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On nonrenewable resource oligopolies: the *asymmetric* case.*

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Abstract

We give a full characterization of the open-loop Nash equilibrium of a nonrenewable resource *asymmetric* game. We show that (i) there *almost* always exists a phase where both supply simultaneously positive quantities, (ii) when the high cost mine is exploited by a number of firms that goes to infinity the equilibrium approaches the cartel-versus-fringe equilibrium with the fringe firms acting as price takers, (iii) the cheaper resource may not be exhausted first.

This last result has an interesting implication: more competition in the industry may be detrimental to social welfare. Increasing the number of high cost firms may be welfare reducing. This is because a larger number of high cost firms may result in an inefficient order of exhaustion of the resources: the cheaper resource being exhausted first.

Key words: nonrenewable resources, Nash equilibrium, cartel-versus-fringe, open-loop.

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1 Introduction

Although there exists an established and very large literature addressing the problem of exploitation of a nonrenewable resource such as oil, there is still no complete treatment of the case where several agents interact at the market level, each owning a stock of the resource. Probably the reason is the technical complexity of treating such a problem in a general formulation. To make progress, this question has been addressed under different assumptions. The assumptions typically specify the type of competition prevailing, the type of information and commitment possibilities, the market conditions and firms' characteristics. In this paper we characterize the extraction equilibrium for a larger set of cases than has been treated in the literature so far (a more specific description follows).

Interest in nonrenewable resource has been at least partly driven by the importance of the oil market worldwide. Salant (1976) and later Ulph and Folie (1980) model the oil market as a market with one coherent cartel and a fringe, consisting of many identical small oil suppliers acting as price takers. Lewis and Schmalensee (1980) and Loury (1986) justify the assumption of price taking behavior of the fringe and show that when the number of fringe members becomes arbitrarily large, the equilibrium converges to the equilibrium obtained under price taking behavior assumption. In these papers the cartel and the fringe act simultaneously. Gilbert (1978), Newbery (1980), Ulph (1980), and Groot, Withagen and de Zeeuw (1992, 2003) consider the case where the cartel has a first mover advantage and the fringe members are followers.

In this literature firms are assumed to choose a production path and therefore are able to commit to a production path at the initial time, they choose open-loop strategies. The alternative modelling choice is to allow the firms to adjust their extraction rates at each moment to the level of the stock at that moment and to time, firms choose closed-loop strategies. Commitment can be achievable for example in the presence of a perfect futures market for the resource. In other circumstances information about the resource stocks of competitors may not be available. In that case modelling each firm's problem as a choice of an open-loop extraction strategy that maximizes its discounted profits is a reasonable assumption[1].

[1] The drawback is that if firms have information about all stocks at future dates and have the flexibility to adjust their production, the equilibrium obtained with open-loop strategies may not

In the present paper we use a differential game framework[2] (see Dockner et al., (2000)) and restrict attention to the open-loop Nash equilibrium. The relationship between open-loop Nash equilibrium and feedback Nash equilibrium is studied in Benchekroun and Withagen (2008). In particular they show that in case of a finite number of players the two equilibria do not coincide, but that they do in the case of a price taking fringe.

The objective of this paper is threefold. We first characterize the open-loop Nash equilibrium in the case where there is an arbitrary number of firms in two groups where firms can differ across groups, in both their resource stocks and their constant marginal extraction costs. We thereby extend the analysis by Loury (1986), where the players are identical with regard to extraction costs. We also extend the work by Lewis and Schmalensee (1980) by including more than one supplier in each cost category. Secondly, we derive the open-loop Nash equilibrium for the cartel-versus-fringe model from the oligopoly model by considering the case of one coherent cartel and the number of fringe members going to infinity. Thirdly, we address the order of exploitation of the resources and its welfare implications. It is shown that the Nash equilibrium might entail situations where the well known Herfindahl rule is violated, in particular, the low cost mines are still being exploited after exhaustion of the more expensive mines. We provide a full account of the phenomenon which was first illustrated in an example in Lewis and Schmalensee (1980). Moreover, we highlight a surprising implication of the inefficiency of the order of use of the resources: we show that an increase in the number of high cost mines might lead to a deterioration of social welfare. This seems paradoxical because one would conjecture that increasing the number of players will bring the economy closer to perfect competition, which, in the case at hand yields the social optimum. However, here, increasing the number of high cost mines may cause a reversal of the order of the exhaustion of stocks: the

be subgame perfect. It is however time-consistent: if all players follow their equilibrium strategies then at no instant of time will a given firm want to revise its production plan. We would like to point out that the predominance of the analysis of open-loop strategies in the literature is not purely motivated by realism. One of the reasons given by Lewis and Schmalensee, who concentrate on the open loop Nash equilibrium, and not on the feedback equilibrium, is that, "The latter concept seems intractable in this model" as they note (p 477). A few exceptions are Eswaran and Lewis (1985), Polasky (1990) and Salo and Tahvonen (2001).

[2] Dockner, Jorgensen, Long and Sorger (2000) covers differential games theory and applications, in particluar to natural resource economics. Jorgensen and Zaccour (2004) contains a tutorial on differential games.

high cost stock is exhausted before the low cost stock. This inefficiency that results from more competition due to an additional high cost firm can outweigh the benefit from extra competition.

In section 2 we present the model and the equilibrium concepts. In section 3 we fully characterize the open-loop Nash equilibrium. Section 4 treats the order of exploitation and the welfare consequences. Section 5 concludes.

2 The model

There are $n(>1)$ firms and two types of mines l and h, distinguished by their marginal extraction costs. Marginal extraction costs are constant: k^l and k^h for the low cost and high cost mines, respectively, with $k^l < k^h$. Each firm exploits one and only one mine. There are n^l firms that exploit the mines of type l and n^h firms that exploit the mines of type h with $n^l + n^h = n$. Firm i of type j $(i = 1, 2, ..., n^j, j = l, h)$ is endowed with an initial stock S^j_{0i}. Extraction rates at time $t \geq 0$ are denoted by $q^j_i(t)$ and are non-negative. Define $q^j(t) = \sum_{i=1}^{n^j} q^j_i(t)$ and $S^j_0 = \sum_{i=1}^{n^j} S^j_{0i}$ for $j = l, h$ as aggregate supply and initial stocks of the firm types. Demand for the resource is stationary and linear with a choke price $\bar{p} : p(t) = \bar{p} - x(t)$, where $p(t)$ is the price[3] at time t, $x(t)$ is the quantity demanded at time t and $\bar{p} > k^h$. We work in continuous time, which starts at time 0. In an equilibrium at each moment $t \geq 0$ the price of the resource is given by $p(t) = \bar{p} - q^l(t) - q^h(t)$. All agents within a group are assumed identical with regard to stocks: $S^j_{0i} = S^j_0/n^j$ for $j = l, h$. An extraction path q^j_i $(i = 1, 2, ..., n^j, j = l, h)$ is said to satisfy the resource constraint if

$$\int_0^\infty q^j_i(s)ds \leq S^j_{i0} \tag{1}$$

[3] More precisely we have $p(t) = Max\{\bar{p} - x(t), 0\}$.Throughout the paper we will focus on cases where the outcome is such that $\bar{p} - x(t) > 0$ for all t. This will be true for example if ceteris paribus, the choke price $\bar{p}$ is large enough.

Firms are oligopolists in the resource market and the objective of each firm i is to maximize the discounted sum of its profits

$$\int_{0}^{\infty} e^{-rs}[\bar{p} - q^l(s) - q^h(s) - k^j]q_i^j(s)ds \tag{2}$$

subject to its resource constraint.

Definition: Open-loop Nash Cournot equilibrium (OLNE)

A vector of functions $q \equiv (q_1^l, .., q_{n^l}^l, q_1^h, .., q_{n^h}^h)$ with $q(t) \geq 0$ for all $t \geq 0$ is an open-loop Nash-Cournot equilibrium (OLNE) if

i. every extraction path satisfies the corresponding resource constraint

ii. for all $i = 1, 2, ..., n^l$

$$\int_{0}^{\infty} e^{-rs}[\bar{p} - q^l(s) - q^h(s) - k^l]q_i^l(s)ds \geq \int_{0}^{\infty} e^{-rs}[\bar{p} - \sum_{j \neq i} q_j^l(s) - \hat{q}_i^l(s) - q^h(s) - k^l]\hat{q}_i^l(s)ds$$

for all $\hat{q}_i^l$ satisfying the resource constraint

iii. for all $i = 1, 2, ..., n^h$

$$\int_{0}^{\infty} e^{-rs}[\bar{p} - q^h(s) - q^l(s) - k^h]q_i^h(s)ds \geq \int_{0}^{\infty} e^{-rs}[\bar{p} - \sum_{j \neq i} q_j^h(s) - \hat{q}_i^h(s) - q^l(s) - k^h]\hat{q}_i^h(s)ds$$

for all $\hat{q}_i^h$ satisfying the resource constraint.

3 Open-loop Nash equilibrium with a finite number of players

Each firm i takes the supply paths of its $n - 1$ competitors as given and maximizes (2) subject to (1). The Hamiltonian associated with the problem of firm i therefore reads

$$H_i^j(q_i^j, \lambda_i^j, t) = e^{-rt}\left(\bar{p} - q^l - q^h - k^j\right) q_i^j + \lambda_i^j \left(-q_i^j\right) \tag{3}$$

Among the necessary conditions we have that the co-state variable λ_i^j is constant because the Hamiltonian does not contain the resource stock. In addition the Hamiltonian is maximized with respect to the firm's own extraction rate, at each moment in time. Writing these necessary conditions for a solution to each of the n individual firms and using symmetry among the players within a group, i.e. $q_i^j = q^j/n^j$ and $\lambda_i^j = \lambda^j$ for $j = l, h$ and all $i = 1, .., n^j$, gives the following.

Along intervals of time $[t_1, t_2]$ with $t_2 > t_1 \geq 0$ where, for all $t \in [t_1, t_2]$, $q_i^l(t) > 0$ for all $i = 1, .., n^l$ and $q_i^h(t) = 0$ for all $i = 1, .., n^h$, we have

$$e^{-rt}\left(\bar{p} - q^l(t) - \frac{1}{n^l}q^l(t) - k^l\right) = \lambda^l \tag{4}$$

$$p(t) = \bar{p} - q^l(t) \leq k^h + e^{rt}\lambda^h \tag{5}$$

$$p(t) = \frac{1}{n^l + 1}\left(\bar{p} + n^l\left(k^l + \lambda^l e^{rt}\right)\right) \tag{6}$$

The first condition follows from the maximization of the Hamiltonian of player i from category l assuming an interior solution. The second condition is necessary in order for players from category h not to supply. The third condition gives the resulting market price.

Along intervals of time $[t_1, t_2]$ with $t_2 > t_1 \geq 0$ where, for all $t \in [t_1, t_2]$, $q_i^l(t) = 0$ for all i and $q_i^h(t) > 0$ for all i, we have

$$e^{-rt}\left(\bar{p} - q^h(t) - \frac{1}{n^h}q^h(t) - k^h\right) = \lambda^h \tag{7}$$

$$p(t) = \bar{p} - q^h(t) \leq k^l + e^{rt}\lambda^l \tag{8}$$

$$p(t) = \frac{1}{n^h + 1}\left(\bar{p} + n^h\left(k^h + \lambda^h e^{rt}\right)\right). \tag{9}$$

Along intervals of time $[t_1, t_2]$ with $t_2 > t_1 \geq 0$ where, for all $t \in [t_1, t_2]$, $q_i^l(t) > 0$ for all i and $q_i^h(t) > 0$ for all i, we have

$$e^{-rt}\left(\bar{p} - q^l(t) - q^h(t) - \frac{1}{n^l}q^l(t) - k^l\right) = \lambda^l \tag{10}$$

$$e^{-rt}\left(\bar{p}-q^h(t)-q^l(t)-\frac{1}{n^h}q^h(t)-k^h\right)=\lambda^h \tag{11}$$

For $q^l(t)$, $q^h\,(t)$ and $p(t)$ we then obtain

$$\frac{n^l+n^h+1}{n^l}q^l(t)=\bar{p}+n^h\left(k^h+\lambda^h e^{rt}\right)-\left(n^h+1\right)\left(k^l+\lambda^l e^{rt}\right) \tag{12}$$

$$\frac{n^l+n^h+1}{n^h}q^h(t)=\bar{p}+n^l\left(k^l+\lambda^l e^{rt}\right)-\left(n^l+1\right)\left(k^h+\lambda^h e^{rt}\right) \tag{13}$$

$$p(t)=\frac{1}{n^l+n^h+1}\left(\bar{p}+n^l\left(k^l+\lambda^l e^{rt}\right)+n^h\left(k^h+\lambda^h e^{rt}\right)\right) \tag{14}$$

By $S,\ C^l$ and C^h we denote intervals of time with simultaneous supply, sole supply by the $l-$type mines and sole supply by the $h-$type mines, respectively.

At points of transition from one regime to another the price trajectory is continuous, a property that will be exploited below. This property is due to the fact that along an optimum the Hamiltonian is continuous. Continuity of the price path at the different possible transitions gives:

- a transition at t from S to C^l or vice versa requires

$$\frac{1}{n^l+1}(\bar{p}+n^l\left(k^l+\lambda^l e^{rt}\right))=k^h+\lambda^h e^{rt} \tag{15}$$

- a transition at t from S to C^h or vice versa requires

$$\frac{1}{n^h+1}(\bar{p}+n^h\left(k^h+\lambda^h e^{rt}\right))=k^l+\lambda^l e^{rt} \tag{16}$$

- a transition at t from C^l to C^h or vice versa requires

$$\frac{1}{n^l+1}(\bar{p}+n^l\left(k^l+\lambda^l e^{rt}\right))=\frac{1}{n^h+1}(\bar{p}+n^h\left(k^h+\lambda^h e^{rt}\right)) \tag{17}$$

The direct procedure to solve for an OLNE is to solve the above system of necessary conditions. Given all the possible alternatives such an approach is very tedious. Instead, we examine different sequences of phases of exploitation and rule out those that violate the above set of conditions, including continuity of the price path. This turns out to be a powerful instrument to characterize the sequence of exploitation along the equilibrium path and it greatly facilitates the characterization of the exploitation path. We establish several lemmata that are helpful in the derivation of a complete characterization of the equilibrium.

Lemma 1:

i. There cannot be a transition from C^l to C^h or vice versa.

ii. There exists a phase S.

iii. A phase C^h cannot precede a phase S.

Proof:

i. Suppose a transition from C^l to C^h or vice versa takes place at time t. Then equation (17) holds. It follows from (5) and (6) that

$$\frac{1}{n^l+1}\left(\bar{p}+n^l\left(k^l+\lambda^l e^{rt}\right)\right) \leq k^h+\lambda^h e^{rt} \tag{18}$$

Moreover, it follows from (8) and (9) that

$$\frac{1}{n^h+1}\left(\bar{p}+n^h\left(k^h+\lambda^h e^{rt}\right)\right) \leq k^l+\lambda^l e^{rt} \tag{19}$$

Using (19) in (18) yields $\bar{p} \leq k^h+\lambda^h e^{rt}$. Therefore

$$p(t)=\frac{1}{n^h+1}\left(\bar{p}+n^h\left(k^h+\lambda^h e^{rt}\right)\right) \geq \frac{1}{n^h+1}\left(\bar{p}+n^h\bar{p}\right)=\bar{p}$$

which implies a too high price to have a positive quantity demanded after the transition.

ii. This follows immediately from statement (i) of the lemma.

iii. Along C^h equations (8) and (9) hold. Hence, for $t \in C^h$

$$\bar{p}+n^h k^h-\left(n^h+1\right)k^l \leq \left(\left(n^h+1\right)\lambda^l-n^h\lambda^h\right)e^{rt} \tag{20}$$

A transition at t_1 from C^h to S or vice versa requires (16) to hold, stated otherwise

$$\bar{p}+n^h k^h-\left(n^h+1\right)k^l = \left(\left(n^h+1\right)\lambda^l-n^h\lambda^h\right)e^{rt_1} \tag{21}$$

Since $\bar{p}>k^h>k^l$ we have $\bar{p}+n^h k^h-\left(n^h+1\right)k^l>0$. Note that if C^h precedes S, $\left(\left(n^h+1\right)\lambda^l-n^h\lambda^h\right)e^{rt}$ is decreasing over time since it is a monotonic function of time and it is larger than $\bar{p}+n^h k^h-\left(n^h+1\right)k^l$ before t_1 and equal to $\bar{p}+n^h k^h-$

$\left(n^h+1\right)k^l$ at t_1. This implies that $\left(n^h+1\right)\lambda^l - n^h\lambda^h < 0$, which along with $\bar{p} + n^h k^h - \left(n^h+1\right)k^l > 0$ contradicts (21) ■

It will prove useful to define

$$\bar{k}^l \equiv \frac{1}{n^l+1}(\bar{p} + n^l k^l).$$

Since $\bar{p} > k^l$ we clearly have $\bar{k}^l > k^l$. Given a cost k^l the equilibrium depends on the extent of the cost disadvantage of the high cost firms.

Lemma 2:

i. Suppose $k^h < \bar{k}^l$, then C^l cannot precede S.

ii. Suppose $k^h > \bar{k}^l$, then S cannot precede C^l.

Proof:

i. Along C^l equations (5) and (6) hold. Hence for $t \in C^l$

$$\bar{p} + n^l k^l - \left(n^l+1\right)k^h \leq \left(\left(n^l+1\right)\lambda^h - n^l\lambda^l\right)e^{rt} \tag{22}$$

A transition at t_1 from C^l to S or vice versa requires (15) to hold, stated otherwise

$$\bar{p} + n^l k^l - \left(n^l+1\right)k^h = \left(\left(n^l+1\right)\lambda^h - n^l\lambda^l\right)e^{rt_1}$$

Now suppose that $\bar{p} + n^l k^l - \left(n^l+1\right)k^h > 0$ or $\bar{k}^l > k^h$ and we have C^l before S. Note that $\left(\left(n^l+1\right)\lambda^h - n^l\lambda^l\right)e^{rt}$ is decreasing over time since it is a monotonic function of time and it is larger than $\bar{p} + n^l k^l - \left(n^l+1\right)k^h$ before t_1 and equal to $\bar{p} + n^l k^l - \left(n^l+1\right)k^h$ at t_1. This implies that $\left(n^l+1\right)\lambda^h - n^l\lambda^l < 0$, which along with $\bar{p} + n^l k^l - \left(n^l+1\right)k^h > 0$ contradicts (22).

ii. If $\bar{p} + n^l k^l - \left(n^l+1\right)k^h < 0$ or $\bar{k}^l < k^h$ and if a transition occurs from S to C^l, then we get a contradiction in the same way. ■

Lemma 3:

i. There is simultaneous supply throughout if only if

$$\frac{S_0^l/n^l}{S_0^h/n^h} = \frac{\bar{p} + n^h k^h - \left(n^h+1\right)k^l}{\bar{p} + n^l k^l - (n^l+1)k^h} \tag{23}$$

ii. If

$$k^h < \bar{k}^l$$

there is no simultaneous supply just before total exhaustion.

Proof :

i. Define T as the time of exhaustion of all resources. If there is simultaneous exploitation just before T, then $q^l(T) = q^h(T) = 0$, and, from (12), (13) and (14), $\bar{p} - k^h = \lambda^h e^{rT}$ and $\bar{p} - k^l = \lambda^l e^{rT}$. Let the interval with simultaneous exploitation start at t_1. Integration of (12) gives

$$\frac{n^l + n^h + 1}{n^l} \int_{t_1}^{T} q^l(t)dt = \int_{t_1}^{T} \left(\bar{p} + n^h \left(k^h + \lambda^h e^{rt}\right) - \left(n^h + 1\right) \left(k^l + \lambda^l e^{rt}\right)\right) dt$$

That is

$$\begin{aligned} \frac{n^l + n^h + 1}{n^l} S^l(t_1) &= \left(\bar{p} + n^h k^h - \left(n^h + 1\right) k^l\right) (T - t_1) + \\ & \left(n^h \lambda^h - \left(n^h + 1\right) \lambda^l\right) e^{rT} \frac{\left(1 - e^{rt_1 - rT}\right)}{r} \end{aligned}$$

Substituting $\bar{p} - k^h = \lambda^h e^{rT}$ and $\bar{p} - k^l = \lambda^l e^{rT}$ gives after algebraic manipulation

$$\frac{n^l + n^h + 1}{n^l} r S^l(t_1) = \left(\bar{p} + n^h k^h - \left(n^h + 1\right) k^l\right) \left(rT - rt_1 - 1 + e^{rt_1 - rT}\right) \tag{24}$$

Similarly, from 13

$$\frac{n^l + n^h + 1}{n^h} r S^h(t_1) = \left(\bar{p} + n^l k^l - \left(n^l + 1\right) k^h\right) \left(rT - rt_1 - 1 + e^{rt_1 - rT}\right) \tag{25}$$

Setting $t_1 = 0$ and dividing (24) by (25) yields (23).

ii. Since $\bar{p} + n^h k^h - \left(n^h + 1\right) k^l > 0$ by assumption, this is immediately clear from (24) and (25) ■

Lemma 4:

Consider the sequence $S \to C^l$, *with* C^l *the final phase before exhaustion and where the transition takes place at instant of time* t_1 *and exhaustion at* T. *Then*

$$\frac{n^l + n^h + 1}{n^h} r S_0^h = \left(\bar{p} + n^l k^l - \left(n^l + 1\right) k^h\right) \left(r t_1 - 1 + e^{-r t_1}\right) \tag{26}$$

$$\begin{aligned} \frac{n^l + n^h + 1}{n^l} r S_0^l &= -\frac{n^h}{n^l + 1} \left(\bar{p} + n^l k^l - \left(n^l + 1\right) k^h\right) \left(r t_1 - 1 + e^{-r t_1}\right) \\ &\quad + \frac{n^l + n^h + 1}{n^l + 1} \left(\bar{p} - k^l\right) \left(rT - 1 + e^{-rT}\right) \end{aligned} \tag{27}$$

Proof: See Appendix A. ■

Lemma 5:

Consider the sequence $S \to C^h$, *with* C^h *the final phase before exhaustion and with the transition taking place at instant of time* t_1 *and exhaustion at* T. *Then*

$$\frac{n^l + n^h + 1}{n^l} r S_0^l = \left(\bar{p} + n^h k^h - \left(n^h + 1\right) k^l\right) \left(r t_1 - 1 + e^{-r t_1}\right) \tag{28}$$

$$\begin{aligned} \frac{n^l + n^h + 1}{n^h} r S_0^h &= -\frac{n^l}{n^h + 1} \left(\bar{p} + n^h k^h - \left(n^h + 1\right) k^l\right) \left(r t_1 - 1 + e^{-r t_1}\right) + \\ &\quad \frac{n^l + n^h + 1}{n^h + 1} \left(\bar{p} - k^h\right) \left(rT - 1 + e^{-rT}\right) \end{aligned} \tag{29}$$

Proof: Due to symmetry the proof is exactly the same as the proof of the previous lemma. ■

We can now exploit the lemmata above to characterize the sequence along an OLNE depending on the parameter values.

Proposition 1

Suppose $k^h > \bar{k}^l$. *For a given* S_0^h, *there exists* $\tilde{S}_0^l > 0$ *such that the equilibrium sequence reads* $C^l \to S \to C^h$ *if* $S_0^l > \tilde{S}_0^l$ *and* $S \to C^h$ *if* $S_0^l \leq \tilde{S}_0^l$.

Proof:

Given the assumption on the costs, $k^h > \overline{k}^l$, we cannot have C^l alone or C^h alone over the entire game: both types of mines eventually exploit and exhaust their stock. According to lemma 1ii, there must be a phase S. According to lemma 3ii, the final phase of the exploitation cannot be S. A phase S cannot precede C^l (lemma 2ii). Therefore, the last phase of the exploitation pattern is C^h. From Lemma 1i there cannot be a transition $C^l \to C^h$. Therefore, the last two phases will be $S \to C^h$. It remains to show that the only phase that can precede S is C^l. This can easily be seen from (16), that implies that there is at most one transition $S \to C^h$ or vice versa. Since there must be one transition $S \to C^h$, the equilibrium reads $C^l \to S \to C^h$ or $S \to C^h$.

We show next that if the initial stock S_0^l is large enough we must have a phase C^l that precedes $S \to C^h$. Suppose that the equilibrium reads $S \to C^h$ with the transition at t_1 and final time T. It follows from (13) that

$$\frac{n^l + n^h + 1}{n^h} q^h(0) = \bar{p} + n^l k^l - \left(n^l + 1\right) k^h + n^l \lambda^l - \left(n^l + 1\right) \lambda^h \tag{30}$$

When S_0^l becomes arbitrarily large it follows from (28) and (29) that t_1 and T also become arbitrarily large. Use $\lambda^h = e^{-rT}\left(\bar{p} - k^h\right)$ and rewrite (16) to obtain

$$\lambda^l = \frac{n^h}{n^h + 1} \lambda^h + e^{-rt_1} \left(\frac{\bar{p} + n^h k^h}{n^h + 1} - k^l\right) \tag{31}$$

Hence, both λ^l and λ^h tend to zero when S_0^l becomes arbitrarily large. Equation (30) then implies a negative value of $q^h(0)$. When S_0^l tends to zero $q^h(0)$ is positive. It is shown in Appendix B that $q^h(0)$ as a function of S_0^l is monotonically decreasing. Therefore, there exists a unique $\hat{S}_0^l > 0$ such that if $S_0^l > \hat{S}_0^l$ the equilibrium reads $C^l \to S \to C^h$ and $S \to C^h$ if $S_0^l \leq \hat{S}_0^l$ ■

From Proposition 1 it is clear that if the cost difference is large enough, i.e. given $k^l > 0$, $k^h > \bar{k}^l$, the low cost resource is exhausted first regardless of the level of the stocks. However, when the cost difference is not too large, i.e. given $k^l > 0$, $k^h \in \left(k^l, \bar{k}^l\right)$, the order of exhaustion of the resource stocks depends on the initial stocks available.

Proposition 2

Suppose $k^h < \bar{k}^l$.

i. If

$$\frac{S_0^l/n^l}{S_0^h/n^h} = \frac{\bar{p} + n^h k^h - \left(n^h + 1\right) k^l}{\bar{p} + n^l k^l - \left(n^l + 1\right) k^h} \tag{32}$$

then the equilibrium reads S

ii. If

$$\frac{S_0^l/n^l}{S_0^h/n^h} < \frac{\bar{p} + n^h k^h - \left(n^h + 1\right) k^l}{\bar{p} + n^l k^l - \left(n^l + 1\right) k^h} \tag{33}$$

then the equilibrium reads $S \to C^h$

iii. If

$$\frac{S_0^l/n^l}{S_0^h/n^h} > \frac{\bar{p} + n^h k^h - \left(n^h + 1\right) k^l}{\bar{p} + n^l k^l - \left(n^l + 1\right) k^h} \tag{34}$$

then the equilibrium reads $S \to C^l$.

Proof:

From Lemma 1ii there always exists an S phase. By Lemma 2i the S phase cannot be preceded by C^l and, from Lemma 1iii, the S phase cannot be preceded by C^h. Therefore, the equilibrium starts with S. Since a transition from C^l to C^h or vice versa is ruled out by Lemma 1i, the equilibrium reads $S \to C^h$ or $S \to C^l$. If condition (32) is satisfied we have simultaneous supply throughout. If conditions (28) and (29) are satisfied with $T > t_1 > 0$ then we must have $S \to C^h$. Otherwise the equilibrium reads $S \to C^l$. If this would not hold, a contradiction is obtained as is straightforward to see ■

We have not yet treated the border cases where either $k^l = k^h$ or $k^h = \bar{k}^l$. This is dealt with in

Proposition 3

i. Suppose $k^h = \bar{k}^l$. Then the result of proposition 1 holds with C^l collapsing

ii. Suppose $k^l = k^h$. Then the results of proposition 2 hold with $k^l = k^h$.

Proof:

i. If the premises of the proposition holds, lemma 1i still holds. This implies from (13) that at the beginning of the $S-$phase $q^h(0) > 0$, regardless of the stocks.

ii. This is evident. This case reduces exactly to the case studied in Loury (1986) ■

These propositions fully characterize the OLNE for a finite number of players. It can be checked that in each of the cases above we have $T > t_1$.

We end this section by considering the case of an infinite number of fringe members and a single cartel. This constitutes what is called the cartel-versus-fringe model. We assume that the cartel owns the resources that are cheap to exploit. So, $n^l = 1$ and $n^h = \infty$. If we take the limits of the equilibrium derived under different cost constellations we arrive at

Proposition 4

If $n^l = 1$ and if $n^h \to \infty$ the open loop Nash equilibrium has the following limits:

i. If $\frac{1}{2}\left(\bar{p} + k^l\right) < k^h$, then the Nash equilibrium sequence is $C^l \to S \to C^h$, with the C^h phase collapsing if S_0^l is smaller than a certain threshold.

ii. If $\frac{1}{2}\left(\bar{p} + k^l\right) = k^h$, then the Nash equilibrium sequence is $S \to C^h$

iii. If $\frac{1}{2}\left(\bar{p} + k^l\right) > k^h$, then the Nash equilibrium sequence is S if $\frac{S_0^l}{S_0^h} = \frac{k^h - k^l}{\bar{p} + k^l - 2k^h}$; $S \to C^h$ if $\frac{S_0^l}{S_0^h} < \frac{k^h - k^l}{\bar{p} + k^l - 2k^h}$ and $S \to C^l$ if $\frac{S_0^l}{S_0^h} > \frac{k^h - k^l}{\bar{p} + k^l - 2k^h}$

iv. If $k^l = k^h$, then the Nash equilibrium sequence is $S \to C^l$

Proof:

This is straightforward from propositions 1, 2 and 3 ■

This outcome corresponds to the outcome of a cartel-fringe model where the fringe firms are assumed to be price takers. See Ulph and Folie (1980). Theoretically it cannot be excluded beforehand that the type of mine with the number of players going to infinity is the cheaper type. According to proposition 1 (with $n^l = \infty$) the equilibrium reads $C^h \to S \to C^l$, with the first phase possibly collapsing.

We conclude that not only in the case of a symmetric oligopoly but also in more general settings behaviour of large groups of similarly placed oligopolists can be represented as price taking behaviour. This gives a sound foundation for the assumption of price taking behavior of the fringe in the open-loop cartel-fringe model.

4 The order of extraction and welfare effects

One feature of the equilibrium is that for certain parameter values the more expensive resource is exhausted before the cheaper one. This occurs when the cost advantage of the cheaper mines is only moderate and their aggregate stock is large (see (34)). This is an instance where the Herfindahl rule does not hold. Violation of the Herfindahl rule in our framework was illustrated in an example with two firms in Lewis and Schmalensee (1980, proposition 6c). We have provided the precise and general conditions under which this phenomenon can occur[4]. From a welfare perspective the violation of the Herfindahl rule is an undesirable outcome because welfare maximization requires the cheaper resources be exploited first. In the present section we exploit the characterization of the conditions under which the Herfindahl rule is violated to determine the welfare effects of having a more competitive industry through a larger number of high cost firms.

Suppose that initially we are in an equilibrium with only simultaneous supply, i.e., (32) holds. Written differently

$$\frac{S_0^l}{S_0^h} = \frac{(\bar{p} + n^h k^h - \left(n^h + 1\right) k^l)/n^h}{(\ \bar{p} + n^l k^l - (n^l + 1)\, k^h)/n^l} \tag{35}$$

Let us keep the aggregate initial stocks fixed and increase the number of high cost mines. Then the right hand side of equality (35) decreases while the left hand side remains unchanged, implying that the new equilibrium becomes $S \to C^l$. Therefore, increasing the number of high cost mines causes an inefficiency. We show that this inefficiency can outweigh the positive impact from having more competition in the market and therefore that having more competitors can be detrimental to social welfare. This possibility cannot be detected in models where firms have the same costs. Indeed, Loury (1986) assumes equal extraction costs (but differing reserves) and finds that increasing the number of players in the oligopoly game increases efficiency. In order to demonstrate the possibility of decreasing social welfare we construct a simple example. Take $\bar{p} = 10$, $r = 0.1$, $n^l = 1$, $k^l = 0$, $k^h = 2.5$, $S_0^l = 100$ and $S_0^h = 150$. Then $k^h < \bar{k}^l$, and for $n^h = 12$ equality (35) holds. Therefore, if $n^h = 12$

[4] The Herfindahl rule is known to be violated in other circumstances as well (e.g., general equilibrium framework, cost uncertainty, ..). See for example Amigues et al. (1998), Chakravorty and Krulce (1994), Gaudet et al. (2001) and Gaudet and Lasserre (2008).

the equilibrium has simultaneous supply throughout and for $n^h < 12$ $\left(n^h > 12\right)$ the equilibrium reads $S \to C^h$ $\left(S \to C^l\right)$. Let W denote social welfare defined as the discounted sum of consumer and producers' surplus:

$$W = \int_0^\infty e^{-rt}[\frac{1}{2}(\bar{p} - p)(q^l + q^h) + (p - k^l)q^l + (p - k^h)q^h]dt$$

which after substitution of the price gives

$$W = \int_0^\infty e^{-rt}[(\bar{p} - k^l)q^l + (\bar{p} - k^h)q^h - \frac{1}{2}(q^l + q^h)^2]dt$$

For the case at hand W is plotted in figure 1 as a function of the number of high cost firms. We observe that social welfare increases as n^h increases from 1 to 2, but that it monotonically decreases thereafter. Therefore, the example shows that social welfare can decrease not only if the number of high cost firms is increased from the level where the equilibrium is S throughout[5] (i.e., $n^h = 12$), but also if we increase that number in part of the $S \to C^h$ regime (i.e., $n^h < 12$). This surprising negative outcome from increased competition is due to the reversal of the Herfindahl rule. Increasing competition at the level of high cost mines only, exacerbates the inefficiency from the order use of resources which can outweigh any gain on the consumer surplus front.

Note that changing the number of high cost firms does not affect the terminal time. This is the case since using (25) and (26) gives

$$r\left(\frac{n^l + 1}{n^l}S_0^l + S_0^h\right) = \left(\bar{p} - k^l\right)\left(rT - 1 + e^{-rT}\right)$$

The shadow price of the stock that is exhausted last doesn't change either. What does change is the date of the transition from the $S-$phase to the monopoly phase (either C^l or C^h).

Some further illustrations are given in figures 2 and 3 below, for the same parameter values as above except for S^h which is now taken equal to 40. These parameter values are such that for $n^h = 1$ we are in S throughout and thus for all $n^h > 1$ we are

[5] we are in a $S \to C^l$ regime for $n^h > 12$.

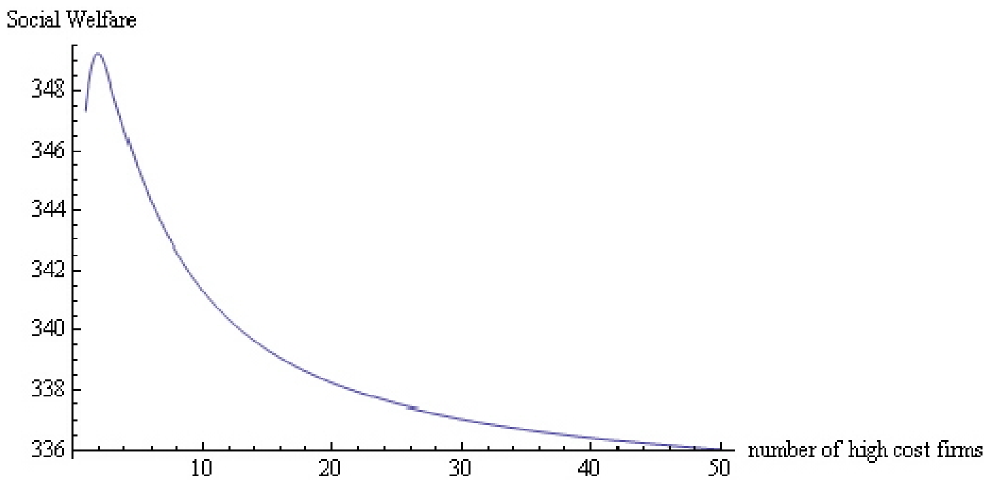

Figure 1: Social welfare as a function of the number of high cost firms

in the $S \to C^l$ regime and social welfare W is a strictly decreasing function of n^h.

For this latter case, figure 2 displays the two price paths corresponding to $n^h = 1$ and $n^h = 2$. The initial price at $n^h = 2$ is lower compared to $n^h = 1$, reflecting increased competition. However the equilibrium price path under $n^h = 2$ eventually passes above the price path under $n^h = 1$. Thus, consumer surplus for $n^h = 2$ is initially larger than for $n^h = 1$ but eventually falls below it. The difference between instantaneous social welfare for $n^h = 2$ and for $n^h = 1$ is depicted in figure 3. We observe that instantaneous social welfare does not decrease uniformly. The example shows that initially and eventually instantaneous welfare is smaller for $n^h = 2$, but there is an intermediate period of time where instantaneous social welfare for $n^h = 2$ is larger than for $n^h = 1$.

The intuition of this result is that the increase of the number of high cost firms creates more competition in the market and leads the firm that owns the low cost mine to reallocate its production through time: extract less initially and more later (see figure 4). Thus the increase of the number of high cost firms, although it increases consumption in the short-run (see figure 5), results in a larger share of consumption coming from high cost mines which in itself is a source of welfare loss. The example shows that, even in the short-run, this welfare loss, can outweigh the gain in consumer surplus enjoyed through larger production and a lower price.

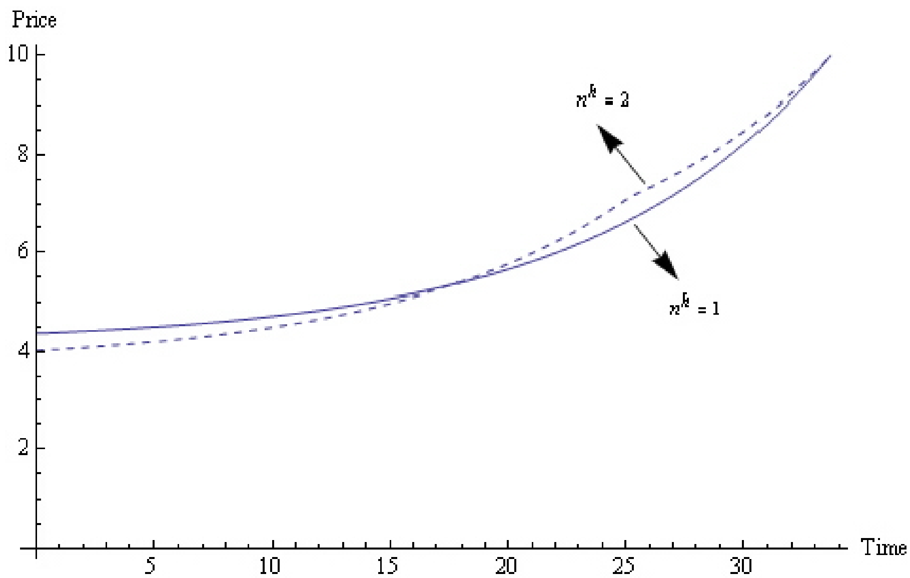

Figure 2: Price path for $n^h = 1$ and $n^h = 2$

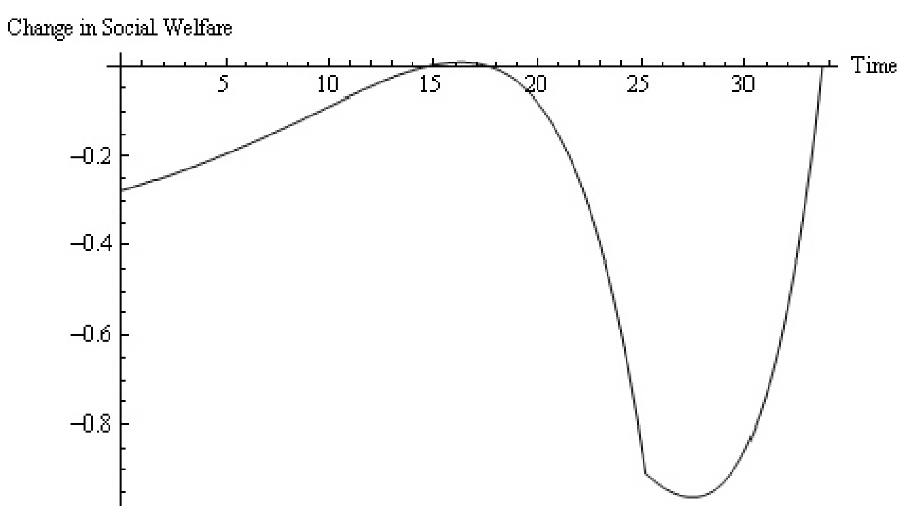

Figure 3: Change in instantaneous social welfare when n^h changes from 1 to 2

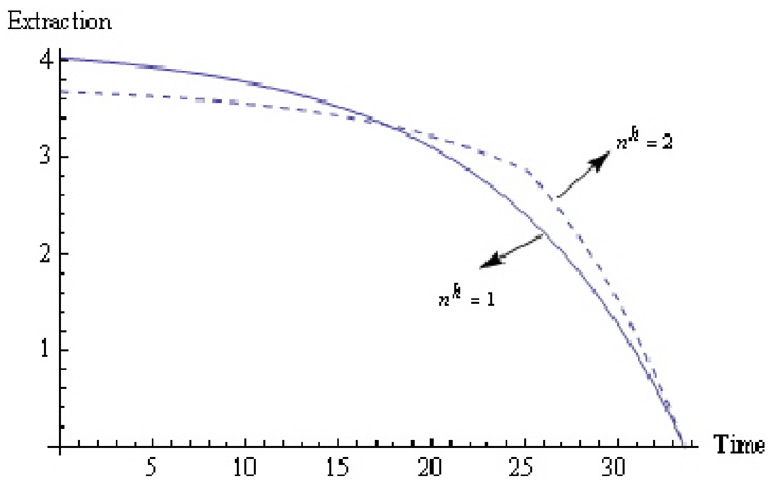

Figure 4: Extraction path of the high costs firm for $n^h = 1$ and $n^h = 2$

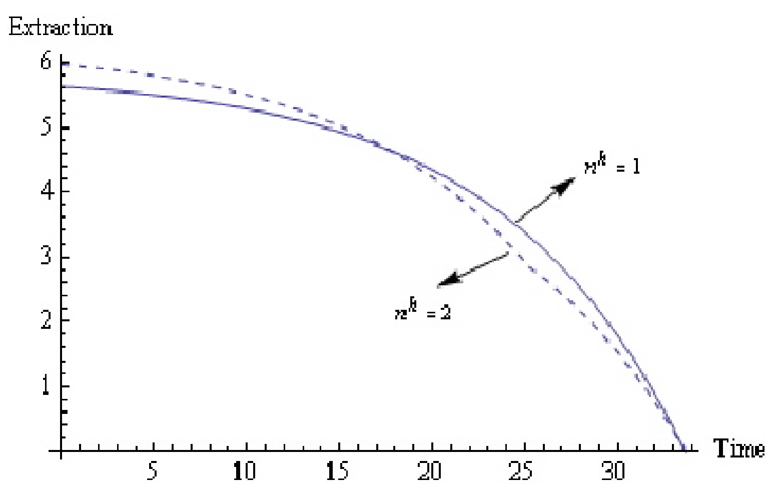

Figure 5: Total extraction path for $n^h = 1$ and $n^h = 2$

The calculations performed here have been done for other parameter sets as well. Qualitatively the results are unaffected.

5 Conclusions

We have fully characterized the open-loop Nash equilibrium in the case of 'oil'igopoly in a general case where firms may have different extraction costs and initial stocks of the resource. We have then used this equilibrium to derive the open-loop Nash equilibrium for the cartel-versus-fringe model by considering one coherent cartel and the number of fringe members going to infinity.

The characterization of an equilibrium in this more general context than was explored so far brings us closer to more realistic modelling of the complex reality of the oil market and nonrenewable resource markets in general. The result we derive regarding the welfare implications of increasing the number of firms shows that the intuition we have from standard, static, microeconomic analysis does not necessarily follow through in the case of a dynamic oligopoly. In particular increasing competition may reduce welfare. Competition policy for resource extracting industries requires a specific analysis.

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Appendix A: Proof of Lemma 4

In the case $S \to C^l$ we have, from (6)

$$p(T) = \frac{1}{n^l + 1}\left(\bar{p} + n^l\left(k^l + \lambda^l e^{rT}\right)\right) = \bar{p}$$

so that

$$k^l + \lambda^l e^{rT} = \bar{p} \tag{36}$$

At the transition time t_1 we have from (15)

$$\bar{p} + n^l\left(k^l + \lambda^l e^{rt_1}\right) = \left(n^l + 1\right)\left(k^h + \lambda^h e^{rt_1}\right) \tag{37}$$

It follows from straightforward integration of (13) that

$$\frac{n^l + n^h + 1}{n^h} r S_0^h = \left(\bar{p} + n^l k^l - \left(n^l + 1\right) k^h\right) rt_1 + \left(n^l \lambda^l - \left(n^l + 1\right)\lambda^h\right)\left(e^{rt_1} - 1\right) \tag{38}$$

Equations (36) and (37) imply

$$\lambda^l = e^{-rT}\left(\bar{p} - k^l\right) \tag{39}$$

$$n^l\lambda^l - \left(n^l + 1\right)\lambda^h = -e^{-rt_1}\left(\bar{p} + n^l k^l - \left(n^l + 1\right)k^h\right) \tag{40}$$

Hence, after substitution into (38) we obtain (26).

We now derive (27). Integration of q^l from (12) yields

$$\int_0^{t_1} rq^l(s)ds = \frac{n^l}{n^l + n^h + 1}\left\{\left(\bar{p} + n^h k^h - \left(n^h + 1\right)k^l\right)rt_1 + \left(n^h\lambda^h - \left(n^h + 1\right)\lambda^l\right)\left(e^{rt_1} - 1\right)\right\} \tag{41}$$

and

$$\int_{t_1}^{T} rq^l(s)ds = \frac{n^l}{n^l + 1}\left\{\left(\bar{p} - k^l\right)(rT - rt_1) - \lambda^l\left(e^{rT} - e^{rt_1}\right)\right\} \tag{42}$$

Summing up the two cumulative extractions gives

$$\begin{aligned}
\int_0^{t_1} rq^l(s)ds + \int_{t_1}^{T} rq^l(s)ds &= rS_0^l = \frac{n^l}{n^l + n^h + 1}\left(\bar{p} + n^h k^h - \left(n^h + 1\right)k^l\right)rt_1 \\
&+ \frac{n^l}{n^l + n^h + 1}\left(n^h\lambda^h - \left(n^h + 1\right)\lambda^l\right)\left(e^{rt_1} - 1\right) \\
&+ \frac{n^l}{n^l + 1}\left\{\left(\bar{p} - k^l\right)(rT - rt_1) - \lambda^l\left(e^{rT} - e^{rt_1}\right)\right\}
\end{aligned} \tag{43}$$

Substituting in the last term λ^l from $\bar{p} - \left(k^l + \lambda^l e^{rT}\right) = 0$ yields

$$\begin{aligned}
rS_0^l &= \frac{n^l}{n^l + n^h + 1}\left(\bar{p} + n^h k^h - \left(n^h + 1\right)k^l\right)rt_1 + \frac{n^l}{n^l + n^h + 1}\left(n^h\lambda^h - \left(n^h + 1\right)\lambda^l\right)\left(e^{rt_1} - 1\right) \\
&+ \frac{n^l}{n^l + 1}\left(\bar{p} - k^l\right)(rT - rt_1) - \frac{n^l}{n^l + 1}\left(\bar{p} - k^l\right)e^{-rT}\left(e^{rT} - e^{rt_1}\right)
\end{aligned} \tag{44}$$

Factorise $\frac{n^l}{n^l+1}\left(\bar{p}-k^l\right)$ in the last two terms

$$\begin{aligned}
rS_0^l &= \frac{n^l}{n^l+n^h+1}\left(\bar{p}+n^hk^h-\left(n^h+1\right)k^l\right)rt_1+\frac{n^l}{n^l+n^h+1}\left(n^h\lambda^h-\left(n^h+1\right)\lambda^l\right)\left(e^{rt_1}-1\right) \\
&\quad +\frac{n^l}{n^l+1}\left(\bar{p}-k^l\right)\left(rT-1+e^{rt_1-rT}-rt_1\right) \qquad (45)
\end{aligned}$$

Adding and substracting e^{-rT} in the last term

$$\begin{aligned}
rS_0^l &= \frac{n^l}{n^l+n^h+1}\left(\bar{p}+n^hk^h-\left(n^h+1\right)k^l\right)rt_1+\frac{n^l}{n^l+n^h+1}\left(n^h\lambda^h-\left(n^h+1\right)\lambda^l\right)\left(e^{rt_1}-1\right) \\
&\quad +\frac{n^l}{n^l+1}\left(\bar{p}-k^l\right)\left(rT-1+e^{-rT}+\left(-1+e^{rt_1}\right)e^{-rT}-rt_1\right) \qquad (46)
\end{aligned}$$

Multiplying both sides by $\frac{n^l+n^h+1}{n^l}$ and splitting the last term yields

$$\begin{aligned}
\frac{n^l+n^h+1}{n^l}rS_0^l &= \left(\bar{p}+n^hk^h-\left(n^h+1\right)k^l\right)rt_1+\left(n^h\lambda^h-\left(n^h+1\right)\lambda^l\right)\left(e^{rt_1}-1\right) \\
&\quad +\frac{n^l+n^h+1}{n^l+1}\left(\bar{p}-k^l\right)\left(\left(-1+e^{rt_1}\right)e^{-rT}-rt_1\right) \\
&\quad +\frac{n^l+n^h+1}{n^l+1}\left(\bar{p}-k^l\right)\left(rT-1+e^{-rT}\right) \qquad (47)
\end{aligned}$$

Note that the last term only includes T and not t_1 and corresponds to the last term in (27). Algebraic manipulation of the first three terms and substituting the $\lambda's$ yields (27). ■

Appendix B:

Consider

$$\frac{n^l+n^h+1}{n^h}q^h(0)=\bar{p}+n^lk^l-\left(n^l+1\right)k^h+n^l\lambda^l-\left(n^l+1\right)\lambda^h \qquad (48)$$

where $\lambda^h=e^{-rT}\left(\bar{p}-k^h\right)$ and

$$\lambda^l=\frac{n^h}{n^h+1}\lambda^h+\frac{e^{-rt_1}\left(\bar{p}+n^hk^h-\left(n^h+1\right)k^l\right)}{n^h+1} \qquad (49)$$

In this appendix we show that $q^h(0)$ is a monotonicaly decreasing function of S_0^l. We have

$$\frac{n^l + n^h + 1}{n^h} \frac{\partial q^h(0)}{\partial S_0^l} = n^l \frac{\partial \lambda^l}{\partial S_0^l} - \left(n^l + 1\right) \frac{\partial \lambda^h}{\partial S_0^l} \tag{50}$$

Subtituting λ^l gives

$$\frac{n^l + n^h + 1}{n^h} \frac{\partial q^h(0)}{\partial S_0^l} = n^l \frac{\partial \left(\frac{n^h}{n^h+1} \lambda^h + \frac{e^{-rt_1}\left(\bar{p} + n^h k^h - \left(n^h+1\right) k^l\right)}{n^h+1} \right)}{\partial S_0^l} - \left(n^l + 1\right) \frac{\partial \lambda^h}{\partial S_0^l} \tag{51}$$

or

$$\frac{n^l + n^h + 1}{n^h} \frac{\partial q^h(0)}{\partial S_0^l} = -\frac{n^l + n^h + 1}{n^h + 1} \frac{\partial \lambda^h}{\partial S_0^l} - n^l r \frac{e^{-rt_1}\left(\bar{p} + n^h k^h - \left(n^h + 1\right) k^l\right)}{n^h + 1} \frac{\partial t_1}{\partial S_0^l} \tag{52}$$

with $\frac{\partial \lambda^h}{\partial S_0^l} = -re^{-rT}\left(\bar{p} - k^h\right) \frac{\partial T}{\partial S_0^l}$.

Moreover, writing (29) in the following form

$$r\left(S_0^h + \frac{n^h}{n^h + 1} S_0^l\right) = \frac{n^h}{n^h + 1}\left(\bar{p} - k^h\right)\left(rT - 1 + e^{-rT}\right) \tag{53}$$

and using (28) we obtain

$$\frac{n^l + n^h + 1}{n^l} \frac{1}{\left(1 - e^{-rt_1}\right)} r = r\left(\bar{p} + n^h k^h - \left(n^h + 1\right) k^l\right) \frac{\partial t_1}{\partial S_0^l} \tag{54}$$

and

$$1 = \left(1 - e^{-rT}\right)\left(\bar{p} - k^h\right) \frac{\partial T}{\partial S_0^l} \tag{55}$$

Substituting both terms into (52) gives

$$\frac{n^l + n^h + 1}{n^h} \frac{\partial q^h(0)}{\partial S_0^l} = \frac{n^l + n^h + 1}{n^h + 1} re^{-rT} \frac{1}{\left(1 - e^{-rT}\right)} - n^l \frac{e^{-rt_1}}{\left(n^h + 1\right)} \frac{n^l + n^h + 1}{n^l} \frac{1}{\left(1 - e^{-rt_1}\right)} r \tag{56}$$

or

$$\frac{\partial q^h(0)}{\partial S_0^l} = \frac{n^h}{n^h + 1} r\left(\frac{e^{-rT}}{1 - e^{-rT}} - \frac{e^{-rt_1}}{1 - e^{-rt_1}}\right) < 0 \blacksquare \tag{57}$$