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Mergers in Nonrenewable Resource Oligopolies and Environmental Policies*

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Abstract

We examine the profitability of horizontal mergers within nonrenewable resource industries, which account for a large proportion of merger activities worldwide. Each firm owns a private stock of the resource and uses open loop strategies when choosing its extraction path. We analytically show that even a small merger (merger of 2 firms) is always profitable when the resource stock owned by each firm is small enough. In the case where pollution is generated by the industry's activity, we show that an environmental policy that increases firms' production cost or reduces the price received by

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firms can deter a merger. This speeds up the industry's extraction and thereby causes emissions to occur earlier than under a laissez-faire scenario.

1 Introduction

This paper examines the incentive to merge in nonrenewable resource industries. This sector constitutes a large proportion of GDP in many economies,[1] and also has a long history of mergers and acquisitions (M&A) activity, starting with Standard Oil's acquisitions in the early 1900's. The volume of M&A has been consistently higher in the exhaustible resource sector relative to many others. Moreover, this sector has experienced a spate of mega-mergers, starting in the late 1990's, including the mergers of BP and Amoco (1998, $63 billion); Exxon and Mobil (1999, $74.2 billion); Total Fina and Elf Aquitaine (1999, $54.2 billion); Chevron and Texaco (2001, $45 billion); and Royal Dutch Petroleum and the Shell Group (2004).[2] A first research question addressed in this paper is understanding why there is so much M&A activity in the exhaustible resource sector.

There exists a vast literature concerned with various aspects of horizontal mergers. Salant, Switzer & Reynolds (1983), henceforth referred to as SSR, is arguably one of the most influential papers in that literature. SSR's important contribution is to show that horizontal mergers can be unprofitable, that is, the profits of the merged entity is smaller than the sum of the pre-merger profits of the individual firms that merge. In particular, in the case of a symmetric oligopoly with linear demand and constant marginal cost of production

[1] For example, exhaustible resource sectors, including oil, gas and minerals and mining, accounted for about 10% of Canadian GDP annually during 2008-2012, according to Statistics Canada.

[2] The global value of M&A in the oil sector rose from $88.99 billion in 1997 (representing about 25% of global income from the oil sector in 1997) to $372 billion in 2007 (representing about 22% of global income from the oil sector in 2007) (see Kumar, 2012, for further details). In Canada, for instance, exhaustible resource extraction industries have seen rising volumes of M&A in recent years. According to the data provided by the Canadian Competition Bureau, during 2012 to 2013, about 20% of the 330 mergers that were reviewed by the Bureau were in this sector, with 16% of mergers being realized in oil and gas extraction industries. The highest value merger transactions in Canada in 2012 were realized in the oil and gas extraction industry in the form of cross-border acquisitions, according to Macleans and Blake Canadian Lawyers, including the C$15-billion acquisition of Nexen by China's CNOOC and the C$5.5-billion acquisition of Progress Energy Resources by Malaysia's Petronas.

where firms compete in quantity, a merger of two firms is never profitable unless it is a merger to form a monopoly. Moreover, the merged entity must be significant enough for the merger to be profitable. The basic intuition driving this result is that, in the case of strategic substitutes such as in Cournot competition, when the merger participants decrease quantity, the non-merging firms respond by increasing their output levels, thereby mitigating the increase in market power of the merger participants. The increase in output of the outsiders more than offsets the benefit the merging firms can get from their reduction of output. SSR's result has proven to be very robust to various modifications of the basic benchmark model (see, e.g., Stigler 1950; Kamien & Zang 1990, 1991, 1993; Gaudet & Salant 1991; Farrell & Shapiro 1990).

The nonrenewable resource sector requires a specific merger analysis to account for the fact that the output of firms, that is, their cumulative extraction over time, is limited by their stock. In that context, we investigate the profitability of mergers. We find that the result of SSR do not carry over to the case of nonrenewable resource industries: even a small merger (merger of 2 firms) is always profitable when the resource stock owned by each firm is small enough.[3]

We then analyze the impact, on the profitability of a merger, of an environmental policy that raises firms' extraction costs (or diminishes the price of the resource). This analysis is motivated by the fact that many important nonrenewable resources' production and/or consumption generate a negative externality (e.g., oil or phosphate). The impact of a tax on the resource has received a lot of attention recently, as a carbon tax on fossil fuels is often viewed as a natural instrument to slow down global warming. An important stream of that literature examines whether a carbon tax may result in the *Green Paradox*, that is, the unintended consequence of speeding up fossil fuel extraction and therefore increasing pollution (see Sinn 2008 and, e.g., Pittel, van der Ploeg & Withagen 2014 and Long 2015).

Two papers that are closely related to ours are Benchekroun & Gaudet 2003, which

[3]It is possible to overturn SSR's result if marginal cost is increasing (see, Perry and Porter, 1985). In this paper, we highlight a new mechanism through which this may occur, namely resource constraints.

examines the impact of an exogenous marginal production restriction in a nonrenewable resource duopoly, and Benchekroun & Gaudet 2015, which considers a renewable, common pool resource. In the case considered in this paper, the production restriction is non-marginal and determined endogenously in equilibrium, while each firm owns a private stock of the non-renewable resource. We find that a tax on extraction may prevent a merger from happening. We show that a merger slows down the industry's extraction rate, and therefore delays emissions. If a higher tax rate deters a merger, it follows that emissions occur earlier under the stricter environmental policy than under a laissez-faire scenario. This result clearly carries a similar flavor and is an instance of a green paradox; however the channel of the increase in pollution, i.e., the merger decision of the players, is novel.

In instances where resource ownwers are countries and not firms, coordination of interests among few resource ownwers is more likely to take the form of partial cartels rather than mergers. In the case of symmetric firms or the case where firms have identical constant marginal costs, all the results derived in our paper naturally extend to the case of a cartel.

We use a dynamic game model where firms compete in quantity in the output market while each firm faces a resource constraint (cumulative extraction over time must not exceed its initial endowment of the resource). We use a continuous time framework with an endogenous time horizon. We follow much of the existing literature on oligopoly models of nonrenewable resource markets, and use open-loop strategies where firms choose a time path of extraction at the beginning of the game (see, e.g., Salant 1976, 1982; Lewis & Schmalensee 1980; Loury 1986; Gaudet & Long 1994; Benchekroun, Halsema & Withagen 2009, 2010).[4,5]

[4] None of these papers analyze the impact of mergers, nor do they examine the interplay of an environmental policy on firms' decisions to merge.

[5] We note that the equilibrium derived using open-loop strategies may not be subgame perfect (see, e.g., Karp & Newberry 1991; Groot, Withagen & de Zeeuw, 1992, 2003). A set of papers use stationary Markovian strategies, that is, strategies that depend on the vector of stocks. The equilibrium within this class of strategies is by construction subgame perfect, but much more challenging to characterize. In an effort to derive analytical solutions, these papers rely on specific functional forms or assumptions, such as isoelastic demand and zero extraction cost (Eswaran & Lewis 1985; Reinganum & Stokey 1985; Benchekroun & Long 2006), economic abandonment where the resource is not exhausted in full (Salo & Tahvonen 2001), an exogenously fixed time horizon (Hartwick & Brolley 2008; Polasky 1996; Wan & Boyce 2014). For instance, Wan & Boyce 2014 offers, in a two-period model, a full characterization of the duopolistic equilibrium in the case of Cournot and Stackelberg games.

We also generalize our results to the case of a marginal cost function that is decreasing in the resource stock remaining.

We proceed as follows. Section 2 presents the model. Section 3 presents the analysis of the profitability of mergers. Section 4 analyzes the impact of a tax on extraction. Section 5 presents the case where the marginal cost function that is decreasing in the resource stock remaining. Section 6 concludes.

2 The Model and Preliminary Analysis

2.1 The Model

We consider an exhaustible resource industry with n firms. There are n_S firms that each own a stock S_{0S} and n_L firms that each own a stock S_{0L} with $n_L + n_S = n$. Without loss of generality, we shall consider that $S_{0S} \leq S_{0L}$ and the firms that own the stock S_{0S} are the first n_S firms. That is, $S_{0i} = S_{0S}$ for $i = 1, .., n_S$ and $S_{0i} = S_{0L}$ for $i = n_S + 1, .., n$. Firms may only differ with respect to the initial stock of the resource owned.

Marginal extraction costs are constant and identical across all firms, and given by c. Let $q_i(t) \geq 0$ denote the extraction rate at time $t \geq 0$ of firm i that owns a stock S_{0i}. Define $Q_S(t) = \sum_{i=1}^{n_S} q_i(t)$ to be the aggregate supply of the firms that own a stock S_{0S}, and $Q_L(t) = \sum_{i=n_S+1}^{n} q_i(t)$ to be the aggregate supply of the firms that own a stock S_{0L}. Demand for the resource is stationary and linear with a choke price $a : p(t) = a - bQ(t)$, where $p(t)$ is the price at time t, $Q(t)$ is the quantity demanded at time t, and $a > c$.[6] The inverse demand at time $t \geq 0$ for the extracted resource is given by $p(t) = a - b[Q_S(t) + Q_L(t)]$. An admissible extraction path for firm i is an extraction path, $q_i(.)$, such that $q_i(t) \geq 0$ for

[6] More precisely, we have $p(t) = \max\{a - bQ(t), 0\}$. Throughout the paper, we will focus on cases where the outcome is such that $a - bQ(t) > 0$ for all t. This is true, for example, as long as, ceteris paribus, the choke price a is large enough.

all $t \geq 0$ and

$$\int_0^\infty q_i(t)\,dt \leq S_{0i}. \tag{1}$$

Firms are oligopolists in the resource market where they compete *à la* Cournot, and the objective of each firm i is to maximize the discounted sum of its profits

$$\int_0^\infty e^{-rt}\left[a - b\left(Q_S(t) + Q_L(t)\right) - c\right] q_i(t)\,dt \tag{2}$$

subject to its resource constraint, as given by (1).

Each firm takes the extraction paths of its competitors as given and chooses an extraction path that maximizes its discounted sum of profits subject to the resource constraint. We characterize an open-loop Nash-Cournot equilibrium (OLNE) of this game. More precisely:

Definition 1 *Open-loop Nash-Cournot equilibrium (OLNE)*

A n-tuple vector of extraction paths $q(.) \equiv (q_1(.), \ldots, q_n(.))$ *with* $q(t) \geq 0$ *for all* $t \geq 0$ *is an open-loop Nash-Cournot equilibrium if:*

(i) the resource constraint is satisfied for all $i = 1, .., n$, *and*

(ii) for all $i = 1, 2, \ldots, n$ *we have*

$$\begin{aligned} &\int_0^\infty e^{-rt}\left[a - b\left(Q_S(t) + Q_L(t)\right) - c\right] q_i(t)\,dt \\ \geq &\int_0^\infty e^{-rt}\left[a - b\left(\hat{q}_i(t) + Q_{-i}(t)\right) - c\right] \hat{q}_i(t)\,dt \end{aligned}$$

for all $\hat{q}_i$ *satisfying the resource constraint.*

Remark: note that by using the open-loop solution concept, we are assuming that firms have the ability to commit to the whole extraction path at the beginning of the game. A Nash equilibrium results in an outcome that is time-consistent. An alternative is to consider, Markovian startegies, extraction strategies that are stock dependent. The resulting equilibrium is then subgame perfect (see Dockner et al. (2000) Ch. 4). The use of Markovian strategies assumes the ability of firms to adjust their production plans. This flexibility may not be a realistic assumption in resources such as oil, for technical reasons (see e.g., Anderson et al. (2017)). Another drawback of such strategies is that the model becomes analytically untractable and one must rely on numerical methods. Moreover, such numerical methods suffer from the curse of dimensionality: they fail to deliver a solution in cases such as ours where the number of state variables can be large. We have opted for the set of open-loop strategies for analytical tractability. Indeed, the possibility of a profitable merger will be proven analytically in the case of stock levels that are small enough and the numerical analysis of the case of arbitrary stock levels can be handled with built-in functions available in softwares such as Mathematica or Maple.

2.2 The Pre-Merger Equilibrium

We now proceed to characterize an OLNE of the game above. We will show that all firms exhaust their stocks in finite time. Let T_S and T_L denote the time at which firms with stocks S_{0S} and S_{0L} exhaust their stocks respectively. We will show that when $S_{0S} < S_{0L}$ we have $T_S < T_L$. The equilibrium then consists of two phases: phase I from date 0 to T_S and phase II from T_S to T_L. During phase I, the extraction of both types of firms is positive except at T_S where the extraction and the stock of each firm $i = 1, .., n_S$ reach zero. During phase II, only firms $i = n_S + 1, .., n$ still own a positive stock except at T_L where the extraction and the stock of each firm reach zero. We denote by q_S and q_L the extraction paths of each firm with stock S_{0S} and S_{0L} respectively.

Proposition 1: *Let*

$$q_S \equiv \begin{cases} \frac{(a-c)}{b(1+n)}\left(1 - e^{-r(T_S - t)}\right) & \text{for} \quad t \in [0, T_S] \\ 0 & \text{for} \quad t \geq T_S \end{cases}, \tag{3}$$

$$q_L \equiv \begin{cases} \frac{(a-c)}{b(1+n)}\left(1-\frac{1+n}{1+n_L}e^{-r(T_L-t)}+\frac{n_S}{1+n_L}e^{-r(T_S-t)}\right) & t\in[0,T_S] \\ \frac{(a-c)}{b(1+n_L)}\left(1-e^{-r(T_L-t)}\right) & t\in[T_S,T_L] \\ 0 & t\geq T_L \end{cases} \tag{4}$$

where T_S and T_L are the unique solutions to:

$$\int_0^{T_S} q_S(t)\,dt = S_{0S} \tag{5}$$

and

$$\int_0^{T_L} q_L(t)\,dt = S_{0L} \tag{6}$$

Then the n-tuple vector q^{eq} where $q_j^{eq} = q_S$ when $j = 1,..,n_S$ and $q_k^{eq} = q_L$ when $k = n_S+1,..,n$ constitutes an OLNE.

Proof:

Each firm i takes the supply paths of its rivals as given and maximizes (2) subject to (1). The current value Hamiltonian associated with the problem of firm i is given by:

$$H_i(q_i, q_{-i}, \lambda_i, t) = [a - b(Q_S + Q_L) - c]\, q_i + \lambda_i(-q_i).$$

where q_{-i} is an $n-1$-tuple vector obtained from the vector q by deleting its ith component q_i.

The maximum principle, and exploiting symmetry, yield interior solutions.

$$a - b\left((1+n_S)\,q_j + n_L q_k\right) - c - \lambda_j = 0 \tag{7}$$

$$a - b\left(n_S q_j + (1+n_L)\,q_k\right) - c - \lambda_k = 0 \tag{8}$$

for $j = 1, .., n_S$ and $k = n_S + 1, .., n$, with

$$\begin{aligned} \dot{\lambda}_j &= r\lambda_j \quad (9) \\ \dot{\lambda}_k &= r\lambda_k \quad (10) \end{aligned}$$

During the phase where only firms $k = n_S + 1, .., n$ have a positive extraction, the maximum principle yields the following:

$$a - b\left(1 + n_L\right) q_k - c - \lambda_k = 0 \quad (11)$$

with

$$\dot{\lambda}_k = r\lambda_k. \quad (12)$$

The terminal times T_S and T_L are endogenous and determined by

$$H_j\left(q_j\left(T_S\right), q_{-j}\left(T_S\right), \lambda_j\left(T_S\right), T_S\right) = 0$$

for $j = 1, .., n_S$ and

$$H_k\left(q_k\left(T_L\right), q_{-k}\left(T_L\right), \lambda_k\left(T_L\right), T_L\right) = 0$$

for $k = 1 + n_S, .., n$. These last conditions along with the maximum principle imply that

$$q_j\left(T_S\right) = 0 \text{ and } q_k\left(T_L\right) = 0. \quad (13)$$

Solving for (q_j, q_k), from (7) and (8), we obtain the following:

$$\begin{aligned} \frac{a - c - \lambda_j}{b} &= \left(1 + n_S\right) q_j + n_L q_k \quad (14) \\ \frac{a - c - \lambda_k}{b} &= n_S q_j + \left(1 + n_L\right) q_k \quad (15) \end{aligned}$$

or,

$$q_j = \frac{\frac{a-c-\lambda_j}{b}\left(1+n_L\right) - \frac{a-c-\lambda_k}{b}\left(n_L\right)}{1+n}$$

which yields,

$$q_j = \frac{\left(a-c-\lambda_j\right)\left(1+n_L\right) - \left(a-c-\lambda_k\right)n_L}{b\left(1+n\right)}.$$

Thus, by symmetry, we have the following:

$$q_j = \frac{a-c-\lambda_j\left(1+n_L\right)+\lambda_k n_L}{b\left(1+n\right)} \tag{16}$$

$$q_k = \frac{a-c-\lambda_k\left(1+n_S\right)+\lambda_j n_S}{b\left(1+n\right)} \tag{17}$$

From (9), (10), (12) and continuity of the costate variable λ_k at T_S, we have

$$\lambda_j = \lambda_{j0}e^{rt} \text{ for all } t \in [0, T_S] \tag{18}$$

$$\lambda_k = \lambda_{k0}e^{rt} \text{ for all } t \in [0, T_L] \tag{19}$$

The costate variables, λ_{j0} and λ_{k0}, are determined using conditions (13), along with (16) and (11). From (11), we have the following:

$$q_k\left(T_L\right) = \frac{a-c-\lambda_{k0}e^{rT_L}}{b\left(1+n_L\right)} = 0$$

That is,

$$\left(a-c\right)e^{-rT_L} = \lambda_{k0}$$

From (16), we have the following:

$$q_j\left(T_S\right) = \frac{a-c-\lambda_{j0}e^{rT_S}\left(1+n_L\right)+\lambda_{k0}e^{rT_S}n_L}{b\left(1+n\right)} = 0$$

and

$$\frac{(a-c)\,e^{-rT_S}+\lambda_{k0}n_L}{(1+n_L)}=\lambda_{j0}$$

or

$$\frac{(a-c)\left(e^{-rT_S}+e^{-rT_L}n_L\right)}{(1+n_L)}=\lambda_{j0}$$

Using λ_{j0} and λ_{k0} and substituting (18) and (19) into (16), (17) and (11) yields (3) and (4) for $t \leq T_S$.

For $t \geq T_S$, we have a symmetric equilibrium between n_L firms that exhaust a stock during a time length of $T_L - T_S$. We can therefore use the analysis above to obtain such equilibrium path.

The equilibrium paths (3) and (4) are determined as functions of the terminal times T_L and T_S. These dates are determined from the resource constraint conditions, i.e., (5) and (6). It can be shown that such a non-linear system in (T_L, T_S) admits a unique solution with $T_L \geq T_S$ ■

We can now compute the value function of each player, which constitute a building block to conduct the analysis of the profitability of a merger. The equilibrium discounted sum of profits of an individual firm is given by:

$$V_S=\int_0^{T_S}\left[a-b\left(n_S q_i^{eq}(t)+n_L q_j^{eq}(t)\right)-c\right]q_i^{eq}(t)\,e^{-rt}dt \tag{20}$$

and

$$V_L=\int_0^{T_L}\left[a-b\left(n_S q_i^{eq}(t)+n_L q_j^{eq}(t)\right)-c\right]q_j^{eq}(t)\,e^{-rt}dt \tag{21}$$

for $i = 1,..,n_S$ and $j = n_S+1,..,n$. It will be useful to explicitly write the equilibrium discounted sum of profits as functions of $(n_S, n_L, S_{0S}, S_{0L})$. Substitution of q^{eq} and integrating gives respectively $V_S(n_S, n_L, S_{0S}, S_{0L})$ and $V_L(n_S, n_L, S_{0S}, S_{0L})$. The expressions of these functions for any $(n_S, n_L, S_{0S}, S_{0L})$ are too cumbersome to report here. Instead we report these value functions for two special cases that will be instrumental in the analysis of a horizontal merger in the following section. First, we have

$$V_S(n-1,1,S_0,S_0)=\frac{a^2e^{-rT}\left(rT(n-1)+e^{rT}+ne^{-rT}-n-1\right)}{br(n+1)^2} \tag{22}$$

where the function $V_S(n-1,1,S_0,S_0)$ is the discounted sum of profits of a single firm in a symmetric n firm oligopoly, where each firm owns a stock S_0. Second, we have

$$V_L(n-m,1,S_0,mS_0)=\frac{(a-c)^2}{4br}\left(e^{r(T_S-2T_L)}\left(1-e^{r(T_L-T_S)}\right)^2+\frac{\Psi}{(2-m+n)^2}\right) \tag{23}$$

with

$$\Psi\equiv 4-e^{-2rT_S}(n-m)^2+4e^{-rT_S}(-2+(m-n)(1-rT_S))+\left(e^{-2rT_L}+e^{-rT_S}-e^{r(T_S-2T_L)}\right)(2-m+n)^2$$

where the function $V_L(n-m,1,S_0,mS_0)$ represents, in an $n-m+1$ firms asymmetric oligopoly, the value function of a single firm that owns a stock mS_0 where $m\in\{2,..,n\}$ while each of the remaining $n-m$ firms owns a stock S_0.

Note that the equilibrium discounted sum of profits, reported in (22) and (23), are expressed in terms of terminal times, T in the case of (22) and T_S and T_L in the case of (23). The stocks do not appear in their expressions. However, the terminal times themselves depend on S_0. The relationship between the terminal times T, T_S and T_L and the stock S_0 will be discussed in more detail in the next section.

3 Profitability of Mergers

We exploit the characterization of the OLNE of the game above to investigate the profitability of mergers of a subset of firms in the industry. We first focus on the case of a merger when all firms' initial stocks are equal, and provide an analytical proof to show that a merger is always profitable when the stock of the resource is small enough. We then examine, through numerical simulations, the impact of a merger on the speed of extraction of the resource as well as the case where the stock of an outsider may differ from the stock of the merging firms.

3.1 Mergers in a symmetric oligopoly

We focus here on the case where all n firms own the same initial stock: $S_0 \equiv S_{0S} = S_{0L}$. We consider a merger of a subset m of the n firms. Since the marginal cost of extraction is constant, the market structure after a merger corresponds to an asymmetric oligopoly competition between $n-m+1$ firms. The merged entity owns an initial stock equal to mS_0 and the $n-m$ outsider firms each own an initial stock equal to S_0. For simplicity, henceforth we set $c=0$, which does not qualitatively affect our results. The OLNE for this post merger oligopoly is readily available from Proposition 1 by setting $n_L = 1$, $S_{0L} = mS_0$, $n_S = n-m$ and $S_{0S} = S_0$.

A merger is profitable when

$$G \equiv V_L\left(n-m, 1, S_0, mS_0\right) - mV_S\left(n-1, 1, S_0, S_0\right) > 0.$$

From (5), it follows that T is given by:

$$\frac{a}{br(1+n)}(e^{-rT} - 1 + rT) = S_0 \tag{24}$$

It can be shown that this condition defines a unique T for each value of S_0. We denote this value by T_{pre}, the terminal time at which the stock is exhausted in the pre-merger game. It can be shown that T_{pre} is a strictly increasing function of S_0 with $T_{pre} = 0$ when $S_0 = 0$. For values of S_0 close to 0 we have the following Lemma.

Lemma 1: When $S_0 \to 0$, we have $T_{pre} \to 0$ with

$$\frac{arT_{pre}^2}{2b\left(1+n\right)} + O\left[T_{pre}\right]^3 = S_0 \tag{25}$$

where $O\left[T_{pre}\right]^3$ is an expression such that $\lim_{T_{pre}\to 0}\frac{O[T_{pre}]^3}{T_{pre}^2}=0$.

Proof: This follows from total differentiation of (24) and using the second order Taylor series expansion of e^{-rT} around 0.■

Remark 1: Lemma 1 also allows us to establish that $\lim_{S_0\to 0}\frac{S_0}{T_{pre}^2}=\frac{ar}{2b(1+n)}$ and that $\lim_{S_0\to 0}\frac{O[T_{pre}]^3}{S_0}=0$.

Similarly for the case of the merger, the times at which the n_S outsider firms and the n_L merging firms respectively exhaust their resources are T_S and T_L that solve (5) and (6). It can be shown that T_S and T_L are determined as the solutions to system below

$$S_0=\frac{(a-c)\left(e^{-rT_S}+rT_S-1\right)}{b\left(2+n-m\right)} \tag{26}$$

and

$$mS_0=\frac{(a-c)\left(-e^{-rT_S}\left(n-m\right)+e^{-rT_L}\left(2+n-m\right)-2+rT_L\left(2+n-m\right)-\left(n-m\right)rT_S\right)}{2b\left(2+n-m\right)r}. \tag{27}$$

Lemma 2: When $S_0\to 0$, we have $T_S\to 0$ and $T_L\to 0$ with

$$\frac{arT_S^2}{2b\left(2+n_S\right)}+O\left[T_S\right]^3=S_0 \tag{28}$$

and

$$\frac{arT_L^2}{2b}-\frac{an_Sr}{2b\left(2+n_S\right)}T_S^2+O\left[T_L\right]^3+O\left[T_S\right]^3=2n_L^2S_0 \tag{29}$$

where $O\left[T_S\right]^3$ and $O\left[T_L\right]^3$ are expressions such that $\lim_{T_S\to 0}\frac{O[T_S]^3}{T_S^2}=0$ and $\lim_{T_L\to 0}\frac{O[T_L]^3}{T_L^2}=0$.

Proof: This follows from total differentiation of (5), (6) and using the second order Taylor series expansion of e^{-rT_S} and e^{-rT_L} around 0.■

Remark 2: Lemma 2 also allows us to establish that $\lim_{S_0\to 0}\frac{S_0}{T_S^2}=\frac{ar}{2b(2+n_S)}$ and that $\lim_{S_0\to 0}\frac{O[T_S]^3}{S_0}=$

0 along with $\lim_{S_0 \to 0} \left(\frac{arT_L^2}{2bS_0} - \frac{an_S r}{2b(2+n_S)} \frac{T_S^2}{S_0} \right) = 2n_L^2$, which yield the following:

$$\lim_{S_0 \to 0} \left(\frac{arT_L^2}{2bS_0} \right) - \lim_{S_0 \to 0} \left(\frac{an_S r}{2b\,(2+n_S)} \frac{T_S^2}{S_0} \right) = 2n_L^2$$

or,

$$\lim_{S_0 \to 0} \left(\frac{arT_L^2}{2bS_0} \right) - \frac{an_S r}{2b\,(2+n_S)} \frac{2b\,(2+n_S)}{ar} = 2n_L^2$$

and after simplification,

$$\lim_{S_0 \to 0} \left(\frac{arT_L^2}{2bS_0} \right) = 2n_L^2 + n_S$$

and that $\lim_{S_0 \to 0} \frac{O[T_L]^3}{S_0} = 0$.

In order to analyze the gains from a merger G when S_0 is close to 0 we first note that G can be expressed as a function of the dates of exhaustion of the stocks (T_{pre}, T_S, T_L), and that a change in the stock S_0 affects G through its impact on (T_{pre}, T_S, T_L). We observe that when $S_0 = 0$, we have $G = 0$ and when $S_0 \to 0$, we have $(T_{pre}, T_S, T_L) \to 0$. We have the following lemmata:

Lemma 3: When $T_S \to 0$ and $T_L \to 0$, we have

(i)

$$V_S\,(n-1, 1, S_0, S_0) = \frac{a^2 r}{2b\,(1+n)} T_{pre}^2 + O\,[T_{pre}]^3$$

(ii)

$$V_L\,(n-m, 1, S_0, mS_0) = \frac{a^2 r}{4b} T_L^2 - \frac{a^2\,(n-m)\,r}{8b + 4b\,(n-m)} T_S^2 + O\,[T_L]^3 + O\,[T_S]^3$$

Proof: This follows from total differentiation of $V_S\,(n-1, 1, S_0, S_0)$ with respect to T_{pre} and $V_L\,(n-m, 1, S_0, mS_0)$ with respect to (T_S, T_L) and using the second order Taylor series expansion of e^{-rT}, e^{-rT_S} and e^{-rT_L} around 0.■

We are now ready to determine the sign of the gains from a merger when S_0 is close to 0.

Proposition 2: *For S_0 positive and sufficiently small, the gains from a merger are always positive.*

Proof: Using Lemma 3, we can write the gains from a merger, $G \equiv V_L(n-m+1, 1, mS_0, S_0) - mV_S(n-1, 1, S_0, S_0)$ when $S_0 \to 0$ as

$$G = \frac{a^2 r}{4b} T_L^2 - \frac{a^2 (n-m) r}{8b + 4b(n-m)} T_S^2 + O[T_L]^3 + O[T_S]^3 - m\left(\frac{a^2 r}{2b(1+n)} T_{pre}^2 + O[T_{pre}]^3\right)$$

which gives, after using Lemma 1 and Lemma 2, the following:

$$G = a\left(m^2 S_0 - mS_0\right) + O[T_{pre}]^3 + O[T_L]^3 + O[T_S]^3$$

or

$$G = am(m-1) S_0 + O[T_{pre}]^3 + O[T_L]^3 + O[T_S]^3$$

where $O[T_{pre}]^3, O[T_L]^3, O[T_S]^3$ are expressions such that $\lim_{S_0 \to 0} \frac{O[T_{pre}]^3}{S_0} = 0$, $\lim_{S_0 \to 0} \frac{O[T_L]^3}{S_0} = 0$, $\lim_{S_0 \to 0} \frac{O[T_S]^3}{S_0} = 0$ (see Remarks 1 and 2 above). Therefore, when S_0 is positive and close enough to 0 we have

$$G \simeq am(m-1) S_0 > 0 \text{ when } m > 1. \blacksquare$$

Proposition 2 provides a sharp contrast to the case of a standard Cournot model without resource stock contraints. Within a similar setting to ours, that is, with linear demand and constant marginal cost, Salant et al (1983) show that unless 80% of the industry participates in a given merger, the merger is not profitable. As $S_0 \to \infty$, our model converges to the standard Cournot setting. However, in the presence of stock constraints, there exists a range of stocks for which even the smallest merger ($n_L \geq 2$ merger participants, where n_S may be arbitrarily large) is profitable. This is because, unlike in the standard Cournot model, where outsiders respond to the merger by increasing output and mitigating the merger participants' gain in market power, in this model, the outsiders are restricted in their response due to their resource constraints. Within our context, the n_S outsiders exhaust their stocks earlier than the merger participants, that is, $T_L > T_S$ in the post-merger equilibrium with symmetric firms. This results in greater merger-induced market power than in the standard Cournot model.[7]

3.2 Mergers and resource extraction

We examine here the extraction paths of a firm in the premerger case, a firm not part of a merger, and a firm that is part of the merger respectively.

Figure 1 below provides a plot of the dates of exhaustion of the stocks (T_{pre}, T_S, T_L), of a firm in the premerger case, a firm not part of a merger and a firm that is part of the merger respectively, for $n = 3$ and $m = 2$. We follow Kagan *et al.* (2015), and choose the following parameter values: $a = 0.855$, $b = a^2$, and $r = 0.1$. These dates are plotted against the stock owned by each firm, S_0 with $S_0 \equiv S_{0S} = S_{0L}$; i.e., firms are symmetric in the premerger game.

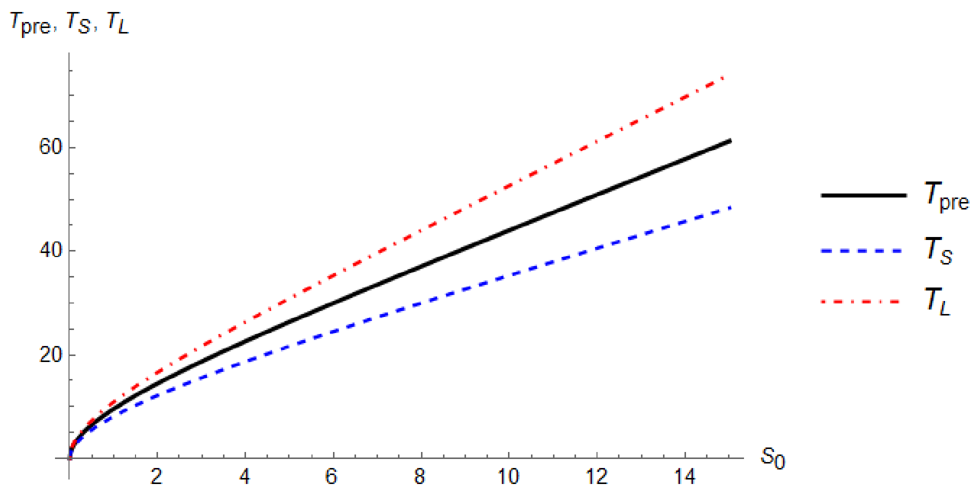

Figure 1: Terminal dates as a function of initial stock

As Figure 1 illustrates, we have $T_S < T_{pre} < T_L$. When a merger occurs, the outsider firm tends to extract its stock faster than in the premerger scenario, whereas each of the merging firms extracts its stock slower than in the premerger case. This is in line with

[7] The mechanism captured by our model, driving the profitability of mergers, also applies to cases of partial cartelizations which have been an important feature of nonrenewable resource markets. In the case of partial cartelization, in response to the cartel's reduction in output, the non-members would be hindered in their attempt to expand output due to limited resource stocks, similar to the outsiders to the merger in our model, making the cartel more profitable.

standard oligopoly theory where for strategic substitutes, a merger induces the merging firms to reduce their output which is met by outsider firms expanding their output. The specificity of a nonrenewable industry is that the overall extraction is constant. However, the rate of extraction of the total industry is a priori undertermined.

Our simulations point to the result that a merger results in an overall slower extraction of the industry's stock. This is illustrated by Figure 2 below which plots the total initial industry's extraction (i.e., at date 0), using the same parameter values as in the previous figure, under a merger and a premerger scenario againts the stock owned by each firm, S_0. Let Q_{pre} denote pre-merger industry extraction and Q_{post} denote post-merger industry extraction.

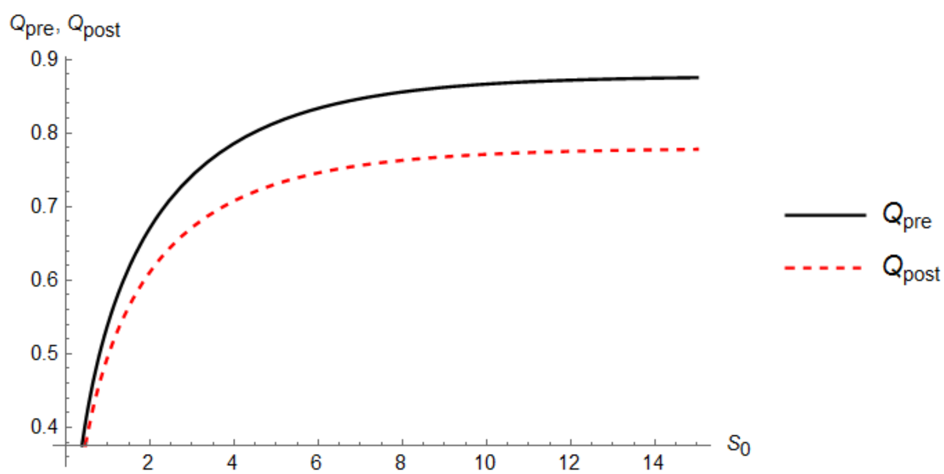

Figure 2: Initial industry extraction as a function of initial stock

It can be observed from Figure 2 that a merger results in an initial industry's extraction rate that is slower than in the premerger case.

As a robustness check we have runs simulations for all combinations of n, m with $n \leq 10$ and $m \in \{2, n-1\}$ and they all delivered the same qualitative conclusion: a merger results in a slower initial extraction of the industry compared to the premerger case[8].

[8]The values of a, b, r have no impact on the conclusions drawn from the simulations.

3.3 The case of an asymmetric oligopoly

Next, we use a numerical example to illustrate that Proposition 2 carries over to cases where the firms have heterogeneous stocks using the same parameter values as in the previous figure. Consider three firms, two with initial stock S_{0L}, and one with an initial stock $S_{0S} = fS_{0L}$, where $f \in (0, 1]$. Figure 3 illustrates the gains from a merger of the two firms with initial stock S_{0L} as a function of S_{0L}. In line with Proposition 2, for a sufficiently small value of S_{0L}, we have that the merger is profitable.

Figure 3: Gains from merger as a function of S_{0L}

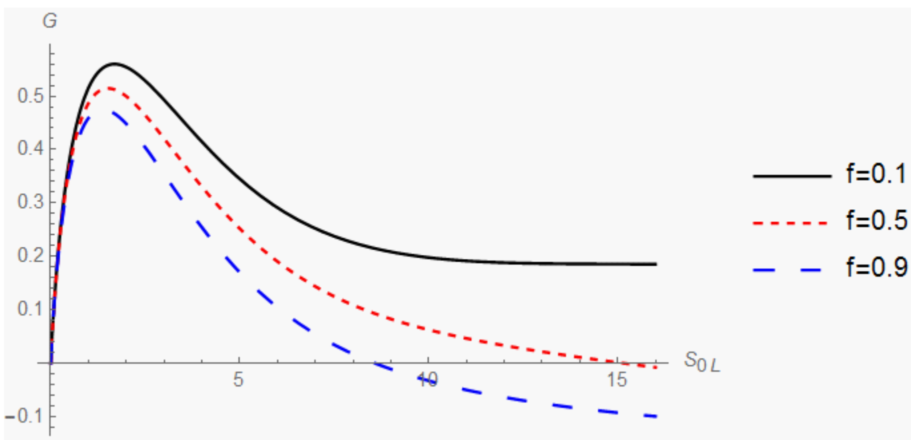

Figure 3: Gains from merger as a function of S_{0L}

We use our numerical example to illustrate two results that are robust to changes in parameter values.

Result 1: *A given merger is more likely to be profitable the smaller the stock of the outsider and the larger the stocks of the merger participants.*

Result 1 is illustrated by Figure 3 where the smaller is f, the greater the range of S_{0L} for which the merger is profitable. This may explain why some of the largest firms in the oil extraction sector have merged since the 1990's. This result is intuitive: we know that a merger of two firms in a duopoly is always profitable. The smaller the outsider firm's stock relative to the stock of the merged firms when two firms merge, the closer the profit of the merged firm to the profit of a merger of firms in a duopoly. Therefore, a merger of two firms in the case of a triopoly, when the outsider firm's stock is small relative to the stock of the merged firms, is more likely to be profitable than when the outsider firm's stock is large relative to the stock of the merged firms.

4 Impact of a tax on extraction

Non renewable resource industries can be important sources of pollution. A natural policy instrument that is often considered is the imposition of a unit tax on the resource produced. We examine here the interplay of such policy on firms' incentives to merge and, thus, on the industry's market structure.

More precisely, we examine the effect of stricter environmental policies. For simplicity, we consider a scenario where each unit of extraction generates a unit of emissions. The policy maker sets a constant tax per unit of extraction, τ. Thus, the objective function of each firm i is given by:

$$\max_{q_i} \int_0^{\infty} e^{-rt} \left[a - b\left(Q_S\left(t\right) + Q_L\left(t\right)\right) - c - \tau\right] q_i\left(t\right) dt \tag{30}$$

subject to the resource constraint. The OLNE and the analysis of profitability above can be directly exploited, where the marginal cost of extraction is simply augmented by τ. Our numerical simulations yield the following result.

Result 2: *A given merger is less likely to be profitable the higher the emission tax.*

Result 2 is illustrated by Figure 4, where the higher is τ, the smaller the range of S_{0L} for which the merger is profitable.

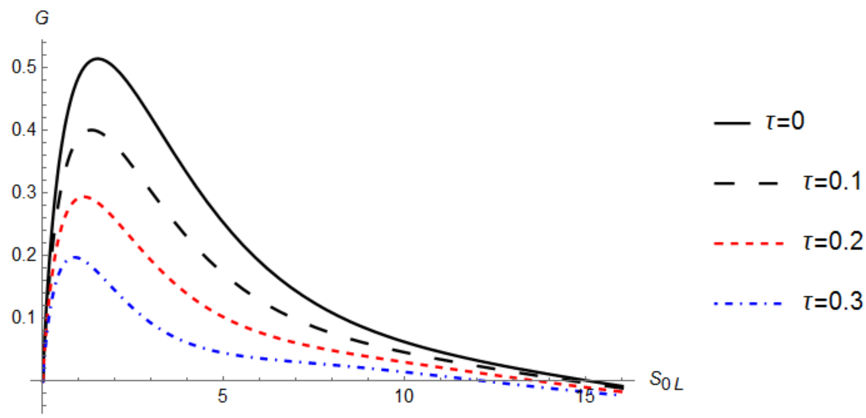

Figure 4: Gains from merger as a function of S_{0L} with $f = 0.5$

An implication of Result 2 is that the emission of pollutants may be affected in a perverse manner due to the imposition of a higher emission tax, resulting in a green paradox. We note that in Hotelling models in which the long-run cumulative extraction is not impacted by the tax, a constant unit tax induces markets to postpone the extraction of the resource. Indeed, a constant tax means that the present-value tax decreases over time, encouraging markets to delay extraction, irrespective of the market structure. Thus, the first effect of the tax policy under study is opposite to a green paradox. At the same time, Result 2 shows that a tax might deter a merger, which increases the speed of extraction. This is the second effect, which drives the green paradox. In what follows, we illustrate that, when a merger is deterred due to a higher emission tax, the second effect dominates within our context.

Indeed as noted above in the analysis of the impact of a merger on the industry's speed of extraction of the resource in Section 3.2, a merger results in a smaller initial extraction of the resource than in the premerger scenario. If the increase in τ prevents a merger from being realized that would otherwise have occured, the path of emissions in equilibrium is altered such that more emissions occur earlier than would have been the case with a lower tax level. This is illustrated with the following example of a symmetric triopoly where we set again $a = 0.8555$, $b = 0.8555^2$, $r = 0.1$ and examine the impact of a unit tax $\tau = 0.05$. Figure 5 contains the plots of the industry's initial extraction rate as a function of the stock in four cases: (i) symmetric triopoly and $\tau = 0$, (ii) symmetric triopoly and $\tau = 0.05$, (iii) merger of 2 firms among 3 and $\tau = 0$ and (iv) merger of 2 firms among 3 and $\tau = 0.05$. We again note that for both $\tau = 0$ and $\tau = 0.05$, for all stock levels a merger results in a decrease of the initial industry's extraction rate. To illustrate the implication of Result 2 we consider the case where $S_0 = S_{0L} = S_{0S} = 7.5$. The gains from a merger when $\tau = 0$ ($\tau = 0.05$) are 0.0154 (-0.0006). That is, a merger is profitable under $\tau = 0$ and unprofitable when $\tau = 0.05$. We, thus, need to compare the extraction rates under case (ii) and case (iii). From Figure 5 we can observe that the industry's extraction rate under case (ii) is above that of case (iii). Figure 6 gives the plots of the path of the cumulative industry's extraction rate under case (ii) and case (iii): when the profitability of a merger is taken into account, a tax of $\tau = 0.05$ results in a faster depletion of the resource than the case of $\tau = 0$.

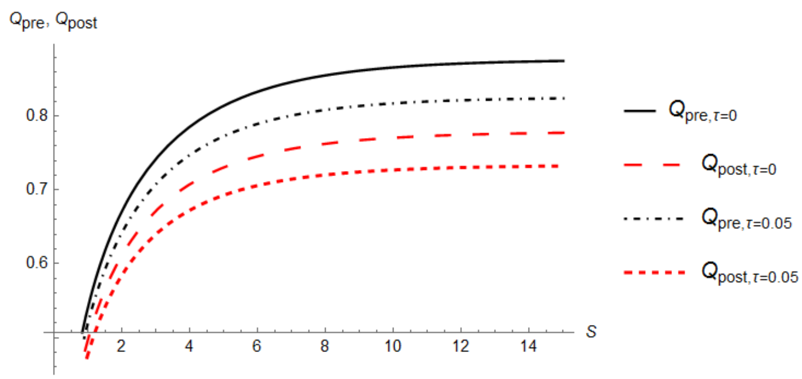

Figure 5: Initial industry extraction as a function of initial stock

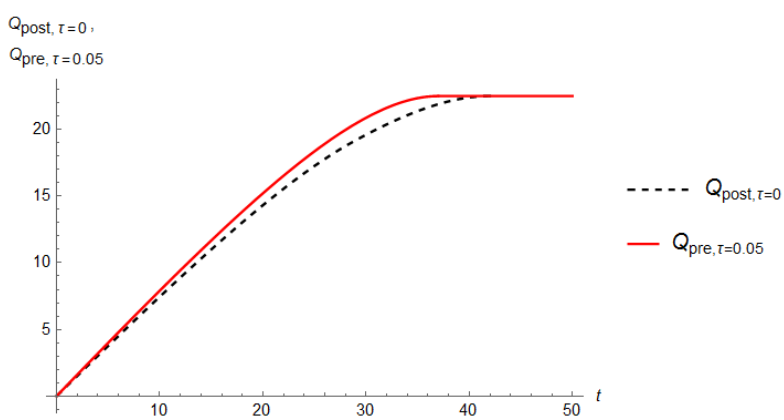

Figure 6: Industry extraction path

In cases where the damage from pollution, which we have not explicitly modeled in this paper, is convex in the emission level and/or where pollution is accumulative, the prevention of the merger due to the higher tax could ultimately adversely affect the environment.

5 The case of economic exhaustion

Thus far, we have considered the case of physical exhaustion of the resource. For some resources, the marginal cost of extraction is a decreasing function of the stock. In this case, it is possible that the cost of extraction increases to levels that render the exploitation of the remaining stock unprofitable: some stock is left in the ground. We examine this case below, and revisit the profitability of a merger in an oligopolistic industry.

The marginal cost of extraction from each mine[9] is given by:

$$c_0 - c_1 S$$

where $c_0 > 0$ and $c_1 > 0$ are such that the demand's choke price $a < c_0$, which implies that a mine will be shut down when the stock reaches the following level:

$$\bar{S} \equiv \frac{c_0 - a}{c_1}.$$

For the purpose of comparison we limit our attention to the case of a symmetric triopoly each firm owns an initial stock S_0. We assume that $S_0 > \bar{S}$. Let $x \equiv S - \bar{S}$, it is straightfoward to rewrite the discounted sum of profits of firm $i = 1, 2, 3$ as

$$\int_0^{\infty} \left(-bQ + c_1 x_i\right) q_i e^{-rt} dt \tag{31}$$

Firm i maximizes (31) subject to

$$\dot{x}_i = -q_i \tag{32}$$

[9] We use the term "mine" to represent more generally a single facility from which the resource is extracted.

and

$$x_i(0) = x_0 \equiv S_0 - \bar{S}. \tag{33}$$

Proposition 3: *Let*

$$q(t) = -\sigma e^{\sigma t} x_0 \tag{34}$$

where

$$\sigma \equiv \frac{1}{2}\left(r - \sqrt{r(c_1 + r)}\right) < 0$$

then the vector (q, q, q) *constitutes an OLNE of the game above.*

Proof: See Appendix.

From Proposition 3, it follows that the equilibrium discounted sum of profits of a single firm is given by:

$$\begin{aligned} \int_0^\infty -(3b\sigma_1 x + c_1 x)\,\sigma_1 x e^{-rt} dt &= -x_0^2 (3b\sigma_1 + c_1)\,\sigma_1 \int_0^\infty e^{-(-2\sigma_1 + r)t} dt \\ &= -\sigma_1 \frac{3b\sigma_1 + c_1}{-2\sigma_1 + r} x_0^2 \end{aligned}$$

We now consider a merger of 2 firms. Since the marginal cost of extraction is no longer constant, we can no longer use the pre-merger equilibrium to deduce the post-merger equilibrium. We characterize below the post-merger equilibrium. More precisely, we consider an industry that consists of two firms, an outsider firm that owns a single mine and a merged entity that owns two mines, mine 1 and mine 2. Let $S_S(.)$, $S_{L1}(.)$ and $S_{L2}(.)$ respectively denote the stock path of the outsider firm and mine 1 and mine 2 of the merged entity with $S_S(0) = S_{0S}$, $S_{L1}(0) = S_{0L1}$ and $S_{L2}(0) = S_{0L2}$. Denote by $q_S(.)$ the extraction path of the outsider firm and by $q_{L1}(.)$ and $q_{L2}(.)$ the extraction paths of the merged entity from mine 1 and mine 2 respectively. We drop the argument of the paths whenever the explicit reference to time is not necessary. While the outsider firm chooses q_S the merged entity chooses a pair of extraction paths $\{q_{L1}, q_{L2}\}$.

Proposition 4: *Suppose* $S_{0S} = S_{0L1} = S_{0L2} = S_0$ *and let* $x_0 = S_0 - \bar{S}$*, then the vector*

$(q_o, \{q_m, q_m\})$ *where*

$$q_o(t) = -\frac{1}{6}((3+\sqrt{3})\mu_1 e^{\mu_1 t} - (-3+\sqrt{3})\mu_3 e^{\mu_3 t})x_0 \tag{35}$$

and

$$q_m(t) = \frac{1}{6}(-(3+2\sqrt{3})\mu_1 e^{\mu_1 t} + (-3+2\sqrt{3})\mu_3 e^{\mu_3 t})x_0 \tag{36}$$

with

$$\mu_1 = \frac{1}{2}r - \frac{1}{2}\sqrt{\frac{1}{3b}r\left(6c_1 - 2\sqrt{3}c_1 + 3br\right)} < 0 \tag{37}$$

and

$$\mu_3 = \frac{1}{2}r - \frac{1}{2}\sqrt{\frac{1}{3b}r\left(6c_1 + 2\sqrt{3}c_1 + 3br\right)} < 0. \tag{38}$$

constitutes an OLNE between the merged entity and the outsider firm.

Proof: See Appendix.

It is now straightforward, using Propositions 3 and 4, to compute the equilibrium discounted sum of profits for each firm under both the pre-merger scenario and when a merger occurs, from which we can then infer the profitability of a merger. Let $W_{ol}(x_0)$ denote the equilibrium discounted sum of profits of a merged entity and $V_{ol}(x_0)$ denote the equilibrium discounted sum of profits for the two firms that merge under the pre-merger scenario, when all firms own identical stocks x_0. It can be shown that

$$V_{ol}(x_0) = 2\frac{\sigma(c_1 + 3\sigma)}{2\sigma - r}x_0^2$$

and that

$$W_{ol}(x_0) = -\frac{1}{6}\left(\Omega c_1 - \frac{\left(19+11\sqrt{3}\right)\mu_1^2}{2\mu_1 - r} + \frac{\left(-19+11\sqrt{3}\right)\mu_3^2}{2d_3 - r} + \frac{2\mu_1\mu_3}{\mu_1 + \mu_3 - r}\right)x_0^2$$

where

$$\Omega \equiv \mu_1 \left(-\frac{7}{(2\mu_1 - r)} - \frac{4\sqrt{3}}{(2\mu_1 - r)} + \frac{1}{\mu_1 + \mu_3 - r} \right)$$
$$+\mu_3 \left(-\frac{7}{(2\mu_3 - r)} + \frac{4\sqrt{3}}{(2\mu_3 - r)} + \frac{1}{\mu_1 + \mu_3 - r} \right).$$

Result 3: *A merger of two firms can be profitable.*

This is illustrated in Figure 7 using a numerical example, where we set $r = 0.05, b = 1$ and $x_0 = 100$, and plot the gains from a merger $W_{ol}(x_0) - V_{ol}(x_0)$ as a function of c_1.

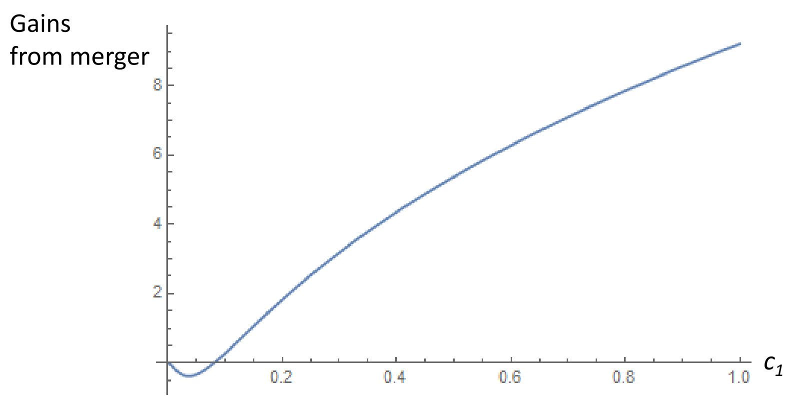

Figure 7: Gains from merger as a function of c_1

We highlight two important qualitative differences with the case of physical scarcity: (i) when firms' initial stocks are identical, whether a merger is profitable or not does not depend on the stock size, (ii) the gains from a merger, even if positive are typically relatively very small.

Figure 8 illustrates how the relative gains from a merger, i.e. $\frac{W_{ol}(x_0) - V_{ol}(x_0)}{V_{ol}(x_0)}$, depend on the parameter c_1. The maximum gain from a merger is approximately 0.3% of the total payoff under no-merger. The larger is c_1 the more important the role of the cost effect in the firm's payoffs, and the less important is the market interaction between firms. In this case, the extraction and payoff of a firm when a given merger occurs and when it does not occur are very close to each other.

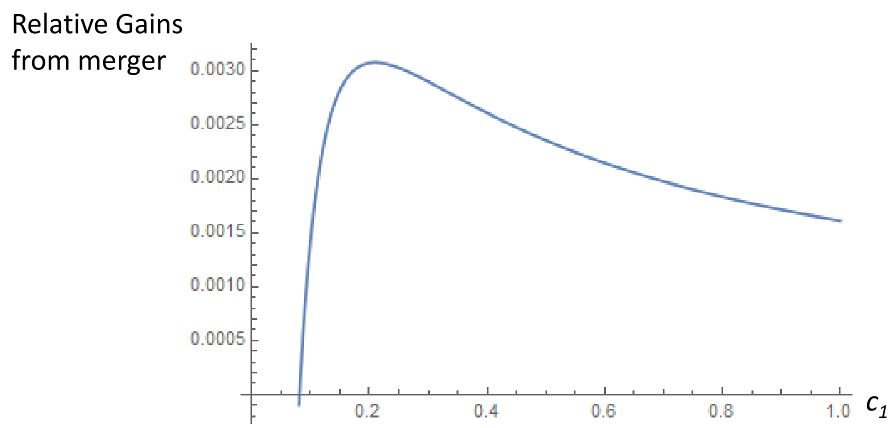

Figure 8: Relative gains from merger as a function of c_1

When c_1 is sufficiently small then the market interaction is relatively more important than the stock effect on costs such that we tend to retrieve the standard result from static oligopoly theory that a merger of two firms is not profitable. These qualitive results are robust to changes in the values of the parameters b and r.

6 Conclusion

We showed that when the resource stocks are small enough, small mergers (mergers of two firms) are profitable under constant marginal costs and may also be profitable when the marginal cost is decreasing in the resource stock. The profitability of the mergers arises because outsiders are limited in their response in terms of increased output due to their finite resource stocks. As such, these mergers allow the merger participant to raise price more than in industries without stock constraints. Therefore, antitrust authorities should be cautious when ruling on mergers in exhaustible resource industries. At the same time, mergers in these industries reduce environmental damage by delaying extraction.

The interplay of environmental regulations and incentives to merge delivered interesting insights: an environmental tax can affect merger profitability. In the case of a polluting resource, a unit tax on extraction can deter mergers and, therefore, may result in causing emissions earlier than under laissez-faire; and a small increase of the tax may result in a non marginal jump-up of the industry's pollution.

In our analysis, we have ignored the possibility of the presence of a perfect substitute to the resource. This has simplified the analysis and at the same time allowed for direct contrast of our results with those obtained in the SSR framework. However, for example in the case of fossil fuels, the existence of a backstop technology that can provide in abundance a substitute to the resource at a given fixed price and is supplied by a perfectly competitive market has received a significant amount of attention. In this case, the resource owner, e.g., OPEC, can resort to limit pricing: charging a constant price during a period of time that is just enough to undercut the backstop substitute. Recent important contributions on limit pricing arising in fossil fuel extraction include Andrade de Sa and Daubane (2016) and Van der Meijden, Ryszka and Withagen (2018)[10], or Salant (1977) and Hoel (1978, 1984) for earlier contributions. A natural and relevant follow-up research question to our analysis is to examine the profitability of a merger or of cartel formation[11] in the presence of a backstop substitute. In that case, if, for example, initial endowments of the nonrenewable resource and/or demand elasticity are such that limit pricing takes place from the initial date until the extinction of the resource, then a merger may not affect the price path of the resource[12] and a specific analysis of cartel or merger profitability is in order. This is left for future research.

[10] Andrade de Sa and Daubane (2016) considers a nonrenewable resource monopoly facing a backstop substitute. They show that, when the price elasticity of demand is smaller than one, the monopolist choses, *at each moment*, a corner solution, i.e., induces the limit price which deters the backstop-substitute production. Van der Meijden, Ryszka and Withagen (2018) consider a non-renewable resource supplier who faces demand from two regions, one of which employs a tax on the imported resource and a subsidy on the available backstop technology, and one that has no environmental policy in place. They show that the resource extraction path possibly contains two limit pricing phases.

[11] As noted above, in instances where resource owners are countries and not firms, coordination of interests among the resource owners takes the form of cartels rather than mergers.

[12] We thank an anonymous referee for pointing to this possibility.

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Appendix:

Proof of Proposition 3

Consider the symmetric triopoly game described by (31-33), given extraction paths q_1 and q_2 of firm 1 and firm 2 the necessary conditions for a positive extraction q_3 of the third firm are given by

$$H_{q_3} = -2bq_3 - q_1 - q_2 + c_1 x - \lambda_3 = 0 \tag{39}$$

$$\dot{\lambda}_3 = r\lambda_3 - c_1 q_3 \tag{40}$$

$$\dot{x} = -q_3 \tag{41}$$

$$x(0) = x_0$$

with

$$\lim_{t\to\infty} x(t) = 0$$

For a symmetric equilibrium and interior solutions we have $q_3 = q_1 = q_2 = q$, and $\lambda_3 = \lambda_1 = \lambda_2 = \lambda$. From (39) , we have the following:

$$\lambda = -4bq + c_1 x$$

and thus

$$\dot{\lambda} = -4b\dot{q} + c_1\dot{x}. \tag{42}$$

From (40) and (42), we have the following:

$$-4b\dot{q} + c_1\dot{x} = r(-4bq + c_1 x) - c_1 q \tag{43}$$

From (41) and (43) , it follows that the symmetric OLNE when firms are identical is given by the solution to the following system:

$$\dot{q} = rq - \frac{c_1}{4b} rx \tag{44}$$

$$\dot{x} = -q \tag{45}$$

$$x(0) = x_0$$

with

$$\lim_{t \to \infty} x(t) = 0$$

Moreover, from (44) and (45), we have the following:

$$\begin{bmatrix} \dot{q} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} r & -\frac{rc_1}{4b} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} q_S \\ x \end{bmatrix}$$

The two eigenvalues of $\begin{bmatrix} r & -\frac{rc_1}{4b} \\ -1 & 0 \end{bmatrix}$ are given by $\sigma_1 = \frac{1}{2}\left(r - \sqrt{\frac{1}{b}\left(br^2 + c_1 r\right)}\right) < 0$ and $\sigma_2 = \frac{1}{2}\left(r + \sqrt{\frac{1}{b} r \left(c_1 + br\right)}\right) > 0$, and the associated eigenvectors are $\begin{Bmatrix} -\sigma_1 \\ 1 \end{Bmatrix}$ and $\begin{Bmatrix} -\sigma_2 \\ 1 \end{Bmatrix}$ respectively.

Given that the trajectory cannot oscillate around the steady state, the solution is given by:

$$\begin{bmatrix} q \\ x \end{bmatrix} = A \begin{bmatrix} \sigma e^{\sigma t} \\ e^{\sigma t} \end{bmatrix} \text{ with } A = x_0 \text{ and } \sigma = \sigma_1$$

Therefore, the OLNE path is given by:

$$\begin{bmatrix} q \\ x \end{bmatrix} = x_0 \begin{bmatrix} -\sigma e^{\sigma t} \\ e^{\sigma t} \end{bmatrix}$$

In feedback form, we have the following:

$$q(x) = -\sigma x \quad \blacksquare$$

Proof of Proposition 4

The merged entity, denoted firm M, controls two stocks denoted, S_{L1} and S_{L2},

$$\dot{S}_{L1} = -q_{L1}$$

$$\dot{S}_{L2} = -q_{L2}$$

q_{Li} is the extraction from stock S_{Li} with $i = 1, 2$. Firm M maximizes the joint profits from both stocks. The outsider controls stock S_S. Thus, we have the following:

$$H^S(q_S, \lambda_S, t) = [a - b(Q_S + Q_L) - c_0 + c_1 S_S] q_S + \lambda_S(-q_S).$$

$$\begin{aligned} H^M(q_{L1}, q_{L2}, \lambda_{L1}, \lambda_{L2}, t) &= [a - b(Q_S + Q_L) - c_0 + c_1 S_{L1}] q_{L1} \\ &\quad + [a - b(Q_S + Q_L) - c_0 + c_1 S_{L2}] q_{L2} \\ &\quad + \lambda_{L1}(-q_{L1}) + \lambda_{L2}(-q_{L2}). \end{aligned}$$

The maximum principle gives

$$H^S_{q_S} = a - c_0 - 2bq_S - bQ_L + c_1 S_S - \lambda_S \tag{46}$$

$$H^M_{q_{L1}} = a - c_0 - 2bq_{L1} - 2bq_{L2} - bq_S + c_1 S_{L1} - \lambda_{L1} \tag{47}$$

$$H^M_{q_{L2}} = a - c_0 - 2bq_{L2} - 2bq_{L1} - bq_S + c_1 S_{L2} - \lambda_{L2} \tag{48}$$

where H_z denotes the partial derivative of H with respect to variable z.

The necessary conditions are

$$H^S_{q_S} \leq 0 \text{ and } H^S_{q_S} q_S = 0$$

$$H^M_{q_{L1}} \leq 0 \text{ and } H^M_{q_{L1}} q_{L1} = 0$$

$$H^M_{q_{L2}} \leq 0 \text{ and } H^M_{q_{L2}} q_{L2} = 0$$

The other Maximum Principle conditions are

$$\dot{\lambda}_S = r\lambda_S - c_1 q_S$$

$$\lambda_{L1} = r\lambda_{L1} - c_1 q_{L1}$$

$$\lambda_{L2} = r\lambda_{L2} - c_1 q_{L2}$$

and

$$\dot{S}_S = -q_S$$

$$\dot{S}_{L1} = -q_{L1}$$

$$\dot{S}_{L2} = -q_{L2}$$

with

$$S_S(0) = S_{0S}$$

$$S_{L1}(0) = S_{0L1}$$

$$S_{L2}(0) = S_{0L2}$$

When $S_{0L1} = S_{0L2} = S_0$, then the Merged entity operates both mines with $q_{L1} = q_{L2}$ and therefore

$$H^S_{q_S} = -2bq_S - bQ_L + c_1 S_S - \lambda_S \tag{49}$$

$$H^M_{q_{L1}} = -4bq_{L1} - bq_S + c_1 S_{L1} - \lambda_{L1} \tag{50}$$

where H_z denotes the partial derivative of H with respect to variable z.

The necessary conditions are

$$H^S_{q_S} \leq 0 \text{ and } H^S_{q_S} q_S = 0$$

$$H^M_{q_{L1}} \leq 0 \text{ and } H^M_{q_{L1}} q_{L1} = 0$$

The other Maximum Principle conditions are

$$\dot{\lambda}_S = r\lambda_S - c_1 q_S$$

$$\dot{\lambda}_{L1} = r\lambda_{L1} - c_1 q_{L1}$$

and

$$\dot{S}_S = -q_S$$

$$\dot{S}_{L1} = -q_{L1}$$

with

$$S_S(0) = S_{0S}$$

$$S_{L1}(0) = S_{0L1}$$

There could be different regimes: Simultaneous regime (Φ_S), Merged entity produces alone (Φ_M) and Outsider produces alone (Φ_O). We focus on the regime where solution is interior, i.e. regime Φ_S :

$$-2bq_S - 2bq_{L1} + c_1 S_S = \lambda_S \tag{51}$$

$$-4bq_{L1} - bq_S + c_1 S_{L1} = \lambda_{L1} \tag{52}$$

Taking the time derivative of the conditions above yields

$$-2b\dot{q}_S - 2b\dot{q}_{L1} + c_1 \dot{S}_S = \dot{\lambda}_S \tag{53}$$

$$-4b\dot{q}_{L1} - b\dot{q}_S + c_1 \dot{S}_{L1} = \dot{\lambda}_{L1} \tag{54}$$

and therefore the other Maximum Principle conditions give

$$-2b\dot{q}_S - 2b\dot{q}_{L1} + c_1 \dot{S}_S = r(-2bq_S - 2bq_{L1} + c_1 S_S) - c_1 q_S$$

$$-4b\dot{q}_{L1} - b\dot{q}_S + c_1 \dot{S}_{L1} = r(-4bq_{L1} - bq_S + c_1 S_{L1}) - c_1 q_{L1}$$

or

$$-2b\dot{q}_S - 2b\dot{q}_{L1} = r(-2bq_S - 2bq_{L1} + c_1 S_S) \tag{55}$$

$$-4b\dot{q}_{L1} - b\dot{q}_S = r(-4bq_{L1} - bq_S + c_1 S_{L1}) \tag{56}$$

and

$$\dot{S}_S = -q_S$$

$$\dot{S}_{L1} = -q_{L1}$$

with

$$S_S(0) = S_{0S}$$

$$S_{L1}(0) = S_{0L1}$$

along with

$$\lim_{t\to\infty} S_S(t) = \frac{c_0 - a}{c_1} \text{ and } \lim_{t\to\infty} S_{L1}(t) = \frac{c_0 - a}{c_1}.$$

From (55) and (56) we get

$$-4b\dot{q}_S - 4b\dot{q}_{L1} = 2r\left(-2bq_S - 2bq_{L1} + c_1 S_S\right)$$

$$-4b\dot{q}_{L1} - b\dot{q}_S = r\left(-4bq_{L1} - bq_S + c_1 S_{L1}\right)$$

Therefore

$$-3b\dot{q}_S = 2r\left(-2bq_S - 2bq_{L1} + c_1 S_S\right) - r\left(-4bq_{L1} - bq_S + c_1 S_{L1}\right)$$

$$6b\dot{q}_{L1} = -2r\left(-4bq_{L1} - bq_S + c_1 S_{L1}\right) + r\left(-2bq_S - 2bq_{L1} + c_1 S_S\right)$$

or

$$-3b\dot{q}_S = r\left(-3bq_S + 2c_1 S_S - c_1 S_{L1}\right)$$

$$6b\dot{q}_{L1} = r\left(6bq_{L1} - 2c_1 S_{L1} + c_1 S_S\right)$$

Or

$$\dot{q}_S = rq_S - r\frac{2c_1}{3b} S_S + r\frac{c_1}{3b} S_{L1} \tag{57}$$

$$\dot{q}_{L1} = rq_{L1} - r\frac{2c_1}{6b} S_{L1} + r\frac{c_1}{6b} S_S \tag{58}$$

along with

$$\dot{S}_S = -q_S \tag{59}$$

$$\dot{S}_{L1} = -q_{L1} \tag{60}$$

and the initial and transversality conditions

$$S_S(0) = S_{0S} \tag{61}$$

$$S_{L1}(0) = S_{0L1} \tag{62}$$

$$\lim_{t\to\infty} S_S(t) = 0 \text{ and } \lim_{t\to\infty} S_{L1}(t) = 0. \tag{63}$$

Solving the system of differential equation (57) – (60) with the conditions (61) – (63) yields (35) and (36) ■

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