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Adaptation and the Allocation of Pollution Reduction Costs*

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We consider a game of abatement of a transboundary pollutant. We use a time-consistent Shapley value allocation of the cost of pollution reduction, and study the sensitivity of such an allocation to countries' adaptation to pollution. A country's adaptation to pollution is captured by a change in its damage function. We show that if there is a reduction in the damage cost of one country only, this can harm the other countries. Some countries may end up worse off even in the case where all countries experience a uniform decrease in their damage from pollution. An important policy implication of our analysis is that the Shapley value approach to the allocation of abatement costs doesn't necessarily provide the right incentives for all players to act on reducing pollution damage. We determine conditions under which a uniform fall in all countries' pollution damage benefits all countries.

JEL classification: C71; Q2; Q54; Q55

Keywords: Adaptation; Shapley value; Transboundary pollution; Climate Change; Time-consistency.

1 Introduction

Dynamic transboundary pollution problems, such as the failure to control greenhouse gas (GHG) emissions constitute one of the major challenges of our time. The consequences of non-cooperation are oftentimes substantial if not catastrophic. For example, climate change, by now a widely recognized consequence of anthropogenic emissions of GHGs, can result in a temperature increase relative to preindustrial levels that is likely to be[1] between 1.9°C and 4.5°C (Intergovernmental Panel on Climate Change IPCC (2007)). This could result in damages estimated at 1 to 2 percent of the world GDP in the case of a 2.5°C increase, and 2 to 4 percent in the case of a 4°C increase[2] (see, e.g., Aldy et al. (2010)).

One way to reduce the severity of a transboundary pollution problem is to invest in technologies that reduce the environment's vulnerability to the pollutant(s) in question. Parties invest in innovative ways to reduce the harm caused by pollution. In the case of climate change, this can take several forms, such as building levees to protect coastal areas, developing disaster-management strategies, adapting the use of agricultural land to the new climatic conditions[3]. Modern adaptation measures also include wildlife reserves to protect biodiversity, as well as drinking-water reservoirs and building codes. The rising importance of adaptation is manifest at both the regional and global levels. For example, in the US, the America's Climate Choices (ACC) Panel on Adapting to the Impacts of Climate Change was mandated to "describe, analyze, and assess actions and strategies to reduce vulnerabilities, increase adaptive capacity, improve resilience, and promote successful adaptation to climate change in different regions, sectors, systems, and populations." In a recent report[4], the panel recommends that "the executive branch, in consultation with Congress, develop a national adaptation strategy". At the international level, several funds run by the United Nations[5], World Bank and European Commission invest in adaptation (see Le Goulven (2008)). The agreement reached at the COP 16[6] meeting, held in Cancun in 2010, includes a "Green Climate Fund," proposed to be worth $100 billion a year by 2020. The recent COP 18 in Doha has maintained that objective.

In this paper we examine the impact of changes in pollution damage that can result from investments in adaptation, within a transboundary pollution game. Transboundary pollution problems such as GHG emissions typically involve a finite number of important actors whose utilities are interdependent and whose actions have lasting impacts. They represent a fertile area of application for dynamic game theory

[1] Assuming a 550 ppm CO_2-equivalent stabilization.

[2] For temperature increases of 6°C and 7.4°C, Nordhaus and Boyer (2000) and Stern (2007) give an expected damage of respectively 10.3 percent and 11.3 percent of the world GDP.

[3] Adaptation options include crop management, ranching practices or more efficient irrigation systems.

[4] US National Research Council. *Advancing the Science of Climate Change.* Washington, DC: The National Academies Press, 2010.

[5] See United Nations Framework Convention on Climate Change, UNFCCC.

[6] Sixteenth Conference of Parties.

in general and cooperative (dynamic) game theory more specifically[7]. In the cooperative game theory approach, the objective is to determine the gains from cooperation and propose an allocation of these gains across the players. Different allocation rules have been examined. Germain et al. (2003) extend the static cooperative framework of Chander and Tulkens (1995) to a closed-loop dynamic setting where cooperation is negotiated at each period (i.e., players re-evaluate their interest in cooperation at every stage) taking into account the current stock of pollutant. Using the core solution concept, they define a transfer mechanism to ensure the stability of the grand coalition over time, under both individual and coalitional rationality. Botteon and Carraro (2001) consider a model of five asymmetric players to study the stability of an international environmental agreement. Applying the Shapley value and Nash bargaining solutions as abatement-cost sharing rules for calibrated parameter estimates, they find that using Shapley value with transfer payment induces higher cooperation. In a transboundary pollution game, Petrosyan and Zaccour (2003) -henceforth, PZ- compute the characteristic function and each player's Shapley value. They propose an allocation over time of the abatement cost resulting from the Shapley value, which allocation is time consistent.

In this paper, we use the Shapley value to allocate the cost of pollution reduction and we use the decomposition of the Shapley value over time, as derived in PZ, to conduct a comparative dynamics exercise with respect to the damage caused by pollution. Our work offers an understanding of the types of support that may be provided by initiatives to improve adaptation to pollution and how such initiatives affect each player's welfare.

Our work is also related to the literature on the economics of adaptation (see Agrawala et al. (2011) for a recent survey) and more precisely, the literature on adaptation within pollution games. This literature examines, within strategic static frameworks, the difficult exercise of balancing investments in adaptation versus emissions control (e.g., Brechet et al. (2013), Buob and Stephan (2011) or Zehaie (2009)). In this paper, the focus is not on the equilibrium efforts of cost reduction and emissions control; rather we examine how a change in damage costs[8] impacts the payoffs of players in the emissions game. Furthermore, in our paper, we consider the case of a stock pollutant and therefore employ a dynamic game framework.

The paper is organized as follows. Section 2 gives a brief description of the model and gives the characteristic function values as well as the Shapley value allocation and its decomposition over time. In Section 3, we analyze the sensitivity of the Shapley values to changes in the damage from pollution. In Section 4, we analyze the decomposition of Shapley values over time, as the damage parameters change. The case where the change in the damage from pollution is endogenous is considered in Section 5. Section

[7] For dynamic non-cooperative games of transboundary pollution, see a recent survey in Long (2011).

[8] The analysis in sections 3 and 4 examines the case of an exogenous change of damage costs. In section 5 we present a setup where the level of efforts in adaptation is modeled and is endogenous.

6 concludes the paper.

2 Model and preliminaries

We use a simple model of an abatement game with a stock pollutant (see, e.g., PZ). Consider n countries indexed by $i \in \{1, 2, ..., n\}$. At each moment $t \geq 0$, country i chooses its emissions rate denoted by $e_i(t)$. The emissions of all countries build up and form a stock pollutant. The stock of accumulated pollution at time t, denoted $S(t)$, evolves according to the following differential equation:

$$\dot{S}(t) = \sum_{i=1}^{n} e_i(t) - \delta S(t), \qquad S(0) = S_0 > 0 \tag{1}$$

where $\delta > 0$ is the pollutant's natural rate of degradation.

From now on, for notational convenience, the time argument is omitted when no ambiguity arises. Let $\bar{e}_i$ denote country i's business-as-usual emissions rate (i.e., in the absence of abatement activities) and $C_i(e_i)$ denote the emissions-reduction cost incurred by country i when limiting its emissions to e_i, with

$$C_i(e_i) = \frac{\gamma}{2}(e_i - \bar{e}_i)^2, \qquad 0 \leq e_i \leq \bar{e}_i, \gamma > 0. \tag{2}$$

The stock of pollution causes damage to country i, which is denoted $D_i(S)$ and given by

$$D_i(S) = \pi_i S, \qquad \text{with } \pi_i \geq 0. \tag{3}$$

Remark 1: For tractability and ease of presentation, we focus on the case of linear damage cost functions. Our results, however, are robust and remain qualitatively true in the case of quadratic damage cost functions.

The objective of each country is to minimize a stream of discounted sum of emissions-reduction cost and damage costs. Therefore the non-cooperative solution is obtained by solving the following problem:

$$V(\{i\}, S) = \min_{e_i} \int_0^\infty (C_i(e_i) + D_i(S))e^{-\rho t} dt \text{ for } i = 1, .., n \tag{4}$$

subject to (1) where $\rho > 0$ is the common discount rate.

The first best emissions path solves the following problem:

$$V(I, S) = \min_{(e_1,..,e_n)} \int_0^\infty \sum_{i=1}^{n} (C_i(e_i) + D_i(S))e^{-\rho t} dt, \tag{5}$$

subject to (1).

Let $I = \{1, .., n\}$ be the set of countries. To economize on notation, we also use i to refer to country i. The problem of a coalition $K \subseteq I$ of size k is

$$V(K,S) = \min_{(e_i)_{i\in K}} \int_0^\infty \sum_{i\in K} (C_i(e_i) + D_i(S))e^{-\rho t}dt, \tag{6}$$

subject to (1).

To compute the value functions for intermediate coalitions in problem (6), where a group of k players, $k < n$, form a coalition, we use the γ-core assumption[9]. The coalition K acts as a singleton whose objective is to minimize the sum of its members' discounted costs, while the rest of the countries respond individually by minimizing their own discounted total costs. We solve for a Nash equilibrium among $n - k + 1$ players: the coalition K and the $n - k$ players that are outside coalition K. An alternative approach is used in PZ: it is assumed that when a coalition is built, non-signatories would play the strategy obtained from the non-cooperative scenario, which is solution to (4). The two approaches are equivalent within the context of the linear-state differential game we consider (see Zaccour (2003))[10]: in equilibrium, each country or coalition of countries plays a dominant strategy. We now give the characteristic value functions.

We compute the value function of a coalition K in Appendix A and obtain

$$V(K,S) = \frac{\sum_{i\in K}\pi_i}{\rho(\rho+\delta)}[\bar{e} - \frac{1}{2\gamma(\rho+\delta)}(2\pi + (k-2)\sum_{i\in K}\pi_i)) + \rho S] \tag{7}$$

where[11]

$$\pi = \sum_{i=1}^{n} \pi_i, \text{ and } \bar{e} = \sum_{i=1}^{n} \bar{e}_i. \tag{8}$$

The value functions for the grand coalition, $V(I,S)$, and Nash equilibrium, $V(\{i\},S)$, are then given by

$$\begin{aligned} V(I,S) &= \frac{\pi}{\rho(\rho+\delta)}\left[\bar{e} - \frac{1}{2\gamma(\rho+\delta)}(n\pi) + \rho S\right] && (9)\\ V(\{i\},S) &= \frac{\pi_i}{\rho(\rho+\delta)}\left[\bar{e} - \frac{1}{2\gamma(\rho+\delta)}(2\pi - \pi_i) + \rho S\right]. && (10)\end{aligned}$$

The cooperative (interior) solution is $e_i = \bar{e}_i - \frac{\pi}{\gamma(\rho+\delta)}$, and the non-cooperative (interior) solution is given by $e_i = \bar{e}_i - \frac{\pi_i}{\gamma(\rho+\delta)}$. We shall focus on interior solutions and therefore assume that

$$\frac{\pi}{\gamma(\rho+\delta)} \leq \bar{e}_i \quad \text{for all } i = 1, .., n. \tag{11}$$

[9] See, e.g., Chander and Tulkens (1995) and Germain et al. (2003).

[10] This equivalence is lost if one considers a strictly convex damage function. When we check that our results remain qualitatively valid in the case of a quadratic damage function, we use the γ-core assumption.

[11] From equation (7), we observe that the value function of a coalition K depends on π_j even for $j \notin K$.

The time path of the stock of pollution under cooperation is given by

$$S^I(t) = S_0 e^{-\delta t} + \frac{1}{\delta}(\bar{e} - \frac{n}{\gamma}\frac{\pi}{\rho+\delta})(1 - e^{-\delta t}). \tag{12}$$

We can now compute the allocation among the players of the total cost of abatement under cooperation (i.e., the total cost if the grand coalition forms), based on the Shapley value allocation. The Shapley value of country j is given by the following formula:

$$\phi_j(V,S) = \sum_{K \subset I \backslash \{j\}} [V(K \cup \{j\}, S) - V(K,S)] \frac{k!(n-k-1)!}{n!} \text{ for } j = 1,..,n \tag{13}$$

where $V(K,S)$ is given by (7). After some algebraic manipulations we have[12]

$$\begin{aligned} V(K \cup \{m\}) - V(K) &= \frac{\pi_m}{\rho(\rho+\delta)}(\bar{e} + \rho S_0) - \\ &- \frac{1}{2\gamma\rho(\rho+\delta)^2}[2\pi\pi_m + 2(k-1)\pi_m \sum_{i\in K}\pi_i + (k-1)\pi_m^2 + (\sum_{i\in K}\pi_i)^2]. \end{aligned} \tag{14}$$

Let $\Gamma(S,t)$ be a subgame starting at time t, with a stock of pollution at t given by $S^I(t)$. PZ show the following: the path $\beta(.)$ given by

$$\beta_j(t) = \rho\phi_j(V, S^I(t)) - \frac{d}{dt}\phi_j(V, S^I(t)) \text{ for } j = 1,..,n \tag{15}$$

decomposes the Shapley value for player j through time

$$\int_0^\infty \beta_j(t) e^{-\rho t} dt = \phi_j(V, S^I(0)) = \phi_j(V, S_0)$$

and is time-consistent:

$$\phi_j(V, S_0) = \int_0^t \beta_j(\tau) e^{-\rho\tau} d\tau + e^{-\rho t}\phi_j(V, S^I(t)).$$

The vector β is called an imputation distribution procedure.

Remark 2: Since the game we consider is a linear-state differential game where the instantaneous payoff functions and state dynamics include no multiplicative interactions between the players' controls, then, from Zaccour (2003), we can infer that (i) the characteristic functions in our problem coincide with those in PZ and (ii) our cooperative game is convex (Proposition 2 in PZ). From the Bondareva-Shapley theorem, Bondareva (1968) and Shapley (1972), we can then infer that the Shapley value allocation of the total cost of pollution reduction is in the core[13].

[12] We omit S from the notation and use $V(K)$ instead of $V(K,S)$ if no ambiguity arises.

[13] We thank an anonymous referee for pointing the reference for this result in Bondareva (1968).

The main objective of this paper is to analyze how efforts in adaptation to pollution, which result in a perturbation of the vector $(\pi_1, .., \pi_n)$, affect the countries' Shapley values as well as their decomposition over time. More specifically, we seek to answer the following question: does each country have an incentive to reduce its vulnerability to pollution when the total cost of pollution reduction is allocated using the Shapley value approach?

3 Adaptation and the Shapley value

It is rather straightforward to see that if a country's (and only that country's) damage decreases, it incurs a smaller cost when under the Shapley value allocation of the total cost from pollution.

Proposition 1 *Player i's Shapley value decreases if its marginal damage cost π_i falls.*

Proof. See Appendix B. ■

Less obvious is the impact on the other countries' Shapley values. The next proposition shows that if one country's damage from pollution falls, then this country gains, but at the expense of all the other countries, even though the total cost under cooperation falls.

Proposition 2 *A fall in player j's marginal damage cost, π_j, will increase the Shapley value of player i where i=1,..,n, with $i \neq j$.*

Proof. See Appendix C. ■

The next two propositions show that when all countries simultaneously experience a fall in their damage from pollution some countries might see their burden of the abatement cost increase. Therefore, the Shapley value approach to the allocation of abatement costs doesn't necessarily give all players the right incentives to reduce the damage from pollution.

We first consider a uniform fall in the marginal damage from pollution, from π_i to $\pi_i - d$ for all $i = 1, .., n$ with $d > 0$ and $d < \pi_s \equiv \min_{i=1,..,n}\{\pi_i\}$.

We seek to determine if the Shapley value of a player m can be smaller when $d > 0$ than when $d = 0$:

$$\sum_{K \subset I \setminus \{m\}} \left([V_d(K \cup \{m\}) - V_d(K)] - [V(K \cup \{m\}) - V(K)]\right) \frac{k!(n-k-1)!}{n!} > 0 \tag{16}$$

where V_d is the value function when the marginal damage from pollution is $\pi_i - d$ for all $i = 1, .., n$. When (16) holds, player m's welfare is smaller when $d > 0$ than when $d = 0$.

We determine conditions under which each term in the summation in (16) is positive. Let

$$G(d, k, K) \equiv V_d(K \cup \{m\}) - V_d(K) - (V(K \cup \{m\}) - V(K)) .$$

Using (14), it can be shown, after some algebraic manipulations, that $G(d,k,K)$ can be written in the following form:

$$G(d,k,K) = d\left(\tilde{d}\left(\pi_m, K\right) - d\right)\frac{\left(3k^2 - k - 1 + 2n\right)}{2\gamma\rho(\rho+\delta)^2} \tag{17}$$

where

$$\tilde{d}\left(\pi_m, K\right) = \frac{Z\left(\pi_m\right) - 2\gamma(\rho+\delta)\left(\bar{e} + \rho S_0\right)}{\left(3k^2 - k - 1 + 2n\right)} \tag{18}$$

and where

$$Z\left(\pi_m, K\right) = 2\pi + 2(2k-1)\sum_{i\in K}\pi_i + 2\left(n + k^2 - 1\right)\pi_m. \tag{19}$$

We note that $3k^2 - k - 1 + 2n > 0$ for all $k = 1, .., n$ and that Z depends on the coalition K. Given that the number of coalitions $K \subset I\backslash\{m\}$ is finite, there exists

$$\tilde{d}_{\min}\left(\pi_m\right) = Min_{K\subset I\backslash\{m\}}\left\{\tilde{d}\left(\pi_m, K\right)\right\}.$$

Therefore for all coalitions $K \subset I\backslash\{m\}$ we have, for $d \in \left(0, \tilde{d}_{\min}\left(\pi_m\right)\right) : G(d,k,K) > 0$.

We can now state the following proposition.

Proposition 3 *Consider a player m with a marginal damage cost π_m; then, for any vector $(\pi_1, .., \pi_n)$ and $(\bar{e}, S_0)$ such that $\tilde{d}_{\min}\left(\pi_m\right) > 0$, a uniform fall in the marginal damage from pollution, from π_i to $\pi_i - d$ for all $i = 1, .., n$ with $d \in \left(0, Min\left(\pi_s, \bar{d}_{\min}\right)\right)$ results in an increase in player m's Shapley value.*

Proof. The proof follows from the fact that for any $d \in \left(0, Min\left(\pi_s, \tilde{d}_{\min}\left(\pi_m\right)\right)\right)$ we have $G(d,k,K) > 0$ for all $K \subset I\backslash\{m\}$, which implies that (16) holds. ■

It can be checked that the condition $\tilde{d}_{\min}\left(\pi_m\right) > 0$ and assumption (11) are not mutually exclusive. We examine this question in the case where all players but player m have identical marginal damage costs π_o. We then have

$$\pi = (n-1)\,\pi_o + \pi_m \tag{20}$$

and therefore

$$\tilde{d}\left(\pi_m, K\right) = \frac{2\left(n + k^2\right)\pi_m + \left(4k^2 - 2k + 2n - 2\right)\pi_o - 2\gamma(\rho+\delta)\left(\bar{e} + \rho S_0\right)}{\left(3k^2 - k - 1 + 2n\right)} \tag{21}$$

We check that if

$$n\,(n-2)\,\pi_o < \pi_m \tag{22}$$

then there exists $(\bar{e}, S_0)$ such $\tilde{d}(\pi_m) > 0$ for all $k = 1, .., n$ and (11) holds. Indeed we have $\tilde{d}(\pi_m, K) > 0$ iff the numerator of (21) is positive. From (11) and for $S_0 \geq 0$, $\tilde{d}(\pi_m, K) > 0$ gives

$$2\left(n+k^2\right)\pi_m + \left(4k^2 - 2k + 2n - 2\right)\pi_o > 2\gamma(\rho+\delta)\left(\bar{e}+\rho S_0\right) \geq 2\gamma(\rho+\delta)\bar{e} > 2n\pi,$$

which holds only if

$$2\left(n+k^2\right)\pi_m + \left(4k^2 - 2k + 2n - 2\right)\pi_o > 2n\left((n-1)\pi_o + \pi_m\right). \tag{23}$$

The left-hand side of (23) is a strictly increasing function k. Therefore if this inequality holds for $k = 1$, it holds for all $k \geq 1$. For $k = 1$, after simplification, this gives (22). Therefore, for π_o and π_m such that $(n-2)\pi_o n < \pi_m$, we have

$$2\left(n+k^2\right)\pi_m + \left(4k^2 - 2k + 2n - 2\right)\pi_o > 2n\pi \text{ for all } k \geq 1$$

and therefore for $(\bar{e}, S_0)$ such that

$$2(n+1)\pi_m + 2n\pi_o > 2\gamma(\rho+\delta)\left(\bar{e}+\rho S_0\right) > 2n\pi,$$

we have both $\tilde{d}(\pi_m, K) > 0$ for all $K \subset I \backslash \{m\}$ (which implies $\tilde{d}_{\min}(\pi_m) > 0$) and assumption (11) satisfied.

Corollary 4 *Consider two players m and l with marginal damage costs π_m and π_l respectively with $\pi_l > \pi_m$ and a uniform decrease of the marginal damage costs from π_i to $\pi_i - d$ for all $i = 1, .., n$. If player m's welfare is smaller when $d > 0$ than when $d = 0$, then so is player l's welfare.*

Proof. This follows from the fact that the change in the Shapley value of a player m, due to a uniform decrease of the marginal damage costs, from π_i to $\pi_i - d$ for all $i = 1, .., n$, given by (16), is an increasing function of π_m. Indeed, each term of the summation in (16), $G(d, k, K)$, is an increasing function of π_m for all $K \subset I \backslash \{m\}$. This is true since by (19), $Z(\pi_m, K)$ is increasing in π_m and by (17) and (18), $G(d, k, K)$ rises with an increase in $Z(\pi_m, K)$. ■

We now examine another type of change in the vector of damage costs, namely, a proportional fall of the marginal damage cost, which falls from π_i to $\alpha\pi_i$ for all $i = 1, .., n$ with $\alpha \in (0, 1)$. We show that some countries may see their Shapley values rise.

Proposition 5 *Consider the case where $\pi_l = \pi_o > 0$ for all $l \neq m$ and player m's marginal damage is $\pi_m > 0$. Let $r \equiv \frac{\pi_m}{\pi_o}$, $\hat{r} \equiv \frac{1}{n}\left(\sqrt{n+1} - 1\right)(n-1)$ and $\hat{\alpha} \equiv \frac{nr^2 + 2r(n-1) - (n-1)^2}{nr^2 + 2(n-1)^2 r + (n-1)^2}$. For all $r > \hat{r}$ we have*

$\hat{\alpha} > 0$; moreover, a fall in all countries' marginal damage costs by a proportion $(1-\alpha)$ with $\alpha \in (0, \hat{\alpha})$ results in an increase in player m's Shapley value.

Proof. See Appendix D. ■

Proposition 6 below provides sufficient conditions under which a proportional fall in all countries' marginal damage costs may result in a decrease of the Shapley values of all the players: all players benefit from the (proportional) decrease in the marginal cost. We give these sufficient conditions for the case where $\pi_l = \pi_o > 0$ for all $l \neq m$.

Proposition 6 *Consider the case where $\pi_l = \pi_o > 0$ for all $l \neq m$ and player m's marginal damage is $\pi_m > 0$. Let $r \equiv \frac{\pi_m}{\pi_o}$; there exists $\tilde{\alpha} = \frac{nr((n-1)+r)}{2(n-1)^2 r + nr^2 + (n-1)^2} < 1$ such that, for all $\alpha < \tilde{\alpha}$, a fall in all countries' marginal damage costs by a proportion $(1-\alpha)$ results in a decrease of the Shapley value of each individual country.*

Proof. See Appendix E. ■

It can be shown that $\tilde{\alpha}$ is strictly increasing in r; therefore, the larger the damage of player m relative to the other players' damage values, the larger is the set $(0, \tilde{\alpha})$.

4 Adaptation and the decomposition of Shapley values over time

We can compute $\beta_i(t)$ from (15) using (12) and (7). The next proposition describes how $\beta_i(t)$ changes when the damage cost parameters change. The expression of $\beta_i(t)$ is cumbersome. To facilitate the presentation, we give the results in the case where $\pi_l = \pi_o > 0$ for all $l \neq m$ and player m's marginal damage is $\pi_m > 0$. We first give the impact of a change in the marginal damage costs on $\beta_i(t)$ in the short run, i.e., at $t = 0$.

Proposition 7a *Consider the case where $\pi_l = \pi_o > 0$ for all $l \neq m$ and player m's marginal damage is $\pi_m > 0$. We have:*

i) $\frac{\partial}{\partial \pi_m} \beta_m(0) > 0$

ii) $\frac{\partial}{\partial \pi_j} \beta_m(0) > 0$ *for $j \neq m$.*

Proof. See Appendix F. ■

While (i) is rather intuitive, given that $\frac{\partial \phi_m}{\partial \pi_m} > 0$, (ii) is less straightforward. In the short run, all countries benefit from a decrease of one of the players' marginal damage costs. This is not trivial since we have seen that the Shapley value of player m may actually fall in the event player j's ($j \neq m$) marginal damage cost falls.

Also surprising is the fact that $\frac{\partial}{\partial \pi_m}\beta_m(t)$ can take negative values. Since $\frac{\partial}{\partial \pi_m}\beta_m(t)$ is a very cumbersome expression, we report this possibility in the limit case where $\pi_o = 0$.

Proposition 7b *Consider the case where $\pi_l = \pi_o$ for all $l \neq m$ and player m's marginal damage is $\pi_m > 0$. We have:*

i) assume $\delta < (2n-1)\rho$ and let $\omega \equiv \frac{4n\rho}{(n+1)\delta}$, $\bar{\pi}_m \equiv \omega - \frac{2(\delta+\rho)}{(n+1)\delta}$ and $R \equiv \frac{\omega - \pi_m}{\omega - \bar{\pi}_m}$. Then $\bar{\pi}_m > 0$, and for $\pi_m \in (0, \bar{\pi}_m)$, we have $R > 1$. Moreover, for $\pi_m \in (0, \bar{\pi}_m)$ and $\bar{e}$ such that

$$\frac{\pi_m}{\gamma(\rho+\delta)} \leq \bar{e} < \frac{\pi_m}{\gamma(\rho+\delta)} R, \tag{24}$$

we have $\lim_{t\to\infty} \frac{\partial}{\partial \pi_m}\beta_m(t)\Big|_{\pi_o=0} < 0$

ii) $\lim_{t\to\infty} \frac{\partial}{\partial \pi_j}\beta_m(t) < 0$ for $j \neq m$.

Proof. See Appendix G. ■

Thus the impact of a unilateral fall in the damage cost on the instantaneous allocation of the Shapley value over time is not the same at all times.

It is reasonable to expect that when π_m increases, $\beta_m(t)$ will also increase. However according to part (i) of Proposition 7b, we see that the opposite can be true as well.

5 Endogenous change in the damage cost

We have so far considered the change in the damage cost to be exogenous; we have ignored the cost of achieving a decrease in the damage cost. We now include the cost of adaptation. The abatement cooperative game that we analyzed above could be viewed as part of a two-stage meta-game, where it is preceded by a first stage in which investments are made to decrease the damage cost from pollution. A similar approach was recently used in Athanassoglu and Xepapadeas (2012) where, prior to the unfolding of an emissions control problem, the decision maker determines at time 0 the investment in damage control.

This meta-game is solved backwards, with the outcome of stage 2 yielding the payoffs given by the Shapley value allocation (13), and where in stage 1, a level of investment is determined.

In stage 1, countries choose a level of damage-control effort. Let ε_i denote the effort toward damage control done by country i, and let $\varepsilon = (\varepsilon_1, .., \varepsilon_n)$ denote a vector of damage-control efforts. The damage incurred by country i during the abatement game when the stock of pollution is S is given by $D_i(S, \varepsilon)$ with $\frac{\partial D_i}{\partial \varepsilon_i} < 0$ and $\frac{\partial D_i}{\partial \varepsilon_k} \leq 0$ for $k \neq i$. When $\frac{\partial D_i}{\partial \varepsilon_k} = 0$ for $k \neq i$, there are no spillovers in damage control, and when $\frac{\partial D_i}{\partial \varepsilon_k} < 0$, we have spillover of the technologies that reduce pollution damage. When $D_j(S, \varepsilon) = (\pi_j - \varepsilon_j) S$ with $\varepsilon_j = d$ and $\varepsilon_k \neq 0$ for $k \neq j$, we obtain the case of a unilateral reduction in pollution damage, as analyzed in Proposition 2. The case covered in Proposition 3 can represent a situation where $D_i(S, \varepsilon) = (\pi_i - \Sigma_{k=1}^n \varepsilon_k) S$ for all $i = 1, .., n$ with $\Sigma \varepsilon_k = d$. Similarly, when $D_i(S, \varepsilon) = (1 - \Sigma_{k=1}^n \varepsilon_k) \pi_i S$ for all $i = 1, .., n$ with $\Sigma_{k=1}^n \varepsilon_k = \alpha$, we obtain the case covered by Proposition 5. In each case ε, is supposed to be such that the damage from pollution is non-negative.

The cost of exerting a given level of damage-control effort ε_i in country i is given by $F_i(\varepsilon_i)$, with $F_i' > 0$ for $\varepsilon_i > 0$, and $F_i'' \geq 0$. We further assume that $F_i'(0) = F_i(0) = 0$. Let $\phi_i(S, \varepsilon)$ denote the payoff of player i in stage 2, using the Shapley value allocation of the cost of pollution reduction.

In stage 1, the objective is to choose a vector of damage reduction ε that solves

$$Min_{\varepsilon} \Sigma_{i=1}^n (\phi_i(S, \varepsilon) + F_i(\varepsilon_i)) \tag{25}$$

subject to

$$\Sigma_{i=1}^n F_i(\varepsilon_i) \leq \bar{F}, \tag{26}$$

where $\bar{F} > 0$ is the total level of funding available for damage reduction.

Consider for example the cases where $D_i(S, \varepsilon) = (\pi_i - \Sigma_{k=1}^n \varepsilon_k) S$ with $\Sigma \varepsilon_k = d$ or where $D_i(S, \varepsilon) = (1 - \Sigma_{k=1}^n \varepsilon_k) \pi_i S$ for all $i = 1, .., n$ with $\Sigma_{k=1}^n \varepsilon_k = \alpha$, and assume $F_i = \frac{\lambda}{2} \varepsilon_i^2$ with $\lambda > 0$. Consider a level of $\bar{F}$ such that the budget constraint is binding at the solution to problem (25). The analysis of exogenous changes in pollution damage (presented in Sections 3 and 4) sheds light on some of the repercussions of relaxing the budget constraint on total investments in damage control (i.e., an increase in $\bar{F}$). This question is clearly relevant in the face of the growing interest in prioritizing investments in adaptation technologies. Propositions 3 and 5 show that some players can be worse off from an increase in $\bar{F}$ even in the limit case where their share in the increase in $\bar{F}$ is zero.

6 Quadratic damage cost from pollution

In this section, instead of assuming a linear damage function (3) we consider the case of a quadratic damage function

$$D_i(S) = \eta_i S^2 \text{ with } \eta_i \geq 0, \text{ for all } i \in \{1, 2, ..., n\}. \tag{27}$$

When there exists at least two countries k and $l \in \{1, 2, ..., n\}$ such that $\eta_k, \eta_l > 0$ and $\eta_k \neq \eta_l$, the derivation of value functions for different coalitions, including the Nash equilibrium, is no longer analytically tractable: while the value functions for each player is quadratic, the parameters of the quadratic term of the value functions for all the players are solutions of a non-linear system and cannot be determined analytically. To make progress, we consider the case of 3 players and the limit scenario where only one country incurs a damage from pollution. This scenario is enough to illustrate that the results obtained in the case of a linear damage function can extend to the case of a quadratic damage function. Therefore, suppose $\eta_2 = \eta_3 = 0$, and $\eta_1 > 0$. We further set $\gamma = 1$ for simplicity.

It is shown in Appendix H that the value function in the case where the three players minimize the sum of their discounted sum of the cost from pollution abatement and damage cost from pollutions (solution to (5)) is given by $W(I, S) = Z_I S^2 + A_I S + B_I$ where the coefficients Z_I, A_I and B_I are found using the undetermined coefficients technique[14]:

$$Z_I = \frac{1}{12} f(12), \; A_I = g(12, 6), \; B_I = u(12, 6, 3) \tag{28}$$

where

$$\begin{aligned} f(x) &= \sqrt{(2\delta + \rho)^2 + \eta_1 x} - (2\delta + \rho) && (29) \\ g(x, q) &= \frac{2\bar{e}\; f(x)}{x(\delta + \rho) + q\; f(x)} && (30) \\ u(x, q, \kappa) &= \frac{1}{2\rho} g(x, q)(2\bar{e} - \kappa g(x, q)). && (31) \end{aligned}$$

Similarly the value function under the Nash equilibrium for player i (solution to (4)) and a coalition of size two (solution to (6)) of players i and j are respectively $W(\{i\}, S) = Z_i S^2 + A_i S + B_i$ and $W(\{i, j\}, S) = Z_{ij} S^2 + A_{ij} S + B_{ij}$ with $i, j \in \{1, 2, 3\}$ and where

$$\begin{aligned} Z_1 &= \frac{1}{4} f(4), \; A_1 = g(4, 2), \; B_1 = u(4, 2, 1) && (32) \\ Z_2 &= Z_3 = A_2 = A_3 = B_2 = B_3 = 0 \\ Z_{12} &= Z_{13} = \frac{1}{8} f(8), \; A_{12} = A_{13} = g(8, 4), \; B_{12} = B_{13} = u(8, 4, 2) \\ Z_{23} &= A_{23} = B_{23} = 0. \end{aligned}$$

[14] Note that we use W to denote the value function in the case of a quadratic damage cost from pollution case and V in case where the damage cost from pollution is linear.

The Shapley value for each player is determined according to the following:

$$\phi_1 = \frac{1}{3}[(Z_I + Z_{12} + Z_1)S^2 + (A_I + A_{12} + A_1)S + (B_I + B_{12} + B_1)] \tag{33}$$

$$\phi_2 = \phi_3 = \frac{1}{6}(2Z_I - Z_{12} - Z_1)S^2 + \frac{1}{6}(2A_I - A_{12} - A_1)S + \frac{1}{6}(2B_I - B_{12} - B_1) \tag{34}$$

The evolution of stock pollutant under cooperation is given by:

$$S^C(t) = \frac{\bar{e} - 3A_I}{6Z_I + \delta} + \left(S_0 - \frac{\bar{e} - 3A_I}{6Z_I + \delta}\right) e^{-(6Z_I + \delta)t} \tag{35}$$

See Appendix H for full derivation of the Shapley values.

The following two propositions describe the changes of the Shapley values with respect to changes in damage cost parameters. The results are in line with the results in the preceding section for linear damage costs.

Proposition 8 *Assume $\eta_1 > 0$ and $\eta_i = 0$ for $i = 2, 3$. A fall in η_1 results in a decrease in Country one's Shapley value.*

Proof. It follows from the fact that each term in (33) is increasing with respect to η_1. ■

Proposition 9 *Assume $\eta_1 > 0$ and $\eta_i = 0$ for $i = 2, 3$. A fall in η_1 results in an increase in Country 1 and 2's respective Shapley values.*

Proof. See Appendix I. ■

We can compute $\beta_i(t)$ in the case of quadratic damage cost function from (15) using (12) and (7). The next proposition describes how $\beta_i(t)$ changes when the damage cost parameters changes. It shows that $\beta_i(0)$ could be increasing or decreasing with respect to η_1.

Proposition 10 *Assume $\eta_1 > 0$ and $\eta_i = 0$ for $i = 2, 3$. Then $\frac{d\beta_1(0)}{d\eta_1} > 0$ and $\frac{d\beta_i(0)}{d\eta_1} < 0$* for $i \neq 1$.

Proof. See Appendix J. ■

7 Conclusion

In a pollution abatement game between asymmetric countries, we considered the Shapley value as a cost-sharing rule to allocate the cost of pollution abatement to each country. We studied the impact of a change in pollution damage on a country's Shapely value and its decomposition over time.

We characterized conditions under which a reduction in damage cost by one or more countries deteriorates the welfare of other countries. We showed that if the damage cost of one country falls, some other countries may have to bear a larger burden of emissions reduction. Moreover we showed that the impact of a decrease in the damage cost on the decomposition of the Shapley value over time is not necessarily uniform with respect to time: it can result in a decrease of the instantaneous allocation of the cost at time zero and an increase of that cost allocation in the long run.

An important implication of our analysis is that the Shapley value approach to the allocation of abatement costs doesn't necessarily provide all players with the right incentives to act on reducing the damage from pollution. We provided sufficient conditions under which a drop in all countries' damage from pollution is welfare improving for all.

It would be interesting to investigate the impact of a change in the damage cost of pollution within the richer framework in Kozlovskaya et al. (2010) which allows for the formation of multiple coalitions and where in a first stage the Nash equilibrium in the game played by coalitions is determined and in the second stage, the value of each coalition is allocated according to the Shapley value as in Petrosyan and Mamkina (2006). This promising extension is left for future research.

Appendix A: Derivation of value function (7)

Computation of the value function of a coalition of size k

Suppose there is a coalition K of size k and that each of the $n-k$ players that are not in the coalition individually chooses its cost-minimizing strategy. Let $V(K,S)$ be the value function for coalition K; and, for each player j who stays out of the coalition, let value function be $J(j,S)$. The Hamilton Jacobi Bellman (HJB) equations associated with the problem of coalition K and of each of the other $n-k$ outsiders are, respectively,

$$\begin{aligned}\rho V(K,S) &= \min_{e_i}\{\sum_{i\in K}[\frac{\gamma}{2}(e_i-\bar{e}_v)^2+\pi_i S]+V^{'}(K,S)(\sum_{i\in K}e_i+\sum_{l\in I\setminus K}e_l-\delta S)\} \text{ for all } i\in K\\ \rho J(j,S) &= \min_{e_j}\{\frac{\gamma}{2}(e_j-\bar{e}_j)^2+\pi_j S+G^{'}(j,S)(\sum_{v\in K}e_i+\sum_{l\in I\setminus K}e_l-\delta S)\} \text{ for } j\notin K\end{aligned}$$

where the $'$ refers to the derivative with respect to the stock S. Note that, to economize on notation, we also use i and j to respectively refer to country i and j.

The first-order conditions give, for an interior solution:

$$\begin{aligned}e_i &= \bar{e}_i-\frac{1}{\gamma}V^{'}(K,S)\\ e_j &= \bar{e}_j-\frac{1}{\gamma}J^{'}(j,S).\end{aligned} \tag{36}$$

It can be shown that V and G of the following form, $V(K,S)=A_K S+B_K$ and $J(j,S)=D_j S+E_j$, satisfy the above system where the coefficients A_K, B_K, D_j and E_j are found using the undetermined coefficients technique (see e.g., Dockner et al. (2000)). Let $\bar{e}=\sum_{i=1}^{n}\bar{e}_i$; we have:

$$\begin{aligned}\rho(A_K S+B_K) &= \frac{k}{2\gamma}{A_K}^2+\sum_{i\in K}\pi_i S+A_K[\bar{e}-\frac{k}{\gamma}A_K-\frac{1}{\gamma}\sum_{i\in I\setminus K}D_j-\delta S]\\ \rho(D_j S+E_j) &= \frac{1}{2\gamma}{D_j}^2+\pi_j S+D_j[\bar{e}-\frac{k}{\gamma}A_K-\frac{1}{\gamma}\sum_{i\in I\setminus K}D_j-\delta S]\end{aligned}$$

for all $S\geq 0$, which gives

$$\begin{aligned}A_K &= \frac{1}{\rho+\delta}\sum_{i\in K}\pi_i\\ D_j &= \frac{1}{\rho+\delta}\pi_j\end{aligned}$$

and

$$\begin{aligned}
\rho B_K &= A_K\left(\frac{k}{2\gamma}A_K + \bar{e} - \frac{1}{\gamma}\sum_{i\in I\setminus K} D_j\right) \\
\rho E_j &= D_j\left(\frac{1}{2\gamma}D_j + \bar{e} - \frac{k}{\gamma}A_K - \frac{1}{\gamma}\sum_{i\in I\setminus K} D_j\right).
\end{aligned}$$

The value of a coalition K of size k is thus given by

$$V(K,S) = \frac{\sum_{i\in K}\pi_i}{\rho(\rho+\delta)}\left(\bar{e} - \frac{1}{2\gamma}\frac{1}{\rho+\delta}\left(k\sum_{i\in K}\pi_i + 2\sum_{i\in I\setminus K}\pi_j\right)\right) + \rho S]$$

or

$$V(K,S) = \frac{\sum_{i\in K}\pi_i}{\rho(\rho+\delta)}[\bar{e} - \frac{1}{2\gamma(\rho+\delta)}(2\pi + (k-2)\sum_{i\in K}\pi_i)) + \rho S]$$

where $\pi = \sum_{i=1}^{n}\pi_i$.

Appendix B: Proof of Proposition 1

Using the Shapley value formula (13), we obtain

$$\frac{\partial\phi_i(V,S_0)}{\partial\pi_i} = \sum_{K\subset I\setminus\{i\}}[\frac{\partial V(K\cup\{i\})}{\partial\pi_i} - \frac{\partial V(K)}{\partial\pi_i}]\frac{k!(n-k-1)!}{n!}.$$

Using (8) and (7), we have, after algebraic manipulations:

For $i \notin K$:

$$\frac{\partial V(K,S_0)}{\partial\pi_i} = \frac{\sum_{j\in K}\pi_j}{\rho(\rho+\delta)}\left[-\frac{1}{\gamma(\rho+\delta)}\right]$$

and

$$\frac{\partial V(K\cup\{i\},S_0)}{\partial\pi_i} = \frac{1}{\rho(\rho+\delta)}\left[\bar{e} + \rho S + \frac{1}{\gamma(\rho+\delta)}\left(-\pi - k\left(\sum_{j\in K}\pi_j + \pi_i\right)\right)\right]$$

Therefore,

$$\begin{aligned}
\frac{\partial\phi_i(V,S_0)}{\partial\pi_i} &= \frac{1}{\rho(\rho+\delta)}\sum_{K\subset I\setminus\{i\}}\left(\bar{e} + \rho S + \frac{1}{\gamma(\rho+\delta)}\left[-\pi + (-k+1)\sum_{j\in K}\pi_j - k\pi_i\right]\right)\frac{k!(n-k-1)!}{n!} \\
&> \frac{1}{\rho(\rho+\delta)}\left(\bar{e} + \rho S + \frac{-k\pi}{\gamma(\rho+\delta)}\right) > 0
\end{aligned}$$

since: $\bar{e} + \rho S > \frac{n\pi}{\gamma(\rho+\delta)}$ by (11). Q.E.D.

Appendix C: Proof of Proposition 2

Using the Shapley value formula (13), we obtain

$$\begin{aligned}\frac{\partial \phi_m(V,S_0)}{\partial \pi_j} &= \sum_{K\subset I\backslash\{m,j\}}[\frac{\partial V(K\cup\{m\})}{\partial \pi_j}-\frac{\partial V(K)}{\partial \pi_j}]\frac{k!(n-k-1)!}{n!}+ \\ &+\sum_{\substack{K\subset I\backslash\{m\}\\ j\in K}}[\frac{\partial V(K\cup\{m\})}{\partial \pi_j}-\frac{\partial V(K)}{\partial \pi_j}]\frac{k!(n-k-1)!}{n!}.\end{aligned} \tag{37}$$

For $i\in K$, using (8) and (7), we have, after algebraic manipulations:

$$\frac{\partial V(K,S_0)}{\partial \pi_i}=\frac{\bar{e}+\rho S_0}{\rho(\rho+\delta)}-\frac{\pi}{\rho\gamma(\rho+\delta)^2}+\frac{-k+1}{\rho\gamma(\rho+\delta)^2}\sum_{i\in K}\pi_i. \tag{38}$$

For $j\notin K$,

$$\frac{\partial V(K,S_0)}{\partial \pi_j}=\frac{-1}{\rho\gamma(\rho+\delta)^2}\sum_{i\in K}\pi_i. \tag{39}$$

Substituting (38) and (39) into (37) yields

$$\begin{aligned}\frac{\partial \phi_m(V,S_0)}{\partial \pi_j} &= \sum_{\substack{K\subset I\backslash\{m\}\\ j\in K}}[\frac{-k}{\rho\gamma(\rho+\delta)^2}(\sum_{i\in K}\pi_i+\pi_m)-\frac{-k+1}{\rho\gamma(\rho+\delta)^2}\sum_{i\in K}\pi_i]\frac{k!(n-k-1)!}{n!}+ \\ &+\sum_{K\subset I\backslash\{m,j\}}[\frac{-1}{\rho\gamma(\rho+\delta)^2}(\sum_{i\in K}\pi_i+\pi_m)-\frac{-1}{\rho\gamma(\rho+\delta)^2}\sum_{i\in K}\pi_i]\frac{k!(n-k-1)!}{n!} \\ &= \sum_{\substack{K\subset I\backslash\{m\}\\ j\in K}}[\frac{-k}{\rho\gamma(\rho+\delta)^2}\pi_m+\frac{-1}{\rho\gamma(\rho+\delta)^2}\sum_{i\in K}\pi_i]\frac{k!(n-k-1)!}{n!}+ \\ &+\sum_{K\subset I\backslash\{m,j\}}[\frac{-1}{\rho\gamma(\rho+\delta)^2}\pi_m]\frac{k!(n-k-1)!}{n!} \\ &< 0.\end{aligned}$$

Q.E.D.

Appendix D: Proof of Proposition 5

We seek to determine if the Shapley value of a player m can be larger when $\alpha<1$ than when $\alpha=1$:

$$\sum_{K\subset I\backslash\{m\}}([V_\alpha(K\cup\{m\})-V_\alpha(K)]-[V(K\cup\{m\})-V(K)])\frac{k!(n-k-1)!}{n!}>0$$

where V_α is the value function when the marginal damage from pollution is $\alpha\pi_i$ for all $i=1,..,n$.

Substituting π_i by $\alpha\pi_i$ for $i = 1, .., n$ in (14) yields

$$
\begin{aligned}
& [V_\alpha(K \cup \{m\}) - V_\alpha(K)] - [V(K \cup \{m\}) - V(K)] \\
= \; & \frac{(\alpha - 1)\,\pi_m}{\rho(\rho+\delta)}(\bar{e} + \rho S_0) - \frac{\left(\alpha^2 - 1\right)}{2\gamma\rho(\rho+\delta)^2}[(k+1)\pi_m^2 + 2\left(n - 1 + k^2 - k\right)\pi_o\pi_m + k^2{\pi_o}^2] \\
= \; & \frac{(\alpha - 1)}{2\gamma\rho(\rho+\delta)^2}\left(2\gamma(\rho+\delta)(\bar{e} + \rho S_0)\pi_m - (\alpha+1)\left[(k+1)\pi_m^2 + 2\left(n - 1 + k^2 - k\right)\pi_o\pi_m + k^2{\pi_o}^2\right]\right),
\end{aligned}
$$

which is positive if

$$
2\gamma(\rho+\delta)(\bar{e} + \rho S_0)\pi_m - (\alpha+1)\left[(k+1)\pi_m^2 + 2\left(n - 1 + k^2 - k\right)\pi_o\pi_m + k^2{\pi_o}^2\right] > 0. \tag{40}
$$

The left-hand side of (40) is decreasing in k. Therefore if it is positive for $k = n-1$, it's positive for all $k \leq n-1$.

For $k = n-1$, condition (40) gives

$$
2\gamma(\rho+\delta)(\bar{e} + \rho S_0)\pi_m - (\alpha+1)\left[n\pi_m^2 + 2\,(n-1)^2\,\pi_o\pi_m + (n-1)^2\,{\pi_o}^2\right] > 0.
$$

By (11) and the fact that $S_0 \geq 0$, a sufficient condition for (40) to hold is that

$$
2n\,((n-1)\,\pi_o + \pi_m)\,\pi_m - (\alpha+1)\left(n\pi_m^2 + 2\,(n-1)^2\,\pi_o\pi_m + (n-1)^2\,{\pi_o}^2\right) > 0
$$

or

$$
2n\left((n-1)\,r + r^2\right) - (\alpha+1)\left(nr^2 + 2\,(n-1)^2\,r + (n-1)^2\right) > 0.
$$

Therefore, we can infer that

$$
\alpha < \frac{nr^2 + 2r\,(n-1) - (n-1)^2}{nr^2 + 2\,(n-1)^2\,r + (n-1)^2}.
$$

For the right-hand side to be positive, we need

$$
nr^2 + 2r\,(n-1) - (n-1)^2 > 0
$$

or

$$
r > \frac{1}{n}\left(\sqrt{n+1} - 1\right)(n-1).
$$

Q.E.D.

Appendix E: Proof of Proposition 6

In this appendix we provide sufficient conditions to have

$$\sum_{K\subset I\backslash\{m\}}([V(K\cup\{m\})-V(K)]-[V_\alpha(K\cup\{m\})-V_\alpha(K)])\frac{k!(n-k-1)!}{n!}>0 \tag{41}$$

where V_α is the value function corresponding to $\{\alpha\pi_i, i\in I\}$ and V is the value function corresponding to $\{\pi_i, i\in I\}$. From (14), after substituting π_i by $\alpha\pi_i$ for all $i=1,..,n$, we have

$$V_\alpha(K\cup\{m\})-V_\alpha(K)=\frac{\alpha\pi_m}{\rho(\rho+\delta)}(\bar{e}+\rho S_0)-\frac{\alpha^2}{2\gamma\rho(\rho+\delta)^2}[2\pi\pi_m+2(k-1)\pi_m\sum_{i\in K}\pi_i+(k-1)\pi_m^2+(\sum_{i\in K}\pi_i)^2]. \tag{42}$$

We first show that

$$\frac{\partial}{\partial\alpha}\left[V_\alpha(K\cup\{m\})-V_\alpha(K)\right]>0, \tag{43}$$

which implies that $V_\alpha(K\cup\{m\})-V_\alpha(K)$, and consequently, that the Shapley value for player m is increasing in α. In particular, the Shapley value will be highest for $\alpha=1$; thus, we can conclude that the Shapley value for player m when $\alpha<1$ is strictly larger than when $\alpha=1$.

Using (42), condition (43) holds iff

$$\frac{\pi_m}{\rho(\rho+\delta)}(\bar{e}+\rho S_0)-\frac{\alpha}{\gamma\rho(\rho+\delta)^2}[2\pi\pi_m+2(k-1)\pi_m\sum_{i\in K}\pi_i+(k-1)\pi_m^2+(\sum_{i\in K}\pi_i)^2]>0 \tag{44}$$

$$\Leftrightarrow$$

$$\alpha[2\pi\pi_m+2(k-1)\pi_m\sum_{i\in K}\pi_i+(k-1)\pi_m^2+(\sum_{i\in K}\pi_i)^2]<\pi_m\gamma(\rho+\delta)(\bar{e}+\rho S_0) \tag{45}$$

In the case $\pi_k=\pi_o$ for all $k\neq m$, this condition becomes

$$\alpha[2\left(\left(n-1\right)\pi_o+\pi_m\right)\pi_m+2(k-1)\pi_m k\pi_o+(k-1)\pi_m^2+(k\pi_o)^2]<\pi_m\gamma(\rho+\delta)(\bar{e}+\rho S_0). \tag{46}$$

The left-hand side of this inequality is a strictly increasing function k, and therefore, if it holds for $k=n-1$, then it holds for all $k\leq n-1$:

$$\alpha[2\left(\left(n-1\right)\pi_o+\pi_m\right)\pi_m+2(n-2)\left(n-1\right)\pi_m\pi_o+(n-2)\pi_m^2+\left(\left(n-1\right)\pi_o\right)^2]<\pi_m\gamma(\rho+\delta)(\bar{e}+\rho S_0)$$

or

$$\alpha[2\left(n-1\right)^2\pi_o\pi_m+2\pi_m^2+(n-2)\pi_m^2+\left(\left(n-1\right)\pi_o\right)^2]<\pi_m\gamma(\rho+\delta)(\bar{e}+\rho S_0).$$

A sufficient condition is

$$\alpha[2\left(n-1\right)^{2}\pi_{o}\pi_{m}+n\pi_{m}^{2}+(\left(n-1\right)\pi_{o})^{2}]<\pi_{m}\gamma(\rho+\delta)\bar{e}.$$

From (11), it is sufficient that

$$\alpha[2\left(n-1\right)^{2}\pi_{o}\pi_{m}+n\pi_{m}^{2}+(\left(n-1\right)\pi_{o})^{2}]<\pi_{m}n\pi,$$

which, after substitution of $\pi=\left(n-1\right)\pi_{o}+\pi_{m}$, gives

$$h\left(\alpha\right)\equiv\alpha\left(2\left(n-1\right)^{2}r+nr^{2}+\left(n-1\right)^{2}\right)-nr\left(\left(n-1\right)+r\right)<0,$$

where $r=\frac{\pi_m}{\pi_o}$. Thus it is sufficient to have $h\left(\alpha\right)<0$ for (41) to be satisfied. The function h is strictly increasing in α with $h\left(0\right)<0$ and $h\left(1\right)>0$. We can therefore conclude that there exists $\tilde{\alpha}=\frac{nr((n-1)+r)}{2(n-1)^2r+nr^2+(n-1)^2}<1$ such that, for all $\alpha<\tilde{\alpha}$, we have, for all $(\bar{e},S_0)$: $\frac{\partial}{\partial\alpha}\left[V_\alpha(K\cup\{m\})-V_\alpha(K)\right]>0$ for all $K\subset I\backslash\{m\}$ and therefore (41) is satisfied. Q.E.D.

Appendix F: Proof of Proposition 7a

We have

$$\frac{\partial}{\partial\pi_m}\beta_m(t)=\rho\frac{\partial}{\partial\pi_m}\phi_m(V,S^I(t))-\frac{\partial}{\partial\pi_m}\frac{d}{dt}\phi_m(V,S^I(t)).$$

To study the sign of $\frac{\partial}{\partial\pi_m}\beta_m(t)$, we separately compute $\frac{\partial}{\partial\pi_m}\phi_m(V,S^I(t))$ and $\frac{\partial}{\partial\pi_m}\frac{d}{dt}\phi_m(V,S^I(t))$.

Using (13), we get, after simplification,

$$\frac{\partial}{\partial\pi_m}\phi_m(V,S^I\left(t\right))=\frac{\bar{e}}{\rho(\rho+\delta)}+\frac{S^I\left(t\right)}{(\rho+\delta)}+\frac{\pi_m}{(\rho+\delta)}\frac{\partial S^I\left(t\right)}{\partial\pi_m}-\frac{1}{\rho\gamma(\rho+\delta)^2}\Omega$$

where

$$\Omega\equiv\pi+\sum_{K\subset I\backslash\{m\}}\left(\pi_m k+\sum_{i\in K}\pi_i\left(k-1\right)\right)\frac{k!(n-k-1)!}{n!}. \tag{47}$$

When $\pi_j=\pi_o$ for all $j\neq m$, we have

$$\Omega=\pi+\frac{1}{n}\left(\frac{1}{2}n\left(n-1\right)\pi_m+\pi_o\left(\frac{1}{3}n\left(n^2-3n+2\right)\right)\right)$$

Moreover differentiating (12) with respect to π_m gives

$$\frac{\partial}{\partial\pi_m}S^I(t)=\frac{1}{\delta}(-\frac{n}{\gamma}\frac{1}{\rho+\delta})(1-e^{-\delta t}),$$

and therefore,

$$\frac{S^I(t)}{(\rho+\delta)} + \frac{\pi_m}{(\rho+\delta)}\frac{\partial S^I(t)}{\partial \pi_m} = \frac{1}{(\rho+\delta)}\left(S_0 e^{-\delta t} + \frac{1}{\delta}(\bar{e} - \frac{n}{\gamma}\frac{1}{\rho+\delta}(\pi+\pi_m))(1-e^{-\delta t})\right). \tag{48}$$

Replacing (48) into(47) and π by $\pi_m + (n-1)\pi_o$ gives

$$\begin{aligned}\frac{\partial}{\partial \pi_m}\phi_m(V,S) &= \frac{\bar{e}}{\rho(\rho+\delta)} + \frac{1}{(\rho+\delta)}\left(S_0 e^{-\delta t} + \frac{1}{\delta}(\bar{e} - \frac{n}{\gamma}\frac{1}{\rho+\delta}((n-1)\pi_o + 2\pi_m))(1-e^{-\delta t})\right) \\ &\quad - \frac{((n-1)\pi_o + \pi_m)}{\rho\gamma(\rho+\delta)^2}\left(\frac{1}{2}(n+1)\pi_m + \pi_o\frac{1}{3}(n^2-1)\right).\end{aligned} \tag{49}$$

We now compute $\frac{\partial}{\partial \pi_m}\frac{d}{dt}\phi_m(V,S^I(t))$:

$$\begin{aligned}\frac{\partial}{\partial \pi_m}\frac{d}{dt}\phi_m(V,S^I(t)) &= \frac{\partial}{\partial \pi_m}\left(\sum_{K\subset I\setminus\{m\}}[\frac{dV(K\cup\{m\})}{dt} - \frac{dV(K)}{dt}]\frac{k!(n-k-1)!}{n!}\right) \\ &= \frac{\partial}{\partial \pi_m}\left(\sum_{K\subset I\setminus\{m\}}\left[\frac{\pi_m}{\rho(\rho+\delta)}[\rho\frac{d}{dt}S^I(t)]\right]\frac{k!(n-k-1)!}{n!}\right).\end{aligned} \tag{50}$$

Using (12), we compute $\frac{d}{dt}S^I(t)$ and substitute it into (50) to obtain

$$\frac{\partial}{\partial \pi_m}\frac{d}{dt}\phi_m(V,S) = \frac{1}{(\rho+\delta)}\left(-\delta S_0 + \bar{e} - \frac{n}{\gamma}\left(\frac{(n-1)\pi_o + 2\pi_m}{\rho+\delta}\right)\right)e^{-\delta t}. \tag{51}$$

Proof of part i):

By (49) and (51):

$$\begin{aligned}\lim_{t\to 0}\frac{\partial}{\partial \pi_m}\beta_m(t) &= \lim_{t\to 0}\left(\rho\frac{\partial}{\partial \pi_m}\phi_m(V,S^I(t)) - \frac{\partial}{\partial \pi_m}\frac{d}{dt}\phi_m(V,S^I(t))\right) \\ &> \frac{\bar{e}}{(\rho+\delta)} + \frac{\rho}{(\rho+\delta)}S_0 e^{-\delta t} - \frac{(n+3)\pi}{2\gamma(\rho+\delta)^2} - \frac{1}{(\rho+\delta)}\left(-\delta S_0 + \bar{e} - \frac{n}{\gamma}\left(\frac{\pi+\pi_m}{\rho+\delta}\right)\right)\end{aligned} \tag{52}$$

which gives, after substitution of π by $(n-1)\pi_o + \pi_m$ and simplifications:

$$\lim_{t\to 0}\frac{\partial}{\partial \pi_m}\beta_m(t) > S_0 + \frac{((n-3)\pi_o + 3\pi_m)(n-1)}{2\gamma(\rho+\delta)^2}, \tag{53}$$

which is clearly positive for $n \geq 3$. For $n = 2$ from (49) and (51) we have

$$\frac{\partial}{\partial \pi_m}\phi_m(V,S) = \frac{\bar{e}}{\rho(\rho+\delta)} + \frac{S^I(t)}{(\rho+\delta)} + \frac{\pi_m}{(\rho+\delta)}\frac{\partial S(t)}{\partial \pi_m} - \frac{\pi+\pi_m}{\rho\gamma(\rho+\delta)^2}$$

and

$$\lim_{t\to 0}\left(\frac{\partial}{\partial \pi_m}\frac{d}{dt}\phi_m(V,S)\right)=\frac{1}{(\rho+\delta)}\left(-\delta S_0+\bar{e}-\frac{2}{\gamma}\left(\frac{\pi+\pi_m}{\rho+\delta}\right)\right).$$

Therefore, using (48),

$$\begin{aligned}
\lim_{t\to 0}\frac{\partial}{\partial \pi_m}\beta_m(t) &= \lim_{t\to 0}\left(\frac{\bar{e}}{(\rho+\delta)}+\frac{\rho S^I(t)}{(\rho+\delta)}+\frac{\rho\pi_m}{(\rho+\delta)}\frac{\partial S(t)}{\partial \pi_m}-\frac{\pi+\pi_m}{\gamma(\rho+\delta)^2}\right)- \\
&\quad -\lim_{t\to 0}\left(\frac{1}{(\rho+\delta)}\left(\bar{e}-\delta S_0-\frac{2}{\gamma}\left(\frac{\pi+\pi_m}{\rho+\delta}\right)\right)\right) \\
&= S_0+\frac{(\pi+\pi_m)}{\gamma(\rho+\delta)^2}>0. \qquad (54)
\end{aligned}$$

Proof of part ii):

Differentiating (13) with respect to π_j yields

$$\begin{aligned}
\frac{\partial}{\partial \pi_j}\phi_m(V,S) &= \sum_{\substack{K\subset I\backslash\{m\}\\ j\in K}}\left[\frac{-k}{\rho\gamma(\rho+\delta)^2}\pi_m+\frac{-1}{\rho\gamma(\rho+\delta)^2}\sum_{i\in K}\pi_i\right]\frac{k!(n-k-1)!}{n!}+ \qquad (55)\\
&\quad +\sum_{K\subset I\backslash\{m,j\}}\left[\frac{-1}{\rho\gamma(\rho+\delta)^2}\pi_m\right]\frac{k!(n-k-1)!}{n!}+ \\
&\quad +\frac{-n\pi_m}{\delta\gamma\left(\rho+\delta\right)^2}(1-e^{-\delta t})
\end{aligned}$$

and

$$\begin{aligned}
\frac{d}{dt}\phi_m(V,S) &= \frac{\pi_m}{(\rho+\delta)}\left(-\delta S_0+\bar{e}-\frac{n}{\gamma}\frac{\pi}{\rho+\delta}\right)e^{-\delta t} \qquad (56)\\
\frac{\partial}{\partial \pi_j}\left(\frac{d}{dt}\phi_m(V,S^I(t))\right) &= -\frac{n\pi_m}{\gamma(\rho+\delta)^2}e^{-\delta t}.
\end{aligned}$$

Therefore, we can obtain the following:

$$\begin{aligned}
\lim_{t\to 0}\frac{\partial}{\partial\pi_j}\beta_m(t) &= \lim_{t\to 0}\left(\rho\frac{\partial}{\partial\pi_j}\phi_m(V,S^I(t))-\frac{\partial}{\partial\pi_j}\frac{d}{dt}\phi_m(V,S^I(t))\right)\\
&= \sum_{\substack{K\subset I\setminus\{m\}\\ j\in K}}\left[\frac{-k}{\gamma(\rho+\delta)^2}\pi_m+\frac{-k}{\gamma(\rho+\delta)^2}\pi_o\right]\frac{k!(n-k-1)!}{n!}+\\
&\quad+\sum_{K\subset I\setminus\{m,j\}}\left[\frac{-1}{\gamma(\rho+\delta)^2}\pi_m\right]\frac{k!(n-k-1)!}{n!}+\frac{n\pi_m}{\gamma(\rho+\delta)^2}\\
&= \frac{(n-2)\,(2n-3)}{6n}\left(\frac{\pi_m+\pi_o}{\gamma(\rho+\delta)^2}\right)-\frac{1}{\gamma(\rho+\delta)^2}\pi_m\left(\frac{1}{2}\right)+\frac{n\pi_m}{\gamma(\rho+\delta)^2}\\
&= \left(\frac{1}{6n}\left(\left(8n^2-10n+6\right)\pi_m+\left(2n^2-7n+6\right)\pi_o\right)\right)\frac{1}{\gamma(\rho+\delta)^2}
\end{aligned}$$

which is always positive for $n\geq 2$. Therefore $\lim_{t\to 0}\frac{\partial}{\partial\pi_j}\beta_m(t)>0$. Q.E.D.

Appendix G: Proof of Proposition 7b

Proof of part i):

When $t\to\infty$ by (51) $\lim_{t\to\infty}\left(\frac{\partial}{\partial\pi_m}\frac{d}{dt}\phi_m(V,S^I(t))\right)=0$, so using (49), we get

$$\begin{aligned}
\frac{\partial}{\partial\pi_m}\phi_m(V,S) &= \frac{\bar{e}}{\rho(\rho+\delta)}+\frac{1}{(\rho+\delta)}\left(S_0e^{-\delta t}+\frac{1}{\delta}(\bar{e}-\frac{n}{\gamma}\frac{1}{\rho+\delta}\left((n-1)\,\pi_o+2\pi_m\right))(1-e^{-\delta t})\right)-\\
&\quad-\frac{((n-1)\,\pi_o+\pi_m)}{\rho\gamma(\rho+\delta)^2}\left(\frac{1}{2}\,(n+1)\,\pi_m+\pi_o\frac{1}{3}\left(n^2-1\right)\right) \qquad (57)
\end{aligned}$$

$$\begin{aligned}
\lim_{t\to\infty}\frac{\partial}{\partial\pi_m}\beta_m(t) &= \lim_{t\to\infty}\left(\rho\frac{\partial}{\partial\pi_m}\phi_m(V,S^I(t))-\frac{\partial}{\partial\pi_m}\frac{d}{dt}\phi_m(V,S^I(t))\right)\\
&= \frac{1}{\delta}\bar{e}-\frac{1}{\delta\gamma(\rho+\delta)^2}\left(\rho n\left((n-1)\,\pi_o+2\pi_m\right)-\delta\,(n+1)\,(\pi_m+(n-1)\,\pi_o)\left(\frac{1}{2}\pi_m+\pi_o\frac{1}{3}\,(n+1)\right)\right).
\end{aligned}$$

Now, at $\pi_o=0$, we have

$$\lim_{t\to\infty}\frac{\partial}{\partial\pi_m}\beta_m(t)\bigg|_{\pi_o=0}=\frac{1}{\delta}\bar{e}+\frac{1}{\delta\gamma(\rho+\delta)^2}\left(-2n\rho\pi_m+\frac{1}{2}\,(n+1)\,\delta\pi_m^2\right)$$

therefore

$$\lim_{t\to\infty}\frac{\partial}{\partial\pi_m}\beta_m(t)\bigg|_{\pi_o=0}<0$$

if and only if

$$\bar{e}<-\frac{1}{\gamma(\rho+\delta)^2}\left(-2n\rho\pi_m+\frac{1}{2}\,(n+1)\,\delta\pi_m^2\right).$$

Using (11), the condition for this to hold is that

$$\frac{1}{\delta}\frac{\pi_m}{\gamma(\rho+\delta)}+\frac{1}{\delta\gamma(\rho+\delta)^2}\left(-2n\rho\pi_m+\frac{1}{2}\left(n+1\right)\delta\pi_m^2\right)<0 \tag{58}$$

or

$$\frac{1}{2\gamma\left(\delta+\rho\right)^2}\left(n+1\right)\pi_m^2+\left(\frac{1}{\gamma\delta\left(\delta+\rho\right)}-2\frac{n}{\gamma\delta}\frac{\rho}{\left(\delta+\rho\right)^2}\right)\pi_m<0 \tag{59}$$

Therefore if $\left(2n-1\right)\rho-\delta>0$ and $\pi_m<\bar{\pi}_m\equiv 2\frac{(2n-1)\rho-\delta}{(n+1)\delta}$, then

$$\frac{1}{\delta}\frac{\pi_m}{\gamma(\rho+\delta)}+\frac{1}{\delta\gamma(\rho+\delta)^2}\left(-2n\rho\pi_m+\frac{1}{2}\left(n+1\right)\delta\pi_m^2\right)<0$$

and for $\bar{e}$, such that

$$\frac{1}{\delta}\frac{\pi_m}{\gamma(\rho+\delta)}\leq\frac{1}{\delta}\bar{e}<\frac{1}{\delta\gamma(\rho+\delta)^2}\left(2n\rho\pi_m-\frac{1}{2}\left(n+1\right)\delta\pi_m^2\right), \tag{60}$$

we have $\lim_{t\to\infty}\frac{\partial}{\partial\pi_m}\beta_m(t)=\frac{1}{\delta}\bar{e}-\frac{1}{\delta\gamma(\rho+\delta)^2}\left(2n\rho\pi_m-\frac{1}{2}\left(n+1\right)\delta\pi_m^2\right)<0$. Condition (60), after simple manipulations, can be rewritten as (24).

Proof of part ii)

Using (55) and (56), we have

$$\begin{aligned}
\lim_{t\to\infty}\frac{\partial}{\partial\pi_j}\beta_m(t) &= \lim_{t\to\infty}\left(\rho\frac{\partial}{\partial\pi_j}\phi_m(V,S^I(t))-\frac{\partial}{\partial\pi_j}\frac{d}{dt}\phi_m(V,S^I(t))\right)\\
&= \sum_{\substack{K\subset I\setminus\{m\}\\ j\in K}}\left[\frac{-k}{\gamma(\rho+\delta)^2}\pi_m+\frac{-k}{\gamma(\rho+\delta)^2}\pi_o\right]\frac{k!(n-k-1)!}{n!}+\\
&\quad+\sum_{K\subset I\setminus\{m,j\}}\left[\frac{-1}{\gamma(\rho+\delta)^2}\pi_m\right]\frac{k!(n-k-1)!}{n!}+\frac{-\rho n\pi_m}{\delta\gamma\left(\rho+\delta\right)}\\
&< 0.
\end{aligned}$$

Q.E.D.

Appendix H—Derivation of the value functions for quadratic damage costs

Let $W(I,S)$ be the value function for the grand coalition. The HJB equation associated with the grand coalition reads:

$$\rho W(I,S) = \min\{\sum_{i=1}^{3}[\frac{1}{2}(e_i - \bar{e}_i)^2] + \frac{1}{2}\eta_1 S^2 + W^{'}(I,S)(\sum_{i=1}^{3} e_i - \delta S)\}$$

where the $'$ refers to the derivative with respect to the stock S.

The first order conditions give for an interior solution:

$$e_i = \bar{e}_i - W'(I,S).$$

A quadratic value function, $W(I,S) = Z_I S^2 + A_I S + B_I$ satisfies the above equation and the coefficients $Z_I,\ A_I,\ B_I$ are found using undetermined coefficient technique. Let $\bar{e} = \sum_{i\in I}\bar{e}_i$, we have:

$$\rho(Z_I S^2 + A_I S + B_I) = \sum_{i=1}^{3}[\frac{1}{2}(2Z_I S + A_I)^2] + \frac{1}{2}\eta_1 S^2 + (2Z_I S + A_I)(\bar{e} - \sum_{j=1}^{3}(2Z_I S + A_I) - \delta S).$$

Equating the coefficients on both sides of the equality provides two sets of solutions:

$$\begin{aligned}
Z_I &= \frac{1}{12}\left(-(2\delta+\rho) \pm \sqrt{(12\eta_1 + 4\delta^2 + \rho^2 + 4\delta\rho)}\right) \\
A_I &= 2\bar{e}\frac{Z_I}{\delta + \rho + 6Z_I} \\
B_I &= \frac{1}{2\rho}A_I(2\bar{e} - 3A_I)
\end{aligned}$$

Evolution of stock of pollution under full cooperation is given by:

$$S^C(t) = \frac{\bar{e} - 3A_I}{6Z_I + \delta} + \left(S_0 - \frac{\bar{e} - 3A_I}{6Z_I + \delta}\right)e^{-(6Z_I+\delta)t}$$

where we should have:

$$6Z_I + \delta \geq 0.$$

Therefore, the coefficient of the value function for the grand coalition are:

$$
\begin{aligned}
Z_I &= \frac{1}{12}\left(-(2\delta+\rho)+\sqrt{12\eta_1+(2\delta+\rho)^2}\right)\\
A_I &= 2\bar{e}\frac{Z_I}{\delta+\rho+6Z_I}\\
B_I &= \frac{1}{2\rho}A_I(2\bar{e}-3A_I)
\end{aligned}
$$

which can be written as:

$$
Z_I = \frac{1}{12}f(12),\ \ A_I = g(12,6),\ \ B_I = u(12,6,3),
$$

where

$$
\begin{aligned}
f(x) &= \sqrt{(2\delta+\rho)^2+\eta_1 x}-(2\delta+\rho)\\
g(x,q) &= \frac{2\bar{e}\ f(x)}{x(\delta+\rho)+q\ f(x)}\\
u(x,q,\kappa) &= \frac{1}{2\rho}g(x,q)(2\bar{e}-\kappa g(x,q)).
\end{aligned}
$$

Let $W(\{1\},S)$ be the value function for player i corresponding to the Nash equilibrium. The HJB equation for player 1 reads:

$$
\rho W(\{1\},S) = \min\{\frac{1}{2}(e_1-\bar{e}_1)^2+\frac{1}{2}\eta_1 S^2+W'(\{1\},S)(\sum_{j=1}^{3}e_j-\delta S)\}
$$

In the case where players 2 and 3 have no damage cost, their emission strategy is constant and equal to $\bar{e}_i$. Therefore, their value function is zero.

In the same manner as computing the value function for the grand coalition, we can find the coefficient of player 1's value function for the Nash equilibrium $W(\{1\},S) = Z_1 S^2 + A_1 S + B_1$:

$$
\begin{aligned}
Z_1 &= \frac{1}{4}(-(2\delta+\rho)+\sqrt{(2\delta+\rho)^2+4\eta_1}) = \frac{1}{4}f(4)\\
A_1 &= 2\bar{e}\frac{Z_1}{\delta+\rho+2Z_1} = g(4,2)\\
B_1 &= \frac{1}{2\rho}A_1(2\bar{e}-A_1) = u(4,2,1).
\end{aligned}
$$

Computing the value function for coalition of countries 1 and 2 :

Denote this coalition by $W(K,S) = W(\{1,2\}, S)$, and the value function for player 3 by $G(j,S)$. HJB equation for this coalition reads:

$$\rho W(K,S) = \sum_{i\in\{1,2\}} [\frac{1}{2}(W'(K,S))^2] + \frac{1}{2}\eta_1 S^2 + W'(K,S)[\sum_{i\in\{1,2\}}(\bar{e}_i - W'(K,S)) + \sum_{i=3}(\bar{e}_i - G^{'}(i,S)) - \delta S] \tag{61}$$

Emission strategy for player 3 is to choose $e = \bar{e}_3$, so $G = 0$. A quadratic value function, $W(\{1,2\}, S) = Z_{12}S^2 + A_{12}S + B_{12}$ satisfies equality (61). The coefficients of this value function are as follows:

$$\begin{aligned}
Z_{12} &= Z_{13} = \frac{1}{8}(-(2\delta+\rho) + \sqrt{(2\delta+\rho)^2 + 8\eta_1}) \\
A_{12} &= A_{13} = 2\bar{e}\frac{Z_{12}}{\delta+\rho+4Z_{12}} \\
B_{12} &= B_{13} = \frac{1}{\rho}A_{12}(\bar{e} - A_{12}) \\
Z_{23} &= A_{23} = B_{23} = 0
\end{aligned}$$

Finally, the Shapley values can be obtained as follows:

$$\begin{aligned}
\phi_1 &= \frac{1}{3}[(Z_I + Z_{12} + Z_1)S^2 + (A_I + A_{12} + A_1)S + (B_I + B_{12} + B_1)] \\
\phi_2 &= \phi_3 = \frac{1}{2}Z_I S^2 - \frac{1}{6}(Z_I + Z_{12} + Z_1)S^2 + \frac{1}{2}A_I S - \frac{1}{6}(A_I + A_{12} + A_1)S) + \frac{1}{2}B_I - \frac{1}{6}(B_I + B_{12} + B_1)) \\
&= \frac{1}{6}(2Z_I - Z_{12} - Z_1)S^2 + \frac{1}{6}(2A_I - A_{12} - A_1)S + \frac{1}{6}(2B_I - B_{12} - B_1)
\end{aligned}$$

Appendix I—Proof of Proposition 9

Denote by ϕ'_{i,η_1} the derivative of ϕ_i $(i = 1,2,3)$ with respect to η_1.

$$\phi'_{2,\eta_1} = \frac{1}{6}(2Z'_I - Z'_{12} - Z'_1)S^2 + \frac{1}{6}(2A'_I - A'_{12} - A'_1)S + \frac{1}{6}(2B'_I - B'_{12} - B'_1)$$

Here $'$ refers to the derivative with respect to η_1.

Let

$$\begin{aligned}
ZZ &= 2Z'_I - Z'_{12} - Z'_1 \\
AA &= 2A'_I - A'_{12} - A'_1 \\
BB &= 2B'_I - B'_{12} - B'_1
\end{aligned}$$

Below we show $ZZ,\ AA,\ BB$ are negative.

Using (29) and (30) we can present $ZZ,\ AA,\ BB$ in the following forms:

$$\begin{aligned} ZZ &= 2h(12) - h(8) - h(4) \\ AA &= 2w(12) - w(8) - w(4) \\ BB &= \bar{e}\left(2w(12) - w(8) - w(4)\right) + 2m(12) - m(8) - m(4) \end{aligned}$$

where $h(x) = f'_x(x)$, and $w(x)$ and $m(x)$ are given by:

$$\begin{aligned} w(x) &= \frac{8\bar{e}(\delta+\rho)}{\left(2(\delta+\rho)+f(x)\right)^2} h(x) \\ m(x) &= \frac{-\bar{e}\ f(x)}{2(\delta+\rho)+f(x)} w(x) \end{aligned}$$

$h(x)$, $w(x)$ and $m(x)$ are decreasing functions of x. Therefore ZZ, AA and BB are negative, and the Shapley values for players 2 and 3 are decreasing in η_1. Q.E.D.

Appendix J—Proof of Proposition 10

Proof of part i):

According to (15), it is enough to show that $\left.\frac{\partial}{\partial\eta_1}\phi_1(t)\right|_{\eta_1=0}$ is positive and $\left.\frac{\partial}{\partial\eta_1}\frac{\partial}{\partial t}\phi_1(t)\right|_{\eta_1=0}$ is negative. First, note that from (35), we can infer that

$$\frac{\partial}{\partial\eta_1}S(t) = \frac{-3}{\delta}\frac{\partial A_I}{\partial\eta_1} + \frac{3}{\delta}\frac{\partial A_I}{\partial\eta_1}e^{-(6Z_I+\delta)t} - 6t\frac{\partial Z_I}{\partial\eta_1}[S_0 - \frac{1}{\delta}(\bar{e}-3A_I)]e^{-(6Z_I+\delta)t} \leq 0$$

In particular at zero:

$$\left.\frac{\partial}{\partial\eta_1}S(t)\right|_{\eta_1=0} = 0 \tag{62}$$

Now:

$$\begin{aligned} \frac{\partial}{\partial\eta_1}\phi_1(t) &= \frac{1}{3}\left(\frac{\partial}{\partial\eta_1}(Z_I+Z_{12}+Z_1)S^2(t) + \frac{\partial}{\partial\eta_1}(A_I+A_{12}+A_1)S(t) + \frac{\partial}{\partial\eta_1}(B_I+B_{12}+B_1)\right) \\ &\quad + \frac{1}{3}\left(2(Z_I+Z_{12}+Z_1)S + (A_I+A_{12}+A_1)\right)\frac{\partial}{\partial\eta_1}S(t) \end{aligned}$$

Derivatives of (29) and (30) with respect to η_1 are positive. This together with (62) implies that $\left.\frac{\partial}{\partial\eta_1}\phi_1(t)\right|_{\eta_1=0}$ is positive.

On the other hand, around $t=0$, we can infer that

$$
\begin{aligned}
\left.\frac{\partial}{\partial \eta_1}\frac{\partial}{\partial t}\phi_1(t)\right|_{\eta_1=0} &= \frac{1}{3}\frac{\partial}{\partial \eta_1}\left.\left((Z_I+Z_{12}+Z_1)\frac{\partial}{\partial t}S^2(t)\right)\right|_{\eta_1=0}+\frac{1}{3}\frac{\partial}{\partial \eta_1}\left.\left((A_I+A_{12}+A_1)\frac{\partial}{\partial t}S(t)\right)\right|_{\eta_1=0} \\
&= \frac{1}{3}\frac{\partial}{\partial \eta_1}\left((Z_I+Z_{12}+Z_1)\left(-2(6Z_I+\delta)\left(S_0-\frac{1}{\delta}(\bar{e}-3A_I)\right)S_0\right)\right)+ \\
&\quad +\frac{1}{3}\frac{\partial}{\partial \eta_1}\left((A_I+A_{12}+A_1)\left(-(6Z_I+\delta)\left(S_0-\frac{1}{\delta}(\bar{e}-3A_I)\right)\right)\right)
\end{aligned}
$$

Therefore, by using (28) and (32), we can obtain that $\frac{\partial}{\partial \eta_1}\frac{\partial}{\partial t}\phi_1(0)$ is negative.

Proof of part ii):

From (34), we can infer that

$$
\begin{aligned}
\left.\frac{\partial}{\partial \eta_1}\phi_2(t)\right|_{\eta_1=0} &= \frac{1}{6}\left(\frac{\partial}{\partial \eta_1}(2Z_I-Z_{12}-Z_1)S^2(t)+\frac{\partial}{\partial \eta_1}(2A_I-A_{12}-A_1)S(t)+\frac{\partial}{\partial \eta_1}(2B_I-B_{12}-B_1)\right) \\
&\quad +\frac{1}{6}\left(2(2Z_I-Z_{12}-Z_1)S(t)+(2A_I-A_{12}-A_1)\right)\frac{\partial}{\partial \eta_1}S(t)
\end{aligned}
$$

by the proof of Proposition 9, Appendix I, $\frac{\partial}{\partial \eta_1}(2Z_I-Z_{12}-Z_1)$, $\frac{\partial}{\partial \eta_1}(2A_I-A_{12}-A_1)$, and $\frac{\partial}{\partial \eta_1}(2B_I-B_{12}-B_1)$ are negative. Implementing (62), we can deduce that $\left.\frac{\partial}{\partial \eta_1}\phi_2(t)\right|_{\eta_1=0}<0$.

It remains to show $\left.\frac{\partial}{\partial \eta_1}\frac{\partial}{\partial t}\phi_2(t)\right|_{\eta_1=0}>0$.

$$
\begin{aligned}
\left.\frac{\partial}{\partial \eta_1}\frac{\partial}{\partial t}\phi_2(t)\right|_{\eta_1=0}. &= \frac{1}{6}\frac{\partial}{\partial \eta_1}\left((2Z_I-Z_{12}-Z_1)\frac{\partial}{\partial t}S^2(t)\right)+\frac{1}{6}\frac{\partial}{\partial \eta_1}\left((2A_I-A_{12}-A_1)\frac{\partial}{\partial t}S(t)\right) \\
&= \frac{1}{6}\frac{\partial}{\partial \eta_1}\left((2Z_I-Z_{12}-Z_1)\left(-2(6Z_I+\delta)\left(S_0-\frac{1}{\delta}(\bar{e}-3A_I)\right)S_0\right)\right)+ \\
&\quad +\frac{1}{6}\frac{\partial}{\partial \eta_1}\left((2A_I-A_{12}-A_1)\left(-(6Z_I+\delta)\left(S_0-\frac{1}{\delta}(\bar{e}-3A_I)\right)\right)\right)
\end{aligned}
$$

After some simple manipulations, we can obtain that $\left.\frac{\partial}{\partial \eta_1}\frac{\partial}{\partial t}\phi_2(t)\right|_{\eta_1=0}>0$. Therefore $\beta_2(t)=\beta_3(t)$ is decreasing in η_1 around zero. Q.E.D.

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