\vec{x}^{\,c}(\tau) = \vec{x}_D = (x_D, x_D, \ldots, x_D). \tag{4}$$

If $\tau = 0$, then $x_D = x_0$ and the problem is simply that of an oligopoly game between N identical resource firms. The resulting equilibrium output path may be denoted $\{q^o(t)| \; t \in [0, T^o(x_0)]\}$ where $T^o(x_0) \in (0, \infty]$ is the endogenous period of time taken by the oligopoly to exhaust the resource. On the other hand, if $\tau = \infty$, then the problem reduces to that of a monopolist that controls N identical resource stocks and, without loss of generality, depletes them simultaneously and symmetrically. The resulting output path will be $\{q^m(t)| \; t \in [0, T^m(x_0)]\}$ where $T^m(x_0) \in (0, \infty]$ is the period of time taken by the monopolist to exhaust the resource. It is easy to show that $T^m(x_0) \geq T^o(x_0)$.

In fact the monopolist outcome occurs for all $\tau \geq T^m(x_0)$. The case where $\tau \in (0, T^m(x_0))$ is the interesting case of the *temporary cartel*, to which we now turn.

3 The temporary cartel equilibrium

As already stated above, the temporary cartel wants to maximize the profit of the representative firm, knowing that after τ, all firms become Cournot rivals. The resulting output path is $\{q^c(t) \mid t \in [0, \tau)\}$ during the cartel phase and $\{q^o(t) \mid t \in [\tau, T^c(x_0) = T^o(x_D) + \tau]\}$ during the oligopoly phase, where $T^c(x_0)$ denotes the total time taken to exhaust the resource stocks x_0 when the temporary cartel is involved and $T^o(x_D)$ is that taken by an oligopoly to exhaust the resources when beginning with stocks x_D.

The current-value Hamiltonian associated with the temporary cartel's problem is:

$$H^c = N[P(Nq^c(t))q^c(t) - C(q^c(t))] - N\mu(t)q^c(t), \tag{5}$$

where $\mu(t)$ is the shadow value associated with the resource stocks.

In addition to (1) and (2), we have as necessary conditions:

$$P(Nq^c(t)) + Nq^c(t)P'(Nq^c(t)) - C'(q^c(t)) - \mu(t) = 0 \tag{6}$$

$$\frac{d\mu(t)}{dt} = r\mu(t) \tag{7}$$

and the transversality condition

$$\mu(\tau) = \frac{dV_i^o(\vec{x}_D)}{dx} \quad \left(= \frac{\partial V_i^o(\vec{x}_D)}{\partial x_i(\tau)} + \sum_{j \neq i} \frac{\partial V_i^o(\vec{x}_D)}{\partial x_j(\tau)} \right). \tag{8}$$

Consider first the transversality condition (8), which provides a boundary condition for (7). It says that the value of leaving to *each* member firm an additional unit of the resource in stock at the dissolution time must equal the value to the firm of itself and each of its rivals beginning the oligopoly game that follows with an additional unit of reserves. The latter will depend crucially on the properties of the value function $V_i^o(\vec{x})$.

It is clear that

$$V_i^o(\vec{0}\,) = 0, \tag{9}$$

since, in that case, none of the firms hold any stock of the resource and there can be no sales. Furthermore, if we denote by π the Cournot-Nash equilibrium profit of the corresponding static oligopoly game, then

$$\lim_{x \to \infty} V_i^o(\vec{x}) = \frac{\pi}{r}, \tag{10}$$

since if each firm were to hold an infinite stock of the resource, the non renewability constraint would be lifted (the shadow value of the stock would be zero) and the oligopoly problem would reduce to a static one repeated forever.

As x approaches infinity, the function $V_i^o(\vec{x})$ may approach π/r from either above or below. If it approaches from above, then the function necessarily attains a maximum value greater than π/r for some $x \in (0, \infty)$. We will restrict attention to this case and assume:

A6 $V_i^o(\vec{x})$ is strictly quasi-concave in x, with a unique interior maximum at $\hat{x}$ and a unique point of inflexion at $\tilde{x} > \hat{x}$, so that:

1. $\dfrac{dV_i^o\left(\vec{\hat{x}}\right)}{dx} = 0$ and $\dfrac{dV_i^o\left(\vec{x}\right)}{dx} > (<)\ 0$ for $x < (>)\ \hat{x}$;
2. $\dfrac{d^2V_i^o\left(\vec{\tilde{x}}\right)}{dx^2} = 0$ and $\dfrac{d^2V_i^o\left(\vec{x}\right)}{dx^2} < (>)\ 0$ for $x < (>)\ \tilde{x}$.

This is illustrated in Figure 1. [2]

Note that the transversality condition (8) is obtained from the first-order condition for the determination of x_D. At equilibrium, x_D must also satisfy the following second-order condition:

$$-\frac{\partial\mu(\tau)}{\partial x} + \frac{d^2V_i^o(\vec{x}_D)}{dx^2} < 0. \tag{11}$$

By condition (7), we can write, for any $t \in [0,\tau]$ and any $s \in [0,t]$:

$$\mu(t) = \bar{\mu}(s, x(s))e^{r(t-s)} \tag{12}$$

where $\bar{\mu}(s, x(s))$ denotes the value attributed at $t = s$ by the cartel to a marginal increase in the stocks of each member at that date, given that their stocks are then $x(s)$. But from the structure of the problem for the cartel phase, we know that, for a given x_D:

$$\bar{\mu}(s, x(s)) = \bar{\mu}(\tau - s, z(\tau - s, x(s))) \tag{13}$$

where $z(\tau - s, x(s)) = x(s) - x_D$ is the stock still to be depleted during the interval of time $\tau - s$ remaining to the cartel phase. Equation (13) simply says that, given x_D, the shadow value to the cartel of adding marginally to the stocks existing at s is the same thing as the shadow value of adding marginally to the stocks $(x(s) - x_D)$ left to deplete during the

[2] It can be verified that, for instance, with a linear demand and constant marginal cost, a symmetric open-loop equilibrium satisfies those assumptions. On the other hand, a constant elasticity demand function does not, since then $V_i^o(\vec{x})$ monotonically approaches π/r from below as x tends to infinity. Recall however that our assumption A4 rules out a constant elasticity demand function.

interval of time $\tau - s$ remaining to dissolution. In particular, for $s = 0$, we have:

$$\bar{\mu}(0, x_0) = \bar{\mu}(\tau, z(\tau, x_0)), \tag{14}$$

and, substituting into (12):

$$\mu(t) = \bar{\mu}(\tau, z(\tau, x_0))e^{rt}, \tag{15}$$

where $\bar{\mu}(\tau, z(\tau, x_0))$ denotes the value to the cartel, at $t = 0$, of the marginal unit of stock depleted by each of its members during the whole cartel phase.

Furthermore, from (13) we know that:

$$\frac{\partial \bar{\mu}}{\partial \tau} = \frac{\partial \bar{\mu}}{\partial (\tau - s)} = -\frac{\partial \bar{\mu}}{\partial s} \tag{16}$$

and

$$\frac{\partial \bar{\mu}}{\partial x_D} = -\frac{\partial \bar{\mu}}{\partial z} = -\frac{\partial \bar{\mu}}{\partial x(s)}. \tag{17}$$

Hence the first and second-order conditions (8) and (11) for the determination of x_D can be rewritten, respectively, as:

$$-\bar{\mu}(\tau, z(\tau, x_0))e^{r\tau} + \frac{dV_i^o(\vec{x}_D)}{dx} = 0 \tag{18}$$

and

$$\frac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial z} e^{r\tau} + \frac{d^2 V_i^o(\vec{x}_D)}{dx^2} < 0. \tag{19}$$

Substituting from (15) into (6), we can write the implicit solution to that condition as

$$q^c(t) = \varphi(t, \bar{\mu}(\tau, z(\tau, x_0)). \tag{20}$$

Thus

$$z(\tau, x_0) = \int_0^\tau \varphi(t, \bar{\mu}(\tau, z(\tau, x_0))dt. \tag{21}$$

Since by assumptions A3 and A5 the profit function is strictly concave and hence marginal profit (the left-hand side of (6)) is strictly decreasing, $q^c(t)$ is well defined and unique.[3]

The following lemmas will now be useful:

Lemma 1 *At equilibrium, (i)* $\dfrac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial \tau} > 0$ *and (ii)* $\dfrac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial z} < 0$.

Proof. Differentiating totally (21), we get

$$\begin{aligned} dz &= \left\{ \varphi(\tau, \bar{\mu}(\tau, z(\tau, x_0)) + \frac{\partial \bar{\mu}}{\partial \tau} \int_0^\tau \frac{\partial \varphi(t, \bar{\mu}(\tau, z(\tau, x_0))}{\partial \bar{\mu}} dt \right\} d\tau \\ &+ \left\{ \frac{\partial \bar{\mu}}{\partial z} \int_0^\tau \frac{\partial \varphi(t, \bar{\mu}(\tau, z(\tau, x_0))}{\partial \bar{\mu}} dt \right\} dz, \end{aligned}$$

where, from (6),

$$\frac{\partial \varphi(t, \bar{\mu}(\tau, x_0))}{\partial \bar{\mu}} = \frac{e^{rt}}{2NP' + N^2 \varphi P'' - C''} < 0, \tag{22}$$

the denominator being negative by assumptions A3 and A5.[4] Therefore

$$\frac{\partial \bar{\mu}}{\partial \tau} = \frac{-\varphi(\tau, \bar{\mu}(\tau, z(\tau, x_0))}{\displaystyle\int_0^\tau \frac{\partial \varphi(t, \bar{\mu}(\tau, z(\tau, x_0))}{\partial \bar{\mu}} dt} > 0,$$

which proves part (i), and

$$\frac{\partial \bar{\mu}}{\partial z} = \frac{1}{\displaystyle\int_0^\tau \frac{\partial \varphi(t, \bar{\mu}(\tau, z(\tau, x_0))}{\partial \bar{\mu}} dt} < 0,$$

which proves part (ii). ■

Lemma 2 $x_D(\tau, x_0)$ *is a monotone decreasing function of* τ.

[3]Note that the assumptions (A3) and (A5) also guarantee that the second-order necessary condition for the maximization of the Hamiltonian in (5) is satisfied.

[4]Note that $\dfrac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial z} < 0$ corresponds to the strict concavity in x of the value function for the cartel phase given x_D, which follows from the strict concavity of the profit function.

Proof. Differentiating (18) keeping x_0 fixed gives:

$$\frac{dx_D}{d\tau} = \frac{\left[\frac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial \tau} + r\bar{\mu}(\tau, z(\tau, x_0))\right] e^{r\tau}}{\frac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial z} e^{r\tau} + \frac{d^2 V_i^o(\vec{x}_D)}{dx_D^2}}.$$

The denominator is negative by the second-order condition (19). As for the numerator, we know that from (12) and (13), we have:

$$\mu(\tau) = \bar{\mu}(\tau - s, z(\tau - s, x(s)))e^{r(\tau - s)}.$$

Differentiating with respect to s and using (1), we find that:

$$-\frac{\partial \bar{\mu}(\tau - s, z(\tau - s, x(s)))}{\partial \tau - s} - q(s)\frac{\partial \bar{\mu}(\tau - s, z(\tau - s, x(s)))}{\partial z} - r\bar{\mu}(\tau - s, z(\tau - s, x(s))) = 0$$

or, and using (16) and setting $s = 0$:

$$\frac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial \tau} + r\bar{\mu}(\tau, z(\tau, x_0)) = -q(0)\frac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial z} > 0.$$

by part (ii) of Lemma 1.

Therefore the numerator is positive, independently of the sign of $\bar{\mu}(\tau, z(\tau, x_0))$. ■

Lemma 3 *For any $x_0 > \hat{x}$, there exists a unique $\hat{\tau}(x_0)$ such that $x_D(\hat{\tau}(x_0), x_0) = \hat{x}$ and $x_D(\tau(x_0), x_0) < (>)\, \hat{x}$ for $\tau > (<)\, \hat{\tau}(x_0)$.*

Proof. From Lemma 2, we know that $\partial x_D(\tau, x_0)/\partial \tau < 0$. Furthermore, $x_D(0, x_0) = x_0$, for there is then no cartel phase, and $\lim_{\tau \to \infty} x_D(\tau, x_0) = 0$, since it is never optimal for the monopolist to leave some resource unexploited at $T^M(x_0)$. For $x_0 > \hat{x}$, it must therefore be the case that $x_D = \hat{x}$ crosses $x_D = x_D(\tau(x_0), x_0)$ from below once and only once. ■

Assume then $x_0 > \hat{x}$. By assumption A6 and Lemma 3, we will have $\frac{dV_i^o(\vec{x}_D)}{dx} = 0$ and

$\dfrac{dV_i^o(\vec{x}_D)}{dx} > (<)\, 0$ for $\tau > (<)\, \hat{\tau}(x_0)$. It follows from this and condition (8) that if $\tau = \hat{\tau}(x_0)$, then $\mu(\tau) = 0$. Hence, to satisfy (7), we must have $\mu(t) = 0$ for all $t \in [0, \tau]$. Similarly, if $\tau > \hat{\tau}(x_0)$, we must have $\mu(t) > 0$ and if $\tau < \hat{\tau}(x_0)$, we must have $\mu(t) < 0$, for all $t \in [0, \tau]$.

Substituting for $\mu(t) = 0$ into condition (6), we get that if $\tau = \hat{\tau}(x_0)$, then the cartel will, while it exists, choose to deplete the resource at a constant rate $q^c(t) = q^{sm}$, where $q^{sm} = (x_0 - \hat{x})/\tau$ represents the constant rate of output that would be optimal for a static monopolist.[5]

Since, by assumption A3 and A5, the left-hand side of (6) is a decreasing function of $q^c(t)$, we will have $q^c(0) > q^{sm}$ if $\mu(0) < 0$ and $q^c(0) < q^{sm}$ if $\mu(0) > 0$.

More generally, we can state:

Proposition 1 *For any $x_0 > 0$, the cartel's rate of extraction, $q^c(t)$, $t \in [0, \tau]$, is a decreasing function of the length of the cartel phase.*

Proof. Differentiating (20) with respect to τ, we get

$$\frac{dq^c(t)}{d\tau} = \frac{\partial \varphi(t, \bar{\mu}(\tau, z(\tau, x_0)))}{\partial \bar{\mu}} \frac{\partial \bar{\mu}(\tau, z(\tau, x_0))}{\partial \tau} < 0,$$

by (22) and part (i) of Lemma 1. ■

Therefore, for any x_0, the shorter the anticipated time to dissolution, the more the cartel's rate of output will diverge from the dynamic monopoly output at any time during the cartel phase.

We may now characterize the output path of the cartel phase when $x_0 > \hat{x}$ as follows:

Proposition 2 *For any $x_0 > \hat{x}$, the rate of depletion of the resource stocks during the cartel phase will be:*

1. *increasing over time starting from $q^c(0) > q^{sm}$ if $\tau < \hat{\tau}(x_0)$;*

2. *constant at $q^c(0) = q^{sm}$ if $\tau = \hat{\tau}(x_0)$;*

[5] This constant monopolist output would constitute the equilibrium extraction path if both τ and x_0 were infinite.

3. decreasing over time starting from $q^c(0) < q^{sm}$ *if* $\tau > \hat{\tau}(x_0)$*;*

Proof. From conditions (6) and (7), we know that when $\tau <(>)\ \hat{\tau}(x_0)$, and hence $\mu(\tau) <(>)\ 0$, marginal profit (the left-hand side of (6)) must be decreasing (increasing) over time for $t \in [0, \tau(x_0))$. By assumptions A3 and A4, this means that $q^c(t)$ must be increasing (decreasing) in t, starting at $q^c(0)$. That $q^c(0) >(<)\ q^{sm}$ when $\mu(\tau) <(>)\ 0$ was established above. For the same reasons, when $\tau = \hat{\tau}(x_0)$ and hence $\mu(\tau) = 0$, marginal profit must be zero and $q^c(t)$ must be constant for $t \in [0, \hat{\tau}(x_0))$, given by $q^c(t) = q^{sm} = (x_0 - \hat{x})/\tau(x_0)$, the static monopoly output. ■

At dissolution, the extraction rate must jump up to the oligopoly path that follows from that time on. Since the oligopoly game is time autonomous, the oligopoly path will in fact be similar to what would have occurred had there not been a cartel phase, only now with initial stocks of $x_D < x_0$ instead of x_0, and hence a shorter period of time to exhaustion. The jump must be upward, since otherwise the resource stocks would never be fully depleted.

The intuition underlying those results is clear. A $\mu(t)$ that is negative, means that the cartel attaches a negative marginal value to the resource stocks. This means that if it were possible, it would choose to destroy, at the outset, some of the stocks held by each cartel member in order to reduce the rivalry between them and hence increase their profits in the ensuing oligopoly phase. This not being possible, it chooses to do the next best thing: it begins extraction not only at a higher rate then it would if the possibility of dissolution did not exist, but at a higher rate than would a static monopolist. Because of discounting and the Hotelling rule, this rate of extraction will increase over time throughout its existence in order to leave the oligopolists with the desired stocks at the time of dissolution. However, in doing this, it foregoes profits during the cartel phase in exchange for greater profits in the oligopoly phase. This arbitrage will be sufficiently profitable to justify a rate of extraction higher than the static monopoly rate only if the dissolution date — and hence the increase in profit from the oligopoly phase — is not too far away ($\tau < \hat{\tau}(x_0)$). If the time to dissolution is farther away ($\hat{\tau}(x_0) < \tau < T^M(x_0)$), then, although it is still profitable to leave smaller stocks at dissolution than it would if it were on the dynamic monopolist's path at that time — which explains why it will extract more at each date than along the monopoly path — the trade off becomes not sufficiently profitable to justify attaching a negative marginal value to the resource stock. This is why $\mu(t)$ is then positive. As a consequence, the rate of extraction is then lower than that of the static monopolist and decreases over time due to discounting. The cartel only behaves as a true dynamic monopolist if it does not have to envisage dissolution at all ($\tau \geq T^M(x_0)$).

What if $x_0 \leq \hat{x}$? Then necessarily $\dfrac{dV_i^o(\vec{x}_D)}{dx} > 0$ and $\mu(t) > 0$ for all $\tau > 0$. For levels of initial stocks that low, although the cartel will still want to extract more during the cartel phase than would the dynamic monopolist over the same time period, the gains from the reduced competition during the oligopoly phase is insufficient to make it worth doing so at a rate greater than that of the static monopolist. We would then observe a decreasing output path during the cartel phase, as is usually expected of a natural resource monopolist.

Because x_D is a function of both τ and the initial stock x_0, it is interesting to consider as well the effect on the cartel behavior of a change in x_0, keeping τ fixed. Differentiating (18) with τ fixed, we get:

$$\frac{dx_D}{dx_0} = \frac{\dfrac{\partial\bar{\mu}(\tau, z(\tau, x_0))}{\partial z} e^{r\tau}}{\dfrac{\partial\bar{\mu}(\tau, z(\tau, x_0))}{\partial z} e^{r\tau} + \dfrac{d^2 V_i^o(\vec{x}_D)}{dx^2}} > 0, \tag{23}$$

since the denominator is negative by the second-order condition (19) and the numerator is negative by part (ii) of Lemma 1. Thus $x_D(\tau, x_0)$ is a monotone increasing function of x_0: for a given dissolution date, the greater the initial stocks held by each member, the more stocks the cartel will wish to leave for the oligopoly phase when it breaks up.

However the portion of the total stocks allocated to the cartel phase does not vary in a monotone fashion with x_0. To see this, let $\tilde{x}_0(\tau)$ denote the solution to $x_D(\tau, x_0) = \tilde{x}$. Such a $\tilde{x}_0(\tau)$ exists and is unique since $\lim_{x_0 \to \infty} = \infty$, $x_D(\tau, 0) = 0$ and $x_D(\tau, x_0)$ is monotone increasing in x_0. We can then state the following:

Proposition 3 *With τ fixed, the stocks allocated to the cartel phase are an increasing (decreasing) function of x_0 if x_0 is smaller (larger) than $\tilde{x}_0(\tau)$.*

Proof. The stocks allocated to the cartel phase in equilibrium are $z(\tau, x_0) = x_0 - x_D(\tau, x_0)$. By the definition of $\tilde{x}$, we will have:

$$\frac{d^2 V_i^o(\vec{x}_D)}{dx^2} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 0 \quad \text{if} \quad x_0 \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \tilde{x}_0.$$

Therefore, from (23):

$$\frac{dz}{dx_0} = \left(1 - \frac{dx_D}{dx_0}\right) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \quad \text{if} \quad x_0 \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \tilde{x}_0.$$

■

Choosing x_D to satisfy the transversality condition (8) determines how the total initial stocks, x_0, is divided between the cartel phase ($z = x_0 - x_D$) and the oligopoly phase (x_D). The transversality condition says that the allocation between the two phases must be such as to equate the marginal value of the stocks allocated to the cartel phase ($\mu(\tau)$) to the marginal value of the stocks allocated to the oligopoly phase ($dV_i^o(\vec{x}_D)/dx$). Such an allocation maximizes the total value of the initial stocks x_0.

As long as $x_0 < \tilde{x}_0(\tau)$, both marginal values are decreasing functions of the stocks allocated to the phase, i.e., $\partial\mu(\tau)/\partial z < 0$ and $d^2V_i^o/dx^2 < 0$. Both marginal values are positive if $x_0 < \hat{x} + \tau q^{sm}$ and both negative if $x_0 > \hat{x} + \tau q^{sm}$ (in which case the initial stocks x_0 are larger than the cartel would like). The transversality condition then dictates that a marginal increase in the initial total stocks be allocated partly to the cartel phase and partly to the post-cartel phase in order to maintain the equality of the marginal values. Therefore, if $x_0 < \tilde{x}_0(\tau)$, we have $dz + dx_D = dx_0 > 0$, with $dz > 0$ and $dx_D > 0$.

However, if $x_0 = \tilde{x}_0(\tau)$ then we have $d^2V_i^o/dx^2 = 0$. The marginal value of stocks to the oligopoly phase is negative and reaches a minimum, while the marginal value of stocks to the cartel phase, also negative, continues to decrease as a function of z. The only way to continue satisfying the transversality condition after a marginal increase in x_0 is to allocate it entirely to the post-cartel phase (i.e., $dx_D = dx_0 > 0$ and $dz = 0$). Allocating some of the increase to the cartel phase would result in a lower total value of the stocks.

When $x_0 > \tilde{x}_0(\tau)$, we are in a situation where the marginal value of stocks to the oligopoly phase is increasing ($d^2V_i^o/dx^2 > 0$), while that of stocks to the cartel phase continues to decrease with z. Therefore, in order to maintain the equality of the two marginal values when x_0 is increased, the cartel will want to *lower* the stocks allocated to the cartel phase and *increase* those allocated to the post-cartel phase, so that $dz + dx_D = dx_0 > 0$, with $dx_D > 0$ but $dz < 0$.

Thus, for any given τ, $z(\tau, x_0)$ reaches a maximum at $\tilde{x}(\tau)$, with $z(\tau, \tilde{x}_0(\tau)) > \tau q^{sm}$. Furthermore, $z(\tau, 0) = 0$, $z(\tau, \hat{x} + \tau q^{sm}) = \tau q^{sm}$ and $z(\tau, x_0)$ must again approach τq^{sm} as x_0 tends to infinity. In the limit, with an infinitely large x_0, the situation is one of static equilibrium: the cartel produces the constant monopoly output q^{sm} until its dissolution at τ, after which production increases to the constant oligopoly output $q^o > q^{sm}$, which is repeated forever.

4 Concluding remarks

We have shown how, under highly reasonable assumptions about the value function associated with an oligopolistic equilibrium, an anticipated threat of dissolution will modify the behavior of a nonrenewable resource cartel. Not only will it in all cases induce the cartel to produce more over the same period than it would in the absence of the threat of dissolution, but if the initial stocks are sufficiently large and the time to dissolution is sufficiently short, it will attach a negative marginal value to the resource stock and hence produce more than it would even as a static monopolist. The rate of extraction must therefore be increasing over time during the cartel phase. This result is due to the fact that the marginal value of the resource stocks is negative and therefore the Hotelling rule implies that its value must decrease over time at the rate of discount, becoming more and more negative as the break-up time approaches.

We have also shown that, given the anticipated time to dissolution, if the cartel wants to maximize the total value of the initial stocks, the quantity it will choose to deplete during its existence will at first increase and then decrease as a function of the total initial stocks. Eventually, if we let the initial stocks become infinite, we find that the anticipated dissolution has no effect on the behavior of the cartel, its rate of output being simply that of a static monopolist throughout its existence. This is what is to be expected, since the cartel then faces no intertemporal problem.

Although all the properties assumed for the value function of the oligopolistic firms in the post-cartel phase will not hold for all profit functions, they should hold for a large class of functions and so should our results.

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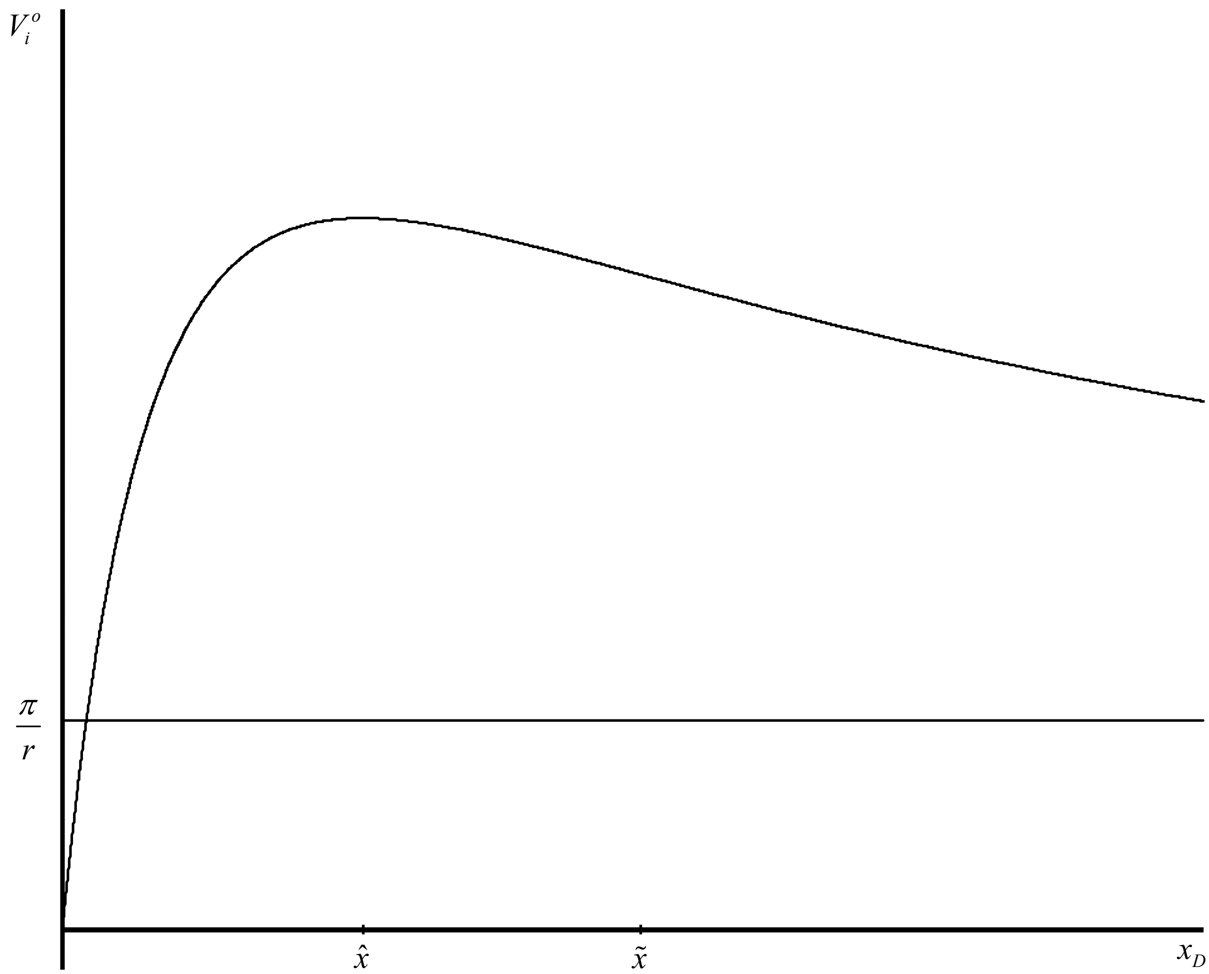

Figure 1 : Individual value function of the oligopolists as a function of x_D